Quiz 3 - Solution
Discrete Structures - SPRING 2024

Time: 20 Mins | Name:
Total Marks: 20 | ID:

Note: Cutting or over-writing is not acceptable.
Show your working otherwise no credit will be given.

Question # 1 (CLO 3 – 2.5*2=5)

a) Develop the formula and List the first 5 terms of the given sequences.
The sequence that begins with 5 and in which each successive term is 4 more than the preceding.

Solution: 5, 9, 13, 17, 21, ….. (This is an arithmetic Sequence)

Where $a = 5$ $d = 4$

So $a_n = a + (n - 1) d$

$a_n = 5 + (n - 1) 4$

$a_n = 5 + 4n – 4$

$$\boxed{\mathbf{a_n = 4n + 1}}$$

b) Find the first five terms of each of the following sequence:

$a_n = n.a_{n-1} + n^2.a_{n-2}$, where as $a_0 = 0$, $a_1 = 1$.

Solution:

$\boldsymbol{a_0 = 0}$

$\boldsymbol{a_1 = 1}$

$a_2 = 2.a_{2-1} + 2^2.a_{2-2}$
$= 2.a_1 + 4.a_0$
$= 2 (1) + 4 (0)$

$\boldsymbol{a_2 = 2}$

$a_3 = 3.a_{3-1} + 3^2.a_{3-2}$
$a_3 = 3.a_2 + 9.a_1$
$a_3 = 3.(2) + 9(1)$

$\boldsymbol{a_3 = 15}$

$a_4 = 4.a_{4-1} + 4^2.a_{4-2}$
$a_4 = 4.a_3 + 16.a_2$
$a_4 = 4.(15) + 16.(2) = 60 + 32$

$\boldsymbol{a_4 = 92}$

So the sequence is 0, 1, 2, 15, 92, ……

Question # 2: Compute the following Sums and Products? (CLO 3 - 3+2=5 marks)

$$\sum_{i=0}^{3}\sum_{j=1}^{2}(3i + 2j)$$

Solution: $\sum_{i=0}^{3}[(3i + 2(1)) + (3i + 2(2))]$

$\sum_{i=0}^{3}[(3i + 2) + (3i + 4)] = \sum_{i=0}^{3}(6i + 6)$

$\sum_{i=0}^{3}(6i + 6) = [(6(0) + 6) + (6(1) + 6) + (6(2) + 6) + (6(3) + 6)]$

$= 6 + 12 + 18 + 24$

$$\boxed{\sum_{i=0}^{3}\sum_{j=1}^{2}(3i + 2j) = 60}$$

$$\prod_{i=0}^{100} i$$

Solution:

$\prod_{i=0}^{100} i = 0 * 1 * 2 * 3 * 4 * 5 \ldots\ldots\ldots * 100$

$\boldsymbol{\prod_{i=0}^{100} i = 0}$

Question # 3 (CLO 3 - 10 marks)

Use mathematical Induction to **prove** that

$$-1+2+5+\ldots\ldots\ldots+3n-4 = \frac{n(3n-5)}{2}$$ for all positive integers ($n \geq 1$)

Solution:

Let P(n) : $-1+2+5+\ldots\ldots\ldots+3n-4 = \frac{n(3n-5)}{2}$

Basic Step:

P(1) is true

L.H.S of P(1) = -1

R.H.S of P(1) $= \frac{n(3n-5)}{2} = \frac{1(3(1)-5)}{2} = \frac{-2}{2} = -1$

Hence the equation is true for n = 1

Inductive Step:

Suppose P(k) is true for some integer $k \geq 1$; i.e.,

$$-1+2+5+\ldots\ldots\ldots+3k-4 = \frac{k(3k-5)}{2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

To prove P(k+1) is true, i.e.,

$$-1+2+5+\ldots\ldots\ldots+3(k+1)-4 = \frac{(k+1)(3(k+1)-5)}{2}$$

$$= \frac{(k+1)(3k+3-5)}{2}$$

$$= \frac{(k+1)(3k-2)}{2}$$

$$= \frac{3k^2-2k+3k-2}{2}$$

$$-1+2+5+\ldots\ldots\ldots+3(k+1)-4 = \frac{3k^2+k-2}{2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Consider LHS of equation (2)

L.H.S. $= -1+2+5+\ldots\ldots\ldots+3(k+1)-4 = -1+2+5+\ldots\ldots\ldots[3k-4]+[3(k+1)-4]$

$= \frac{k(3k-5)}{2} + (3k-1)$ $\therefore$Put values from Equation 1

$= \frac{3k^2-5k+6k-2}{2}$

$= \frac{3k^2+k-2}{2}$ = R.H.S.

Conclusion:

Hence, P(k+1) is true and consequently by mathematical induction the given propositional function is true for all integers $n \geq 1$.