RULES OF INFERENCE

Lecture

RULES OF INFERENCE & PROPOSITIONAL LOGIC

- We always use a **truth table** to show that an argument form is **valid**.

- We do this by showing that whenever the **premises** are **true**, the **conclusion** must also be **true**. This can be a tedious approach.

- **For Example:**

  When an argument form involves 10 different propositional variables, to use a truth table to show this argument form is valid requires $2^{10} = 1024$ rows.

RULES OF INFERENCE

- **Inference rules** are **templates** for **valid arguments**.

- These rules of inference can be used as **building blocks** to construct more **complicated valid argument forms**.

- There are different kind of rules of inference:
  - Modus Ponens
  - Modus Tollens
  - Hypothetical Syllogism
  - Disjunctive Syllogism
  - Addition
  - Simplification
  - Conjunction
  - Resolution

Valid Arguments in Propositional Logic

$$p \to q$$
$$\underline{p \qquad}$$
$$\therefore q$$

$((p \to q) \wedge p) \to q$ is a tautology (always true)

p	q	$p \to q$	$(p \to q) \wedge p$	$((p \to q) \wedge p) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

This is another way of saying that

$\therefore$ *therefore*

The 1st law

$$\begin{array}{l} p \rightarrow q \\ \underline{p \qquad\quad} \\ \therefore q \end{array}$$

modus ponens
aka
law of detachment

modus ponens (Latin) translates to "***mode that affirms***"

MODUS PONENS

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore\ q \end{array}$$

Alternatively, $((p \wedge (p \to q)) \to q)$ is **Tautology.**

Modus Ponens tells us that is a **conditional statement** and **hypothesis** of this **conditional statement** are **both true**, then the **conclusion** must also be **true**.

EXAMPLE

If you have a current password, then you can log on to the network.

You have the password.

Therefore,

You can log on to the network.

Solution:

Let **p** = you have a current password.

q = you can log on to the network

Symbolically:

$$\begin{array}{l} p \rightarrow q \\ \underline{\quad p \quad} \\ \therefore\ q \end{array}$$

(this form of argument is called **modus ponens**)

MODUS TOLLENS

$$\begin{array}{l} \sim q \\ p \to q \\ \hline \therefore \sim p \end{array}$$

Alternatively, $((\sim q \wedge (p \to q)) \to \sim p)$ is **Tautology**.

Modus Tollens tells us that a conditional statement is true, conclusion of this conditional statement is false, then the hypothesis will also be false.

EXAMPLE

You can't log on to the network.

If you have a current password, then you can log on to the network.

Therefore,

You don't have a current password.

Solution:

Let **p** = you have a current password.

q = you can log on to the network

Symbolically:

$$\begin{array}{l} \sim q \\ p \to q \\ \hline \therefore \sim p \end{array}$$

The rules of inference

Rule of inference	Tautology	Name
$p \to q$ $\underline{p \qquad}$ $\therefore q$	$[p \wedge (p \to q)] \to q$	Modus ponens
$\neg q$ $\underline{p \to q}$ $\therefore \neg p$	$[\neg q \wedge (p \to q)] \to \neg p$	Modus tollen
$p \to q$ $\underline{q \to r}$ $\therefore p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\underline{\neg p \quad}$ $\therefore q$	$((p \vee q) \wedge \neg p) \to q$	Disjunctive syllogism
$\underline{p \qquad}$ $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$\underline{p \wedge q}$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p $\underline{q \qquad}$ $\therefore p \wedge q$	$((p) \wedge (q)) \to (p \wedge q)$	Conjunction
$p \vee q$ $\underline{\neg p \vee r}$ $\therefore q \vee r$	$[(p \vee q) \wedge (\neg p \vee r)] \to (p \vee r)$	Resolution

Exercise:

- State which **rule of inference** is the basis of the following argument:

  "It is below freezing now. Therefore, It is either below freezing or raining now."

- **Solution:**

  Let **p** = It is below freezing.

  q = It is raining now.

  Symbolically,

$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$

This is an argument that uses Addition rule of inference.

Exercise:

- State which **rule of inference** is the basis of the following argument:

  "It is below freezing and raining now. Therefore, it is below freezing now."

- **Solution:**

  Let **p** = It is below freezing.

  q = It is raining now.

  Symbolically,

$$\frac{p \wedge q}{\therefore p}$$

This argument uses simplification rule of inference.

Exercise:

- State which **rule of inference** is the basis of the following argument:
- "If it rains today, then we will not have a barbecue today. If we do not have a barbecue today, then we will have a barbecue tomorrow. Therefore, if it rains today, then we will have a barbecue tomorrow."

- **Solution:**

Let **p** = If it rains today.

q = we will not have a barbecue today.

r = we will have a barbecue tomorrow.

- Symbolically,

$$\begin{array}{r} p \to q \\ q \to r \\ \hline \therefore\; p \to r \end{array}$$

Hence, the argument is **hypothetical syllogism.**

Valid Arguments in Propositional Logic

Is this a valid argument?

