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Answer 1

a)

To find expected values we can use the formula : $E(X) = \sum_x xP(x)$
All faces of the dices will have equal probabilities so we can say each will have 1/(no of faces) probability.
For Blue dice the Random variable X will take the values $1, 2, 3, 4, 5, 6$. $P(X = x) = 1/6$ for each x.

$$E(X) = \frac{1}{6} \cdot \sum_{n=1}^{6} n = 3.5$$

For yellow dice X will take the values 1,3,4,8. With the probabilities:

$$P(1) = 3/8, P(3) = 3/8, P(4) = 1/8, P(8) = 1/8$$

$$E(X) = 1 \cdot \frac{3}{8} + 3 \cdot \frac{3}{8} + 4 \cdot \frac{1}{8} + 8 \cdot \frac{1}{8} = 3$$

For red dice X will take the values 2,3,4,6. With the probabilities:

$$P(2) = 5/10, P(3) = 2/10, P(4) = 2/10, P(6) = 1/10$$

$$E(X) = 2 \cdot \frac{5}{10} + 3 \cdot \frac{2}{10} + 4 \cdot \frac{2}{10} + 6 \cdot \frac{1}{10} = 3$$

b)

3 blue dice because from the expected values in part a I would more likely get a bigger result like this. Expected total values will be 9.5 for one of each and 10.5 for 3 blue dice.

c)

A single die of each color. Now the expected total value will increase to 14.5 since the expected value of the yellow dice becomes 8 hence it becomes larger than 10.5.

d)

Let's say that probability of getting a 3 is $P(T)$ and getting red, blue and yellow is $P(R), P(B), P(Y)$ respectively.
The question want us to find $P(R|T)$
By Baye's Rule

$$P(R|T) = \frac{P(T|R) \cdot P(R)}{P(T)}$$

where

$$P(T|R) = \frac{2}{10} = \frac{1}{5}$$

$$P(R) = \frac{1}{3}$$

$$P(T) = P(T \cap B) + P(T \cap Y) + P(T \cap R)$$

$$= P(T|B) \cdot P(B) + P(T|Y) \cdot P(Y) + P(T|R) \cdot P(R) = \frac{1}{3} \cdot (\frac{1}{6} + \frac{3}{8} + \frac{1}{5})$$

So

$$P(R|T) = \frac{24}{89} = 0.2697$$

e)

There are 3 different cases where we can get the total value of 5.
These are (Blue 1, Yellow 4), (Blue 2, Yellow 3), (Blue 4, Yellow 1).
We know their probabilities from part a. Since blue and yellow dice are independent we take their product and take the sum of the results.
(Blue 1, Yellow 4) $= \frac{1}{6} \cdot \frac{1}{8} = \frac{1}{48}$
(Blue 2, Yellow 3) $= \frac{1}{6} \cdot \frac{3}{8} = \frac{1}{16}$
(Blue 4, Yellow 1) $= \frac{1}{6} \cdot \frac{3}{8} = \frac{1}{16}$
Sum $= \frac{1}{48} + \frac{1}{16} + \frac{1}{16} = \frac{7}{48} = 0.1458$

Answer 2

a)

We need to find the probability $\{X \geq 4\}$ such that X is the random variable denoting the number of distributors that offer a discount. This is the number of successes in 80 Bernoulli trials

We can use binomial distribution with parameters $n = 80, p = 0.025$.
Since $F(3) = P\{X \leq 3\}$
Using octave we can calculate $F(3) = 0.8594$
$P\{X \geq 4\} = 1 - F(3) = 0.1406$

b)

Using Bernoulli and Binary distributions.
Let's calculate the probability that we don't get a phone in 2 days and later take the complement of it to get the answer.
Not getting a discount from company A = $0.975^{80} = 0.1319$, company B = 0.9.
Since both the companies are independent we will take the product of them.
Not getting a phone in 1 day $= 0.1319 \cdot 0.9 = 0.1187$
Days are also independent so again take the product.
So for 2 days $= 0.1187 * 0.1187 = 0.0141$
To find the probability we at least get one phone, we can take the complement.
$= 1 - 0.0141 = 0.9859$

Answer 3

Code and output:

```
blue = [1,2,3,4,5,6];
yellow = [1,1,1,3,3,3,4,8];
red = [2,2,2,2,2,3,3,4,4,6];

blue1 = blue(randi(6,1,1000));
blue2 = blue(randi(6,1,1000));
blue3 = blue(randi(6,1,1000));
yellow1 = yellow(randi(8,1,1000));
red1 = red(randi(10,1,1000));


sum1 = blue1+yellow1+red1;
sum2 = blue1 + blue2 + blue3;
disp(sum(sum1)/1000)
disp(sum(sum2)/1000)
disp(sum(sum2>sum1)/10)
```

```
octave:256> blue = [1,2,3,4,5,6];
yellow = [1,1,1,3,3,3,4,8];
red = [2,2,2,2,2,3,3,4,4,6];

blue1 = blue(randi(6,1,1000));
blue2 = blue(randi(6,1,1000));
blue3 = blue(randi(6,1,1000));
yellow1 = yellow(randi(8,1,1000));
red1 = red(randi(10,1,1000));


sum1 = blue1+yellow1+red1;
sum2 = blue1 + blue2 + blue3;
disp(sum(sum1)/1000)
disp(sum(sum2)/1000)
disp(sum(sum2>sum1)/10)
9.4580
10.529
57.400
```

The output is very close to our calculations but not the same since what we calculate is just an approximation and will not be the exact same value in real experiments. And also the output will be more close to our results as we increase the number of experiments.