

Digital Systems

CE212

HW 5

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Question 1

Step 1: the number of J-K F.Fs that are needed is 4

Step 2 : State table

Q_A	Q_B	Q_C	Q_D	Q_A^+	Q_B^+	Q_C^+	Q_D^+
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	0	X	X	X	X
1	0	1	1	X	X	X	X
1	1	0	0	X	X	X	X
1	1	0	1	X	X	X	X
1	1	1	0	X	X	X	X
1	1	1	1	X	X	X	X

Step 3: transition table

[illegible]

AB \ CD	00	01	11	10
00	X	X	X	X
01	0	0	1	0
11	X	X	X	X
10	X	X	X	X

$$K_B = Q \cdot C \cdot QD$$

AB \ CD	00	01	11	10
00	0	0	1	X
01	0	0	1	X
11	X	X	X	X
10	0	0	X	X

$$\delta C = Q'A \cdot QD$$

AB \ CD	00	01	11	10
00	X	X	1	0
01	0	X	1	0
11	X	X	X	X
10	X	X	X	X

$$K_C = QD$$

AB \ CD	00	01	11	10
00	1	X	X	1
01	1	X	X	1
11	X	X	X	X
10	1	X	X	X

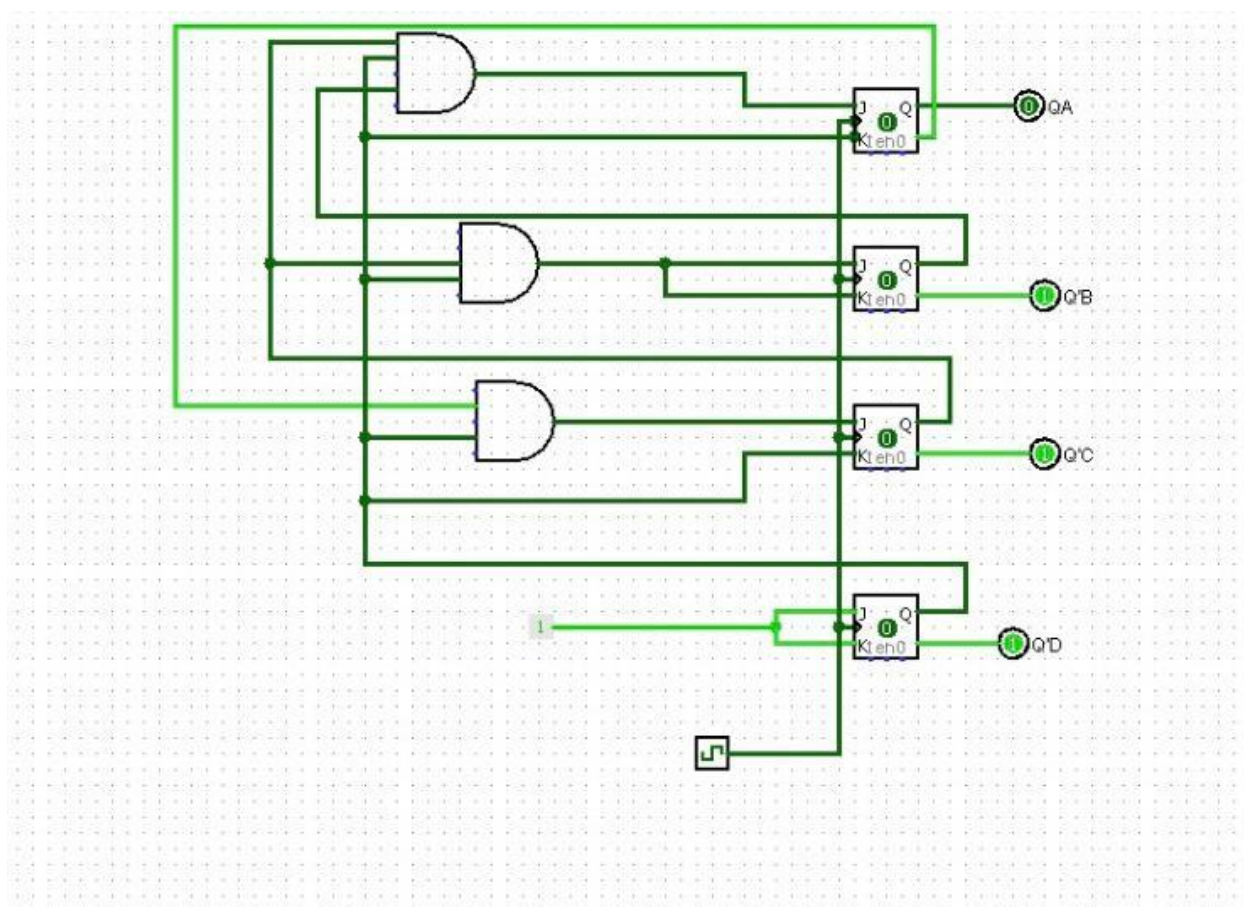
$$J_D = 1$$

AB \ CD	00	01	11	10
00	X	X	X	X
01	X	1	1	X
11	X	X	X	X
10	X	1	X	X

$$K_D = 1$$

AB \ CD	00	01	11	10
00	X	X	X	X
01	X	X	X	X
11	X	X	X	X
10	0	1	X	X

$$K_A = QD$$



Question 2

a.

