Vavuniya Campus of the University of Jaffna

First Examination in Information and Communication Technology - 2018

First Semester - September/October 2019

ICT1113 Discrete Structures (Old Syllabus)

Answer Five Questions Only

Time Allowed : **Three hours**

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1. (a) Write down the elements in each of the following sets:

   i. $A=\{x \in \mathbb{N} \mid x \text{ is even}, x < 11\}$;

   ii. $B=\{x \in \mathbb{N} \mid 3 < x < 9\}$;

   iii. $C=\{x \in \mathbb{N} \mid 4 + x = 3\}$;

   iv. $D=\{x \in \mathbb{R} \mid x^2= 4 \text{ OR } x^2= 9\}$. [20%]

   (b) Suppose **A** is the set of distinct letters in the word *ELEPHANT*, **B** is the set of distinct letters in the word *SYCOPHANT*, **C** is the set of distinct letters in the word *FANTASTIC*, and **D** is the set of distinct letters in the word *STUDENT*. The universe **U** is the set of *English alphabet*. Find:

   i. $A \cup B$;

   ii. $A \cap B$;

   iii. $A \cap (C \cup D)$;

   iv. $(A \cup B \cup C \cup D)^c$. [30%]

[This question is continued on the next page]

(c) A={ 1, 2 } , B={ a, b } and C={ 7, 8, 9 } find:

i. P(C), the power set of C;

ii. A × B × C. [20%]

(d) Five hundred researchers at the National Security agency were surveyed. It was found that 300 have at least one degree in Computer Science, 173 have a Bachelor's degree in Computer Science, 123 have a Master's degree in Computer Science and 99 have a Ph.D. in Computer Science. Additionally, it was found that 63 have both Bachelor's and Master's degree in Computer Science, 22 have both Master's and Ph.D. degrees in Computer Science and 19 have both Bachelor's and Ph.D. degrees in Computer Science. One employee is selected at random. What are the probabilities for each of the following events?

i. She/he has all three degrees in Computer Science.

ii. She/he has the Bachelor's and Ph.D. degrees, but not the Master's. [30%]

2. (a) Find the domain and range of the relation R, where R is defined as below:

i. A={1, 2, 3, 4, 5}, B={1, 2, 3, 10}, aRb if and only if $2a = b$

ii. R on the set X={1, 2, 3, 4, 5} is defined by $(x,y) \in R$ if $x = y-1$, $x,y \in X$. [20%]

(b) Let R be a relation on the set A= {1, 2, 3, 4, 5} . R = {(1, 1), (2, 2), (2, 3), (3, 2), (4, 2), (4, 4)}. Determine whether R is *reflexive*, *symmetric* or *transitive*. [30%]

(c) Determine whether each of the following functions is a `bijection` from $\mathbb{R}$ to $\mathbb{R}$. Justify your answer:

i. $f(x) = 2x + 1$;

ii. $f(x) = x^2 + 1$;

iii. $f(x) = x^3$;

iv. $f(x) = (x^2 + 1)/(x^2 + 2)$. [40%]

(d) Let f: $\mathbb{R} \to \mathbb{R}$ and g: $\mathbb{R} \to \mathbb{R}$ be defined by $f(x)=4x-3$ and $g(x)=x^2+2$ for all $x \in \mathbb{R}$. Find $g \circ f$, and $f \circ g$. [10%]

3. (a) Let p be the proposition *"I will do every exercise in this book"* and q be the proposition *"I will get a grade A in this course."* Express each of these as a combination of p and q.

   i. I will get a grade A in this course only if I do every exercise in this book.

   ii. I will get a grade A in this course and I will do every exercise in this book.

   iii. Either I will not get a grade A in this course or I will not do every exercise in this book.

   iv. For me to get a grade A in this course it is necessary and sufficient that I do every exercise in this book. [30%]

   (b) Show that each of following conditional statements is a tautology by using truth tables.

   i. $(p \wedge q) \longrightarrow (p \rightarrow q)$;

   ii. $\neg (p \rightarrow q) \longrightarrow \neg q$. [30%]

   (c) State the *converse*, *contrapositive*, and *inverse* of the following conditional statement.

