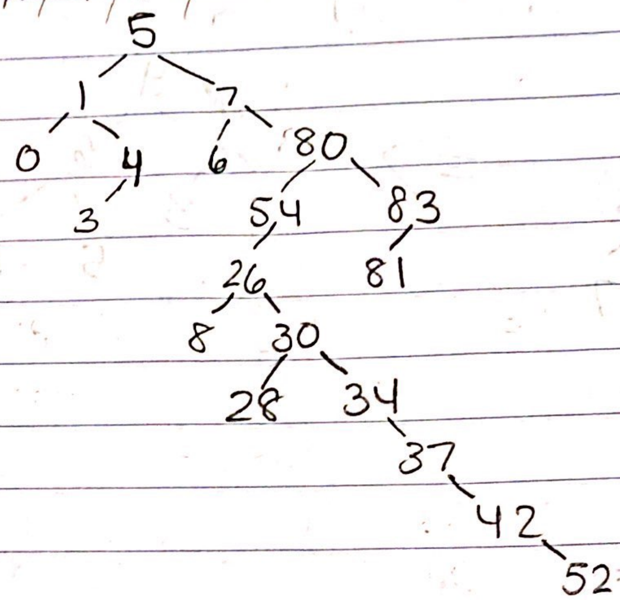
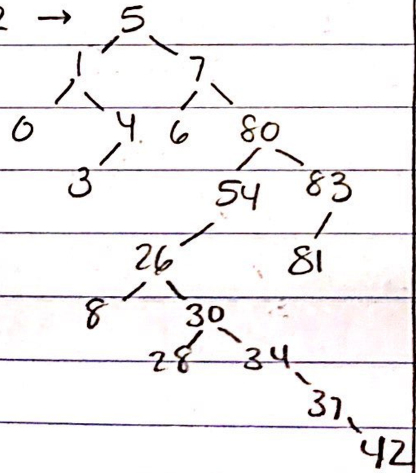
HW #8

