1

$$\boxed{\min_{x \in \mathbb{R}^d} f(x)}$$

Grad descent in our case

1) Define parameter
2) Define constraints
3) Define $f(x)$
4) Define the grad descent
5) define grad descent on our problem

$$X^* = \underset{X}{\operatorname{argmin}} \left(C_{Arap}(X) \right) \qquad \text{size}\left(V_{deform}\right) = \text{size}(X)$$

$$X_1 = X_0 - \alpha \frac{\partial C_{Arap}(X)}{\partial X_0}$$

$$\vdots$$

$$X_N = X_{n-1} - \alpha \frac{\partial C_{Arap}(X)}{\partial X}$$

Imp $\nabla f(x_n)$ is the vector $\left[\frac{\partial f}{\partial x}(x_n), \frac{\partial f}{\partial y}(x_n), \frac{\partial f}{\partial z}(x_n) \right]$

$\frac{\partial f}{\partial x_n}$ implies the grad wrt to the axis

$\therefore$ n is replaced by x, y, z.

$$\therefore \nabla f(x_n) \neq \frac{\partial f}{\partial x_n}$$

$$\frac{\partial f}{\partial x} = \nabla_x f$$

$V_{b_i} = x_{b_{i,1}}, y_{b_{i,2}}, z_{b_{i,3}}$

$V_j = x_{j,1}, y_{j,2}, z_{j,3}$

$$C_1(V) = \sum_{i=1}^{N_u} \| V'_{c_i} - P_i \|^2$$

$$C_2(V) = \sum_{i=1}^{N_m} \sum_{j \in ring(b_i)} \Big| \|V_{b_i} - V_j\| - \|V'_{b_i} - V'_j\| \Big|$$

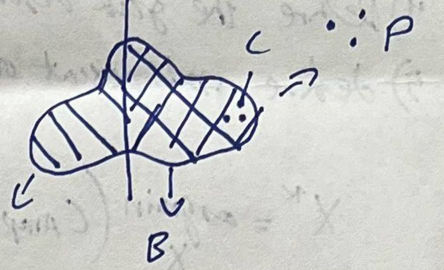
$V_i \in \mathbb{R}^3 \mid V = \{V_1 \ldots V_N\}$

A, $A = (a_1 \cdots a_{nf})$

$B \subset [1, N]$ $B = \{b_1 \ldots b_{N_m}\}$

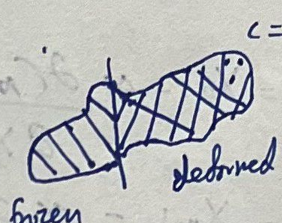
$C \subset B$, $C = (C_1 \ldots C_{N_u})$

$P = (P_1 \ldots P_{N_u})$



$$\boxed{\begin{array}{c} C_{total} = C_1(V) + \lambda C_2(V) \\ \min C_1(V) + \lambda C_2(V) \end{array}}$$

optimise to compute 'X'



$$L(X) = C_1(X) + \lambda C_2(X)$$

$$\cancel{X^* = \arg\min C_2(X)}$$

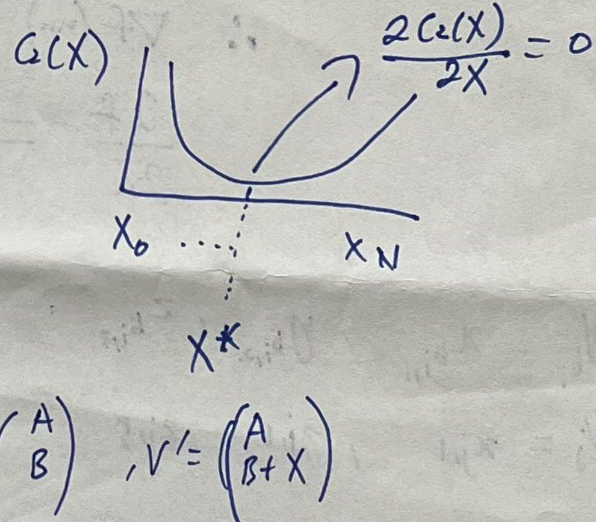
$$X^* = \arg\min_X C_2(X)$$

$$X_1 = X_0 - \alpha \frac{\partial C_2(X)}{\partial X}$$

$\vdots$

$$X_N = X_{n-1} - \alpha \frac{\partial C_2(X)}{\partial X}$$

$$X_K = X_{K-1} - \alpha \frac{\partial C_2(X)}{\partial X}$$



$$X = size(B)$$

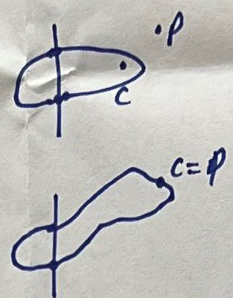
$$V = \begin{pmatrix} A \\ B \end{pmatrix}, \quad V' = \begin{pmatrix} A \\ B + X \end{pmatrix}$$

1) So whats happening intuitively

$V_i - V_j$ = edge length

$V_i' - V_j'$ = " " deformed

C_2 = Loss function



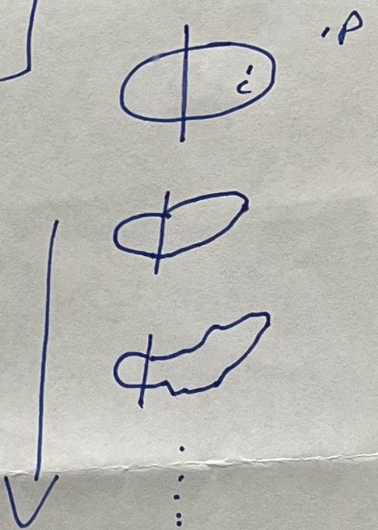
Kitni edge length maintain resemblance ho

isi liye 'X' use karte hain ke kitna rigid rakhna hai

∴ optimize sirf C_2 ke karna hai

basically loss funct

C_2 = [edge length stationary] − [edge length deformed]



So the main questions

$\min_{x \in \mathbb{R}^p} f(x)$

| variables unknowns

x?

f?

