ARGUMENTS

Discrete Structures

In Today's Lecture

▶ **Logic:**

Logic rules and principles is to distinguish an argument is valid or invalid.

▶ **Examples of Arguments:**

  ▶ You have a intuitive idea about argument. When you are talking with you friend you give argument.

  ▶ Sometimes you say to your friend "what are you saying has no logic" it means you are saying that your argument is not valid.

  ▶ Lawyer in court during the trail to defend client, give argument. Judge decision is also based on lawyer argument if its argument is valid then decision will be in his favor.

▶

Example:

An interesting teacher keeps me awake.

I stay awake in discrete structure class.

Therefore, my discrete structure teacher is interesting.

- 1st two statements we are declaring(*premises*) on this basis 3rd line is giving *conclusion*.

- The argument presenting is, its valid or not?

ARGUMENT:

▸ An **argument** is a list of statements called **premises** (or **assumptions** or **hypotheses**) followed by a statement called the **conclusion**.

P1   Premise

P2   Premise

P3   Premise

…….……

Pn   Premise

_____

∴ C   Conclusion

▸ **NOTE** :

▸ The symbol ∴ read "therefore," is normally placed just before the conclusion.

VALID ARGUMENT

▸ An argument is **valid** if the conclusion is true when all the premises are true.

▸ Alternatively, an argument is valid if conjunction of its premises imply conclusion.

That is $(P1 \wedge P2 \wedge P3 \wedge \ldots \wedge Pn) \rightarrow C$ is a tautology.

▸ In that case if argument becomes tautology then we say the argument is valid.

▸

p implies q is mostly true.

p implies q is false whenever
p is true and q is false

That is

P true, q false then p → q is false

Note:

- In argument, if one **premise** is false then we say argument is valid. Because premise implies conclusion, the premise is false implies conclusion is whatever the implication of argument becomes true.

- If all the premise is true then the conjunction of all the premise is also true. And if the conclusion becomes also true, then the whole implication becomes true.

Therefore = thus = so that

INVALID ARGUMENT

▸ An argument is **invalid** if the conclusion is false when all the premises are true.

▸ Alternatively, an argument is invalid if conjunction of its premises does not imply conclusion.

▸ **Validity of argument:**

▸ When our premises conjunction is false, and conclusion is whatever true or false, the argument is valid

▸ When our premises conjunction is true, and conclusion is false, then we say argument is invalid.

▸

Argument Form

▸ **If** the premises and the conclusion

are statement forms

instead of statements,

then the resulting form is called

argument form.

▸ *Ex:*   If p then q;

p;

$\therefore$ q.

Checking the validity of an argument form

1) Construct truth table for the premises and the conclusion;

2) Find the rows in which all the premises are true (*critical rows*);

3) a. *If* in each critical row the conclusion

   is true

   then the argument form is valid;

   b. *If* there is a row in which conclusion

   is false

   then the argument form is invalid.

EXAMPLE:

▶ Show that the following argument form is valid:

$$p \rightarrow q \qquad \textbf{premise}$$

$$p \qquad \textbf{premise}$$

$$\therefore \; q \qquad \textbf{conclusion}$$

premises

conclusion

p	q	p → q	p	q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

critical row

• To validity we will not analyze the whole table.
• We will analyze those rows where in premise we have T value, and if corresponding conclusion also have T value, then we say it's a valid argument.

EXAMPLE OF INVALID ARGUMENT
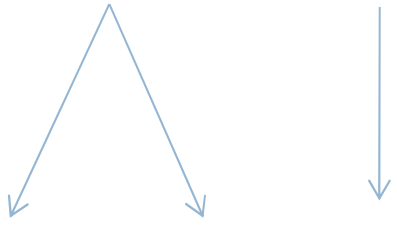
▸ Show that the following argument form is invalid:

$$p \rightarrow q \qquad \text{premise}$$

$$q \qquad \text{premise}$$

$$\therefore \quad p \qquad \text{conclusion}$$
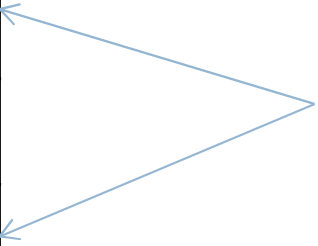
premises   conclusion

p	q	p → q	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

critical row

This argument is not valid.