Step 1 : truth table

G	L	Q(t)	Q(t+1)
0	0	Q(t-1)	Q'(t-1)
0	1	Q(t-1)	Q'(t-1)
1	0	0	1
1	1	1	0

Step 2 : Next state table

G	L	Q(t)	Q(t+1)
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Deriving the characteristic equation using K-map

	00	01	11	10
0	0	1	1	0
1	0	0	1	1

$$Q(t+1) = G'Q(t) + G.L$$

b.

Step 1 : Build the next state table of the G-L f.f

G	L	Q(t)	Q(t+1)
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

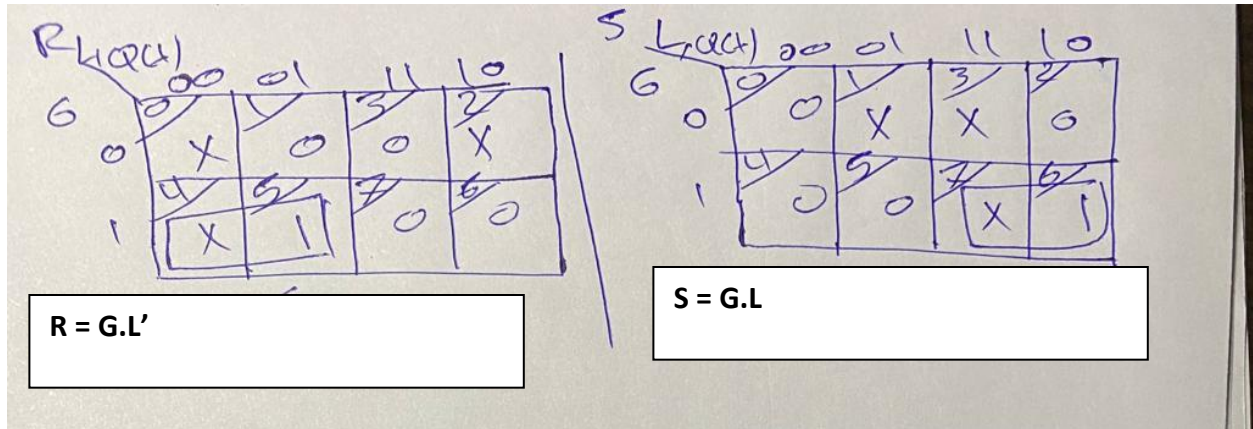
Step 2 : Build the excitation table of the S-R f.f

Q(t)	Q(t+1)	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

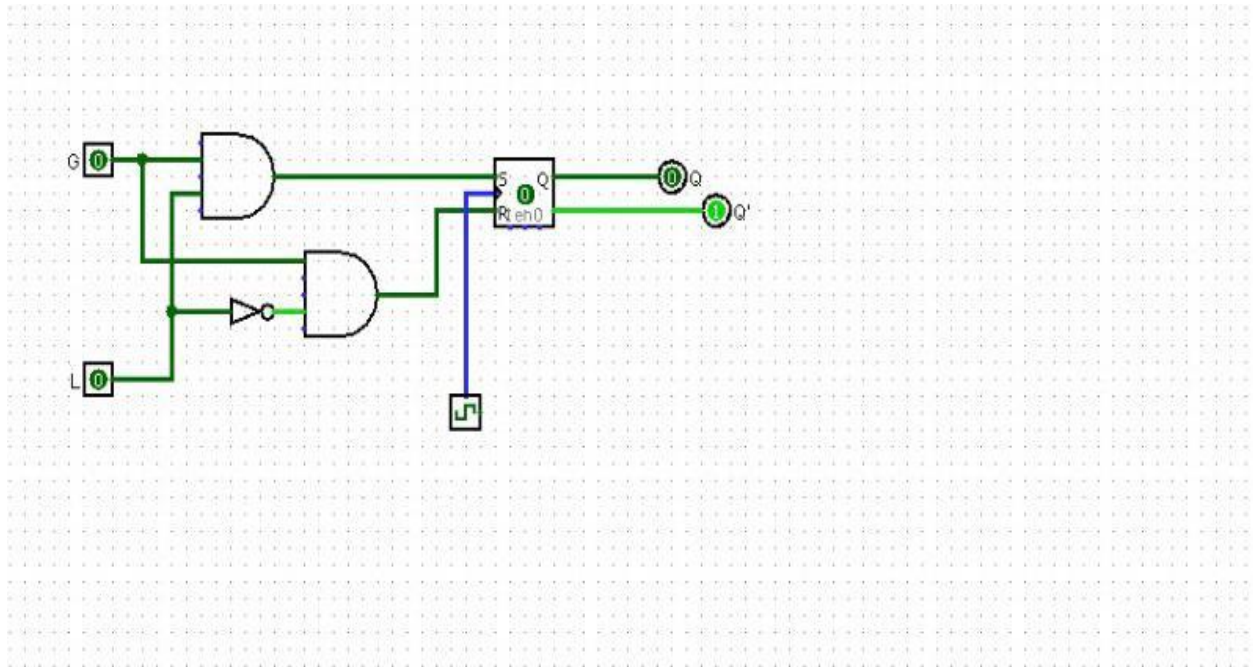
Step 3 :

