Counting sort for a:

[1 2 0 -3 5 -7 10] add +7

[8 9 7 4 12 0 17]

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1				1			1	1	1			1					1

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
				1			2	3	4			5					6
~~1~~	1	1	1	~~2~~	2	2	~~3~~	~~4~~	~~5~~	5	5	~~6~~	6	6	6	6	~~7~~

0	1	2	3	4	5	6
-7	-3	0	1	2	5	10

✓

Radix Sort for a:

[8 9 7 4 12 0 17]

0	1	2	3	4	5	6	7	8	9
							7		
0		12		4			17	8	9

[0 12 4 7 17 8 9]

0	1	2	3	4	5	6	7	8	9
09									
08									
07									
04	17								
0	12								

[0 4 7 8 9 12 17] ✓

328 HW#5

1. Sort numbers using

a. Counting sort:

1	2	0	-3	5	-7	10
0	1	2	3	4	5	6

max # = 10

positive int

1	1	1			1					1
0	1	2	3	4	5	6	7	8	9	10

negative ints

			1				1
0	1	2	3	4	5	6	7

1	~~2~~	3	3	3	4	4	4	4	4	5
0	1	2	3	4	5	6	7	8	9	10

0	0	0	1	1	1	1	2
0	1	2	3	4	5	6	7

-7	-3	0	1	2	5	10
0	1	2	3	4	5	6

First Page for a

b.

0	2	3	8	9	16
0	1	2	3	4	5

1		1	1					1	1							1
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0		1	2					3	4							5

~~1~~	~~1~~	~~2~~	~~3~~	3	3	3	3	~~4~~	8	5	5	5	5	5	5	~~6~~
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

0	2	3	8	9	16
0	1	2	3	4	5

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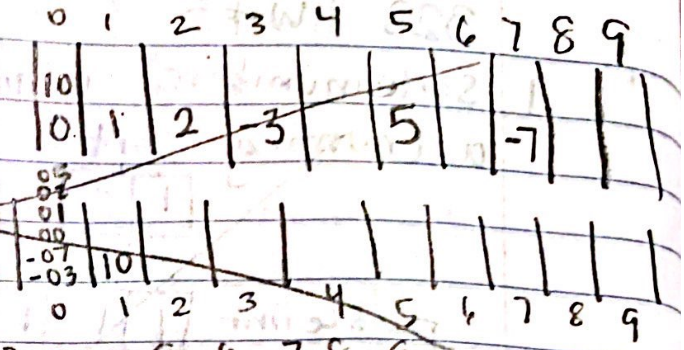
First page

