

# Proof – Total Nodes of a Perfect Binary Tree

- Geometric Progression (finite)

- $a, ar, ar^2, ar^3, ar^4, \dots, ar^{n-1}$

- Sum of Geometric Progression

- $S_n = a + ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-1}$

- $S_n = a \left[ \frac{r^n - 1}{r - 1} \right]$  if  $r > 1$  and  $r \neq 1$

Trees

- Total no. of nodes = sum of internal + external nodes

- Internal nodes =  $2^h - 1$

- External nodes =  $2^h$

- Total nodes =  $2^h - 1 + 2^h$

$$= 2^h + 2^h - 1 \quad ; \text{ let say; } a = 2^h$$

$$= a + a - 1$$

$$= 2a + 1$$

$$= 2 \cdot 2^{h+1} - 1 \quad ; \text{ replacing } a \text{ with } 2^h$$

$$= 2^{h+1} - 1$$