



Theory of Programming Languages

Describing Syntax and Semantics

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Introduction

- **Syntax:** the *form or structure* of the expressions, statements, and program units
- **Semantics:** the *meaning* of the expressions, statements, and program units
- Syntax and semantics provide a language's definition
 - » Users of a language definition
 - Other language designers
 - Implementers
 - Programmers (the users of the language)

Describing Syntax: Fundamentals

- A *sentence* is a string of characters *over some alphabet*
- A *language* is a set of sentences
- A *lexeme* is the *lowest level syntactic unit* of a language (e.g., *, sum, begin)
- A *token* is a category of lexemes (e.g., identifier)

Describing Syntax: Fundamentals

■ Recognizers

- » A recognition device reads input strings *over the alphabet* of the language and decides whether the input strings belong to the language
- » Example: syntax analysis part of a compiler

■ Generators

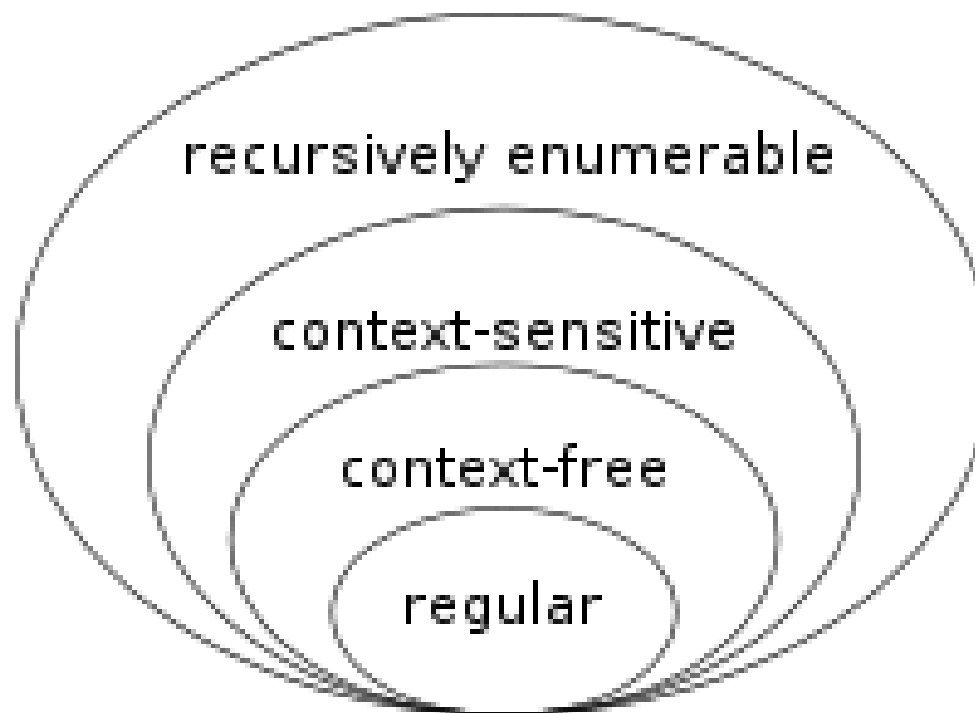
- » A device that generates sentences of a language
- » One can determine if the syntax of a *particular sentence* is syntactically correct by comparing it to the structure of the generator

BNF and Context-Free Grammars

- Context-Free Grammars
 - » Developed by Noam Chomsky in the mid-1950s
 - » Language generators, meant to describe the *syntax* of natural languages
 - » Define a class of languages called context-free languages

- Backus-Naur Form (1959)
 - » Invented by John Backus to describe the syntax of Algol 58
 - » BNF *is equivalent* to context-free grammars

BNF and Context-Free Grammars



BNF Fundamentals

- In BNF, *abstractions* are used to represent classes of syntactic structures
 - » *nonterminal*
 - » *terminals*
- *Terminals* are lexemes or tokens
- A statement in BNF is called *production rule* and it has a:
 - » left-hand side (LHS), which is a nonterminal,
 - » A right-hand side (RHS), which is a string of terminals and/or nonterminals.