B. Radix Sort

1. a. 1, 2, 0, -3, 5, -7, 10

-3, -7, 0, 10, 1, 2, 5

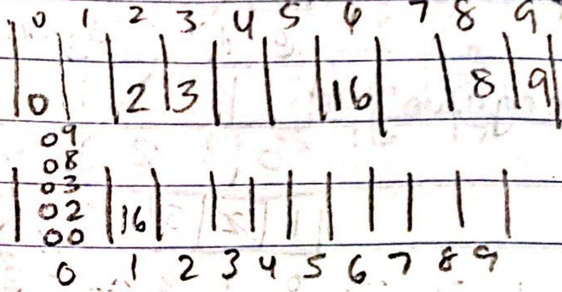
-3, -7, 0, 1, 2, 5, 10 ✓



2. b. 0, 2, 3, 8, 9, 16

0, 2, 3, 16, 8, 9

0, 2, 3, 8, 9, 16 ✓



C. Insertion Sort

1. $\boxed{1}$, 2, $\boxed{0}$, -3, 5, -7, 10

$\boxed{0}$, 1, 2, $\boxed{-3}$, 5, -7, 10

$\boxed{-3}$, 0, 1, 2, 5, $\boxed{-7}$, 10

$\boxed{-7}$, -3, 0, 1, 2, 5, 10 ✓

2. 0, 2, 3, 8, 9, 16 ✓

D. Bubble Sort

1. 1, 2, 0, -3, 5, -7, 10

1, 0, 2, -3, 5, -7, 10

0, 1, 2, -3, 5, -7, 10

0, 1, -3, 2, 5, -7, 10

0, -3, 1, 2, 5, -7, 10

-3, 0, 1, 2, 5, -7, 10

-3, 0, 1, 2, -7, 5, 10

-3, 0, 1, -7, 2, 5, 10

-3, 0, -7, 1, 2, 5, 10

-3, -7, 0, 1, 2, 5, 10

-7, -3, 0, 1, 2, 5, 10 ✓

2. 0, 2, 3, 8, 9, 16 ✓

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E. Selection Sort

1. 1, 2, 0, -3, 5, -7, 16
-7, 1, 2, 0, -3, 5, 10
-7, -3, 1, 2, 0, 5, 10 ✓

2. 0, 2, 3, 8, 9, 16 ✓

2. Running Time of Insertion Sort if all elements equal?
[2, 2, 2, 2]
The run time will have to be at least o(n) because all the elements will need to be checked. The function to move the elements won't bee needed since everything is where it needs to be so we don't need to worry about anything except the loop that goes through input size n.

3. a. Merge Sort

1. 8, 0, 2, -1, -2, 2, 3, 7, -6, -9

| 0 | 8 | | -1 | 2 | | -2 | 2 | | 3 | 7 | | -6 | -9 |

| -1 | 0 | 2 | 8 | | -2 | 2 | 3 | 7 | | -6 | -9 |

| -2 | -1 | 0 | 2 | 2 | 3 | 7 | 8 | | -6 | -9 |

| -9 | -6 | -2 | -1 | 0 | 2 | 2 | 3 | 7 | 8 |

2. 19, 7, 6, 3, 2, -1, -7, -18

| 7 | 19 | | 3 | 6 | | -1 | 2 | | -7 | -1 | | -18 |

| 3 | 6 | 7 | 19 | | -7 | -1 | -1 | 2 | | -18 |

| -7 | -1 | -1 | 2 | 3 | 6 | 7 | 19 | | -18 |

| -18 | -7 | -1 | -1 | 2 | 3 | 6 | 7 | 19 |

b. Quicksort

1. 8, 0, $\boxed{2}$, -1, -2, 2, 3, 7, -6, -9 pivot = 2

-9, 0, $\boxed{2}$, -1, -2, 2, 3, 8

-9, -2, $\boxed{2}$, -1, 0, 2, 3, 8

L R

| -9 | -2 |

2

| -1 | 0 | 2 | 3 | 8 |

~~pivot = 3~~

2. 19, 7, 6, 3, 2, -1, -7, -18 pivot = 7

-18, 7, 6, 3, 2, -1, -7, 19

-18, 7, -7, 3, 2, -1, 6, 19

-18, 7, -7, -1, 2, 3, 6, 19

-18, 2, -7, -1, 7, 3, 6, 19 swap arr[n+1] and pivot

pivot = -7

| -18 | -1 | -7 | 2 |

| -18 | -7 | -1 | 2 |

L R

| -18 | -7 | -1 | 2 |

| 3 | 6 | 7 | 19 |

pivot = 6

| 6 | 3 | 7 | 19 |

| 3 | 6 | 7 | 19 |

| -18 | -7 | -1 | 2 | 3 | 6 | 7 | 19 |

1. 8, 0, 2, -1, -2, 2, 3, 7, -6, -9 pivot = 2

-9, 0, 2, -1, -2, 2, 3, 7, -6, 8

-9, -6, 2, -1, -2, 2, 3, 7, 0, 8

p = -1 -9, -6, 2, -1, -2, 2, 3, 7, 0, 8

L R p = 3

| -9 | -6 | -2 | -1 | 2 |

| 2, | 3 | 0 | 7, | 8 |

| 2 | 0 | 3 | 7 | 8 |

p = 0

| 0 | 2 | 3 | 7 | 8 |

| -9 | -6 | -2 | -1 | 0 | 2 | 2 | 3 | 7 | 8 |

4 Partitioning algorithm using median-of-three heuristic

1. [1, 2, 6, -3, 20, -61, 7, 8, 19, 100]
L L L L L R R R R R

median-of-three
{1, 20, 100}

1 < 100 so keep

[1, 2, 6, -3, -61]
pivot = (1, 6, -6) = 1
1 -3 6 2
[-6, ~~2~~, ~~6~~, 3, ~~1~~]

p = (-6, -3, 2) = -3
[-6, -3, 1, 6, 2]
p = (-6, 1, 2) = 2
[-6, -3, 1, 2, 6]

[20, 7, 8, 19, 100]
pivot = (20, 8, 100) = 8
7 8 19 20
[8, ~~7~~, ~~20~~, ~~19~~, 100]
p = (7, 19, 100) = 7
[7, 8, 19, 20, 100]

