Chapter 2:

Basic Structures: Sets, Functions, Sequences, Sums, and Matrices

Section 2.2 : Set Operations

Set operations

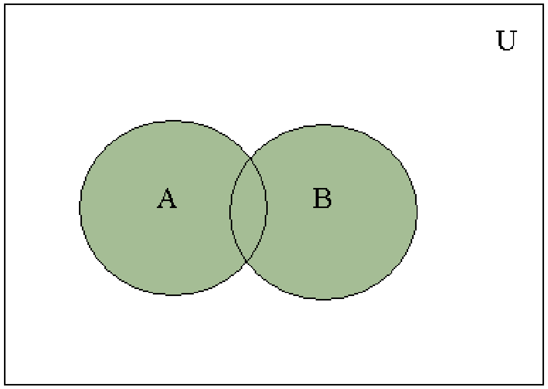
- Two sets can be combined in many different ways.
- Set operations can be used to combine sets.

Union

Let A and B be sets.

- The **union** of A and B, denoted by **A ∪ B**, is the set containing those elements that are either in A or in B, or in both.

$$A \cup B = \{x \mid x \in A \lor x \in B\}.$$



- Shaded area represents $A \cup B$.

Union example

Example:

Find union of the sets {1, 3, 5} and {1, 2, 3}.

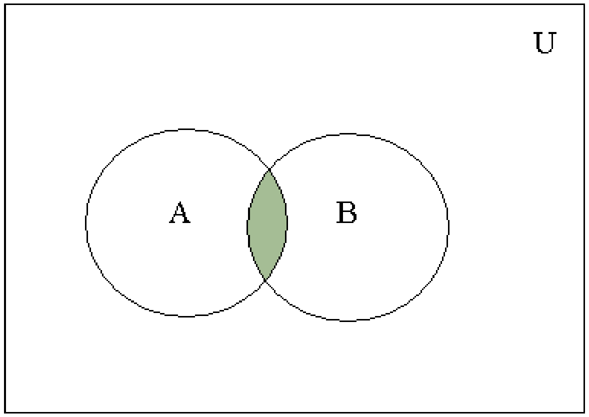
Solution: $\{1, 3, 5\} \cup \{1, 2, 3\} = \{1, 2, 3, 5\}$.

$A \cup B = B \cup A$

Intersection

Let A and B be sets.

- The **intersection** of A and B, denoted by **A ∩ B**, is the set containing those elements in both A and B.
- $A \cap B = \{x \mid x \in A \land x \in B\}$



- Shaded area represents $A \cap B$.

Intersection example

Example:

Find intersection of the sets {1, 3, 5} and {1, 2, 3}

Solution:

$\{1, 3, 5\} \cap \{1, 2, 3\} = \{1, 3\}$.

$A \cap B = B \cap A$

Set operations

A= {1, 3, 5} and B= {1, 2, 3}.

1) Find A ∪ B
2) Find A ∪ B=B ∪ A
3) Find *A* ∩ *B*
4) *Is A* ∩ *B*= *B*∩ A

Solution:

1) {1, 3, 5} ∪ {1, 2, 3} = {1, 2, 3, 5}.
2) Yes
3) {1,3}
4) yes

Disjoint

Two sets are called *disjoint* if their intersection is the empty set.

Example;

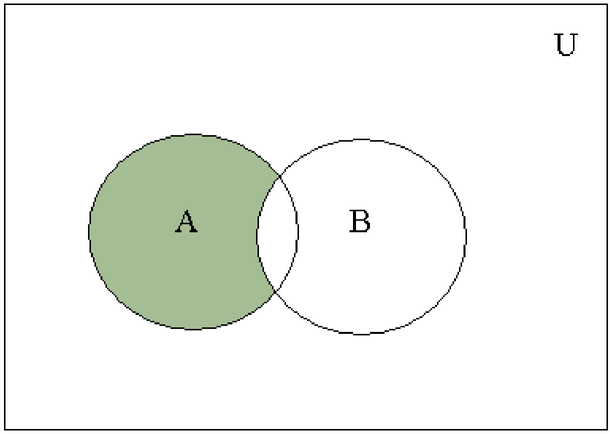
Let $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 4, 6, 8, 10\}$.

