National University of Computer and Emerging Sciences
Lahore Campus

Simulation and Modeling (CS 4056)

Sessional-II

Date: May 04, 2025
Course Instructor(s)
Dr. Mirza Mubasher Baig

Total Time (Hrs): 1
Total Weight: 15
Total Questions: 2

Roll No Section Student Signature

Instructions: Answer in the space provided. Attach extra sheets if really needed
In Case of any missing values state your assumptions clearly

CLO 2: Use simulation models to solve real world problems that involve computing

Question No 1: **[2 + 6 Marks]**

i) Let f(x) = 12 at x = 0. Find the value of this function at x = 12 using Euler's method assuming that derivative of f at x = 0 is i.e. f (*x=0*) = 1

~~Let h = 3~~

~~$x_0 = 0,\ y_0 = 12,\ h = 3$~~ $f(12) = ?$

~~$y_{n+1} = y_n + h$~~

$f(12) = 12 + 12(1)$
$= 24$

1.6

ii) In a forest, the population of rabbits (x) and foxes (y) interact according to the following system of simultaneous equations:

$$\frac{dx}{dt} = 0.6x - 0.02xy$$

2

$$\frac{dy}{dt} = 0.001xy - 0.4y$$

The populations change continuously over time t, measured in **months**.

Assuming X(0) = 40 and Y(0) = 9, estimate the population of rabbits and foxes at time 3 using both the Euler's and RK4 method with a step size h = 3

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$E_i = n p_i$, $CDF = 1 - e^{-\lambda x}$

$p E_i = CDF(b) - CDF(a)$

$E_1 = (1 - e^{-\lambda(1)}) - (1 - e^{-\lambda(0)})n$

$= (1 - e^{-1/2}) - (1 - e^{0})10$

$= 3.93$

$E_2 = (1 - e^{-1/2(2)}) - (1 - e^{-1/2})10$

$= 2.38$

$E_3 = (1 - e^{-1/2(3)}) - (1 - e^{-1/2(2)})10$

$= 1.45$

$E_4 = (1 - e^{-1/2(4)}) - (1 - e^{-1/2(3)})10$

$= 0.88$

$E_5 = (1 - e^{-1/2(5)}) - (1 - e^{-1/2(4)})10$

$= 0.53$

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O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$(O_i - E_i)^2 / E_i$
4	3.93	0.07	4.9×10^{-3}	1.25×10^{-3}
2	2.38	-0.38	0.1444	0.0606
2	1.45	0.55	0.3025	0.2086
1	0.88	0.12	0.0144	0.0164
1	0.53	0.47	0.2209	+ 0.4168
				0.7037

p-value is 0.951 and the value that chi square test gave us (0.7037) is less than the p-value so, yes it does provide a reasonable fit.

4.9

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Question No 2: [1 + 1 + 5 Marks]

i) What are the main steps involved in input data analysis and modeling for simulation?

1) Data Collection

2) Data Plotting

3) Parameter Estimation

4) Goodness of fit Tests

1

ii) MLE of a Uniform random variable that takes value in the interval [a b] are min and max of the sample respectively. Use the following sample to estimate the values of parameters of a Uniform distribution [a b]

Sample: {2.3 1.9, 3, 0.1, 9,1, 0.5, 2,2]

$a = 0.1$, $b = 9$

$MLE = [0.1 \quad 9]$

1

iii) The following data represent the time between failures (in hours) for a sample of 10 identical electronic components:

0.12, 0.45, 0.90, 1.50, 1.90, 2.70, 3.20, 0.30, 4.80, 2.10

It is assumed that the time between failures follows an exponential distribution with mean 2 hours.

Using a Chi-square goodness-of-fit test, determine whether the exponential distribution provides a reasonable fit to the observed data.

Use class intervals of width 1:

[0,1), [1,2), [2,3), [3,4), [4,5) and a 5% level of significance assuming that the p-value for this data at 5% significance is 0.951

$\boxed{\lambda = 1/2}$

Interval	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5
O_i	4	2	2	1	1
E_i	3.93	2.38	1.45	0.88	0.53

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Rabbits

Euler

$x_0=0, y_0=40, h=3$

$\frac{dx}{dt} = 0.6x - 0.02xy$

$\boxed{y_{n+1} = y_n + hf(x_n, y_n)}$

$y_1 = 40 + 3(0)$
$= 40$

$x_1=3, y_1=40$

$= 40+3(-0.6)$
$= 38.2$

$x_2=6, y_2=38.2$

$= 38.2 + 3(-0.98)$

$\boxed{= 35.26}$

RK4 $x_0=0, y_0=40$

$k_1 = hf(x_0, y_0)$

$k_2 = hf(x_0+h/2, y_0+k_1/2)$

$k_3 = hf(x_0+h/2, y_0+k_2/2)$

$k_4 = hf(x_0+h, y_0+k_3)$

$y_{n+1} = y_n + \frac{1}{6}(k_1+2k_2+2k_3+k_4)$

$k_1 = 3(0) = 0$

$k_2 = 3(f(3/2, 40))$
$= 3(-0.3)$
$= -0.9$

$k_3 = 3(f(3/2, 39.5))$
$= -0.85$

$k_4 = 3(f(3/2, 39.15))$
$= -0.82$

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Foxes

Euler

$x_0=0, y_0=9, h=3$

$\frac{dy}{dt} = 0.001xy - 0.4y$

$y_1 = 9 + 3(-3.6)$
$= -1.8$

$x_1=3, y_1=-1.8$

$y_2 = -1.8 + 3(0.715)$
$= 0.345$

$x_2=6, y_2=0.345$

$y_3 = 0.345 + 3(-0.135)$

$\boxed{= -0.06}$

RK4

$x_0=0, y_0=9$

$k_1 = -10.8$

$k_2 = \cancel{-1.435}\ -4.3$

$k_3 = 3(f(3/2, 6.85))$

$k_4 = 3(f(3, y_0+k_3))$