AstroTech MOOC supplementary material

Geostationary orbits

In the third video of week 3, Andy mentioned the idea of a "geostationary orbit". This was dealt with fairly briefly, so for those who are interested, here is a little more detail. (Warning! Maths coming.) Lets take this in steps... To keep life simple we will think about nice simple circular orbits around the equator...

Earth rotation velocity

The Earth rotates once a day. How fast is a point on the surface of the Earth at the equator moving? We can work this out. The radius of the Earth is $R_E = 6370$km (thats an average) so the circumference is $2\pi R = 4.0 \times 10^7$ metres. So the rotation speed is

$$v_{rot} = \frac{2\pi R_E}{24 \text{ hrs}} \times 3600s = 463 \text{ m/sec} = 667 \text{ kph}$$

Orbital velocity

That may seem fast, but in fact you need to go much faster to get into orbit. When you are in orbit, the force of gravity exactly balances the centripetal force you need to keep bending your motion round into a circle. The answer that Newtonian Dynamics gives us is that if you are orbiting a large mass M at radial distance R, you need to have orbital speed

$$v_{orb} = \left(\frac{GM}{R}\right)^{1/2}$$

Any slower and you fall back down; any faster and you fly off into space. Here G is Newton's constant, M is the mass of the Earth, and R is the radial distance from the centre of the Earth (not the height above the surface of the Earth).

Now consider something in low Earth orbit. If the height is say $h = 500$km, then $R = 6370 + 500$km. With $M = 5.97 \times 10^{24}$ kg, we get

$$v_{orb} = 7.6 \text{ km/s}$$

- much faster than Earth rotation. When you launch a spacecraft, most of the energy cost is simply getting it up to speed. Note however that the R is on the bottom .. so as we go further from the Earth, the orbital speed gets slower. By the time you get to 9 Earth radii, the orbital speed its 3 times slower... The same is true for planets going round the Sun by the way. The outer planets go around the Sun more slowly than the inner planets.

Orbital period

However ... a spacecraft that is further away from the Earth also has further to travel in its orbit - the length of the orbit is $2\pi R$. So how long does it take to go round the Earth once, i.e. what is the orbital period? If we divide $2\pi R$ by the orbital velocity we worked out above, and put the constants in, we find that the orbital period is

$$P = 84.35 \text{ minutes} \times \left(\frac{R}{R_E}\right)^{1.5}$$

where R is the radial distance from the centre of the Earth, and R_E is the radius of the Earth, so R/R_E is the radial distance in units of Earth-radii.

Geostationary orbit

So as you move out to larger radii, the orbital period gets longer; eventually you reach a distance where the orbital period is *exactly the same* as the Earth rotation period. Putting P =23 hours 56 minutes in the formula above and solving for R we find

$$R/R_E = 6.62$$

So if a spacecraft is in a circular orbit above the equator at this distance, it will appear to hover above a single spot on the Earth - so it can be in??? permanent contact with a ground station. (BTW ... don't mix up the radial distance from the centre of the Earth and the height above the Earth's surface..)

Geosynchronous orbits

More generally, orbits might be inclined with respect to the equator, or might be elliptical rather than circular. As long as the semi-major axis of the orbit is $6.62R_E$, the orbital period is the same, but the spacecraft will not hover over exactly the same spot on Earth - rather it will appear to oscillate in a pattern known as an analemma. An orbit like this is known as geosynchronous, rather than geostationary.