= [-6, -3, 1, 2, 6, 7, 8, 19, 20, 100]

2. 0, 7, -6, 23, 12, 30, -71, 19   p = (0, 23, 19) = 0
-6, 7, ~~0~~, 23, 12, 30, ~~-71~~, 19

-71
[-6, 7, -71, 23]
p = (-6, 7, 23) = -6
[-71, 7, -61, 23]
p = (-71, 7, 23) = 7
[-71, -61, 7, 23] ✓

0
[12, 30, 0, 19]
p = (12, 30, 19) = 12
[0, 19, 12, 30]
p = (0, 19, 30) = 19
[0, 12, 19, 30] ✓

[-71, -61, 0, 7, 12, 19, 23, 30]

Pick pivot then move all values smaller to left and greater values to right then break into two partitions and repeat w/ new pivot and sort.

☆
[1,2,3]

5. Worst Case for Quicksort if pivot is arr[0]
The worst case for quicksort if the first pivot is randomly the first element would probably be $O(n^2)$ since all the elements checked will be from the first element so n, Then we would need to partition and check all the inputs once more to get all the numbers checked and sorted. The other time complexities would just be constant so we don't check it.

6. The avg size of a_{left} is $(n-1)/2$ since the array will be divided in two with a left and right so $(n-1)$ will be divided in 2, and so will the time to find a median. The $(n-1)$ is so that we don't count the last term so second to last when sorting with quicksort.

7. Three recursive calls w/ input size $n/2$ each and $5n^2$ steps
Calculate running time.
↳ $3T(n/2) + 5n^2$

$2^k = n \rightarrow \log_2 n = k$

Steps — Size — tree

0 — n — $5n^2$ — 3 branches

1 — $\frac{n}{2}$ — $5\left(\frac{n}{2}\right)^2$

2 — $n/2^2$ — $5\left(\frac{n}{4}\right)^2$

↓ — ↓ — $\Theta(1) \longrightarrow \Theta(1)$

k — $1 = \frac{n}{2^k}$

$$T(n) = 5n^2 + 5\left(\frac{n}{2}\right)^2 + 5\left(\frac{n}{2^2}\right)^2 + \dots + 5\left(\frac{n}{2^{k-1}}\right)^2 + 3^k$$

$$\sum_{i=0}^{k-1} 5\left(\frac{n}{2^i}\right)^2 + 3^k \rightarrow n\sum_{i=0}^{k-1}\left(\frac{5}{2^i}\right)^2 \rightarrow n\sum_{i=0}^{k-1}\frac{25}{2^k-1} + 3^k$$

$= n$

8. a. $T(n) = T(n/2) + 1$ $2^k = n \rightarrow k = \log_2 n$

Step	size	tree
0	$n/0$	1
1	$n/2$	1
↓	↓	↓
k	$1 = \frac{n}{2^k}$	1

$T(n) = 1 + 1 + 1 \dots + 1$ $\sum_{i=0}^{k} 1 = k+1 = \log_2 n$

$\boxed{\Theta(\log_2 n)}$

b. $T(n) = T(n-1) + 1$

Step	size	tree
0	$n-0$	1
1	$n-1$	1
↓	↓	↓
k	$1 = n-k$	1
	↳ $k = n-1$	

$T(n) = 1 + 1 + \dots + 1 =$

$\sum_{i=0}^{k} 1 = k+1$

$n-1+1 = \boxed{\Theta(n)}$

c. $T(n) = 2T(n-1) + 1$

Step	size	tree
0	$n-0$	1
1	$n-1$	1 1
↓	↓	∧ ∧ ∧ ∧
K	$1 = n-k$	$\Theta(1) \longrightarrow$
	$k = n-1$	

$T(n) = 1 + 2 + 2^2 + 2^3 \dots + 2^k$

$\sum_{i=0}^{k} 2^i = \frac{2^{k+1} - 1}{2-1} \simeq 2^{k+1}$

$= 2^{n-1+1} = \boxed{\Theta(2^n)}$

d. $T(n) = 3T(n/4) + n$

Step	size	tree
0	$n/4^0$	n
1	$n/4$	$\frac{n}{4}$ $n/4$ $n/4$
2	$n/4^2$	∧ ∧ ∧
↓	↓	∧ ∧ ∧ ∧ ∧ ∧
k	$1 = \frac{n}{4^k}$	$\Theta(1) \longrightarrow$