   "If you are good in Mathematics then you are good in Logic". [20%]

   (d) Express following statements formally using quantifiers, variables and predicates.

   i. No one is perfect.

   ii. Some of the children didn't apologize.

   iii. Some men are genius.

   iv. For any integer there exists an integer such that their sum is 0. [20%]

4. (a) Convert each of the following numbers to base ten:

   i. 1010.10101_2;

   ii. 21.21_8;

   iii. $EF.B1_{16}$. [15%]

[This question is continued on the next page]

(b) Convert the following binary IP address to a dotted-decimal notation.

10011110.10101011.01101001.01011101 [15%]

(c) How many 3 digit numbers greater than 200 can be made from the set of numbers $\{1, 2, 3, 7, 8\}$, where numbers are NOT allowed to repeat? [20%]

(d) Find the sum of the first six terms in the sequence of $\{a_n\}$, $n > 2$ where $a_n = 2a_{n-1} + a_{n-2}$, $a_1 = 1$, and $a_2=1$. [20%]

(e) Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$, $n > 2$ if

i. $a_n = 4^n$

ii. $a_n = 2.4^n + 3n.4^n$ [30%]

5. (a) Use the properties of Boolean algebra, simplify the following boolean expressions:

i. A(B+C) + A +AB

ii. $(A + C)$ (AD+A$\overline{D}$)+AC+C [30%]

(b) Use a Karnaugh map to simplify the following Boolean expression:

$\overline{A}$BC+A$\overline{B}$C+AB$\overline{C}$+ABC [20%]

(c) Design a simplify logic circuit with four input variables that will produce logic value 1 as an output when more than two input variables have logical value 1. [30%]

(d) A system used 3 switches A, B and C; a combination of switches determines whether an alarm, X, sounds: If switch A or switch B are in the ON position and if switch C is in the OFF position then a signal to sound an alarm, X is produced. Design a circuit that produced alarm X sound. [20%]

6. (a) Consider the graph G1 in Figure 1:

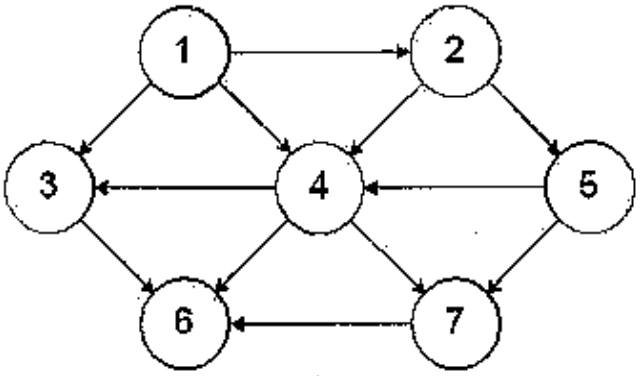

Figure 1

i. Find the in-degree and out-degree of each vertex. [20%]

ii. Represent the graph G1 in an adjacency matrix. [15%]

(b) Represent the following graph in an incidence matrix.

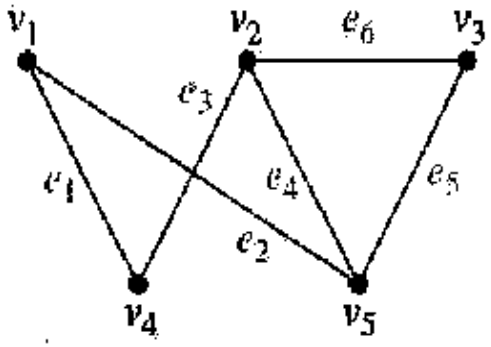

Figure 2 [20%]

(c) A couple has two children; the sample space is S = { bb, bg, gb, gg }, where b represents boy and g represents girl.In the set elements, first letter and second letter represent elder child and younger child respectively. Find the probability p:

i. at least one of the children is a boy

ii. the elder child is a boy [20%]

[This question is continued on the next page]

(d) Consider the finite-state machine M defined by the state table shown in Table 1.

State	Input	
	0	1
S_0	$S_1, 0$	$S_0, 0$
S_1	$S_2, 1$	$S_0, 1$
S_2	$S_0, 0$	$S_3, 1$
S_3	$S_1, 1$	$S_2, 0$

Table 1:

i. What are the states of M? [05%]

ii. What are the input symbols of M? [05%]

iii. Draw the state diagrams for the finite-state machine M. [15%]