Student Information

Full Name : Hasan Küreli
Id Number : 2580751

Answer 1

Let T_A denote the lifetime of A and T_B denote the lifetime of B. Then $T_A \sim Uniform(0, 100)$ and $T_B \sim Uniform(0, 100)$.

a)

$$F(t_A) = \frac{t_A}{100}, F(t_B) = \frac{t_B}{100}$$

To find their joint cdf we can simply take their product since they are independent.

$$F(t_A, t_B) = P(T_A < t_A \cap T_B < t_B) = P(T_A < t_A) * P(T_B < t_B) = F(T_A) * F(T_B) = \frac{t_A * t_B}{10^4}$$

And we can differentiate the cdf to find pdf:

$$f(t_A, t_B) = \frac{\partial^2}{\partial t_A \partial t_B} \frac{t_A * t_B}{10^4} = \frac{1}{10^4}$$

b)

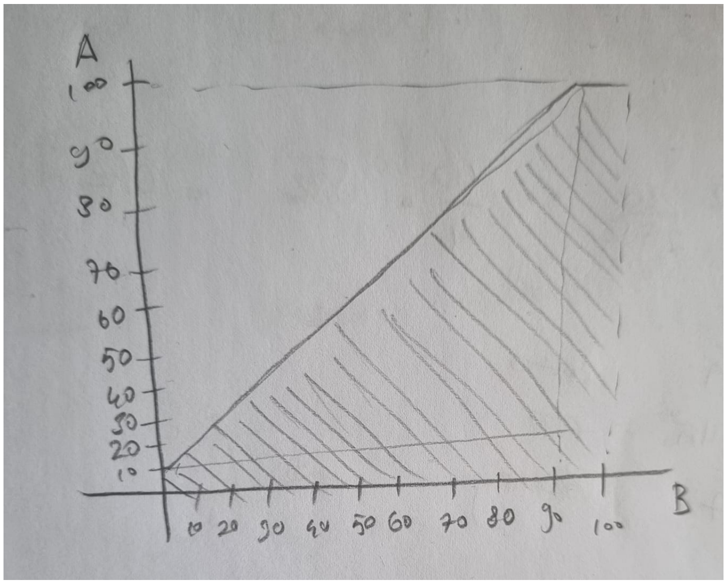
This is given by the probability: $P(T_A < 30 \cap 40 < T_B < 60)$

$$P(T_A < 30 \cap 40 < T_B < 60) = P(T_A < 30) * P(40 < T_B < 60)$$

$$= F(30) * (F(60) - F(40)) = 0.3 * 0.2 = 0.06$$

c)

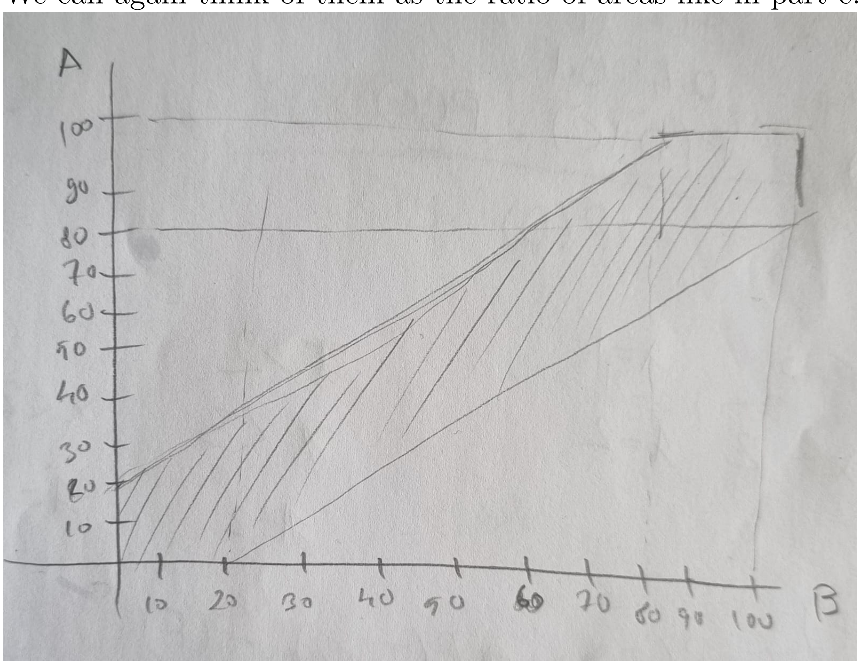
This is given by the probability: $P(T_A < T_B + 10 \cap T_B < 90) + P(T_A < 100 \cap T_B > 90)$
If we think of them as as [0,100]x[0,100] square, we can think of this as the ratio of the area highlighted to the whole area.