$4^k = n \rightarrow k = \log_4 n$

$T(n) = n + 3\frac{n}{4} + 3^2\left(\frac{n}{4^2}\right) + \dots + 3^k$

$\sum_{i=0}^{k-1} 3^i \left(\frac{n}{4^i}\right) \rightarrow n \sum_{i=0}^{k-1} \left(\frac{3}{4}\right)^i + 3^k$

$= n \frac{\cancel{(3/4)^k}^{\to 0} - 1}{(3/4) - 1} + 3^k$ next page

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$n+3^{\log_4 n} = n + n^{\log_4 3} = n + n^{\frac{\log 3}{\log 4}} = n + n^{\frac{1.\sim}{2}} = \boxed{\Theta(n+n^{\log_4 3})}$

e. $T(n) = 3T(n/3) + \sqrt{n}$

Steps	size	tree
0	n	$\sqrt{n}$
1	$n/3$	$\frac{n}{3}$ $n/3$ $n/3$
2	$n/3^2$	$n/3^2$ $\longrightarrow$
$\downarrow$	$\downarrow$	
k	$1 = \frac{n}{3^k}$	$\Theta(1) \longrightarrow$

$T(n) = \sqrt{n} + 3\frac{n}{3} + 3^2\left(\frac{n}{3^2}\right) + 3^k$

$\sum_{i=0}^{k-1} 3^i\left(\frac{n}{3^i}\right) + 3^k$

$n\sum_{i=0}^{k-1}\left(\frac{3}{3}\right)^i + 3^k$

$= \cancel{n\left(\frac{1^k-1}{1-1}\right)} + 3^k$

$3^k = n \rightarrow k = \log_3 n$

$= \cancel{3}^{\log_3 n} \rightarrow n \quad \boxed{\Theta(n)}$

f. $T(n) = T(n/2) + n^2$ $\quad f(n) = n^2 \rightarrow f(n-1) = n^2 - 1$

Steps	size	tree
0	n	n^2
1	$n/2$	$n^2/2$
2	$n/2^2$	$n^2/4$
$\downarrow$	$\downarrow$	$\downarrow$
k	$1 = \frac{n}{2^k}$	$\Theta(1)$

$2^k = n \rightarrow k = \log_2 n$

$T(n) = n^2 + n^2/2 + n^2/4 + \ldots + 1^k$

$\sum_{i=0}^{k-1} \frac{n^2}{2^i} + 1^k$

$= n^2 \sum_{i=0}^{k-1}\left(\frac{1}{2}\right)^i + 1^k$

$\rightarrow n^2 \frac{\frac{1}{2}^k - 1}{\frac{1}{2} - 1} + 1^k$ $\quad 0$

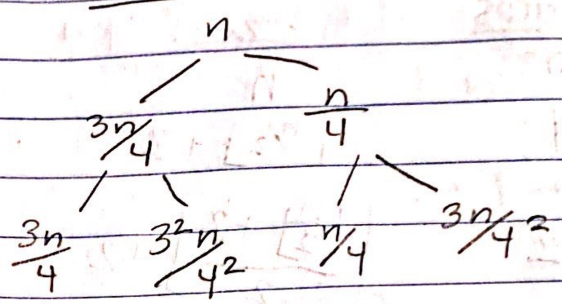
$= n^2 + 1^{\log_2 n} \rightarrow n^2 + n^{\log_2 1}$

$\boxed{= \Theta(n^2)}$

g. $T(n) = T\left(\frac{3n}{4}\right) + T\left(\frac{n}{4}\right) + n$

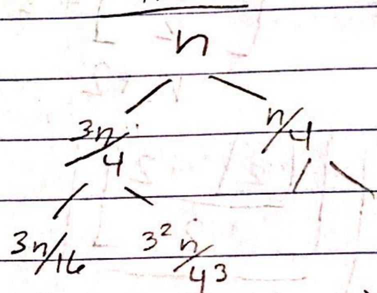
$\Rightarrow T(\square) = T\left(\frac{3\square}{4}\right) + T\left(\frac{\square}{4}\right) + \square$

Step	size	tree
0	n	
1	?	



