Chapter 2:

Basic Structures: Sets, Functions, Sequences, Sums, and Matrices

Section 2.1 : Sets

Set

Sets are used to group objects together. Often the objects in a set have similar properties.

A set is an unordered collection of objects.

Example:
Z is the set of integers.

Set membership

The objects in a set are called the elements, or members, of the set.

- a is an element of the set A, denoted by $a \in A$.
- a is not an element of the set A, denoted by $a \notin A$.

Example:

Set D: all students taking Discrete Mathematics course.

Assume Bill is taking Discrete Mathematics course and George is not taking Discrete Mathematics course.

Bill $\in$ D

George $\notin$ D

Expressing set

There are many ways to express the sets

1. Listing all the elements
2. Set builder notation
3. Venn diagrams

Listing all elements

S = {e1 , e2 , e3 ,…,en }

where e_i is element in the set

Example

All vowels in the English alphabet.

- V = {a, e, i , o, u}

Odd positive integers < 10.

O = {1, 3, 5, 7, 9}

Positive integers less than 100.

{1, 2, 3, …, 99}

Set builder notation

Describe the properties the elements must have to be members

$$S = \{x \mid P(x)\}$$

S contains all the elements which make the predicate P true

Example:

- R = { x | x is integer < 100 and > 40}
- Z+ = { x | x is a positive integer}
- O= {1,3,5,7,9} write set builder notation.

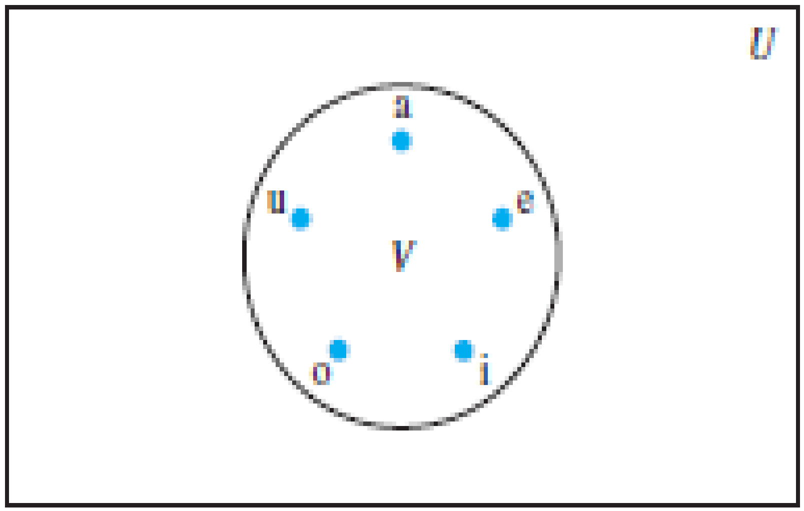
Important Sets

- N = {0,1,2,3,…} , Set of natural numbers
- Z = {…,-2,-1,0,1,2,…}, Set of integers
- Z + = {1,2,3,…} , Set of positive integers
- Q = {p/q | p ∈ Z, q ∈ Z, and q = 0}, Set of rational numbers
- R , Set of real numbers

- Sets can have other sets as members
  Example: The set {N, Z, Q, R} is a set containing four elements, each of which is a set.

Venn diagrams

Sets can be represented graphically using Venn diagram.



- Universal set U contains all objects under consideration say English alphabets is represented by a rectangle
- Other geometric figures are used to represent sets. Say set of vowels
- Points are used to represent particular elements of sets. i.e. a, e, i, o, u

Show set A={a,b,c} using Venn diagram

Set equality

Two sets are equal if and only if they contain exactly the same elements.

denoted by $A = B$.

Mathematically: $A = B$ iff $\forall x\ (x \in A \leftrightarrow x \in B)$

Example: Are the following sets equal? Why &why not?

1. {1, 2, 3, 4} and {1, 2, 3, 4}
2. {1, 2, 3, 4} and {4, 1, 3, 2}
3. {a, b, c, d, e} and {a, a, c, b, e, d}
4. {a, e, i, o} and {a, e, i, o, u}

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Example: Are the following sets equal? Why &why not?

1. {1, 2, 3, 4} and {1, 2, 3, 4} yes
2. {1, 2, 3, 4} and {4, 1, 3, 2} yes
3. {a, b, c, d, e} and {a, a, c, b, e, d} yes
4. {a, e, i, o} and {a, e, i, o, u} } no

Empty set, Singleton Set

A set that has no elements called empty set, or null set.

Denoted by {} , Ø.

Example:

The set of all positive integers that are less than and equal to 0.

$S = \{x \mid x \in Z+ \text{ and } x \leq 0\} = \{\} = \emptyset$

Singleton set: a set with one element

- are Ø and {Ø} equal? No

Φ: an empty set. Think of this as an empty folder

{ϕ}: a set with one element. The element is an empty set. Think of this as a folder with an empty folder in it.

Subset

Let A and B be sets.

- A is a **subset** of B if and only if every element of A is also an element of B, denoted by $A \subseteq B$.