G	L	Q(t)	Q(t+1)	S	R
0	0	0	0	0	X
0	0	1	1	X	0
0	1	0	0	0	X
0	1	1	1	X	0
1	0	0	0	0	X
1	0	1	0	0	1
1	1	0	1	1	0
1	1	1	1	X	0

Step 4: find the equation of the S-R



Step 5 : Build the circuit



Question 3

Step1 : # of f.fs that are needed is 3

Step 2: State table

Q_A	Q_B	Q_C	Q_A^+	Q_B^+	Q_C^+
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	0	0	0
1	0	0	1	1	1
1	0	1	0	0	0
1	1	0	0	0	0
1	1	1	0	0	0

Step 3 : Transition table

Q_A	Q_B	Q_C	Q_A^+	Q_B^+	Q_C^+	J_A	K_A	J_B	K_B	J_C	K_C
0	0	0	0	0	1	0	X	0	X	1	X
0	0	1	0	1	0	0	X	1	X	X	1
0	1	0	1	0	0	1	X	X	1	0	X
0	1	1	0	0	0	0	X	X	1	X	1
1	0	0	1	1	1	X	0	1	X	1	X
1	0	1	0	0	0	X	1	0	X	X	1
1	1	0	0	0	0	X	1	X	1	0	X
1	1	1	0	0	0	X	1	X	1	X	1

$$J_A = \sum_m(2) + \sum_d(4,5,6,7)$$

$$K_A = \sum_m(5,6,7) + \sum_d(0,1,2,3)$$

$$J_A = \sum_m(1,4) + \sum_d(2,3,6,7)$$

$$J_A = \sum_m(2,3,6,7) + \sum_d(0,1,4,5)$$

$$J_A = \sum_m(0,4) + \sum_d(1,3,5,7)$$

$$J_A = \sum_m(1,3,5,7) + \sum_d(0,2,4,6)$$

Handwritten Karnaugh maps and Boolean expressions for three variables A, B, and C.

Top Left: Karnaugh Map for J_A

	00	01	11	10
QA	0	0	0	1
1	1	1	1	1

$J_A = \overline{A}B\overline{C}$

Top Right: Karnaugh Map for K_A

	00	01	11	10
QA	0	1	1	1
1	1	1	1	1

$K_A = \overline{A}B + \overline{A}C$

Middle Left: Karnaugh Map for J_B

	00	01	11	10
QA	0	1	1	1
1	1	1	1	1

$J_B = \overline{A}B\overline{C} + \overline{A}B\overline{C}$
 $J_B = \overline{A}B\overline{C}$

