

# Differential Equations

With Boundary-Value Problems

SEVENTH  
EDITION

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# REVIEW OF DIFFERENTIATION

## Rules

1. **Constant:**  $\frac{d}{dx} c = 0$

2. **Constant Multiple:**  $\frac{d}{dx} cf(x) = c f'(x)$

3. **Sum:**  $\frac{d}{dx} [f(x) \pm g(x)] = f'(x) \pm g'(x)$

4. **Product:**  $\frac{d}{dx} f(x)g(x) = f(x)g'(x) + g(x)f'(x)$

5. **Quotient:**  $\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

6. **Chain:**  $\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$

7. **Power:**  $\frac{d}{dx} x^n = nx^{n-1}$

8. **Power:**  $\frac{d}{dx} [g(x)]^n = n[g(x)]^{n-1} g'(x)$

## Functions

### Trigonometric:

9.  $\frac{d}{dx} \sin x = \cos x$

10.  $\frac{d}{dx} \cos x = -\sin x$

11.  $\frac{d}{dx} \tan x = \sec^2 x$

12.  $\frac{d}{dx} \cot x = -\csc^2 x$

13.  $\frac{d}{dx} \sec x = \sec x \tan x$

14.  $\frac{d}{dx} \csc x = -\csc x \cot x$

### Inverse trigonometric:

15.  $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

16.  $\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$

17.  $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$

18.  $\frac{d}{dx} \cot^{-1} x = -\frac{1}{1+x^2}$

19.  $\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$

20.  $\frac{d}{dx} \csc^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}$

### Hyperbolic:

21.  $\frac{d}{dx} \sinh x = \cosh x$

22.  $\frac{d}{dx} \cosh x = \sinh x$

23.  $\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$

24.  $\frac{d}{dx} \coth x = -\operatorname{csch}^2 x$

25.  $\frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \tanh x$

26.  $\frac{d}{dx} \operatorname{csch} x = -\operatorname{csch} x \coth x$

### Inverse hyperbolic:

27.  $\frac{d}{dx} \sinh^{-1} x = \frac{1}{\sqrt{x^2+1}}$

28.  $\frac{d}{dx} \cosh^{-1} x = \frac{1}{\sqrt{x^2-1}}$

29.  $\frac{d}{dx} \tanh^{-1} x = \frac{1}{1-x^2}$

30.  $\frac{d}{dx} \coth^{-1} x = \frac{1}{1-x^2}$

31.  $\frac{d}{dx} \operatorname{sech}^{-1} x = -\frac{1}{x\sqrt{1-x^2}}$

32.  $\frac{d}{dx} \operatorname{csch}^{-1} x = -\frac{1}{|x|\sqrt{x^2+1}}$

### Exponential:

33.  $\frac{d}{dx} e^x = e^x$

34.  $\frac{d}{dx} b^x = b^x (\ln b)$

### Logarithmic:

35.  $\frac{d}{dx} \ln|x| = \frac{1}{x}$

36.  $\frac{d}{dx} \log_b x = \frac{1}{x(\ln b)}$

# BRIEF TABLE OF INTEGRALS

1.  $\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$
2.  $\int \frac{1}{u} du = \ln|u| + C$
3.  $\int e^u du = e^u + C$
4.  $\int a^u du = \frac{1}{\ln a} a^u + C$
5.  $\int \sin u du = -\cos u + C$
6.  $\int \cos u du = \sin u + C$
7.  $\int \sec^2 u du = \tan u + C$
8.  $\int \csc^2 u du = -\cot u + C$
9.  $\int \sec u \tan u du = \sec u + C$
10.  $\int \csc u \cot u du = -\csc u + C$
11.  $\int \tan u du = -\ln|\cos u| + C$
12.  $\int \cot u du = \ln|\sin u| + C$
13.  $\int \sec u du = \ln|\sec u + \tan u| + C$
14.  $\int \csc u du = \ln|\csc u - \cot u| + C$
15.  $\int u \sin u du = \sin u - u \cos u + C$
16.  $\int u \cos u du = \cos u + u \sin u + C$
17.  $\int \sin^2 u du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$
18.  $\int \cos^2 u du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$
19.  $\int \tan^2 u du = \tan u - u + C$
20.  $\int \cot^2 u du = -\cot u - u + C$
21.  $\int \sin^3 u du = -\frac{1}{3}(2 + \sin^2 u)\cos u + C$
22.  $\int \cos^3 u du = \frac{1}{3}(2 + \cos^2 u)\sin u + C$
23.  $\int \tan^3 u du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$
24.  $\int \cot^3 u du = -\frac{1}{2}\cot^2 u - \ln|\sin u| + C$
25.  $\int \sec^3 u du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln|\sec u + \tan u| + C$
26.  $\int \csc^3 u du = -\frac{1}{2}\csc u \cot u + \frac{1}{2}\ln|\csc u - \cot u| + C$
27.  $\int \sin au \cos bu du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$
28.  $\int \cos au \cos bu du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$
29.  $\int e^{au} \sin bu du = \frac{e^{au}}{a^2 + b^2}(a \sin bu - b \cos bu) + C$
30.  $\int e^{au} \cos bu du = \frac{e^{au}}{a^2 + b^2}(a \cos bu + b \sin bu) + C$
31.  $\int \sinh u du = \cosh u + C$
32.  $\int \cosh u du = \sinh u + C$
33.  $\int \operatorname{sech}^2 u du = \tanh u + C$
34.  $\int \operatorname{csch}^2 u du = -\coth u + C$
35.  $\int \tanh u du = \ln(\cosh u) + C$
36.  $\int \coth u du = \ln|\sinh u| + C$
37.  $\int \ln u du = u \ln u - u + C$
38.  $\int u \ln u du = \frac{1}{2}u^2 \ln u - \frac{1}{4}u^2 + C$
39.  $\int \frac{1}{\sqrt{a^2 - u^2}} du = \sin^{-1} \frac{u}{a} + C$
40.  $\int \frac{1}{\sqrt{a^2 + u^2}} du = \ln\left|u + \sqrt{a^2 + u^2}\right| + C$
41.  $\int \sqrt{a^2 - u^2} du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1} \frac{u}{a} + C$
42.  $\int \sqrt{a^2 + u^2} du = \frac{u}{2}\sqrt{a^2 + u^2} + \frac{a^2}{2}\ln\left|u + \sqrt{a^2 + u^2}\right| + C$
43.  $\int \frac{1}{a^2 + u^2} du = \frac{1}{a}\tan^{-1} \frac{u}{a} + C$
44.  $\int \frac{1}{a^2 - u^2} du = \frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right| + C$

**Note:** Some techniques of integration, such as integration by parts and partial fractions, are reviewed in the *Student Resource and Solutions Manual* that accompanies this text.