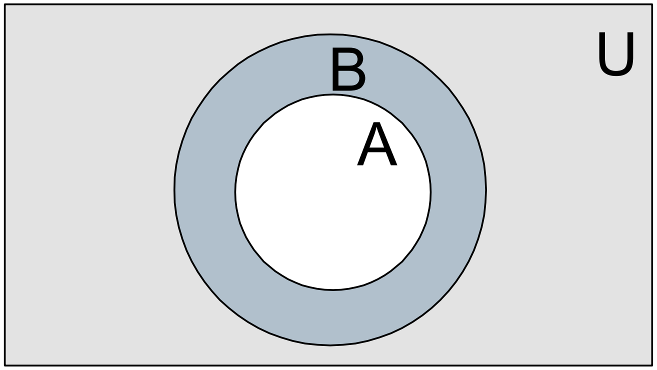
- $A \subseteq B$ if and only if $\forall x(x \in A \rightarrow x \in B)$

Subset equality:

$A \subseteq B = \forall x(x \in A \rightarrow x \in B)$ and

$B \subseteq A = \forall x(x \in B \rightarrow x \in A)$

then $A = B$, $\forall x(x \in A \leftrightarrow x \in B)$



Proper subset

Let A and B be sets.

- A is a subset of a set B but that $A \neq B$, we write $A \subset B$ and say that A is a **proper subset** of B

- For $A \subset B$ to be true, it must be the case that $A \subseteq B$ and there must exist an element x of B that is not an element of A, i.e.

  $$\forall x(x \in A \rightarrow x \in B) \wedge \exists x(x \in B \wedge x \notin A)$$

Cardinality of Set

Let S be a set. If there are exactly n distinct elements in S where n is a nonnegative integer, then S is a finite set.
n is the cardinality of S, denoted by $|S|$.

A set is said to be infinite if it is not finite.

Example:

1. A be the set of odd positive integers less than 10, $|A| = 5$
2. S be the set of letters in the English alphabet, $|S| = ?$

The power set

Let S be a set.

- The power set of S is the set of all subsets of S, denoted by P(S).

If a set has n elements, then its power set has
2^n elements

Example:
P({a ,b }) = {Ø,{a},{b},{a ,b }}

Practice

How many elements do they have?

1. What is P({1,2,3})?
2. What is P({a, b, c, d})?

Ordered n-tuple

The order of elements in a collection is often important

Ordered n-tuple (a1 ,a2 ,... ,an) is the ordered collection that has :

a1 as its first element

a2 as its second element

. . .

an as its nth element

Example: (a ,b) is an ordered 2-tuple (ordered pair).

Ordered n-tuple

Let A=(a1, a2, …, an) and B=(b1, b2, …, bn) be ordered n-tuples.

A and B are equal if and only if each corresponding pair of their elements are equal, denoted by A=B.

A=B if and only $a_i = b_i$ and for i = {1,2,…,n}

Example: Assume c ≠ b.

Are ordered 3-tuples (a ,b ,c) and (a ,c ,b) equal?

Cartesian product

Let A and B be sets.

- The *Cartesian product* of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. Hence,

  $$A \times B = \{(a, b) \mid a \in A \land b \in B\}.$$

Example:

Cartesian product of $A = \{1, 2\}$ and $B = \{a, b, c\}$?

$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}.$

Practice

Given A = {1, 2} and B = {a, b, c}

What are A x B and B x A?

$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$.

$B \times A = \{(a,1),(a,2),(b,1),(b,2),(c,1),(c,2)\}$

- Are A x B and B x A equal?
- A x B and B x A are equal if A = Ø or B = Ø (so that A x B = Ø) or A = B

Cartesian product

The *Cartesian product* of sets A_1, A_2, …, A_n, denoted by $A_1 \times A_2 \times \ldots \times A_n$ is the set of ordered n-tuples (a_1, a_2, …, a_n), where a_i belongs to A_i for $i = 1,2, \ldots, n$.

In other words,

$A_1 \times A_2 \times \ldots \times A_n = \{(a1, a2, \ldots, an) \mid a_i \in A_i \text{ for } i = 1, 2, \ldots, n\}$.

Cartesian Products (example)

Example:

What is the Cartesian product of A × B × C where A= {0,1}, B = {1,2}, and C = {0,1,2}?

Solution:

$A \times B \times$ C= {(0,1,0), (0,1,1), (0,1,2), (0,2,0), (0,2,1), (0,2,2), (1,1,0), (1,1,1), (), (1,2,0), (1,2,1), (1,2,2)}1,1,2

Relation

Let A and B be sets.

A subset R of the Cartesian product A x B is called a relation from the set A to the set B.

The elements of R are ordered pairs, where the first element belongs to A and the second to B

Example

What are the ordered pairs in the less than or equal to relation, which contains *(a, b)* if $a \leq b$, on the set {0, 1, 2, 3}?

Solution:

$R = \{ (0,0), (0,1), (0,2), (0,3), (1,1), (1,2), (1,3), (2,2), (2, 3), (3, 3)\}$.

Practice questions

2.1: 2,3,5,7,13,15,17,19,23,29,33,35