Green Tree : $T(n) = (\frac{3n}{4}) + T(\frac{n}{4}) + n$

Step	size of leftmost	tree
0	n	
1	$3n/4$	
2	$3n/4^2$	
↓	↓	
k	$1 = \frac{3n}{4^k}$	$\theta(1) \longrightarrow$



$\frac{\log 3n}{\log 4} =$

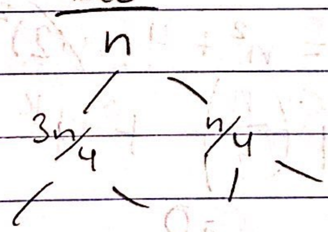
$4^k = 3n \rightarrow k = \log_4 3n$ $\quad \therefore T(n) \geq n + n + \dots + n$

$\geq (k_1 + 1)n \simeq \log_4 3n$

$\Rightarrow T(n) = \boxed{\Omega(n \log n)}$

Red Tree

Step	size of right most	tree
0	n	
1	$n/4$	
2	$n/4^2$	
↓	↓	
K	$1 = \frac{n}{4^k}$	$\Theta(1) \longrightarrow$



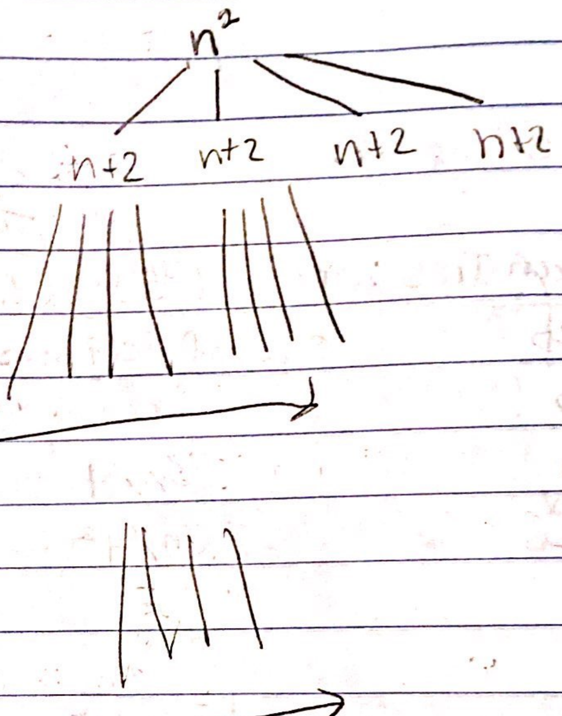
$4^k = n \Rightarrow \log_4 n = k$

$T(n) \leq n + n + \dots + n$

$\leq (k_2 + 1)n \simeq nk_2 \rightarrow T(n) = O(n \log_4 n) \boxed{= O(n \log n)}$

h. $T(n) = 4T(\lfloor n/2 \rfloor + 2) + n^2$

Steps	size	tree
0	n	n^2
1	$\lfloor n/2 \rfloor + 2$	
2	$\left\lfloor \frac{\lfloor \frac{n}{2} \rfloor + 2}{2} \right\rfloor + 2$	
	$\left\lfloor \frac{\frac{n}{2} + 2}{2} \right\rfloor + 2$	
	$n + 2$	
3	$\left\lfloor \frac{\left\lfloor \frac{\lfloor \frac{n}{2} \rfloor + 2}{2} \right\rfloor}{2} \right\rfloor + 2$	
	$\left\lfloor \frac{\frac{\frac{n}{2}+2}{2}}{2} \right\rfloor = \lfloor \frac{n}{2} + 2 \rfloor$	
K	$5 = \frac{n}{k} \rightarrow k = \frac{n}{5}$	



can get rid of 2

$T(n) = n^2 + 4(n\cancel{+2})^2 + 4^2(\frac{n}{2}\cancel{+2}) + \ldots + 4^k$

$\sum_{i=0}^{k-1} 4^i \left(\frac{n}{i}\right) + 4^k$

$= n \cancel{\sum_{i=0}^{k-1}} \frac{\frac{4^i}{i} - 1}{\frac{4^i}{i} - 1} \rightarrow 0 + 4^k \rightarrow n + 4^{n/5} \Rightarrow \boxed{\Theta(4^n)}$?