1. BST : [5, 7, 1, 4, 3, 80, 54, 26, 30, 28, 8, 34, 37, 42, 0, 6, 52, 83, 81]

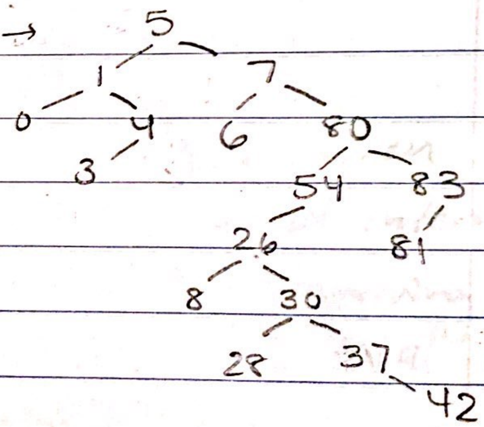


2. Delete following nodes:

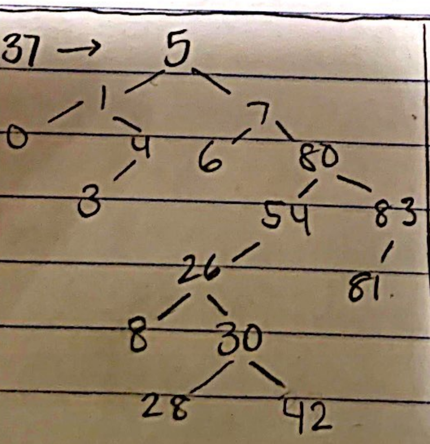
a. 52 →



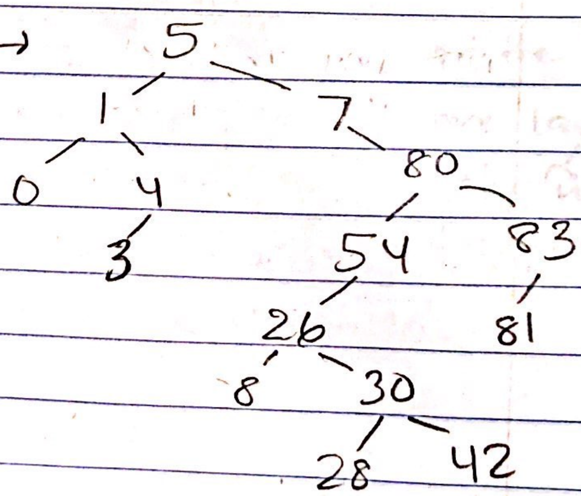
b. 34 →



c. 37 →

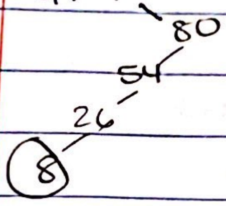


d. 6 →

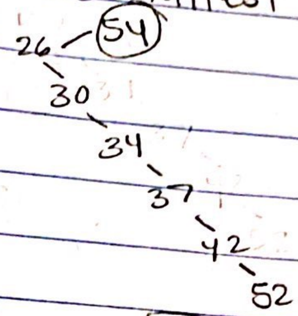


3. Find Successor: (smallest bigger number)

a. 7



b.



c. 34

37

d. 30

34

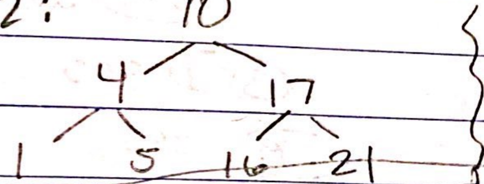
e.

28 → 30

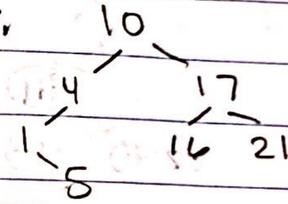
f. 83 → No successor's bc largest number in BST.

4. Keys [1, 4, 5, 10, 16, 17, 21]

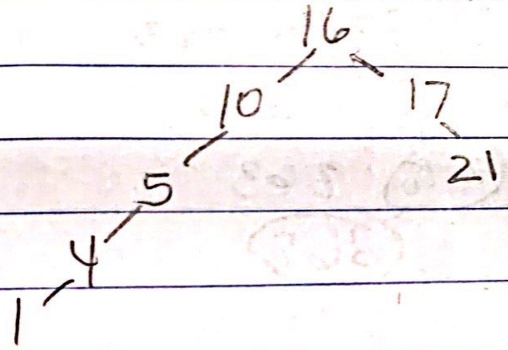
Height 2:



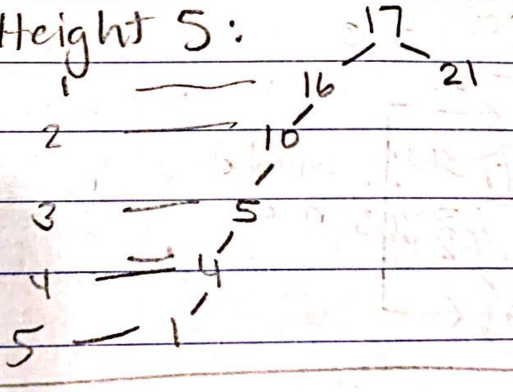
Height 3:



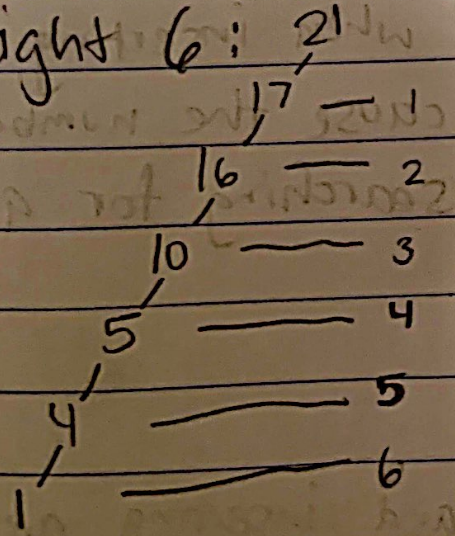
Height 4:



Height 5:



Height 6:



5. key = 363

a.
2 363
252 363
401 363
398 363
330 363
344 363
397

b.
924 363
220 363
911 363
244 363
898 363
258 363
362 363
363

c.
925 363
202 363
911 363
240 363
912

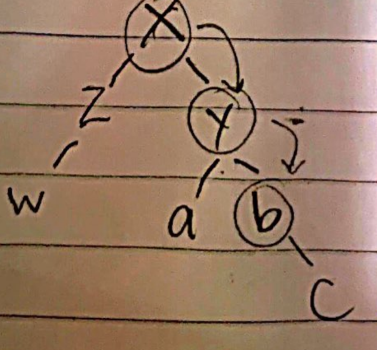
doesn't work

d.
2 363
399 363
387 363
219 363
266 363
382 363
381 363
278 363
368

e.
935 363
278 363
347 363
621 363
299

invalid node

6. The number of nodes examined when inserting that key would also be K because the number of nodes we iterate through when searching for a key.



inserting c

Searching and inserting are proportional in that when we insert we would have to search the same amount of times.

7. Maximum number of nodes in binary tree w/ height h is $2^{h+1}-1$
Induction step: height is longest path from root to leaf.
Base Case = 0 → 0 (one node at h=0)

$$2^{h+1}-1 = 2^{0+1}-1 = 1 \checkmark$$

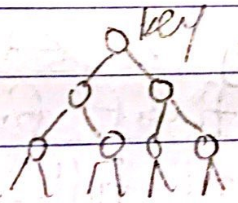
Assume → h = k+1 so we get $2^{k+1}-1$
Inductive hypothesis:

$$\overset{\text{left}}{\left[2^{k+1}-1\right]} + \overset{\text{right}}{\left[2^{k+1}-1\right]} + 1$$

$$= 2\left(2^{k+1}\right) - 2 + 1$$

$$= 2^{(k+1)+1} - 1 \checkmark \rightarrow h = k+1$$

Since we knew that with the base case h=0 and we proved the next element h=k+1 was also true, the max num. of nodes in binary tree with height h is $2^{h+1}-1$.

8. 

The lower bound or best case for the number of steps for the nth key would be $\Omega(\log n)$

$$\sum_{i=1}^{n} \log i \Rightarrow \Theta\left(\int_1^n \log x\,dx\right)$$

$$= \Theta\left(x \log x - x\Big]_1^n\right)$$

$$= \Theta(n \log n)$$

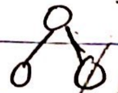
int. by parts:
$u = \log x \quad du = \frac{1}{x}dx$
$v = x \quad dv = 1dx$

∴ Using integral theorem we see that the time complexity of $\sum_{i=1}^{n} \log i$ is equal to $\Theta(n \log n)$.

9. Prove: Binary tree w/ n nodes using links to left/right child total of null links is n+1.

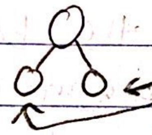
Base Case: n=1 

Assuming this we will try to prove n+1:

 $\Rightarrow c-1=n+1 \rightarrow c=(n+1)+1 \Rightarrow \boxed{c=n+1}$ ✓

We proved with induction that since the base case with 2 null links was n=1 and that when we added one the total of n+1 null links didn't change and was n+1.

10. Induction of number of full nodes → n = # of full nodes

Base Case: n=1  n+1 = 2 leaves ✓

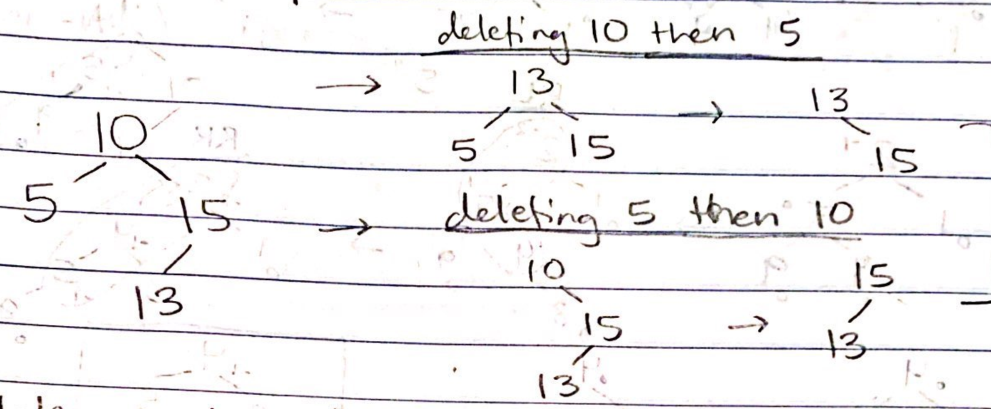
Prove for P(k) → n = k+1

Since we know that each k is equal to "k+1," we could write out the sum making n = k+1+1 = k+2.