EXERCISE

▶ Use truth table to determine the argument form

p ∨ q	premise
p → ~q	premise
p → r	premise
∴ r	conclusion

▶ Is valid or invalid?

p ∨ q   premise
p → ~q   premise
p → r   premise
r   conclusion

Is valid or invalid?

p	**q**	**r**	**p ∨ q**	**p →~q**	**p → r**	**r**
T	T	T	T	F	T	T
T	T	F	T	F	F	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	T	T	T	T
F	T	F	T	T	T	F
F	F	T	F	T	T	T
F	F	F	F	T	T	F

premises

conclusion

p	q	r	p ∨ q	p → ~q	p → r	r
T	T	T	T	F	T	T
T	T	F	T	F	F	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	T	T	T	T
F	T	F	T	T	T	F
F	F	T	F	T	T	T
F	F	F	F	T	T	F

critical rows

The argument is not valid because all corresponding values in conclusion are not T

WORD PROBLEM

▸ If Tariq is not on team A, then Hameed is on team B.

If Hameed is not on team B, then Tariq is on team A.

∴ Tariq is not on team A or Hameed is not on team B.

▸ **SOLUTION:**

Let

$$t = \text{Tariq is on team A}$$

$$h = \text{Hameed is on team B}$$

Then the argument is

$$\sim t \rightarrow h$$

$$\sim h \rightarrow t$$

$$\therefore \sim t \vee \sim h$$

▸

t	h	~t → h	~h → t	~t ∨ ~h
T	T	T	T	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

Argument is invalid. Because there are three critical rows (Remember that the critical rows are those rows where the premises have truth value T) and in the first critical row conclusion has truth value F. (Also remember that we say an argument is valid if in all critical rows conclusion has truth value T)

EXERCISE

▸ If at least one of these two numbers is divisible by 6, then the product of these two numbers is divisible by 6.

Neither of these two numbers is divisible by 6.

$\therefore$ The product of these two numbers is not divisible by 6.

▸ SOLUTION:

Let p = at least one of these two numbers is divisible by 6.

q = product of these two numbers is divisible by 6.

Then the argument become in these symbols

$$p \rightarrow q$$
$$\sim p$$
$$\therefore \quad \sim q$$

▸

▸ We will made the truth table for premises and conclusion as given below:

p	q	p → q	~p	~q
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

▸ **The Argument is invalid.**

EXERCISE

▸ If I got an Eid bonus, I'll buy a stereo.

  If I sell my motorcycle, I'll buy a stereo.

  ∴ If I get an Eid bonus or I sell my motorcycle, then I'll buy a stereo.

SOLUTION:

   Let

   e = I got an Eid bonus

   s = I'll buy a stereo

   m = I sell my motorcycle

▸ The argument is

$$e \rightarrow s$$

$$m \rightarrow s$$

$$\therefore \quad e \vee m \rightarrow s$$

e	s	m	e →s	m →s	e ∨ m	e ∨ m →s
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	F	T	F
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	T	F	T	F
F	F	F	T	T	F	T

The argument is valid. Because there are five critical

▸ rows

REMEMBER:

- (Remember that the critical rows are those rows where the premises have truth value T) and in all critical row conclusion has truth value T. (Also remember that we say an argument is valid if in all critical rows conclusion has truth value T)

EXERCISE

▶ An interesting teacher keeps me awake. I stay awake in Discrete Mathematics class. Therefore, my Discrete Mathematics teacher is interesting.

Solution:

t: my teacher is interesting

a: I stay awake

m: I am in Discrete Mathematics class

▶ The argument to be tested is

$$t \rightarrow a,$$

$$a \wedge m$$

Therefore $\quad\quad\quad\quad\quad m \wedge t$

t	a	m	t $\rightarrow$ a	a $\wedge$ m	m $\wedge$ t
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	F	T
T	F	F	F	F	F
F	T	T	T	T	F
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	T	F	F

Argument is not valid.