Which is equal to $= \frac{5950}{10000} = 0.595$

d)

This is given by the probability: $P(T_B - 20 \geq T_A \geq T_B + 20 \cap 20 \geq T_B \geq 80) + P(T_B - 20 < T_A < 100 \cap T_B > 80) + P(0 < T_A < T_B + 20 \cap T_B < 20)$
We can again think of them as the ratio of areas like in part c.



The ratio is equal to $= \frac{3600}{10000} = 0.36$

Answer 2

a)

Let X denote the number of frequent shoppers. X is a binomial random variable with parameters $n = 150, p = 0.6$
At least 65% means that at least 97.5 customers are frequent shoppers so $P(X \geq 98)$
Since n is large we can use normal approximation to find the probability using Central Limit Theorem.
So here X approximately follows normal distribution with parameters $\mu = 90, \sigma = 6$
The probability that more then 97.5 customers are frequent shoppers is calculated as follows:

$$P(X \geq 98) = 1 - P(X < 98) = 1 - P(X \leq 97.5) = 1 - P(Z \leq \frac{97.5 - 90}{6})$$

$$= 1 - \phi(1.25) = 1 - 0.8944 = 0.1056$$

From Table A4 in the book.

b)

Let Y denote the number of rare shoppers. Y is a binomial random variable with parameters $n = 150, p = 0.1$.
No more than 15% means that there are less than or equal to 22.5 rare shoppers so $P(Y \leq 22)$.
As in the example before since n is large we can use Central Limit Theorem in the same way.
Here Y follows normal distribution with parameters $\mu = 15, \sigma = 3.6742$.
The probability that no more than 15% of customers are rare shoppers is calculated as follows:

$$P(Y \leq 22) = P(Y < 22.5) = P(Z < \frac{22.5 - 15}{3.6742}) = \phi(2.04) = 0.9793$$

From Table A4 in the book.

Answer 3

Let X denote the heights of adults. X is Normal random variable with $\mu = 175, \sigma = 7$.
The adults between 170 and 180 cm is denoted as follows:

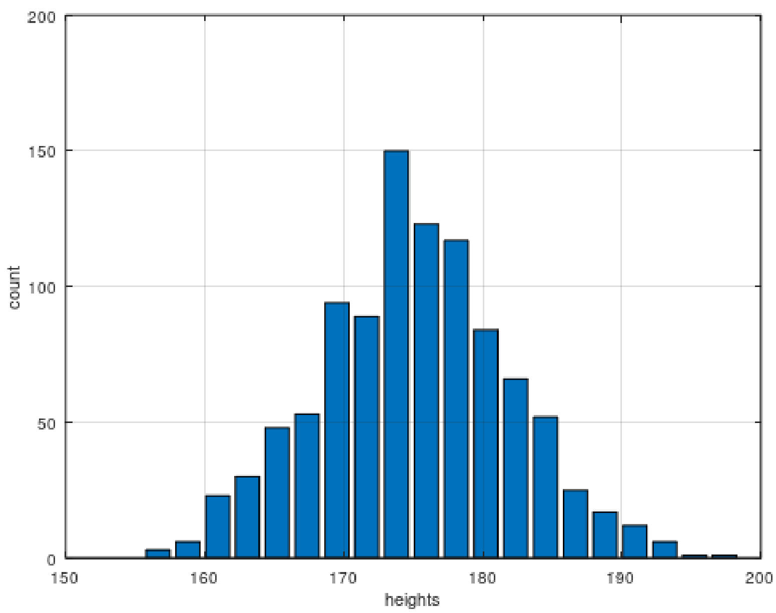
$$P(170 < X < 180) = P(X < 180) - P(X < 170) = P(\tfrac{X-\mu}{\sigma} < \tfrac{180-175}{7}) - P(\tfrac{X-\mu}{\sigma} < \tfrac{170-175}{7})$$