BNF Fundamentals (continued)

- Nonterminals are often enclosed in angle brackets
 - » Examples of BNF rules:
`<ident_list> → identifier | identifier, <ident_list>`
`<if_stmt> → if <logic_expr> then <stmt>`
- Grammar: a *finite* non-empty set of rules
- A *start symbol* is a special element of the nonterminals of a grammar

An Example Grammar

$\langle \text{program} \rangle \rightarrow \mathbf{begin} \langle \text{stmts} \rangle \mathbf{end}$

$\langle \text{stmts} \rangle \rightarrow \langle \text{stmt} \rangle \mid \langle \text{stmt} \rangle ; \langle \text{stmts} \rangle$

$\langle \text{stmt} \rangle \rightarrow \langle \text{var} \rangle = \langle \text{expr} \rangle$

$\langle \text{var} \rangle \rightarrow a \mid b \mid c \mid d$

$\langle \text{expr} \rangle \rightarrow \langle \text{term} \rangle + \langle \text{term} \rangle \mid \langle \text{term} \rangle - \langle \text{term} \rangle$

$\langle \text{term} \rangle \rightarrow \langle \text{var} \rangle \mid \text{const}$

Generate $a=b;$

An Example Derivation

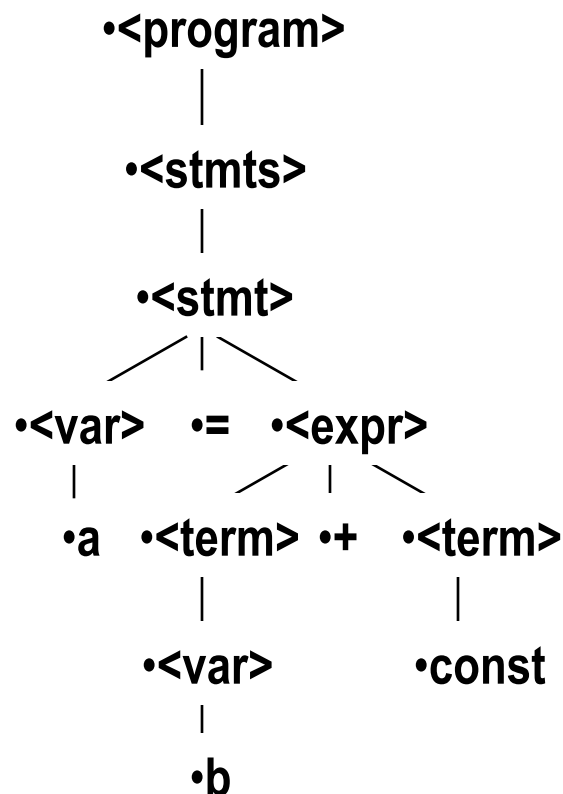
$\langle \text{program} \rangle \Rightarrow \langle \text{stmts} \rangle \Rightarrow \langle \text{stmt} \rangle$
 $\Rightarrow \langle \text{var} \rangle = \langle \text{expr} \rangle$
 $\Rightarrow a = \langle \text{expr} \rangle$
 $\Rightarrow a = \langle \text{term} \rangle + \langle \text{term} \rangle$
 $\Rightarrow a = \langle \text{var} \rangle + \langle \text{term} \rangle$
 $\Rightarrow a = b + \langle \text{term} \rangle$
 $\Rightarrow a = b + \text{const}$

Derivations

- Every string of symbols in a derivation is a *sentential form*
- A *sentence* is a sentential form that has only *terminal symbols*
- A *leftmost derivation* is one in which the leftmost nonterminal in each sentential form is the one that is expanded
- A derivation may be neither leftmost nor rightmost

