CC141

Discrete Structures

IQRA JAVED

iqra.javed@umt.edu.pk

SDT-403

Course Objectives

Discrete Structures course provides students with a foundational understanding of mathematical concepts and structures that are crucial in computer science and related fields.

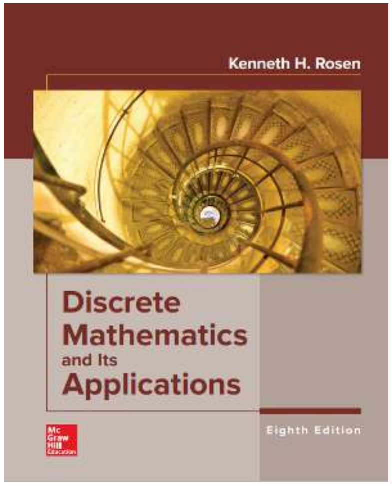
Topics typically covered include set theory, logic, proof techniques, combinatorics, graph theory, trees, and relations.

Through this course, students develop analytical and problem-solving skills, enabling them to tackle complex computational problems, design algorithms, and reason rigorously about discrete systems, forming a solid basis for further study in computer science and related disciplines.

Books

Text Book

- Discrete Mathematics and Its Applications by Kenneth H. Rosen (Latest Edition).



Reference Material

- Discrete Mathematics with Applications by Susanna S. Epp
- Discrete Mathematics by Richard Johnsonbaugh

Assessment Instruments with Weights

Assignments: 10%
Quizzes: 15%
Midterm Exam: 25%
Final Exam: 40%
Class Participation/Attendance: 5%
Class Activities: 5%

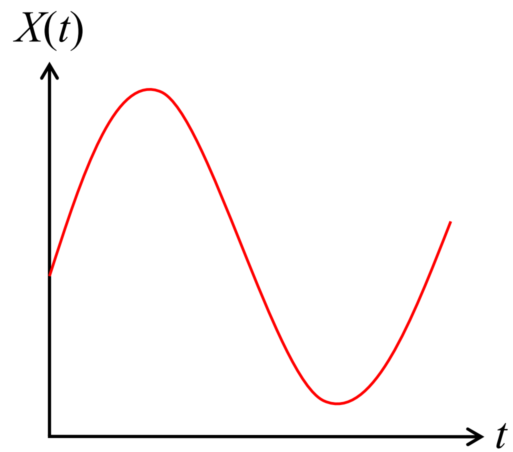
What is Discrete mathematics / structures?

- The word **discrete** is essentially the opposite of **continuous**, **discontinuous** or **segregated**.

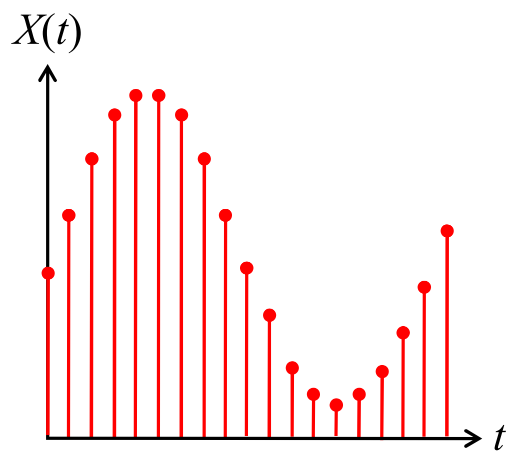
Definition:

"**Discrete Mathematics/Structure** concerns processes that consist of a sequence of individual steps."

Discrete and Continuous



Continuous Signals



Discrete Signals

Discrete and Continuous

<u>Set of Real Numbers:</u>

-3 -2 -1 0 1 2 3

Continuous / Analog

<u>Set of Integers:</u>

3 -2 -1 0 1 2

Discrete / Digital

Logic

- Logic rules and principles is to distinguish an argument is valid or invalid.

- **Def:**

  "Logic is the study of the principles and methods that distinguishes between a valid and an invalid argument."

Logic

- Crucial for mathematical reasoning
- Important for program design
- Used for designing electronic circuitry

- (Propositional)Logic is a system based on **propositions**.

- A proposition is a (declarative) statement that is either true or false (not both).
- We say that the truth value of a proposition is either true (T) or false (F).
- Corresponds to 1 and 0 in digital circuits

The Statement/Proposition Game

"Elephants are bigger than mice."

