第 4 章 不定积分

4.1 不定积分的概念与性质

1. $\int \dfrac{1}{x^4}\mathrm{d}x = $ _____ .

2. $\int x\sqrt[3]{x}\,\mathrm{d}x = $ _____ .

3. $\int \dfrac{1}{x^2\sqrt{x}}\mathrm{d}x = $ _____ .

4. $\int x^m\mathrm{d}x = $ _____ .

5. $\int \left(x^2 + 4x - 7\right)\mathrm{d}x$.

6. $\int \mathrm{e}^x\left(1 - \dfrac{\mathrm{e}^{-x}}{1+x^2}\right)\mathrm{d}x$.

7. $\int \dfrac{\left(1+x\right)^2}{x}\mathrm{d}x$.

8. $\int \left(3\mathrm{e}^x + \dfrac{1}{x^3}\right)\mathrm{d}x$.

9. $\int \dfrac{x^2}{1+x^2}\mathrm{d}x$.

10. $\int \left(\cos x - \dfrac{2}{\sqrt{1-x^2}}\right)\mathrm{d}x$.

11. $\int \left(2e^x + 3\cos x \right) dx$

12. 设曲线方程为 $y = f(x)$ 已知在曲线的任意一点 (x, y) 处满足 $y'' = 6x$，且在曲线上的点 $(0, -2)$ 处的曲线的切线方程为 $2x - 3y = 6$，求 $y = f(x)$.

4.2 换元积分法

1. $\int e^{5x} dx$.

2. $\int \dfrac{e^x}{e^{2x} + 1} dx$.

3. $\int (3 - 2x)^2 dx$.

4. $\int \dfrac{\sin \sqrt{x}}{\sqrt{x}} dx$.

5. $\int \dfrac{1}{x \ln x \ln \ln x} dx$.

6. $\int \dfrac{2}{1-2x}dx$.

7. $\int x\cos\left(x^2\right)dx$.

8. $\int \dfrac{1}{\left(2x-3\right)^3}dx$.

9. $\int \dfrac{1}{\left(x+1\right)\left(x-5\right)}dx$.

10. $\int \dfrac{\sin x}{\cos^3 x}dx$.

11. $\int \dfrac{10^{\arcsin x}}{\sqrt{1-x^2}}dx$.

12. $\int \dfrac{\sqrt{\ln x}}{x}dx$.

13. $\int e^{\sin x}\cos xdx$.

14. $\displaystyle\int \frac{\left(\arctan x\right)^2}{1+x^2}\mathrm{d}x$.

15. $\displaystyle\int x\mathrm{e}^{-x^2}\mathrm{d}x$.

16. $\displaystyle\int \frac{\mathrm{d}x}{\sqrt[3]{2-3x}}$.

17. $\displaystyle\int \frac{1}{\cos^2 x\sqrt{\tan x}}\mathrm{d}x$.

18. $\displaystyle\int \frac{\arctan\sqrt{x}}{\sqrt{x}\left(1+x\right)}\mathrm{d}x$.

19. $\displaystyle\int \frac{\mathrm{d}x}{1+\sqrt{2x}}$.

4.3 分部积分法

1. $\int x\cos x \mathrm{d}x$.

2. $\int x^2 \ln x \mathrm{d}x$.

3. $\int x\mathrm{e}^{-2x}\mathrm{d}x$.

4. $\int \arctan x \mathrm{d}x$.

5. $\int \mathrm{e}^{-x}\cos x \mathrm{d}x$.

6. $\int x\cos^2 x \mathrm{d}x$.

7. $\int x^5 \mathrm{e}^{x^3}\mathrm{d}x$.

8. $\int \ln^2 x \mathrm{d}x$.

9. $\int \dfrac{\ln \ln x}{x} \mathrm{d}x$.

10. $\int \mathrm{e}^{\sqrt[3]{x}} \mathrm{d}x$.

11. $-2 \int t \cdot e^{-2t} dt$

4.4 有理函数的积分

1. 计算 $\int \dfrac{x+1}{x^2-5x+6} \mathrm{d}x$.