∴ by induction, we proved that for n full nodes, we will have n+1 full nodes that we can add one to to find the number of leaves of the binary tree.

11. Deleting x and y from a BST would NOT be commutative because there is a reason why we delete items a certain way such as deleting a parent with two children, we would need to replace it with the successor, otherwise we would be left with an inconsistent tree that wouldn't have the same conditions as a

binary search tree,
Counterexample:

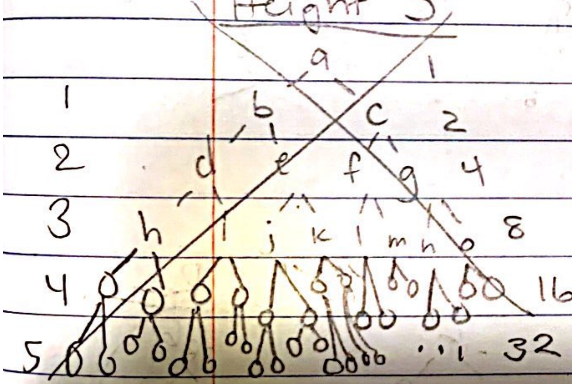


Produce different results

We see that since 10 doesn't have two children in the second scenario, we take the only child it has rather than the successor.

12. Min. # of nodes for balanced tree of height 5, 10, 15?

Height 5