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$$\boxed{\sum_{i=0}^{(\log_b n)-1} a^i f\left(\frac{n}{b^i}\right) + n^{\log_b a}}$$

9. Use formula to find growth of T(n)

a. $T(n) = T(n/2) + 1$

$a = 1$

$b = 2$

$f(n) = 1$

$K = \log_b n = \log n$

$$T(n) = \sum_{i=0}^{K-1} 1^i \cancel{f\left(\frac{n}{2^i}\right)} + n^{\cancel{\log 1}}$$

$$= 0 + 1 \rightarrow \boxed{\Theta(1)}$$

$$\cancel{= n \sum_{i=0}^{K-1} \left(\frac{1}{2}\right)^i + 1}$$

$$\cancel{= n \frac{(1/2)^K - 1}{1/2 - 1} + 1} \quad \boxed{\cancel{= \Theta(n+1)}}$$

b. $T(n) = 3T(n/3) + n$

$a = 3$

$b = 3$

$f(n) = n$

$K = \log_b n = \log_3 n$

$$T(n) = \sum_{i=0}^{K-1} 3^i f\left(\frac{n}{3^i}\right) + n^{\cancel{\log_3 3}}$$

$$= n \sum_{i=0}^{K-1} \left(\frac{3}{3}\right)^i + n$$

$$= n \frac{1^K - 1}{1 - 1} + n \rightarrow 2n = \boxed{\Theta(n)}$$

c. $T(n) = 4T(n/3) + n$

$a = 4$

$b = 3$

$f(n) = n$

$K = \log_b n = \log_3 n$

$$T(n) = \sum_{i=0}^{K-1} 4^i f\left(\frac{n}{3^i}\right) + n^{\log_3 4}$$

$$= n \sum_{i=0}^{K-1} \left(\frac{4}{3}\right)^i + n^{\log_3 4} \qquad \frac{\log 4}{\log 3} \approx 2$$

$$= n \frac{\left(\frac{4}{3}\right)^K \cancel{-1}}{\frac{4}{3} - 1} + n^{\log_3 4}$$

$$= n\left(\frac{4}{3}\right)^K + n^{\log_3 n} \rightarrow n\left(\frac{4}{3}\right)^{\log_3 n} + n^2$$

$$\boxed{\Theta\, n^2} ?$$

d. $T(n) = 3T(n/4) + \sqrt{n}$

$a = 3$

$b = 4$

$f(n) = \sqrt{n}$

$K = \log_b n = \log_4 n$

$$T(n) = \sum_{i=0}^{K-1} 3^i f\left(\frac{n}{4^i}\right) + n^{\log_4}$$

$$= \sqrt{n} \sum_{i=0}^{K-1} \sqrt{\left(\frac{3}{4}\right)^i} + n^{\log_4}$$

$$= \sqrt{n} \frac{\cancel{\left(\frac{3}{4}\right)^K} - 1}{\frac{3}{4} - 1} + n^{\log_4 3} \rightarrow \frac{\log 3}{\log 4} = \frac{1.6}{2}$$

$$\boxed{\Theta(\sqrt{n})}$$

$$= \sqrt{n} + n^{\log_4 n} \Rightarrow \sqrt{n} + n^{\log_4 3} \rightarrow \boxed{\cancel{\Theta(n^{\log_4 3})}}$$

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e. $T(n) = 5T(n/7) + n^2$

$a = 5$

$b = 7$

$f(n) = n^2$

$K = \log_b n = \log_7 n$

$$T(n) = 5^i \left(\frac{n}{7^i}\right)^2 + n^{\log_7 5}$$

$$= n^2 \sum_{i=0}^{k-1} \left(\frac{5}{49}\right)^i + n^{\log_7 5}$$

$$= n^2 \frac{\left(\frac{5}{49}\right)^k - 1}{\left(5/49\right) - 1} + n^{\log_7 5} \rightarrow 5^{\log_7 n}$$

$$\rightarrow \frac{\log 5}{\log 7} = \frac{2.5}{2.8}$$

$$= n^2 + n^{\log_7 5} \rightarrow \boxed{\Theta(n^2)}$$

f. $T(n) = 6T(n/5) + n^3$

$a = 6$

$b = 5$

$f(n) = n^3$

$K = \log_b n = \log_5 n$

$$T(n) = 6^i f\left(\frac{n}{5^i}\right) + n^{\log_5 6}$$

$$= 6^i \left(\frac{n}{5^i}\right)^3 + n^{\log_5 6}$$

$$= n^3 \sum_{i=0}^{K-1} \left(\frac{6}{125}\right)^i + n^{\log_5 6}$$

$$= n^3 \frac{\left(\frac{6}{125}\right)^k - 1}{\frac{6}{125} - 1} + n^{\log_5 6}$$

$$= n^3 + n^{\log_5 6} \Rightarrow \boxed{\Theta(n^3)}$$