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3.21

a) ROC → $|z| > \frac{1}{6}$

$y[n] = 0 \quad n < 0$

$y[n] = \sum x[k] h[n-k] = \frac{1-a^{n+1}}{1-a}$

$y[n] = \sum x[k] h[n-k] = a^{n+1} \cdot \frac{1-a^{-N}}{a-1}$

b) $H(z) = \sum a^n z^{-n} = \frac{1}{1-az^{-1}}$

$X(z) = \sum z^{-n} = \frac{1-z^{-N}}{1-z^{-1}}$

c) $Y(z) = \frac{1-z^{-N}}{(1-az^{-1})(1-z^{-1})} = \frac{1}{(1-az^{-1})(1-z^{-1})} - \frac{z^{-N}}{(1-a^{z^{-1}})(1-z^{-1})}$

d) $\frac{1}{(1-az^{-1})(1-z^{-1})} = \frac{\frac{1}{1-z^{-1}}}{1-az^{-1}} = \left(\frac{1}{1-a}\right)\left(\frac{1}{1-z^{-1}} - \frac{a}{1-az^{-1}}\right)$

f) $y[n] = \begin{cases} 0 & n < 0 \\ \frac{1-a^{n+1}}{1-a} & 0 \le n \le N-1 \\ a^{n+1}\left(\frac{1-a^{-N}}{a-1}\right) & n \ge N \end{cases}$

3.22

a) $y[n] = \sum h[k] x[n-k] = \sum 3\left(-\frac{1}{3}\right)^k u[k]) u[n-k]$

$Y(z) = H(z) X(z) = \frac{3}{1+\frac{1}{3}z^{-1}} \cdot \frac{1}{1-z^{-1}} =$

$= \frac{3}{1+\frac{1}{3}z^{-1}} \cdot \frac{1}{1-z^{-1}} = \frac{\frac{3}{4}}{1+\frac{1}{3}z^{-1}} + \frac{\frac{9}{4}}{1-z^{-1}}$

$y[n] = \frac{9}{4}\left(1-\left(-\frac{1}{3}\right)^{n+1}\right) u[n]$

3.23

a) $H(z) = \frac{1-\frac{1}{2}z^{-2}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})} = -4 + \frac{5+\frac{7}{2}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} =$

$= -4 - \frac{2}{1-\frac{1}{2}z^{-1}} + \frac{7}{1-\frac{1}{4}z^{-1}}$

b) $y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n] - \frac{1}{2}x[n-2]$

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3.27

a) $a(n) = \sum \delta[n-4u] \Rightarrow A(z) = \sum z^{-4n}$

b) $b(n) = \frac{1}{2}\{e^{j\pi n} + \cos(\frac{\pi}{2}n) + \sin(\frac{\pi}{2} + 2\pi n)\} u(n) =$

$= \frac{1}{2}\{(-1)^n + \cos(\frac{\pi}{n}) + 1\} u(n) = \frac{3/2 + 1/2 z^{-2}}{1 - z^{-4}}$

3.28

a) $X(z) = \frac{xz}{(x-a)(z-b)} = \frac{xz}{z^2 - (a+b)z + ab} =$

$= x^2 - (a+b)z + ab \left\lfloor \frac{1}{x^2} \right.$

$$\frac{\overline{\qquad\qquad}}{\frac{x^2 - (a+b)z + ab}{(a+b)z + ab}}$$

b) $X(z) = 1 + \frac{(a+b)z - ab}{(z-a)(z-b)} = 1 + \frac{\frac{(a+b)z - ab}{a-b}}{z-a} + \frac{\frac{(a+b)b - ab}{b-a}}{z-b} =$

$= \delta[n] \cancel{+} \frac{1}{a-b}(a^{n+1} - b^{n+1}) u[n-1]$

3.26

a) $1 + \frac{1}{3}z^{-1} \left| \frac{1 - \frac{2}{3}z^{-1} + \frac{2}{9}z^{-2}}{1 - \frac{1}{3}z^{-1}} \right.$

$1 - \frac{1}{3}z^{-1}$

$-\frac{3}{2}z^{-1} - \frac{3}{9}z^{-2}$

$+\frac{2}{9}z^{-2}$

b) $X(z) = \frac{3}{z - \frac{1}{4} - \frac{1}{8}z^{-1}} = \frac{3z^{-1}}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{4}z^{-1})} = \frac{4}{(1-\frac{1}{2}z^{-1}} - \frac{4}{1-\frac{2}{4}z^{-1}}$

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3.27

a) $X(z) = \dfrac{1}{(1+\frac{1}{2}z^{-1})^2(1-2z^{-1})(1-3z^{-1})} = \dfrac{\frac{1568}{1225}}{1-2z^{-1}} + \dfrac{\frac{2700}{1225}}{1-3^{-2}}$

b) $X(z) = e^{z^{-1}} = 1 + z^{-1} + \dfrac{z^{-2}}{2!} + \dfrac{z^{-3}}{3!} + \dfrac{z^{-4}}{4!} + \ldots$

c) $X(z) = \dfrac{x^3 - 2x}{x-2} = x^2 + 2x + \dfrac{2}{1-2x^{-1}}$

d) $x(n) = \delta[n+2] + 2\delta[n+1] - 2(2)^n u[-n-1]$

3.28

a) $n\,x[n] \Leftrightarrow -z\dfrac{d}{dx}X(z)$

$x[n-n_0] \Leftrightarrow z^{-n_0}(X(z))$

$x[n] = -12(n-2)\left(\frac{1}{4}\right)^{n-2} u[-n+1]$

b) $X(z) = \sin(z) = \sum \dfrac{(-1)^n}{(2n+1)!} z^{2n+2}$ ROC $=1$

$x[n] = \sum_{n=0}^{\infty} \dfrac{(-1)^n}{(2n+1)!} \delta[n+2n+1]$

c) $X(z) = \dfrac{z^7 - 2}{1-z^{7}} = z^7 - \dfrac{1}{1-z^{-7}} = z^{-7} - \sum z^{-7n}$

$x[n] = \delta[n+7] - \sum \delta[n-7n]$

3.29

a) $X(z) = \sum \dfrac{1}{n!} x^n + \sum \dfrac{1}{n!}\left(\frac{1}{2}\right)^n = \dfrac{1}{(-n)!} z^{-n} + \dfrac{1}{n!} z^{-n} =$

$= x[n] = \dfrac{1}{(n)!} + \delta[n]$

b) $X(z) = \log(1-2z) = -\sum \dfrac{(2z)^l}{l} = -\sum \dfrac{1}{-l}(2z)^{-l} = \dfrac{1}{l}\left(\frac{1}{2}\right)^l z^{-l}$

c) $-z\dfrac{d}{dz}\log(1-2x) = -z\left(\dfrac{1}{1-2z}\right)\cdot(-2) = z^{-1}\left(\dfrac{-1}{1-\frac{1}{2}z^{-1}}\right)$