Is this a statement? yes

Is this a proposition? yes

What is the truth value of the proposition? true

The Statement/Proposition Game

"520 < 111"

Is this a statement? yes

Is this a proposition? yes

What is the truth value of the proposition? false

The Statement/Proposition Game

"y > 5"

Is this a statement? yes

Is this a proposition? no

Its truth value depends on the value of y, but this value is not specified.

We call this type of statement a propositional function or open sentence.

The Statement/Proposition Game

"Today is March 8, 2024 and 99 < 5."

Is this a statement? yes

Is this a proposition? yes

What is the truth value of the proposition? false

The Statement/Proposition Game

"Please do not fall asleep."

Is this a statement? no

It's a request.

Is this a proposition? no

Only statements can be propositions.

The Statement/Proposition Game

"If the moon is made of cheese, then I will be rich."

Is this a statement? yes

Is this a proposition? yes

What is the truth value of the proposition? probably true

The Statement/Proposition Game

"x < y if and only if y > x."

Is this a statement? yes

Is this a proposition? yes

... because its truth value does not depend on specific values of x and y.

What is the truth value of the proposition? true

Combining Propositions

- As we have seen in the previous examples, one or more propositions can be combined to form a single **compound proposition**.

- We formalize this by denoting propositions with letters such as **p, q, r, s**, and introducing several **logical operators or logical connectives.**

Logical Operators (Connectives)

- We will examine the following logical operators:

- Negation (**NOT**, $\neg$)
- Conjunction (**AND**, $\wedge$)
- Disjunction (**OR**, $\vee$)
- Exclusive-or (**XOR**, $\oplus$)
- Implication (**if – then**, $\rightarrow$)
- Biconditional (**if and only if**, $\leftrightarrow$)

- Truth tables can be used to show how these operators can combine propositions to compound propositions.

Negation (NOT)

- Unary Operator, Symbol: $\neg$

P	$\neg P$
true (T)	false (F)
false (F)	true (T)

Conjunction (AND)

- Binary Operator, Symbol: $\wedge$

P	Q	P $\wedge$ Q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (OR)

- Binary Operator, Symbol: $\vee$

P	Q	P $\vee$ Q
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive Or (XOR)

- Binary Operator, Symbol: $\oplus$

P	Q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F

Implication (if - then)

- Binary Operator, Symbol: $\rightarrow$

P	Q	P→Q
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional (Bi-implications) (if and only if)

- Binary Operator, Symbol: $\leftrightarrow$

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

EXAMPLES:

p = "Islamabad is the capital of Pakistan"

q = "17 is divisible by 3"

$p \wedge q$ = "Islamabad is the capital of Pakistan **and** 17 is divisible by 3"

$p \vee q$ = "Islamabad is the capital of Pakistan **or** 17 is divisible by 3"

$\sim p$ = "It is **not** the case that Islamabad is the capital of Pakistan"
or simply "Islamabad is **not** the capital of Pakistan"

Translating from English to Symbols:

- Let **p** = "It is hot", and **q** = "It is sunny"

SENTENCE	SYMBOLIC FORM
It is **not** hot.	$\sim p$
It is hot **and** sunny.	$p \wedge q$
It is hot **or** sunny.	$p \vee q$
It is **not** hot **but** sunny.	$\sim p \wedge q$
It is **neither** hot **nor** sunny.	$\sim p \wedge \sim q$

TRANSLATING FROM SYMBOLS TO ENGLISH

Let ***m*** = "Ali is good in Mathematics"

c = "Ali is a Computer Science student"

SYMBOLIC FORM	**STATEMENTS**
$\sim c$	Ali is **not** a Computer Science student
$c \vee m$	Ali is a Computer Science student **or** good in Math's
$m \wedge \sim c$	Ali is good in Math's **but not** a Computer Science student

EXAMPLES:

p = "Maria learns discrete structures"

q = "Maria will find a good job"

$p \to q$

"if p, then q"
"if p, q"
"p is sufficient for q"
"q if p"
"q when p"
"a necessary condition for p is q"
"q unless $\neg p$"
"p implies q"
"p only if q"
"a sufficient condition for q is p"
"q whenever p"
"q is necessary for p"
"q follows from p"
"q provided that p"

"**If** Maria learns discrete structures, **then** Maria will find a good job"

"Maria will find a good job **when** She learns discrete structures"

"For Maria to get a good job, **it is sufficient** for her learns discrete structures"

EXAMPLES:

p **=** "You can take the flight"

q **=** "You buy a ticket"

$$\boldsymbol{p \leftrightarrow q}$$

"You can take the flight **if and only if** you buy a ticket"

"p is necessary and sufficient for q"
"if p then q, and conversely"
"p iff q."
"p exactly when q."

