Quiz 1 - Solution
Discrete Structures - SPRING 2024

Time: 20 Mins | Name:
Total Marks: 10 | ID:

Note: Cutting or over-writing is not acceptable.
Show your working otherwise no credit will be given.

Question # 1 (CLO 1 – 2 marks)

Show following statement it into a logical expression?

"You can graduate only if you have completed the requirements of your major and you do not owe money to the university and you do not have an overdue library book."

Express your answer in terms of

w: *"You can graduate."*
x: *"You owe money to the university."*
y: *"You have completed the requirements of your major."*
z: *"You have an overdue library book."*

Answer:

$$w \rightarrow y \wedge (\neg x) \wedge (\neg z)$$

Question # 2 (CLO 1 - 2 marks)

Write Negation of the following statement:

∀ real numbers x, if x > 3 then x^2 > 9.

Answer:

P(x): x>3 Q(x) : $x^2 > 9$ $\forall x\,(P(x) \rightarrow Q(x))$

Apply Negation: $\neg \forall x\,(P(x) \rightarrow Q(x)) \equiv \exists x\, \neg(P(x) \rightarrow Q(x)) \equiv \exists x\, \neg(\neg P(x) \vee Q(x))$

$\equiv \exists x\, \neg(\neg P(x)) \wedge \neg Q(x) \equiv \exists x\,(P(x) \wedge \neg Q(x))$

For some real number x, x > 3 and $x^2 \leq 9$.

Question # 3 (CLO 1 - 2 marks)

Consider the following statement:

$\exists x \in R$ such that $x^2 = 2$.

Which of the following are **equivalent ways** of expressing this statement?

a) The square of each real number is 2.
b) Some real numbers have square 2.
c) The number x has square 2, for some real number x.
d) If x is a real number, then $x^2 = 2$.
e) Some real number has square 2.
f) There is at least one real number whose square is 2.

Question # 4 (CLO 1 - 2 marks)

Show that the following compound proposition is a tautology by using truth tables.

$$[\neg p \wedge (p \vee q)] \rightarrow q$$

Answer:

p	**q**	**¬ p**	**p v q**	**¬ p ∧ (p v q)**	**[¬ p ∧ (p v q)] → q**
T	T	F	T	F	**T**
T	F	F	T	F	**T**
F	T	T	T	T	**T**
F	F	T	F	F	**T**

Question # 5 (CLO 1 - 2 marks)

Prove using equivalence laws. (Provide justification for each step.)

$$(p \wedge \sim q) \vee (p \wedge q) \equiv p$$

Answer:

L.H.S. = $(p \wedge \sim q) \vee (p \wedge q)$

$\equiv p \wedge (\sim q \vee q)$ ∴ Distributive law of conjunction over disjunction

$\equiv p \wedge T$ ∴ Negation Law

$\equiv p$ ∴ Identity Law

= R.H.S.

As L.H.S = R.H.S, Hence Proved.