Middle Right: Karnaugh Map for K_B

	00	01	11	10
QA	0	1	1	1
1	1	1	1	1

$K_B = 1$

Bottom Left: Karnaugh Map for J_C

	00	01	11	10
QA	0	1	1	1
1	1	1	1	1

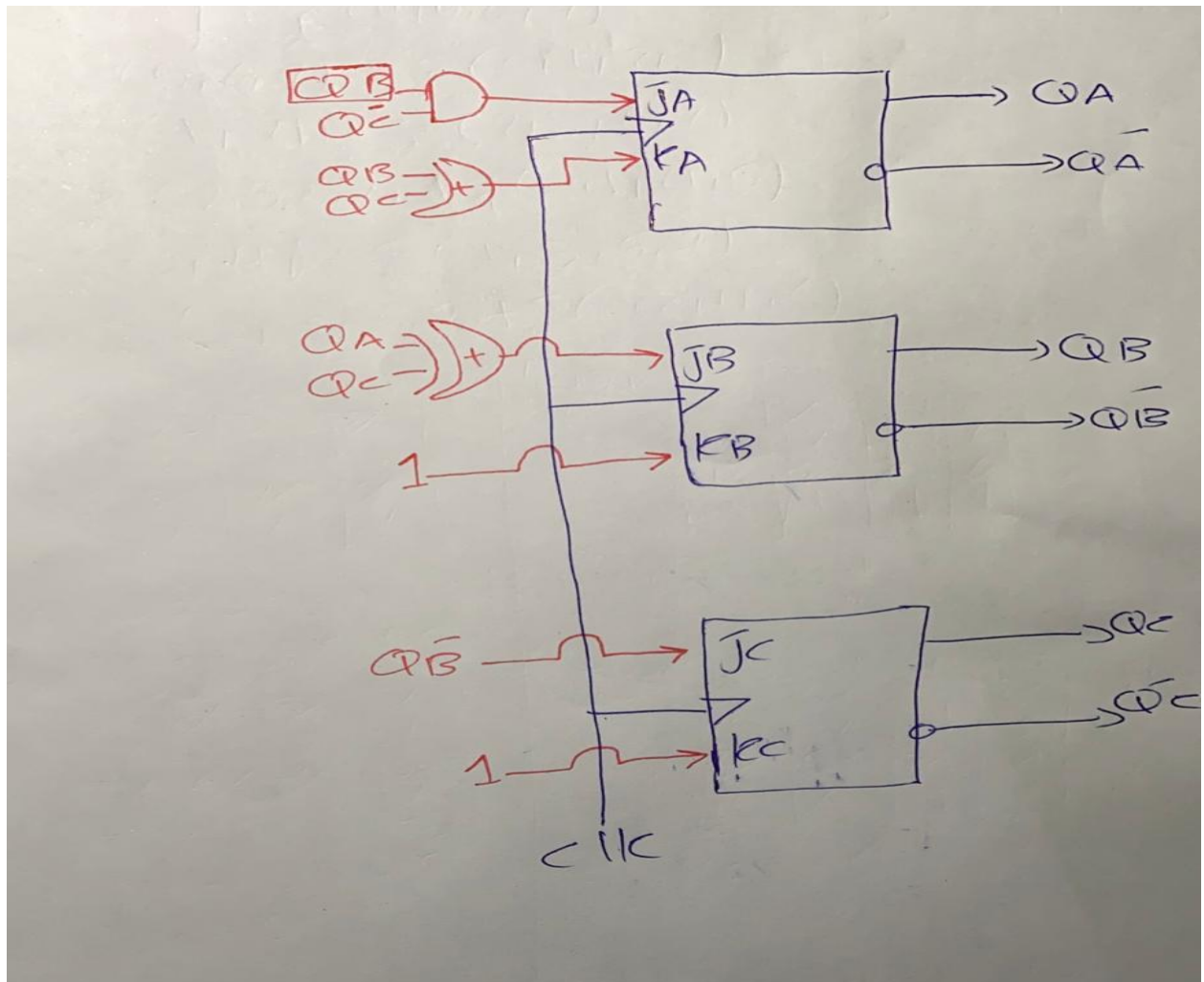
$J_C = \overline{A}B$

Bottom Right: Karnaugh Map for K_C

	00	01	11	10
QA	0	1	1	1
1	1	1	1	1

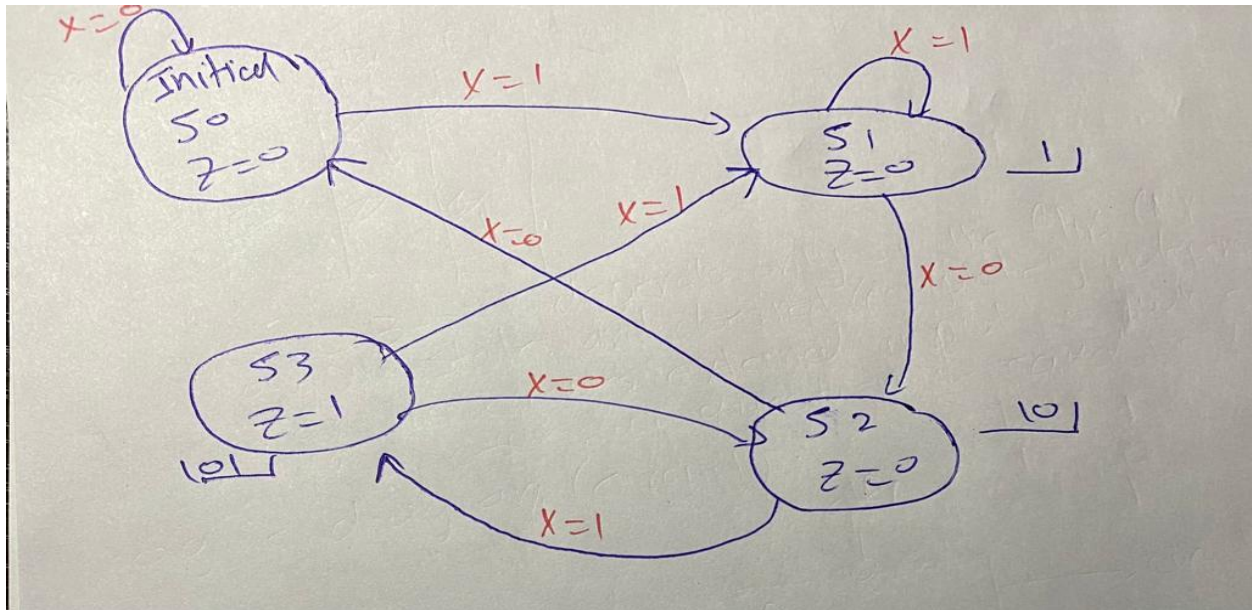
$K_C = 1$

Step 5 : logic circuit



Question 4

a.



b. The number of D-F.F that are needed is equal to:

$$\text{\#of F.Fs} = \log_2 (\text{\#of states}) = \log_2 4 = 2.$$

c.