Parse Tree

- A hierarchical representation of a derivation

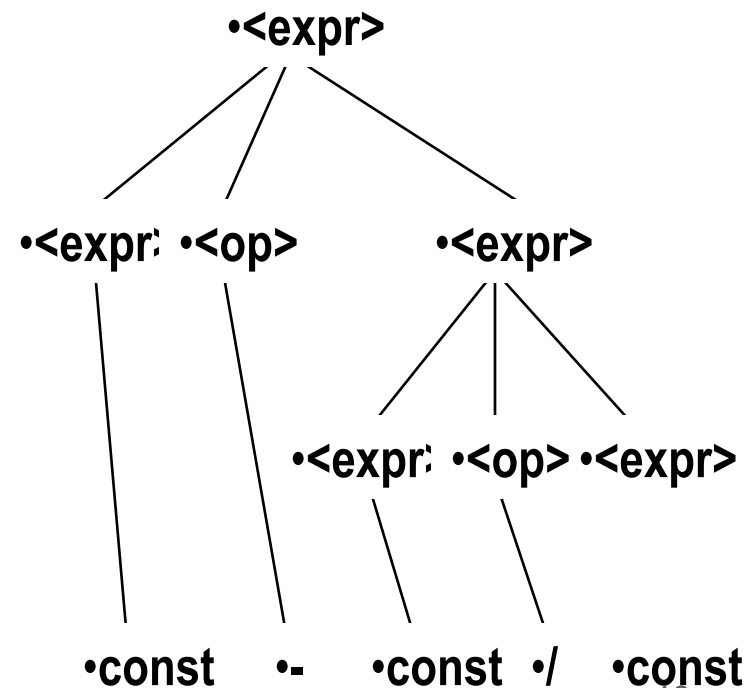
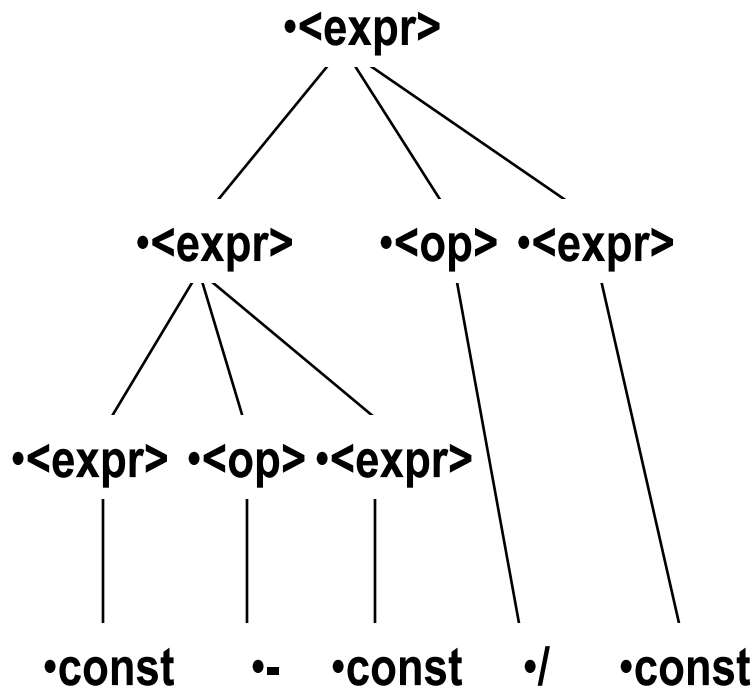


An Ambiguous Expression Grammar

- A grammar is ambiguous if and only if it generates a sentential form that has two or more distinct parse trees

$\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \quad | \quad \text{const}$

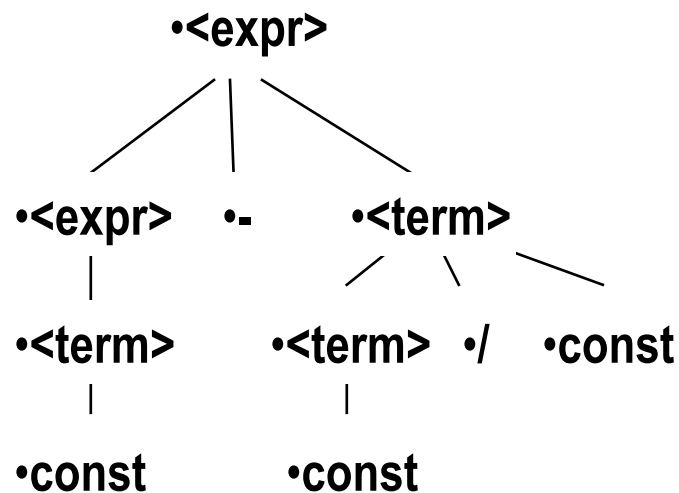
$\langle \text{op} \rangle \rightarrow / \quad | \quad -$