$$= \phi(0.71) - \phi(-0.71) = 0.7611 - 0.2389 = 0.5222$$

From Table A4 in the book.

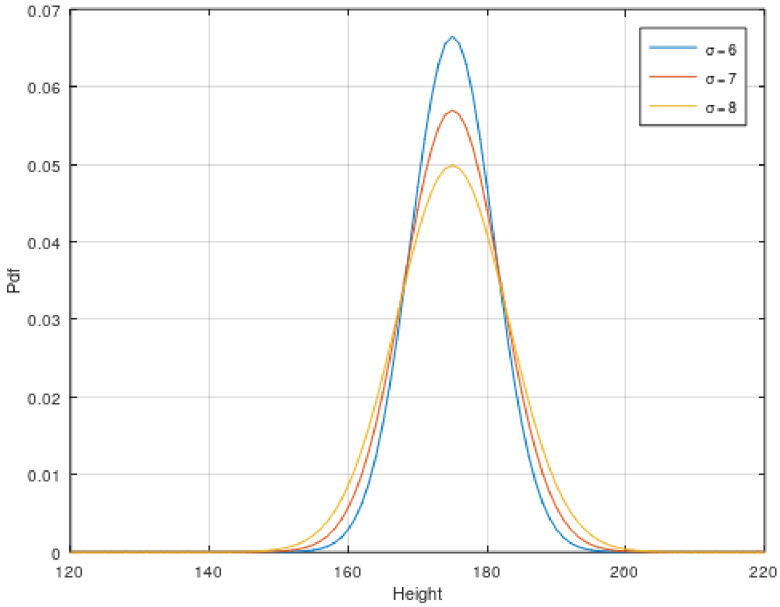
Answer 4

a)



The bar graph is centered in our mean and looks symmetric. It is similar to normal distribution but since we only have 1000 iterations it is still not perfect and I get graphs that look different every time i run the code. If we increase the number of our iterations we will get a distribution that is closer to the normal distribution.

b)



As sigma gets larger our plots get wider since the standard deviation increases but the area under them is always equal to 1.

c)

```
atleast45 = 0.9660
atleast50 = 0.7230
atleast55 = 0.2380
```

These results are as expected but not perfect because of the low number of iterations. As the number increase the probability decreases as expected.

CODE:

```
pkg load statistics

x = normrnd(175,7,1000,1);
[nn,xx] = hist(x,20);

figure (1)
bar(xx,nn)
xlabel('heights')
ylabel('count')
grid on

rng = 120:220;
pdf_sigma_6 = normpdf(rng, 175, 6);
pdf_sigma_7 = normpdf(rng, 175, 7);
pdf_sigma_8 = normpdf(rng, 175, 8);

figure (2)
plot(rng, pdf_sigma_6);
hold on;
plot(rng, pdf_sigma_7);
plot(rng, pdf_sigma_8);
legend('\sigma = 6', '\sigma = 7', '\sigma = 8');
xlabel('Height');
ylabel('Pdf');
grid on;
hold off;

count45=0;
count50=0;
count55=0;

for i=1:1000
  %select 150 random
  indices = randi(1000, 150, 1);
  heights = x(indices);
  probability = mean(170<heights & heights<180);
  if probability>=0.45
    count45++;
  endif
  if probability>=0.50
    count50++;
  endif
  if probability>=0.55
    count55++;
  endif
end
atleast45 = count45/1000
atleast50 = count50/1000
atleast55 = count55/1000

```