$A \cap B = \emptyset$, thus A and B are disjoint

Difference

Let A and B be sets.

- The **difference** of A and B, denoted by **A-B,** is the set containing those elements that are in A but not in B. (also called **complement of B with respect to A**)
- $A - B = \{x \mid x \in A \wedge x \notin B\}$



- $A - B$ is shaded.

Difference (example)

$\{1,2,3\} - \{2,4\} = \{1,3\}$

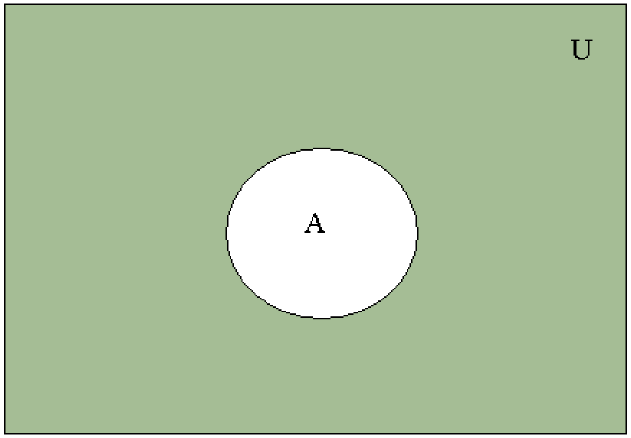
- Find difference of {1, 3, 5} and {1, 2, 3}.
- Find difference of {1, 2, 3} and {1, 3, 5},

- $A - B \neq B - A$

Complement

Let U be the universal set and A be a set.

- The **complement** of A, denoted by $\bar{A}$, is the complement of A with respect to U (which is U-A).
- $\bar{A} = \{x \in U \mid x \notin A\}$.



- $\bar{A}$ is shaded.

Example

- A={a,b,c,d} and U is the set of English alphabet

  $\bar{A}$ = {e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x,y,z}

- Let A be the set of positive integers greater than 10 (with universal set the set of all positive integers). Find $\bar{A}$.

  $\bar{A}$= = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

TABLE 1 **Set Identities.**

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

Membership table

A	B	AUB	A∩B	A-B	$\overline{A}$
1	1	1	1	0	0
1	0	1	0	1	0
0	1	1	0	0	1
0	0	0	0	0	1

Membership tables

Set identities can also be proved using **membership tables**

Example

Use a membership table to show that

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Membership tables example

TABLE 2 A Membership Table for the Distributive Property.

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

Generalized Union

The *union* of a collection of sets is the set that contains those elements that are members of at least one set in the collection.

We use the notation

$$A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i$$

to denote the union of the sets *A1, A2, . . . , An*.

Generalized Intersection

The *intersection* of a collection of sets is the set that contains those elements that are members of all the sets in the collection.

We use the notation

$$A_1 \cap A_2 \cap \cdots \cap A_n = \bigcap_{i=1}^{n} A_i$$

Using Set Notation with Quantifiers

Using Set Notation with Quantifiers

- $\forall x \in S\ (P(x))$ is shorthand for $\forall x(x \in S \rightarrow P(x))$.
- $\exists x \in S\ (P(x))$ is shorthand for $\exists x(x \in S \land P(x))$.

Example

What do the statements $\forall\, x \in R(x^2 \geq 0)$ and $\exists x \in A\ (x^2 = 1)$ where A= {1,2,3,4,5} mean?

Solution:

$\forall x \in \mathbf{R}\ (x^2 \geq 0)$= for every real number *x*, $x^2 \geq 0$.

"The square of every real number is nonnegative." true

$\exists x \in \mathbf{Z}\ (x^2 = 1)$ = there exists an positive integer x less than 5 such that $x^2 = 1$.

True

"For x=1 from the given set A."

Truth Sets and Quantifiers

Let P be a predicate and D is a domain.

The truth set of P is the set of elements x in D for which P(x) is true.

The truth set of P is $\{x \in D \mid P(x)\}$.

Truth Sets and Quantifiers

Example:

Let $P(x)$ be $|x| = 1$ where the domain is the set of integers. What is the truth set of $P(x)$?

Solution: $\{-1,1\}$

Practice

- Let Q(x) be $x^2 = 2$ where the domain is the set of integers. What is the truth set of Q(x)?