Q_A	Q_B	X	Q_A^+	Q_B^+	Z	D_A	D_B
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	1
0	1	0	1	0	0	1	0
0	1	1	0	1	0	0	1
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	0	1	1	0
1	1	1	0	1	1	0	1

d.

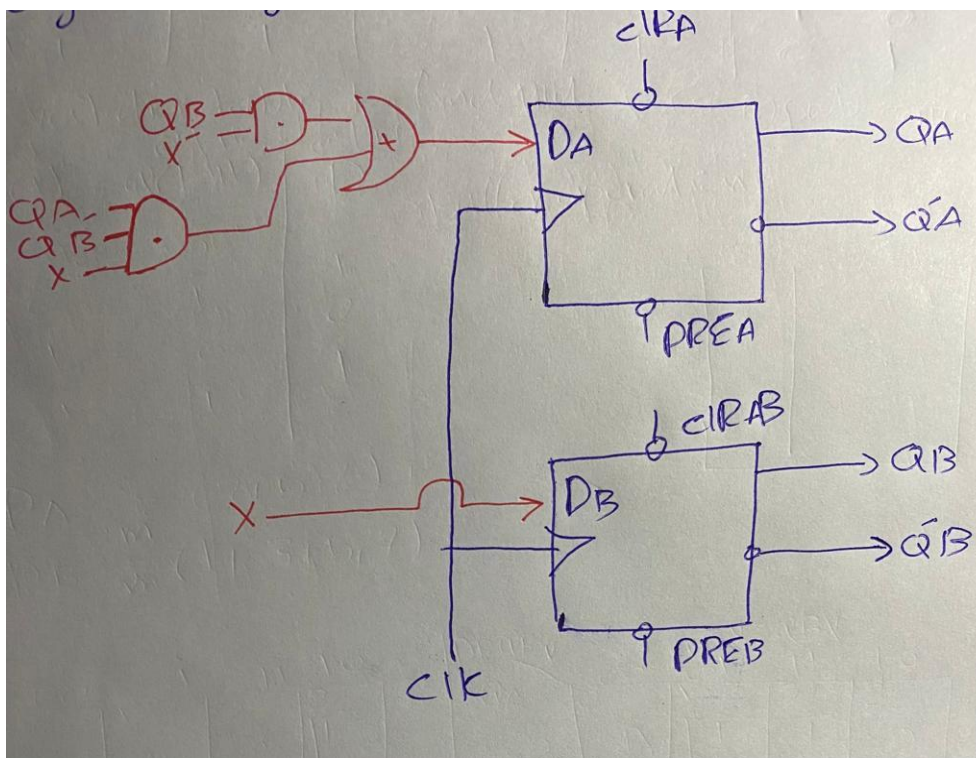
QA \ QB X	00	01	11	10
00	0	0	0	1
01	0	1	1	0
11	0	1	1	0
10	1	0	0	0

DB \ QB X	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	0	1	1	0
10	1	0	0	0

$$D_A(QA, QB, X) = QB.X' + QA.Q'B.X$$

$$D_B(QA, QB, X) = X$$

e.



Question 5

a.

Q(t)	Q(t+1)	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

b.

c. The number of the F.Fs that are required to implement this circuit is 4 since we it is a 4-bit counter and each F.F represent one bit(each F.F has one stable output).

d.

[illegible]

e.

