Quiz 2 - Solution
Discrete Structures - SPRING 2024

Time: 20 Mins Name:
Total Marks: 20 ID:

Note: Cutting or over-writing is not acceptable.
Show your working otherwise no credit will be given.

Question # 1 (CLO 2 – 5 marks)

Apply rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained."

Answer:

Let		$\sim r \vee \sim f \rightarrow s$	(Simplification, i) iv
r= it rains.		$\sim r \vee \sim f \rightarrow t$	(Hypothetical Syllogism, ii & iv) v
f= it is foggy		$\sim (\sim r \vee \sim f)$	(Modus Tollen, iii, v) vi
s= the sailing race will help		$\sim(\sim r) \wedge \sim(\sim f)$	(DeMorgan's Law, vi) vii
l= lifesaving demonstration will go on		$r \wedge f$	(Double Negation Law, vi) viii
t= the trophy will be awarded		r	(Simplification, viii)
$\sim r \vee \sim f \rightarrow s \wedge l$	Premise 1 i	Hence conclusion is that "it rained"	
$s \rightarrow t$	Premise 2 ii		
$\sim t$	Premise 3..... iii		
∴r	**Conclusion**		

Question # 2 (CLO 1 - 2 marks)

Show is the cardinality of each of these sets?

a) {{a}} : The cardinality of the set {{a}} is |{{a}}| = 1.

b) {a, {a}} : The cardinality of the set {a, {a}} is |{a, {a}}| = 2.

Translate each of these quantifications into English and determine its truth value. (CLO 1 - 3 marks)

$\exists x \in Z\ (x^2 = 2)$

Answer:

There exists an integer *x* such that $x^2 = 2$.

OR For some integer x, the square of x is equal to 2.

Truth value: False (No integer *x* satisfying the equation)

Show the power set of the following set S, where a and b are distinct elements. (CLO 1 - 2 marks)

S = {a, b}

Answer:

P(*S*)= {{}, {*a*}, {*b*}, {*a*, *b*}}

Question # 3

Let A = {a, b, c, d, e} and B = {a, b, c, d, e, f, g, h}. **Solve** to find (CLO 2 - 2 marks)

a) A ∪ B.
$=\{a, b, c, d, e, f, g, h\}$

b) A ∩ B.
$=\{a, b, c, d, e\}$

c) A − B.
= {No elements from A that are not in B} = ∅

d) B − A.
$= \{f, g, h\}$

Identify that f(x) well defined function from R to R or not? Give reason. (CLO 2 - 2 marks)

f (x) = 1/x

Answer:

f (x) = 1/x is not a well-defined function from R to R because it is undefined at $x = 0$.

Determine that the following function is a bijective function from R to R using formula.

f (x) = −3x + 4 (CLO 2 - 2 marks)

Answer:

a) f is one-to-one Let $x_1 = x_2 \in$ Let $f(x_1) = f(x_2)$ for $x_1, x_2 \in R$ $\Rightarrow -3x_1 + 4 = -3x_2 + 4$ (by definition of f) $\Rightarrow -3x_1 = -3x_2$ (subtracting 4 on both sides) $\Rightarrow x_1 = x_2$ (dividing -3 on both sides) Hence f is one-to-one.	b) f is on-to Let y ∈R. We search for an x ∈R such that f(x) = y. $\Rightarrow$ $-3x + 4 = y$ (by definition of f) $\Rightarrow$ $x = (y - 4) / (-3)$ $\Rightarrow$ $x = (4 - y) / 3$ Thus for each y ∈R, there exists $x = (4 - y) / 3$ ∈R such that $f(x) = f((4 - y) / 3)$ $= -3((4 - y) / 3) + 4$ (by definition of f) $= (4 - y) + 4 = y$ Hence f is onto
As this function is one-to-one and on-to, so this is a bijective function.	

Identify the inverse of the following function:

$Y(x) = x^{1/2} + 12$ (CLO 2 - 2 marks)

Answer:

$Y = x^{1/2} + 12$

$Y - 12 = x^{1/2}$

$x = (Y - 12)^2$

$Y^{-1}(Y) = x$

$Y^{-1}(y) = (Y - 12)^2$