

Proof – Total Nodes of a Perfect Binary Tree

- Sum of finite Geometric Progression

$$- S_n = a + ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-1} = a \left[\frac{(r^n - 1)}{r - 1} \right] \text{-----Eq. (1)}$$

- Series we have in perfect binary trees

$$- \text{Total nodes} = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + \dots + 2^h$$

$$- \text{Total nodes} = 1.2^0 + 1.2^1 + 1.2^2 + 1.2^3 + 1.2^4 + \dots + 1.2^h$$

$$- a = 1, r = 2 \quad \text{and}$$

$$- n-1 = h \quad \text{OR} \quad n = h+1$$

Putting above values in in the formula of Eq.(1)

$$- \text{Total nodes} = a \left[\frac{(r^n - 1)}{r - 1} \right]$$

$$- \text{Total nodes} = 1 \cdot \left[\frac{(2^{h+1} - 1)}{2 - 1} \right]$$

$$- \text{Total nodes} = 2^{h+1} - 1$$