JA \ CD	00	01	11	10
AB 00	X	X	0	0
AB 01	X	0	1	X
AB 11	X	X	X	X
AB 10	X	X	X	X

$JA = QB \cdot QC$

JB \ CD	00	01	11	10
AB 00	X	X	1	0
AB 01	X	X	X	X
AB 11	X	X	X	X
AB 10	X	X	1	X

$JB = QD$

JC \ CD	00	01	11	10
AB 00	X	X	X	X
AB 01	X	1	X	X
AB 11	X	1	X	X
AB 10	X	X	X	X

$JC = 1$

JD \ CD	00	01	11	10
AB 00	X	X	X	1
AB 01	X	X	X	X
AB 11	X	X	X	X
AB 10	X	X	X	X

$JD = 1$

KA \ CD	00	01	11	10
AB 00	X	X	X	X
AB 01	X	X	X	X
AB 11	X	1	X	X
AB 10	X	X	0	X

$KA = QB$

OR $KB = Q'C$

KB \ CD	00	01	11	10
AB 00	X	X	X	X
AB 01	X	0	1	X
AB 11	X	1	X	X
AB 10	X	X	X	X

$KB = QA + QC$

KC \ CD	00	01	11	10
AB 00	X	X	1	0
AB 01	X	X	0	X
AB 11	X	X	X	X
AB 10	X	X	1	X

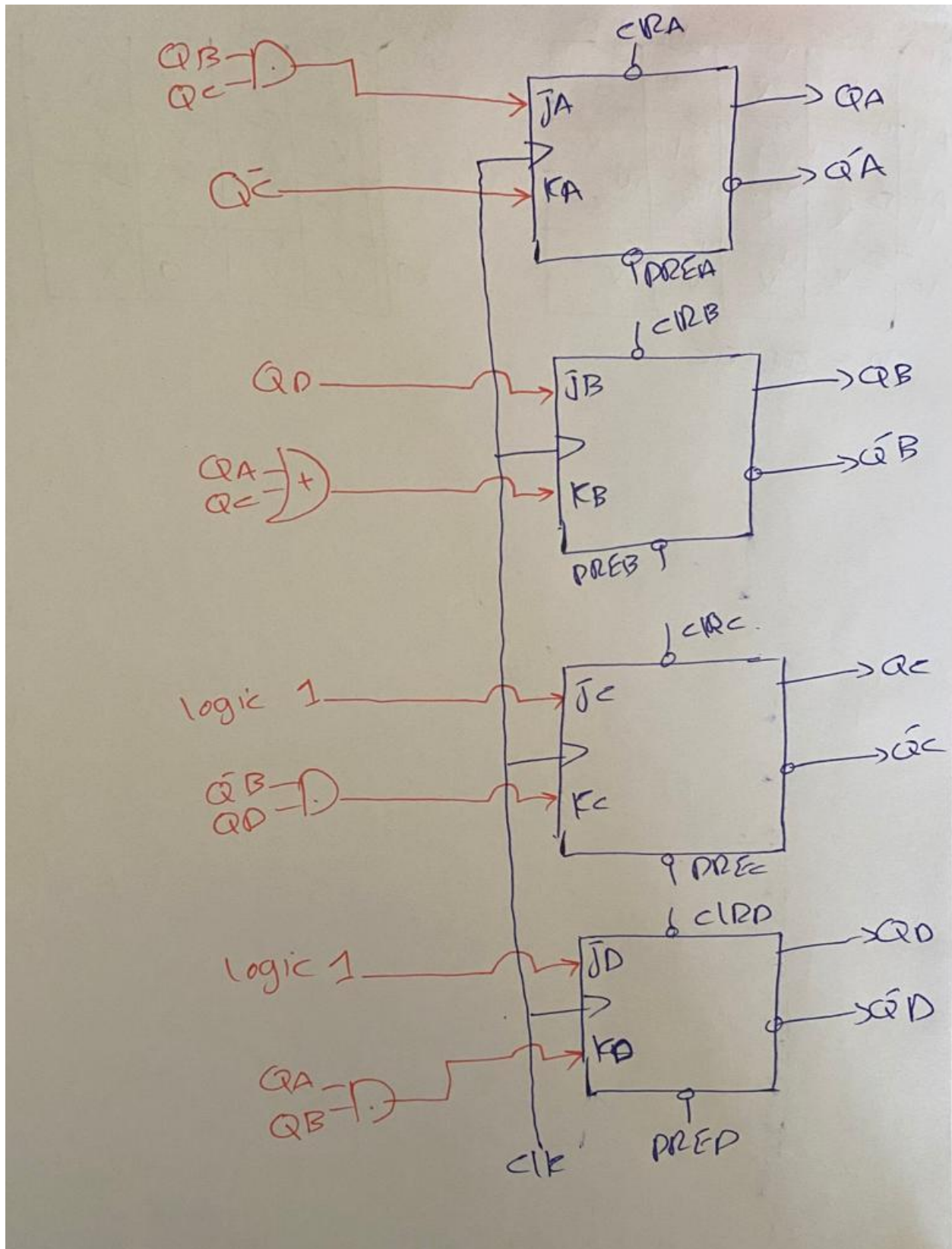
$KC = Q'B \cdot QD$

KD \ CD	00	01	11	10
AB 00	X	X	0	X
AB 01	X	0	0	X
AB 11	X	1	X	X
AB 10	X	X	0	X

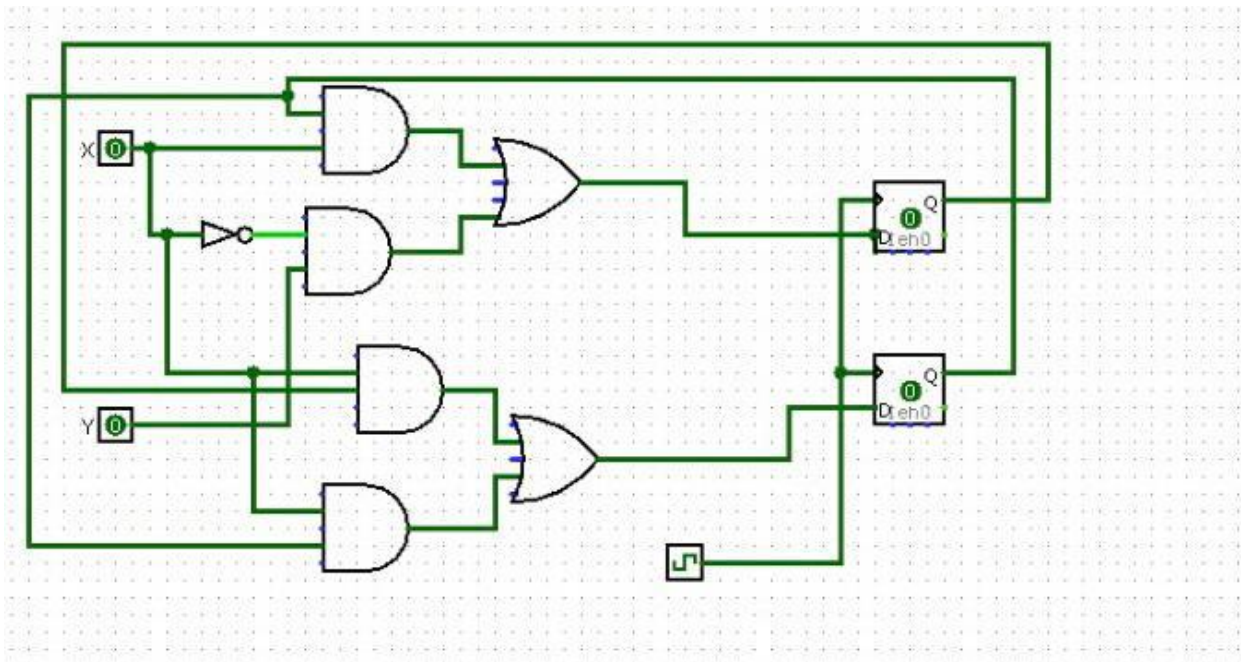
$KD = QA \cdot QB$

OR $KD = QA \cdot Q'C$

f.



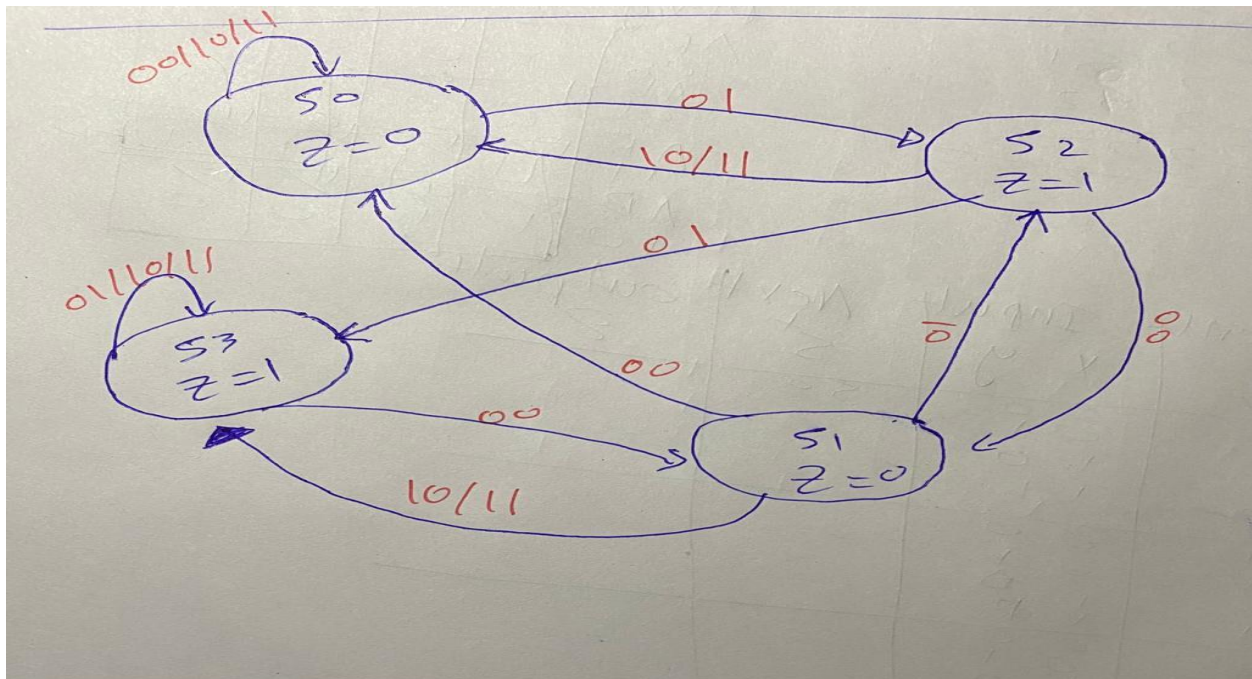
Question 6 .a.



b.