What optimal shape?

Grad descent

$f : \mathbb{R}^d \to \mathbb{R}$

stationary point $\nabla f(x^{*}) = 0$

we're trying to identify

$\min_{x \in \mathbb{R}^d} f(x)$ or

x^* = global solution or ?

$\frac{x^*}{\hat{x}} = \arg\min_{x \in \mathbb{R}^d} f(x)$

Algo G.D (f, x_{start})

initialise $x^{(0)} = x_{start} \in \mathbb{R}^d$

repeat

$x^{(k+1)} := x^{(k)} - \gamma_k \nabla f(x^{(k)})$

until $(\| \nabla f(x^{(k)}) \| \leq \tau)$

return $x^{(k)}$

τ = tolerance for algo

cuz we dunno the end

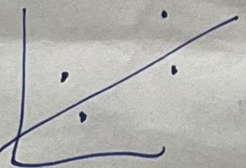
or run for T steps

Leibniz $\frac{dy}{dx}$

Lagrange y'

Newton $\dot{y}$

$$\nabla f(x,y) = \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} \\ \frac{\partial f(x,y)}{\partial y} \end{bmatrix}$$

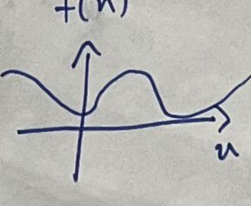


∇f at all points for $\boxed{\text{grad}} \to \sum_i \nabla f(x_i)$

if too many points we use SGD

Adv

$\hat{x}_{t+1} \leftarrow \hat{x}_t - \alpha f'(\hat{x}_t)$



1

$$\text{Cost}_u(V) = \sum_{i=1}^{N_m} \| V'_{c_i} - P_i \|^2$$

$$\text{Cost}_{iso}(V) = \sum_{i=1}^{N_m} \sum_{j \in ring(b_i)} \left| \| V_{b_i} - V_j \| - \| V'_{b_i} - V'_j \| \right|$$

$$C_2(V') = \sum_{i=1}^{N} \sum_{j \in ring(i)} \left| \|V_i - V_j\| - \|V_i' - V_j'\| \right|$$

$\frac{\partial V}{\partial x}$ while V_i, V_j are in eq + initial val] for all 1000 points

~~$-2(\|\cdot\| - \|\cdot\|)$~~

$\therefore$ All expression for points $\sum x_i$ = grad

then $\alpha \cdot$ grad

then old $- \alpha \cdot$ grad = new

$V^0_{b_i} \in \mathbb{R}^3$

but ours is $f(V)$ cuz Cost_u + Cost_{iso}

$$V^{k+1}_{b_i} \leftarrow V^k_{b_i} - \alpha \left[\nabla f(V^k_{b_i}) \right]$$

$$k \leftarrow k+1$$

Para: $V = x, y, z = x \in \mathbb{R}^d$

So $f: \mathbb{R}^p \to \mathbb{R}$

$\therefore f_k(x) = f(x_{k-1}, x, x_{k+1})$

$\therefore \min_{x \in \mathbb{R}^d} f(x)$

~~$x_{k+1} := x_j - \alpha \frac{\partial}{\partial x_j} f(x_0, x_1, x_2 \ldots x_n)$~~

$$x_k := x_k - \alpha \frac{\partial}{\partial x_k} f(x_0, x_1 \ldots x_n)$$

Simultaneous update x_k for $k = 0, 1, \ldots n$

~~6/9/2022~~ $x \in \mathbb{R}^d$ ~~⇒~~ $x = \begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix}$

or $x = [x_i]$ $1 \le i \le d$

$x^T = [x_1 \dots x_d]$

$[x^T]^T = x$

Scalar $a, b \dots$

Vec $\mathbf{a}, \mathbf{b}$ bold

Mat A, B

Set $\mathcal{A}, \mathcal{B}$

Global min

$$\forall w \in \mathbb{R}^d, \ f(w^*) \le f(w)$$

Local min

$$\forall w \in \mathbb{R}^d, \ [\|w - w^*\| \le \delta] \Rightarrow [f(w^*) \le f(w)]$$

Unconstrained min

$$\min_{w \in \mathbb{R}^d} f(w)$$

first order condition

$$w^* \text{ local min of } f \implies \|\nabla f(w^*)\| = 0$$

Second order "

$$[\|\nabla f(w^*)\| = 0 \text{ and } \nabla^2 f(w^*) \succeq 0]$$

$f: \mathbb{R}^d \to \mathbb{R}$ in $C^2(\mathbb{R}^d)$ is ~~conv~~ convex only if

$$\forall w \in \mathbb{R}^d, \ \nabla^2 f(w) \succeq 0$$

C / C^2 continuous diff function set

- good cuz if 'f' convex then every ~~loc~~ local min of f is global min

- Grad descent / steepest descent
  1) Cont diff functi
  2) w^* $\nabla f(w^*) = 0$

  $\therefore$ $w \in \mathbb{R}^d$ two property holds
  1) either $\nabla f(w) = 0$ and ~~so~~ ~~could~~ can be min
  2) $\nabla f(w) \neq 0$, 'f' decreses locally from w in direction of $-\nabla f(w)$