An Unambiguous Expression Grammar

- If we use the parse tree to indicate precedence levels of the operators, we cannot have ambiguity

$\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle - \langle \text{term} \rangle \mid \langle \text{term} \rangle$
 $\langle \text{term} \rangle \rightarrow \langle \text{term} \rangle / \text{const} \mid \text{const}$

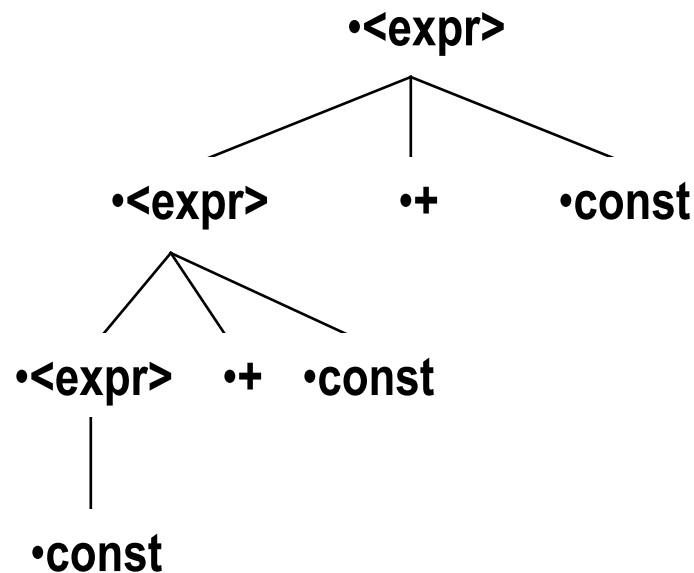


Associativity of Operators

- Operator associativity can also be indicated by a grammar

$\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle + \langle \text{expr} \rangle \mid \text{const}$ (ambiguous)

$\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle + \text{const} \mid \text{const}$ (unambiguous)



Extended BNF

- Optional parts are placed in brackets []

`<proc_call> -> ident [(<expr_list>)]`

- Alternative parts of RHSs are placed inside parentheses and separated via vertical bars

`<term> → <term> (+|-) const`

- Repetitions (0 or more) are placed inside braces { }

`<ident> → letter {letter|digit}`

BNF and EBNF

- BNF

$$\begin{aligned}\langle \text{expr} \rangle &\rightarrow \langle \text{expr} \rangle + \langle \text{term} \rangle \\ &\quad | \langle \text{expr} \rangle - \langle \text{term} \rangle \\ &\quad | \langle \text{term} \rangle \\ \langle \text{term} \rangle &\rightarrow \langle \text{term} \rangle * \langle \text{factor} \rangle \\ &\quad | \langle \text{term} \rangle / \langle \text{factor} \rangle \\ &\quad | \langle \text{factor} \rangle\end{aligned}$$

- EBNF

$$\begin{aligned}\langle \text{expr} \rangle &\rightarrow \langle \text{term} \rangle \{ (+ \mid -) \langle \text{term} \rangle \} \\ \langle \text{term} \rangle &\rightarrow \langle \text{factor} \rangle \{ (* \mid /) \langle \text{factor} \rangle \}\end{aligned}$$

Semantics

- There is *no single widely acceptable notation* or formalism for describing semantics
- Several needs for a methodology and notation for semantics:
 - » Programmers need to know *what statements mean*
 - » *Compiler writers* must know exactly what language constructs do
 - » *Correctness proofs* would be possible
 - » *Compiler generators* would be possible
 - » Designers could *detect* ambiguities and inconsistencies

Operational Semantics

- Operational Semantics
 - » Describe the meaning of a program by executing its *statements on a machine*, either simulated or actual. The change in the state of the machine (memory, registers, etc.) defines the meaning of the statement
- A *hardware* pure interpreter would be too expensive
- A *software* pure interpreter also has problems
 - » The detailed characteristics of the particular computer would make actions difficult to understand
 - » Such a semantic definition would be *machine-dependent*
- To use operational semantics for a high-level language, *a virtual machine* is needed.

