CC1041-DISCRETE STRUCTURES

UNIVERSITY OF MANAGEMENT AND TECHNOLOGY

Assignment # 1 – Solution

Q1. Express predicates using quantifiers for the following: **[CLO-1]**

i. All sportsmen are tall.
S(x) : x is a sportsman
T(x) : x is tall

$$\forall x\ (S(x) \rightarrow T(x))$$

ii. Some people like apples
P(x) : x is a person
A(x) : x likes apples

$$\exists x(P(x) \wedge A(x))$$

iii. No one likes medicine.
M(x) : x likes Medicine

$$\forall x\ \neg M(x)$$

iv. If all animals had wings they would fly
W(x) : x has wings
F(x) : x would fly

$$\forall x\ (W(x) \rightarrow F(x))$$

v. Every student either likes discrete math or likes calculus.
S(x) : x is student
D(x) : x likes discrete math
C(x) : x likes calculus

$$\forall x\ (S(x) \rightarrow D(x) \vee C(x))$$

Q2. Given the following Predicates. **[CLO-1]**

- C(x) is x is a comedian.
- F(x) is x is funny.

Domain/Universe of x is all people.

Translate into English:

(i) $\forall x\,(C(x) \rightarrow F(x))$
Every comedian is funny.

(ii) $\forall x\,(C(x) \wedge F(x))$
Every person is a funny comedian.

(iii) $\exists x(C(x) \rightarrow F(x))$
There exists a person such that if she or he is a comedian, then she or he is funny.

(iv) $\exists x(C(x) \wedge F(x))$
Some comedians are funny.

Q3. [CLO-2]

For each of these collections of premises, what relevant conclusion or conclusions can be drawn? **Identify** the rules of inference used to obtain each conclusion from the premises.

a) "Randy works hard," "If Randy works hard, then he is a dull boy," and "If Randy is a dull boy, then he will not get the job"

w = Randy works hard
d = Randy is a dull boy
j = Randy will get the job

Hypotheses:

w	Premise 1	(i)
$w \rightarrow d$	Premise 2	(ii)
$d \rightarrow \neg j$	Premise 3	(iii)

d	Using (i) and (ii) and applying Modus Ponens	(iv)
$\neg j$	Using (iii) and (iv) and applying Modus Ponens	(v)

So the desired final conclusion is "Randy will not get the job".

b) "If I play hockey, then I am sore the next day." "I use the whirlpool if I am sore." "I did not use the whirlpool."

p= I play hockey
s=I am sore
w=I use whirlpool

Hypotheses:

$p \to s$	Premise 1	(i)
$s \to w$	Premise 2	(ii)
$\neg w$	Premise 3	(iii)

$p \to w$	Using (i) and (ii) and applying Hypothetical Syllogism	(iv)
$\neg p$	Using (iii) and (iv) and applying Modus Tollens	(v)

So the desired final conclusion is "I do not play hockey"

Q4: Construct the truth tables to prove the following equivalences [CLO-3]

a) $(p \to r) \lor (q \to r)$ and $(p \land q) \to r$

P	Q	R	$p \to r$	$q \to r$	$(p \to r) \lor (q \to r)$	$p \land q$	$(p \land q) \to r$
0	0	0	1	1	1	0	1
0	0	1	1	1	1	0	1
0	1	0	1	0	1	0	1
0	1	1	1	1	1	0	1
1	0	0	0	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	0	0	0	1	0
1	1	1	1	1	1	1	1

b) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

P	Q	R	¬P	Q → R	¬P → (Q → R)	P ∨ R	Q → (P ∨ R)
0	0	0	1	1	1	0	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	0	1	1	1
1	1	1	0	1	1	1	1

Q5: Construct a combinatorial circuit for the below expression using inverters, OR, and AND gates. Also produces the output where p=T, q=F and r =T. **[CLO-3]**

$((\neg p \vee r) \wedge \neg q) \vee (\neg p \wedge \neg (q \vee r))$

Solution: