

## Simulation and Modeling (CS 4056)

Date: Before Midterm Exam

Course Instructor(s)

Dr. Mirza Mubasher Baig

## Sessional-II Exam (Assignment)

Total Time (Hrs): 1

Total Weight: 5%

Total Questions: 7

Read Chapter 9 and Material on **Numerical Solution of Differential Equations**

### Question No 1:

Use Euler's method to estimate  $y(0.2)$  for the initial value problem:

$$\frac{dy}{dx} = x + y, \quad y(0) = 1, \quad h = 0.1$$

Compute and compare the results after two steps.

### Question No 2:

Solve the following using both Euler's method and RK4:

$$\frac{dy}{dx} = y - x^2 + 1, \quad y(0) = 0.5, \quad h = 0.2$$

Compute  $y(0.2)$  and  $y(0.4)$ .

Compare your answers with the exact solution:

$$y(x) = (x + 1)^2 - 0.5e^x$$

and compute the absolute error for both methods.

### Question No 3:

Derive Euler's method using Taylor series expansion of  $f(x + h)$

### Question No 4:

Will the Euler's method give exact answer for a differential equation of the form for an arbitrary step size

$$\frac{dy}{dx} = 1, \quad y(0) = 0.5, \quad h$$

In case your answer is YES, Give reason and give a counter example otherwise

### Input Modeling

#### Question No 5: Theory:

- i) What are the main steps involved in input data analysis and modeling for simulation?
- ii) Differentiate between a theoretical and an empirical distribution giving one example of each.
- iii) Name three different tests used to test the goodness of fit while modeling the inputs
- iv) Which test uses cumulative distribution of sample with that of proposed model to check goodness of fit?
- v) Which test uses observed sample frequencies with the expected frequencies to test goodness of fit?

#### Question No 6: Parameter Estimation:

- i) Use MLE principle to derive an expression for estimating the value of parameter  $\lambda$  of the exponential distribution using a sample  $\{x_1, x_2, \dots, x_n\}$  of  $n$  number. The pdf of exponential distribution is

$$f(x; \lambda) = \lambda e^{-\lambda x}, \quad x \geq 0,$$

- ii) Finally, use your expression to find the MLE of  $\lambda$  for the following sample  $\{1, 1, 2, 3, 3\}$

#### Question No 7: Numerical:

- i) A dataset of 30 service times (minutes) has the following frequency distribution:

Interval	0–2	2–4	4–6	6–8	8–10
Observed Frequency	5	10	9	4	2

A fitted exponential distribution has parameter  $\lambda = 0.25/s$

Perform a Chi-square test at  $\alpha = 0.05$  to check if this exponential model is appropriate.

- ii) Given a sample of inter-arrival times:  $\{1.2, 2.0, 2.2, 3.1, 3.4, 4.0, 4.8, 5.6\}$  observed over a period of 30 seconds and a hypothesized exponential distribution with  $\lambda = 0.25$ , perform a K–S test to find the goodness of fit