Quiz-1 Solutions CLO-1

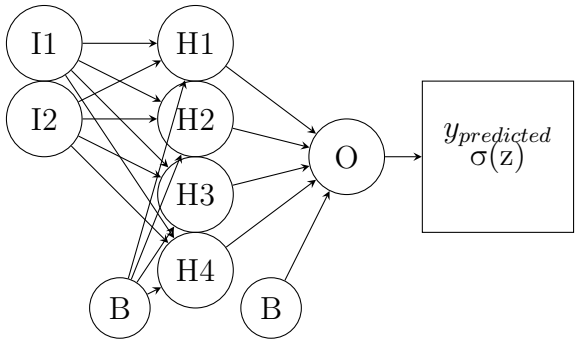
Duration: 45 minutes
Total Marks: 20

Question #1:[10 marks] CLO 1

The general structure of a basic Neural Network is as follows:

Suppose you have a simple neural network with one input layer, one hidden layer, and one output layer. The input layer has 2 neurons, the hidden layer has **4 neurons**, and the output layer has 1 neuron. The activation function for all neurons is the sigmoid function. The network has already been initialized with the following weights and biases:

Network Structure



Network Parameters

Hidden layer weight matrix:

$$W_{hidden} = \begin{bmatrix} 0.3 & 0.8 \\ 0.5 & 0.1 \\ 0.9 & 0.7 \\ 0.2 & 0.6 \end{bmatrix}$$

Hidden layer bias vector:

$$b_{hidden} = \begin{bmatrix} 0.2 \\ 0.4 \\ 0.5 \\ 0.3 \end{bmatrix}$$

Hidden Layer output vector :

$$H_{output} = \begin{bmatrix} 0.3 \\ 0.5 \\ 0.9 \\ 0.7 \end{bmatrix}$$

Output layer bias:

$$b_{output} = \begin{bmatrix} 0.2 \end{bmatrix}$$

The sigmoid activation function is applied to the output neuron to ensure that the output remains between 0 and 1. It is mathematically defined as:

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

where z is the weighted sum of inputs plus bias.

Tasks

1. Given the input vector ($I1 = 0.5, I2 = 0.8$), perform a forward pass through the network to compute the predicted output.
2. Suppose that the true output for the given input is 0.6. Calculate the mean square error (MSE) between the true output and the predicted output.

Solution

Task 1: Forward Pass

1. **Input to Hidden Layer:** - Input vector: $X = \begin{bmatrix} 0.5 \\ 0.8 \end{bmatrix}$ - Weight matrix: $W_{hidden} = \begin{bmatrix} 0.3 & 0.8 \\ 0.5 & 0.1 \\ 0.9 & 0.7 \\ 0.2 & 0.6 \end{bmatrix}$ - Bias vector: $b_{hidden} = \begin{bmatrix} 0.2 \\ 0.4 \\ 0.5 \\ 0.3 \end{bmatrix}$ - Compute the weighted sum for the hidden layer:

$$Z_{hidden} = W_{hidden} \cdot X + b_{hidden}$$

$$Z_{hidden} = \begin{bmatrix} 0.3 \cdot 0.5 + 0.8 \cdot 0.8 + 0.2 \\ 0.5 \cdot 0.5 + 0.1 \cdot 0.8 + 0.4 \\ 0.9 \cdot 0.5 + 0.7 \cdot 0.8 + 0.5 \\ 0.2 \cdot 0.5 + 0.6 \cdot 0.8 + 0.3 \end{bmatrix}$$

$$Z_{hidden} = \begin{bmatrix} 0.15 + 0.64 + 0.2 \\ 0.25 + 0.08 + 0.4 \\ 0.45 + 0.56 + 0.5 \\ 0.1 + 0.48 + 0.3 \end{bmatrix}$$

$$Z_{hidden} = \begin{bmatrix} 0.99 \\ 0.73 \\ 1.51 \\ 0.88 \end{bmatrix}$$

- Apply the sigmoid activation function:

$$H_{output} = \sigma(Z_{hidden}) = \begin{bmatrix} \frac{1}{1+e^{-0.99}} \\ \frac{1}{1+e^{-0.73}} \\ \frac{1}{1+e^{-1.51}} \\ \frac{1}{1+e^{-0.88}} \end{bmatrix}$$

$$H_{output} = \begin{bmatrix} 0.729 \\ 0.675 \\ 0.819 \\ 0.707 \end{bmatrix}$$

2. **Hidden Layer to Output Layer:** - Weight vector: $W_{output} = \begin{bmatrix} 0.3 & 0.5 & 0.9 & 0.2 \end{bmatrix}$ - Bias: $b_{output} = 0.2$ - Compute the weighted sum for the output layer:

$$Z_{output} = W_{output} \cdot H_{output} + b_{output}$$

$$Z_{output} = 0.3 \cdot 0.729 + 0.5 \cdot 0.675 + 0.9 \cdot 0.819 + 0.2 \cdot 0.707 + 0.2$$

$$Z_{output} = 0.2187 + 0.3375 + 0.7371 + 0.1414 + 0.2$$

$$Z_{output} = 1.6347$$

- Apply the sigmoid activation function:

$$y_{predicted} = \sigma(Z_{output}) = \frac{1}{1 + e^{-1.6347}} = 0.837$$

Task 2: Mean Square Error (MSE)

Given the true output $y_{true} = 0.6$ and the predicted output $y_{predicted} = 0.837$, the MSE is calculated as:

$$MSE = \frac{1}{2}(y_{true} - y_{predicted})^2$$

$$MSE = \frac{1}{2}(0.6 - 0.837)^2 = \frac{1}{2}(-0.237)^2 = \frac{1}{2} \cdot 0.0562 = 0.0281$$

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Question #2:[10 marks] CLO 1

A single-layer neural network must have six inputs and two outputs. The outputs are limited to and continuous over the range 0 to 1. Analyze the network architecture with respect to the following aspects:

1. How many neurons are required?
2. What are the dimensions of the weight matrix?
3. What kind of activation functions could be used for continuous output or classification task?
4. Why a bias is required?

Solution

1. **Number of Neurons:** - The network requires 2 neurons in the output layer (one for each output).

2. **Dimensions of the Weight Matrix:** - The weight matrix will have dimensions 2×6 (2 outputs and 6 inputs).

3. **Activation Functions:** - For continuous output in the range [0, 1], the sigmoid activation function is suitable. - For classification tasks, softmax activation can be used for multi-class classification.

4. **Bias Requirement:** - A bias is required to shift the activation function and provide flexibility to the model, allowing it to fit the data better.