Using fibonacci sequence to get

$N(h) = N(h-1) + N(h-2) + 1$

$N(0) = 1$ and $N(1) = 2$

$N(2) = N(1) + N(0) + 1 = 4$

$N(3) = N(2) + N(1) + 1 = 7$

$N(4) = N(3) + N(2) + 1 = 12$

$N(5) = N(4) + N(3) + 1 = 20$

Min. # for height 5 = 20

$N(6) = N(5) + N(4) + 1 = 33$

$N(7) = N(6) + N(5) + 1 = 54$

$N(8) = N(7) + N(6) + 1 = 88$

$N(9) = N(8) + N(7) + 1 = 143$

$N(10) = N(9) + N(8) + 1 = 232$

Min. # for height 10 = 232

$N(11) = N(10) + N(9) + 1 = 376$

$N(12) = N(11) + N(10) + 1 = 609$

$N(13) = N(12) + N(11) + 1 = 986$

$N(14) = N(13) + N(12) + 1 = 1596$

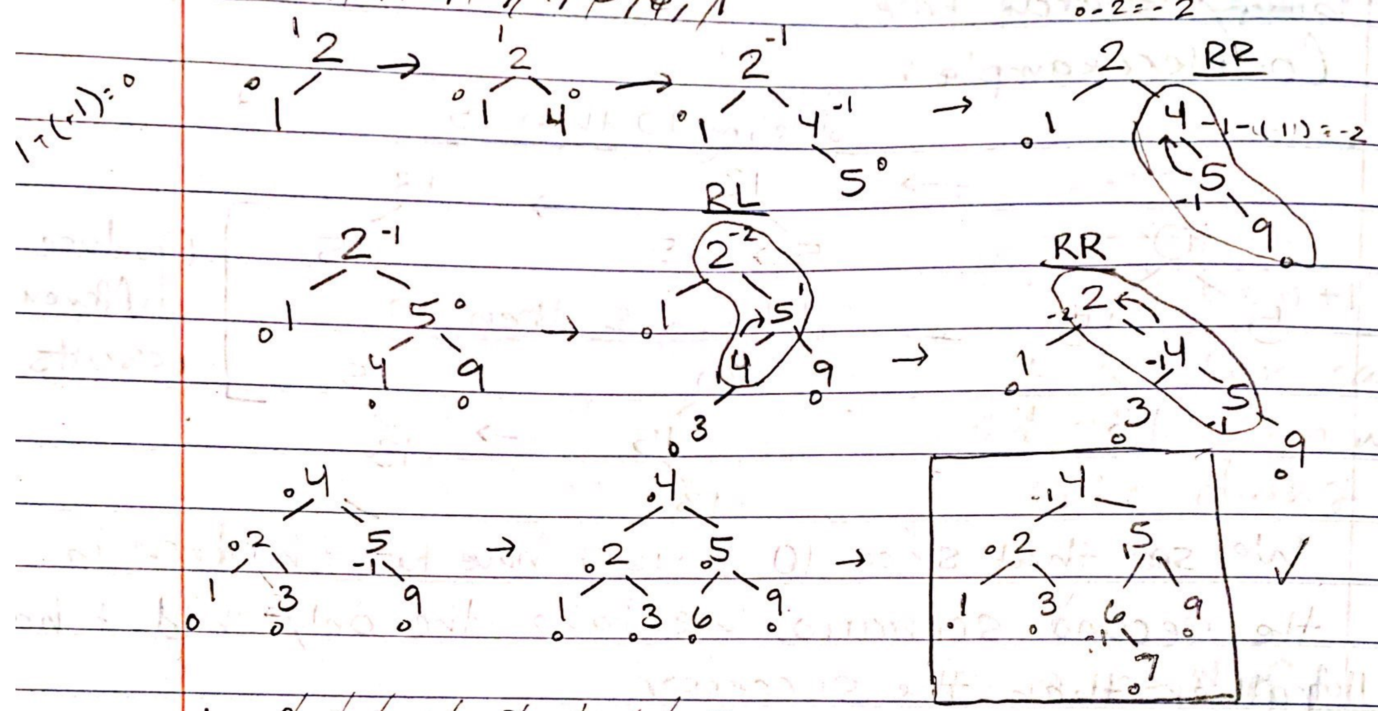
$N(15) = N(14) + N(13) + 1 = 2583$

Min # for height 15 = 2583

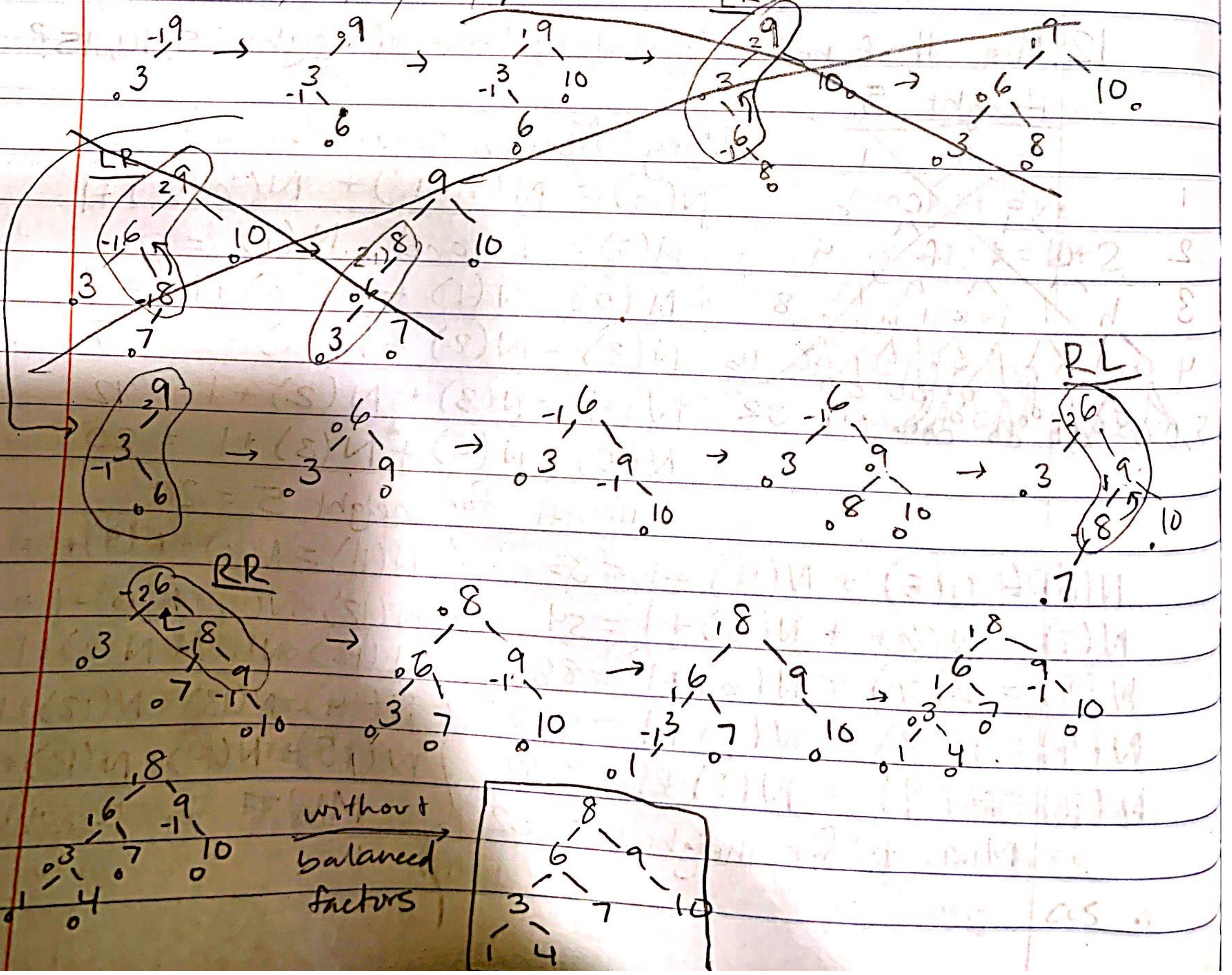
Scanned with CamScanner

Draw AVL tree

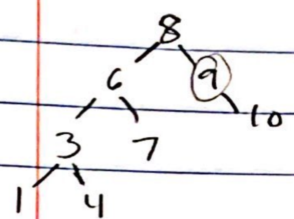