Exercises

- To take discrete mathematics, you must have taken calculus or a course in computer science.
  - P: take discrete mathematics
  - Q: take calculus
  - R: take a course in computer science
- $P \to \mathbf{Q} \lor R$

Exercises

- You can access the Internet from campus only if you are a computer science major or you are not a freshman.

  - P: You can access the Internet from campus
  - Q: you are a computer science major
  - R: you are a freshman

- $P \rightarrow (Q \vee \neg R)$

Exercises

- School is closed if more than 2 feet of snow falls or if the wind chill is below -100.
  - P: School is closed
  - Q: 2 feet of snow falls
  - R: wind chill is below -100
- $Q \vee R \to P$

Statements and Operators

- Statements and operators can be combined in any way to form new statements.

P	Q	$\neg P$	$\neg Q$	$(\neg P)\vee(\neg Q)$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Precedence of Logical Operators

Operator	Precedence
$\neg$	1
$\land$	2
$\lor$	3
$\rightarrow$	4
$\leftrightarrow$	5

Statements and Operations

Statements and operators can be combined in any way to form new statements.

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$	$(\neg P) \vee (\neg Q)$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Logical Bit Operations

00000110011

Logic and Bit Operations

- Computers represents information using bits.
- A bit is a symbol with two possible values, namely,
- 0 (zero) and 1 (one).
- This meaning of the word bit comes from binary digit.

Truth Value	Bit
T	1
F	0

- Information is often represented using bit strings,

  which are lists of zeros and ones.
- The length of this string is the number of bits in the string.
- 101010011 is a bit string of length nine.

P	Q	$P \wedge Q$	$P \vee Q$	$P \oplus Q$
1	1	1	1	0
1	0	0	1	1
0	1	0	1	1
0	0	0	0	0

Exercises

- Find the bitwise OR, bitwise AND, and bitwise XOR of the bit strings 01 1011 0110 and 11 0001 1101

01 1011 0110

<u>11 0001 1101</u>

11 1011 1111 bitwise OR

01 0001 0100 bitwise AND

10 1010 1011 bitwise XOR

Note: bit strings is splited into blocks of four bits to make them easier to read.

CLASS ACTIVITY 1

Q1: Which of the following are propositions and which are not propositions? Give your reason in one line.

- Grass is green.
- Z is greater than 2.
- Close the door.
- 4+2 = 6
- 2+4 = 7
- There are four fingers in a hand.
- He is rich. (Note* [He is a universal variable like integer variable Z])
- Bill Gates is poor.
- Sky is yellow.
- x=1 and x >2
- May I come in?

CLASS ACTIVITY 1

Q2: Find logical connectives in the following statements and name them (AND(conjunction), OR(disjunction), NOT(negation)).

"3+2=5 and Lahore is a city of Pakistan."	
" The pizza is not tasty "	
"Discrete structures is an easy subject and I will work hard in it"	

CLASS ACTIVITY 1

Q3: Write the sentences composed from this propositional logic:

p = "Queen of England is 91 years old"

q = "London is cloudy today"

$p \wedge q$	
$p \vee q$	
$\sim p$	
$\sim p \wedge \sim q$	
$p \wedge \sim(p \wedge q)$	

CLASS ACTIVITY 1

Q4: Translate the following compound statements into their symbolic forms:

p = "John is healthy"

q = "John is wealthy"

r = "John is wise"

John is healthy and wealthy but not wise.	
John is not wealthy but he is healthy and wise	
John is neither healthy, wealthy nor wise.	

CLASS ACTIVITY 1

Q5: Fill the Truth Tables for the following logical statements:

$\sim p \wedge q$

$\sim p \wedge (q \vee \sim r)$

$(p \vee q) \wedge \sim (p \wedge q)$

CLASS ACTIVITY 1

Q6: Evaluate each of these expressions.

a) $1\,1000 \wedge (0\,1011 \vee 1\,1011)$

b) $(0\,1111 \wedge 1\,0101) \vee 0\,1000$

c) $(0\,1010 \oplus 1\,1011) \oplus 0\,1000$

d) $(1\,1011 \vee 0\,1010) \wedge (1\,0001 \vee 1\,1011)$

Summary

Chapter 1 : The Foundations: Logic and Proofs

1.1 Propositional Logic

- Statement, Proposition, Truth value
- Propositional symbol, Open proposition
- Compound Propositions
- Operators
- Conditional statements
- Truth tables of compound propositions
- Precedence of logical operators
- Logic and bit operation

1.2 Application of Propositional Logic

- Translating English Statement