Student Information

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Answer 1

a)

The sample size is $n = 16$
The sample mean is:

$$\frac{1}{16}\Sigma_{i=1}^{n} X_i = \overline{X} = 6.81$$

.
The sample standard deviation is:

$$\sqrt{\frac{1}{15}\Sigma_{i=1}^{n}(X_i - 6.81)^2} = s = 1.06$$

.
Since σ is unknown and $n \leq 30$ we will use T distribution. The question asks for a 98% confidence interval $\alpha = 1 - 0.98 = 0.02$. The critical value with $n - 1 = 15$ degrees of freedom is $t_{\alpha/2} = t_{0.01} = 2.60$. Then the 98% confidence interval for the mean per 100 km gasoline consumption of your car after the improvement is:

$$\overline{X} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 6.81 \pm 2.60 \cdot \frac{1.06}{4} = 6.81 \pm 0.689 = [6.12; 7.50]$$

b)

We test the null hypothesis $H_0 : \mu = 7.5$ against a one sided left tail alternative $H_A : \mu < 7.5$. Because we only want to know whether the consumption has decreased.
Step 1: Test statistic. Since we don't know the population variance and our sample size is smaller than 30 we use the sample variance for test statistic. We know that $s = 1.06$, $\overline{X} = 6.81$, $\mu_0 = 7.5$, $n = 16$ and $\alpha = 0.05$. The test statistic is:

$$t_0 = \frac{\overline{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{6.81 - 7.5}{\frac{1.06}{\sqrt{16}}} = -2.62$$

Step 2: Acceptance and rejection regions. For this case we use T-distribution to find rejection region.
The rejection region for this left tail test is $R = (-\infty, -1.753]$ where we used T-distribution with $n - 1 = 15$ degrees of freedom and $\alpha = 0.05$.
Since $t_0 \in R$ we reject the null hypothesis and conclude that our consumption has decreased and the improvement is effective with a 95% confidence.

c)

Since we have a one sided left tail alternative we need a negative test statistic for it to be rejected. 6.5 is less then our sample mean so our test statistic will result in a positive value which means it can not be rejected.

Answer 2

a)

The null hypothesis is $H_0 : \mu = 5000$ and the alternative hypothesis is a one sided right tail alternative $H_A > 5000$. Ali claims the prices remain same which is the null hypothesis. Ahmet claims that the prices will increase which is the alternative hypothesis.

b)

Yes he can. The mathematical evidence is shown below.
I will use the hypothesis in part (a).
Step 1: Test statistic. We are given: $\sigma = 2000$, $\alpha = 0.05$, $n = 100$, $\mu_0 = 5000$ and $\overline{X} = 5500$. The test statistic is:

$$Z = \frac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{5500 - 5000}{\frac{2000}{\sqrt{100}}} = 2.5$$

Step 2: Acceptance and rejection regions. The critical value is:

$$z_\alpha = z_{0.05} = 1.645$$

(don't divide α by 2 because it is a one-sided test). With the right-tail alternative, we

$$\begin{cases} \text{Reject} & Z \geq 1.645 \\ \text{Accept} & Z < 1.645 \end{cases}$$

Step 3: Result. Our test statistic Z = 2.5 belongs to the rejection region; therefore, we reject the null hypothesis. The data provided sufficient evidence in favor of the alternative hypothesis, so Ahmet can claim that there is an increase in the rent prices compared to the last year

c)

We can use our test statistic in part (b) for Z_{obs} below. From Table A4, we find that the P-value for the right-tail alternative is

$$P = \mathbf{P}\{Z \geq Z_{obs}\} = \mathbf{P}\{Z \geq 2.5\} = 1 - \phi(2.5) = 1 - 0.9938 = 0.0062$$

Since $p < \alpha$, $0.0062 < 0.05$ so we can reject the null hypothesis which makes Ahmet's claim acceptable.

d)

We test the null hypothesis $H_0 : \mu_a - \mu_i = 0$ against a one sided left tail alternative $H_A : \mu_a - \mu_i < 0$ (where μ_a is for Ankara and μ_i is for Istanbul).
We are given:
Ankara: $\overline{X}_a = 5500$, $\sigma_a = 2000$, $n_a = 100$
Istanbul :$\overline{X}_i = 6500$, $\sigma_i = 3000$, $n_i = 60$
Step 1: Test statistic. The test statistic is:

$$Z = \frac{(\overline{X}_a - \overline{X}_i) - (\mu_a - \mu_i)}{\sqrt{\frac{\sigma_a^2}{n_a} + \frac{\sigma_i^2}{n_i}}} = \frac{(-1000) - (0)}{\sqrt{\frac{2000^2}{100} + \frac{3000^2}{60}}} = -2.29$$

Step 2: Acceptance and rejection regions. The critical value is:

$$z_\alpha = z_{0.01} = -2.326$$

(don't divide α by 2 because it is a one-sided test). With the left-tail alternative, we

$$\begin{cases} \text{Reject} & Z \leq -2.326 \\ \text{Accept} & Z > -2.326 \end{cases}$$

Step 3: Result. Our test statistic Z = -2.29 does not belong to the rejection region; therefore, we can not reject the null hypothesis. The data did not provide sufficient evidence in favor of the alternative hypothesis, so we can't claim that Ankara is lower than the prices in Istanbul with 1% level of significance.

Answer 3

Testing H_0: "The number of rainy days in Ankara is independent on the season" vs H_A : "The number of rainy days in Ankara is dependent on the season". Calculate the estimated expected counts.

$$Expected(1,1) = Expected(1,2) = Expected(1,3) + Expected(1,4) = \frac{90 \cdot 100}{360} = 25$$
$$Expected(2,1) = Expected(2,2) = Expected(2,3) + Expected(2,4) = \frac{90 \cdot 2600}{360} = 65$$
$$x^2_{obs} = \frac{(34-25)^2}{25} + \frac{(32-25)^2}{25} + \frac{(15-25)^2}{25} + \frac{(19-25)^2}{25} + \frac{(56-65)^2}{65} + \frac{(58-65)^2}{65} + \frac{(75-65)^2}{65} + \frac{(71-65)^2}{65} = 14.73$$

From Table A6 with $(4-1)(2-1) = 3$ d.f., we find that the P-value is $0.001 < P < 0.005$ which is $P < 0.01$. So we have significant evidence that The number of rainy days in Ankara is dependent on the season.

Answer 4

```
X = [34 32 15 19; 56 58 75 71];
Row = sum(X, 2); % Sum of each row
Col = sum(X); % Sum of each column
Tot = sum(Row); % Total sum
k = length(Col); % No of columns
m = length(Row); % No of rows
e = zeros(size(X)); % Expected counts

for i = 1:m
    for j = 1:k
        e(i,j) = Row(i)*Col(j)/Tot;
    end
end

chisq = (X - e).^2 ./ e;
chistat = sum(sum(chisq));
df = (k - 1) * (m - 1);
Pvalue = 1 - chi2cdf(chistat, df);

fprintf("Xobs^2 = %.8f\n", chistat); % Display the Xobs^2
fprintf("Pvalue = %.8f\n", Pvalue); % Display the P-value
```

Output:

```
octave:1> source("my_script.m")
Xobs^2 = 14.73230769
Pvalue = 0.00206031
```