13. a. 2, 1, 4, 5, 9, 3, 6, 7



b. 9, 3, 6, 10, 8, 7, 1, 4, 5

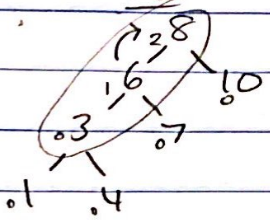


14. Delete 9 and 6

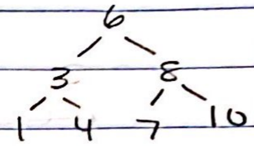


deleting 9 →

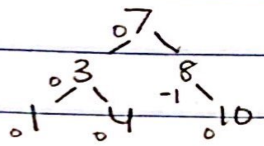
LL



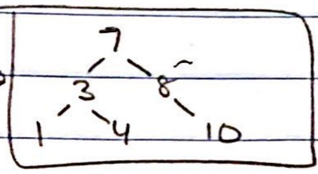
→



deleting 6 →



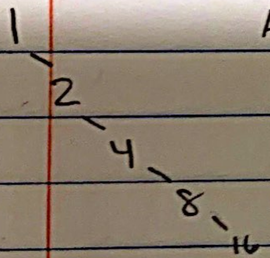
→



15. EC: keys $1, 2, \ldots, 2^{k+1}$ inserted into AVL tree, would b… per…

Base Case $k = 1 \rightarrow 1, 2, \ldots, 4$

Right node: $2^{k+1} + 2$ Left node: $2^{k+1} + 1$

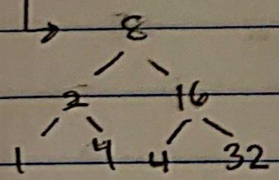


Assuming $P(k) = k+1 \rightarrow 2^{(k+1)+1} \rightarrow \sum_{i=3}^{n} 2^n$

$\int_3^n 2^x \Rightarrow \frac{2^x}{\ln 2}$ ↳ $2^3, 2^4, 2^5$

8, 16, 32

↳



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