National University of Computer and Emerging Sciences 0007

Lahore Campus

Simulation and Modeling (CS 4056)

Sessional-I Exam

Date: Sep 23 2025

Course Instructor(s)

Dr. Mirza Mubasher Baig

Total Time (Hrs): 1

Total Marks: 25(10%)

Total Questions: 3

17.8

22L-7881 — Roll No

SE-7A — Section

Student Signature

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Instructions: Answer in the space provided. Attach extra sheets if needed

CLO 1: **Demonstrate** basic understanding of Probabilistic Models and their use in Simulation.

Question 1: [Statistical Models in Simulation] **[4 + 2 Points]**

Part a) What is the primary difference between a probability mass function (PMF) and a probability density function (PDF)? List/name two PMF and two density functions that might be used to model a random phenomenon in a queuing system.

PMF is used for discrete random variable and PDF is used for continuous random variable.

PMF

1) Bernoulli → success(1), $P(x=1) = p$; failure(0) – $P(x=0) = (1-p)$

2) Binomial → $P(x=k) = \binom{n}{k} p^k (1-p)^{n-k}$

PDF

1) Exponential → $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$; $f(x) = 0$ otherwise

2) ~~Poisson~~ → $\cancel{P(x=k) = \frac{\alpha e^{-\alpha}}{k!}}$; $f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$; $f(x) = 0$ otherwise

Part b) What is a cumulative distribution function (CDF), and how is it used in simulation?

CDF represents a formula which we get by integrating the PDF. It basically tells us the probability that a random variable takes, given a specified range.

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Question 2: [Random Number and Random Variable Generation] **[3 + 3 + 2 + 4 Points]**

Part a) Given an LCG with parameters $a = 7$, $c = 5$ and m=16, generate three uniform random numbers using a least significant digit of your four digit university ID as initial seed. Show working.

$RN_1 = (a x + c) \bmod m$ LSD = 1

$= ((7)(1) + 5) \bmod 16$

$= 12 \bmod 16 = \boxed{12}$

$RN_2 = ((7)(12) + 5) \bmod 16$

$= 89 \bmod 16 = \boxed{9}$

$RN_3 = ((7)(9) + 5) \bmod 16$

$= 68 \bmod 16 = \boxed{4}$

2.7

Part b) Derive the formula for the CDF of an exponential distribution with $\lambda = 3$ and use it to find the probability that random variable takes a valueless than 5?

$f(x) = 3e^{-3x}$ for $x \geq 0$

$f(x) = 0$ otherwise

General formula

$\int_0^x 3e^{-3x}dx = 3\left|\frac{e^{-3x}}{-3}\right|_0^x$

$= 3\left(\frac{e^{-3x}}{-3} - \left(-\frac{1}{3}\right)\right)$

$= \boxed{1 - e^{-3x}}$

CDF formula $= \int_0^5 3e^{-3x}dx$

$= 3\left|\frac{e^{-3x}}{-3}\right|_0^5 = 3\left(\frac{e^{-15}}{-3} - \left(\frac{1}{-3}\right)\right)$

$= -e^{-15} + 1 = \boxed{1 - e^{-15}}$ → Probability

2.7

Part c) Use your uniform random numbers of previous part to generate three exponential random numbers with $\lambda = 3$

1) $1 - e^{-12} = 0.999$

2) $1 - e^{-9} = 0.999$

3) $1 - e^{-4} = 0.982$

0.6

Part d) A discrete random variable X has the following PMF

x	1	2	5
P(X = x)	0.1	0.5	0.4

Use your uniform random numbers of part a to generate random numbers that have a distribution similar to that of X

$x = a + (b - a)u$

$= ~~0 + (1)~~$

0

x	12	9	4
P(x=x)	0.6	0.3	0.1

(7)

Question 2: [Simulation of Queue Model] [4 + 1 + 2 Points]

Part a) The surgical emergency of services hospital Lahore has two doctors for the initial examination of incoming patients and an unlimited waiting area. Patients arrive randomly and are served in the order they arrive. Identify the key components of this queuing system (Population, arrival pattern, service discipline, number of servers, system capacity, etc.).

Population → Patients
Arrival Pattern → Random
Service Rate → Random
Number of Servers → 2
Capacity → Unlimited

The waiting area can be represented as a queue where the patients are arriving and leaving when they have been called for service. ~~We can also~~ Infact we can have two waiting queues, one for each doctor.

Part b) Which parts of this queuing system must be modeled using random variables?

Arrival Rate and Service Rate.

Part c) Differentiate between finite and infinite calling populations. Provide an example of each. Is the population in first part of this question finite or infinite? Justify

Calling population refers to the categories of people. Finite calling population would mean that ~~only spe~~ finite categories of people arrive e.g ~~Students and Faculties in a Library (University)~~. ~~&~~ in this case patients because only patients will come to see the doctor. Infinite calling population would mean that everyone can arrive e.g a public library. Population in the first part is finite.