If you listen you will hear what I'm saying
You are listening
Therefore, you hear what I am saying

Let p represent the statement "you listen"
Let q represent the statement "you hear what I am saying"

The argument has the form:

$$\begin{array}{l} p \to q \\ \underline{p \qquad\quad} \\ \therefore q \end{array}$$

USING RULES OF INFERENCE TO BUILD ARGUMENTS

- When there are many premises, several rules of inference are often needed to show that an argument is valid.

- You translate the statement into argument form using propositional variables.

- You then want to get from premises/hypothesis(A) to the conclusion (B) using rules of inference.

EXAMPLE

- "It is not sunny this afternoon and it is colder then yesterday." "We will go swimming only if it is sunny." "If we do not go to swimming, then we will take a canoe trip," and "if we take a canoe trip, then we will be home by sunset" lead to the conclusion "we will be home by sunset."

- **Solution:**

Let **p** = It is sunny this afternoon.
q = It is colder then yesterday.
r = We will go to swimming.
s = We will take a canoe trip.
t = We will be home by sunset.

- Symbolically,

$$\left.\begin{array}{ll} 1. & \neg p \wedge q \\ 2. & r \to p \\ 3. & \neg r \to s \\ 4. & s \to t \end{array}\right\} \textbf{Premises}$$

$$\left.\begin{array}{ll} 5. & t \end{array}\right\} \textbf{Conclusion}$$

- We construct an argument that our premise lead to desired conclusion as follows:

$\sim p \wedge q$	Premise
$\sim p$	Simplification
$r \longrightarrow p$	Premise
$\sim r$	Modus Tollens
$\sim r \longrightarrow s$	Premise
s	Modus Ponens
$s \longrightarrow t$	Premise
t	Modus Ponens

Rule of inference	Tautology	Name
$p \to q$ p $\therefore q$	$[p \wedge (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \wedge (p \to q)] \to \neg p$	Modus tollen
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \to q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p q $\therefore p \wedge q$	$((p) \wedge (q)) \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$[(p \vee q) \wedge (\neg p \vee r)] \to (p \vee r)$	Resolution

EXAMPLE

- "If you send me an e-mail message, then I will finish writing the problem," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed." lead to the conclusion "If I do not finish writing the problem, then I will wake up feeling refreshed."

- **Solution:**

Let **p** = You send me an e-mail message.
q = I will finish writing the problem.
r = I will go to sleep early.
s = I will wake up feeling refreshed.

- Symbolically,

$$p \to q$$
$$\sim p \to r$$
$$r \to s$$

The **desired conclusion** is:

$$\sim q \to s$$

- We construct argument that our premise lead to desired conclusion as follow:

$p \to q$	Premise
$\sim q \to \sim p$	Contrapositive
$\sim p \to r$	Premise
$\sim q \to r$	Hypothetical Syllogism
$r \to s$	Premise
$\sim q \to s$	Hypothetical Syllogism

Rule of inference	Tautology	Name
$p \to q$, p, $\therefore q$	$[p \land (p \to q)] \to q$	Modus ponens
$\neg q$, $p \to q$, $\therefore \neg p$	$[\neg q \land (p \to q)] \to \neg p$	Modus tollen
$p \to q$, $q \to r$, $\therefore p \to r$	$[(p \to q) \land (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \lor q$, $\neg p$, $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
p, $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$, $\therefore p$	$(p \land q) \to p$	Simplification
p, q, $\therefore p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction
$p \lor q$, $\neg p \lor r$, $\therefore q \lor r$	$[(p \lor q) \land (\neg p \lor r)] \to (p \lor r)$	Resolution

RESOLUTION

- The Resolution law is:

$$\begin{array}{l} p \vee q \\ \underline{\neg p \vee r} \\ \therefore q \vee r \end{array}$$

- Alternatively, ((p ∨ q) ∧ (~p ∨ r))→ (q ∨ r) is **Tautology**.

Example:

- Using the resolution rule to show that the hypothesis:

  "Jasmine is skiing or it is not snowing" and "It is snowing or Bart is playing hockey" imply that "Jasmine is skiing or Bart is playing hockey."

- **Solution:**

  Let **p** = It is snowing.

  q = Jasmine is skiing.

  r = Bart is playing hockey.

- Symbolically,

$$\begin{array}{rl} & \sim p \vee q \\ & p \vee r \\ \hline \therefore & q \vee r \end{array}$$

Using Resolution the $q \vee r$ follows.

EXAMPLE

- **In the back of an old cupboard you discover a note signed by a pirate famous for his sense of humor and love of logical puzzles. In the note he wrote that he had hidden treasure somewhere on the property. He listed five true statements (a–d below) and challenged the reader to use them to figure out the location of the treasure using inference rules for propositional logic**
  a) If this house is next to a lake, then the treasure is not in the kitchen.
  b) This house is next to a lake.
  c) If the tree in the front yard is an elm, then the treasure is in the kitchen.
  d) The tree in the front yard is an elm or the treasure is buried under the flagpole.
- **Where is the treasure hidden?**