

\*Problems with preexisting editorials are excluded.

### Problem B:

Let **B** = the number of black squares.

Then the number of white squares, **W = N-B**.

Let  $w_i$  be the number of white blocks between block  $i$  and  $(i+1)$ . Let  $w_0$  be the no of white squares to the left of the first black square and  $w_{k+1}$  be the no of white square to the right of the last black square.

Then  $w_0 + w_1 + w_2 + \dots + w_{k-1} + w_k = W$

where,  $w_0, w_k \geq 0$  and  $w_1, \dots, w_{k-1} \geq 1$ .

We want to count the number of solutions to the above equation. That can be done with stars and bars.

### Problem D:

Straightforward Game Theory. Let  $dp[i] = 1$  if first player wins, 0 otherwise.

Then  $dp[i] = 0$  if and only if for all  $j$ ,  $dp[i-fibo[j]]$  is 1.

### Problem E:

Let us call a person special if he has one of the four special roles. Let's fix the number of special persons,  $x$ . Then  $x$  can be between 1 and 4. The Special persons can be chosen  $C(n, x)$  ways and their roles of the special persons can be chosen  $S(4, x) \cdot x!$  Ways where  $S(n, x)$  is the Sterling no of the second kind. Finally for each of the rest  $n-x$  people we have 2 choices for each of them, include or exclude.

So, the answer is 
$$\sum_{i=1}^4 C(n, i) * S(4, i) * i! * 2^{n-i}$$

Instead of using stirling numbers we can also count the number of ways to assign roles manually.

**Problem F:**

The answer is the catalan numbers. The [proof](#) is somewhat hard. But this can also be solved with dp.

**Problem H:**

Easy inclusion exclusion. For each subset of  $\{n, n+d, n+2*d, n+3*d, n+4*d\}$ . Calculate how many numbers are divisible by all of the elements. Then add or subtract depending on the parity.

**Problem I:**

Let's fix the number of fixed points,  $x$ . Then the first  $m$  positions will have  $k$  fixed points, the other  $n-m$  positions will have  $x-k$  fixed points. We want a derangement of the remaining  $n-x$  elements. So, the number of valid permutations with exactly  $x$  fixed points is  $C(m, k) * (n-m, x-k) * D[n-x]$ . Where  $D[i]$  is the number of derangements. Sum over all possible values of  $x$ .

**Problem J:**

This was discussed in class.  $C(n, r)$  is odd if and only if  $r$  is a submask of  $n$ .

**Problem L:**

Let the no of unique positions be  $k$ . So, we have to divide  $n$  horses into  $k$  groups with no empty groups. This can be done in  $S(n, k) * k!$  Ways.

So, the answer is  $\sum_{i=1}^k S(n, i) * i!$ . This is known as Ordered Bell Number.

**Problem M:**

Sort the numbers. The  $i$ th smallest one can be the largest number of a subset in  $C(i-1, k-1)$  ways. So, the answer is  $\sum_{i=1}^k a[i] C(i-1, k-1)$ .

**Problem P:**

<https://toph.co/p/pleasant-permutations/editorial>