- Let *R(x)* is ***"|x| = x."*** where the domain is the set of integers. What is the truth set of R(x)?

Representation of Sets

Represent a subset A of U with the bit string of length n, where the ith bit in the string is 1 if a_i belongs to A and is 0 if a_i does not belong to A.

Example:
Let $U = \{1,2,3,4,5,6,7,8,9,10\}$, and the ordering of elements of U has the elements in increasing order; that is $a_i = i$.

- What bit string represents the subset of all odd integers in U?
- Solution: 10 1010 1010
- What bit string represents the subset of all even integers in U?
- Solution: 01 010 10101

Representation of Sets

Example:

Let U = {1,2,3,4,5,6,7,8,9,10}, and the ordering of elements of U has the elements in increasing order; that is $a_i = i$.

- What bit string represents the subset of all integers not exceeding 5 in U?
- Solution: 11 1110 0000
- What bit string represents the complement of the set {1,3,5,7,9}?
- Solution: 01 0101 0101

Representation of Sets

- Union of bit string is bitwise OR
- Intersection of bit string is bitwise AND of the bit strings for the two sets.

Example:

Bit strings for

{1,2,3,4,5} = 11 1110 0000 and

{1,3,5,7,9}=10 1010 1010.

Use bit strings to find the union and intersection of these sets.

Union:

11 1110 0000 $\vee$ 10 1010 1010 = 11 1110 1010, {1,2,3,4,5,7,9}

Intersection:

11 1110 0000 $\wedge$ 10 1010 1010 = 10 1010 0000, {1,3,5}

Shading Venn diagrams with 3 sets

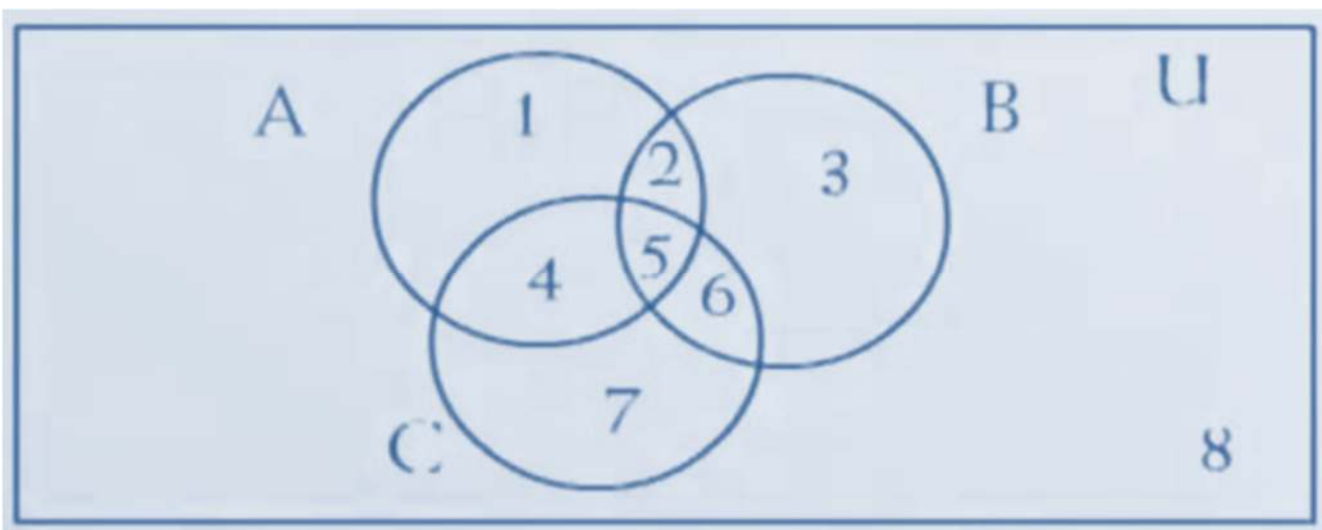
Shade the given diagram for the following expressions

1. $(A \cap B) \cap C'$

2. $A' \cup (B \cup C)$

3. $(A - B) \cap C$

4. $(A \cap B') \cup C'$



Practice questions

2.2: 3,5,9,13,17,27,29