Operational Semantics (continued)

- A better alternative: A complete computer simulation
- The process:
 - » Build a translator (translates source code to the *machine code of an idealized computer*)
 - » Build a simulator for the idealized computer

C Statement

```
for (expr1; expr2; expr3) {  
    ...  
}
```

Meaning

```
    expr1;  
loop: if expr2 == 0 goto out  
    ...  
    expr3;  
    goto loop  
out: ...
```

Denotational Semantics

- Based on recursive function theory
- The most abstract semantics description method
- Originally developed by Scott and Strachey (1970)

- The process of building a denotational specification for a language:
 - » Define a *mathematical object* for each language entity
 - » Define a *function* that *maps instances* of the language entities onto instances of the corresponding mathematical objects

- The meaning of language constructs are defined by only the values of the program's variables
- Denotational vs operational semantics?

Denotational Semantics

$$\begin{aligned} \langle \text{bin_num} \rangle &\rightarrow '0' \\ &| '1' \\ &| \langle \text{bin_num} \rangle '0' \\ &| \langle \text{bin_num} \rangle '1' \end{aligned}$$

$$M_{\text{bin}}('0') = 0$$

$$M_{\text{bin}}('1') = 1$$

$$M_{\text{bin}}(\langle \text{bin_num} \rangle '0') = 2 * M_{\text{bin}}(\langle \text{bin_num} \rangle)$$

$$M_{\text{bin}}(\langle \text{bin_num} \rangle '1') = 2 * M_{\text{bin}}(\langle \text{bin_num} \rangle) + 1$$

Decimal Numbers

`<dec_num>` → '0' | '1' | '2' | '3' | '4' | '5' |
 '6' | '7' | '8' | '9' |
 `<dec_num>` ('0' | '1' | '2' | '3' |
 '4' | '5' | '6' | '7' |
 '8' | '9')

$M_{\text{dec}}('0') = 0, \quad M_{\text{dec}}('1') = 1, \quad \dots, \quad M_{\text{dec}}('9') = 9$

$M_{\text{dec}}(<\text{dec_num}> '0') = 10 * M_{\text{dec}}(<\text{dec_num}>)$

$M_{\text{dec}}(<\text{dec_num}> '1') = 10 * M_{\text{dec}}(<\text{dec_num}>) + 1$

...

$M_{\text{dec}}(<\text{dec_num}> '9') = 10 * M_{\text{dec}}(<\text{dec_num}>) + 9$

Axiomatic Semantics

- Based on *formal logic* (predicate calculus)
- Original purpose: *formal program verification*
- Axioms or inference rules are defined for each statement type in the language (to allow transformations of logic expressions into more formal logic expressions)
- The logic expressions are called *assertions*

Axiomatic Semantics (continued)

- An assertion before a *statement (a precondition)* states the relationships and constraints among variables that are true at that point in execution
- An assertion *following a statement is a postcondition*
- A *weakest precondition* is the least restrictive precondition that will guarantee the postcondition
- Pre-, post form: $\{P\} \text{ statement } \{Q\}$
- An example
 - » $a = b + 1 \quad \{a > 1\}$
 - » One possible precondition: $\{b > 10\}$
 - » Weakest precondition: $\{b > 0\}$

Program Proof Process

- The postcondition for the entire program is the desired result
 - » Work back through the program to the first statement. If the precondition on the first statement is the same as the program specification, the program is correct.

Axiomatic Semantics: Assignment

- An axiom for assignment statements
 $(x = E): \{Q_{x \rightarrow E}\} \ x = E \ \{Q\}$
- The Rule of Consequence:

$$\frac{\{P\} S \{Q\}, P' \Rightarrow P, Q \Rightarrow Q'}{\{P'\} S \{Q'\}}$$

Evaluation of Axiomatic Semantics

- Developing axioms or inference rules for all of the statements in a *language is difficult*
- It is a good tool for *correctness proofs*, and an excellent framework for reasoning about programs, but it is not as useful for language users and compiler writers
- Its usefulness in describing the meaning of a programming language is limited for language users or compiler writers