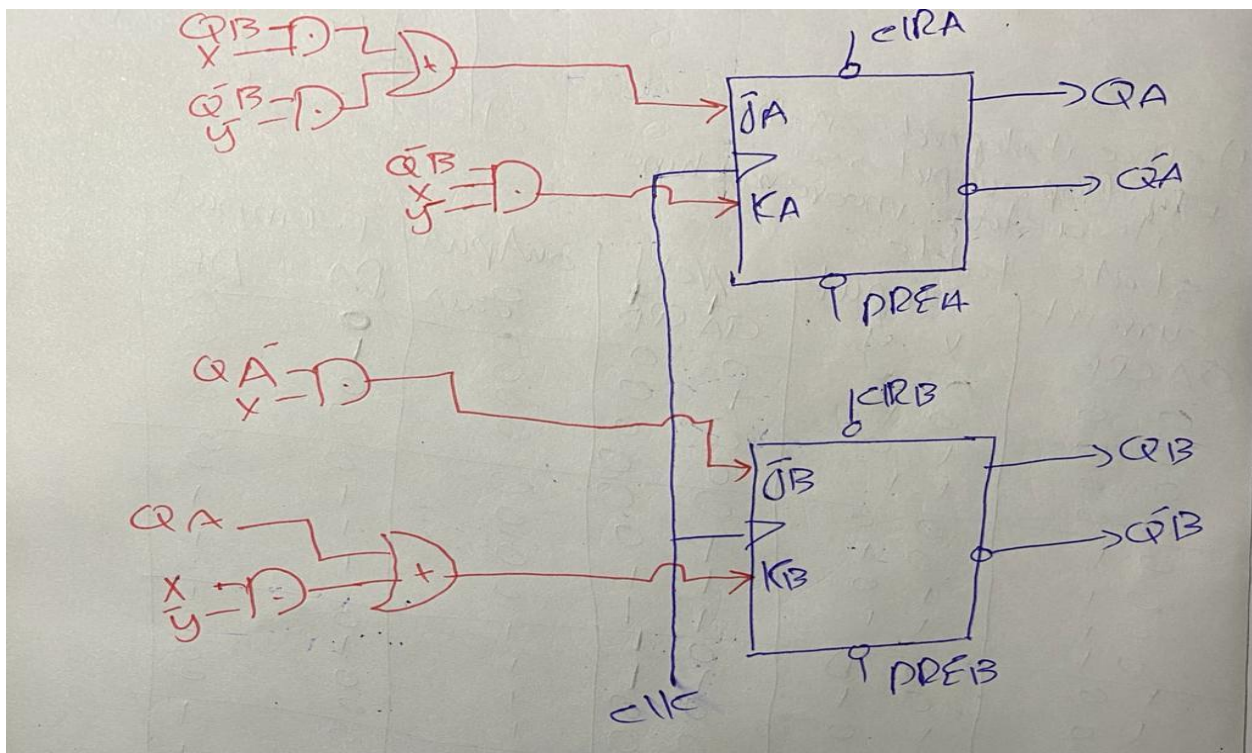
[illegible]

c.



Question 7

a.



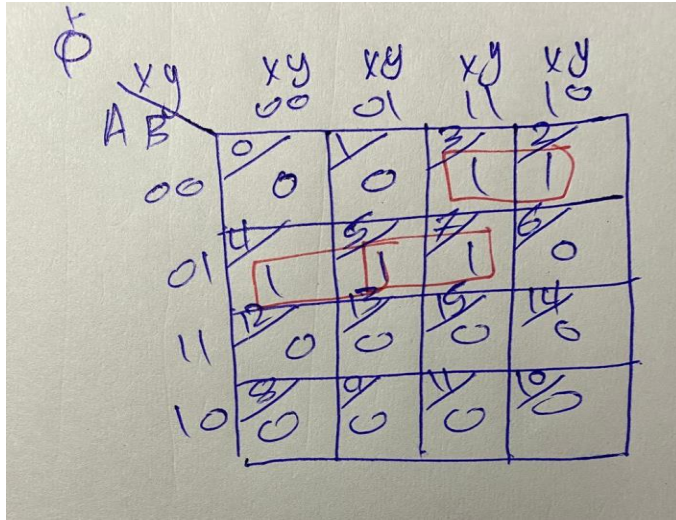
b.

Q_A	Q_B	X	Y	Q_A^+	Q_B^+	Z	J_A	K_A	J_B	K_B
0	0	0	0	1	0	0	1	X	0	X
0	0	0	1	0	0	0	0	X	0	X
0	0	1	0	1	1	0	1	X	1	X
0	0	1	1	0	1	0	0	X	1	X
0	1	0	0	0	1	1	0	X	X	0
0	1	0	1	0	1	0	0	X	X	0
0	1	1	0	1	0	0	1	X	X	1
0	1	1	1	1	1	0	1	X	X	0
1	0	0	0	1	0	0	X	0	0	X
1	0	0	1	1	0	0	X	0	0	X
1	0	1	0	0	0	0	X	1	0	X
1	0	1	1	1	0	0	X	0	0	X
1	1	0	0	1	0	1	X	0	X	1
1	1	0	1	1	0	0	X	0	X	1
1	1	1	0	1	0	0	X	0	X	1
1	1	1	1	1	0	1	X	0	X	1

c.

Handwritten Karnaugh map for $Q_A(t+1)$. The map is a 4x4 grid with rows labeled AB (00, 01, 11, 10) and columns labeled XY (00, 01, 11, 10). The cells contain values for $Q_A(t+1)$ with red boxes highlighting groups of 1s. The groups are: a 2x2 square in the top-left (AB=00, XY=00-01), a 2x2 square in the top-right (AB=00, XY=11-10), a 2x2 square in the bottom-left (AB=11, XY=00-01), a 2x2 square in the bottom-right (AB=11, XY=11-10), and a 2x2 square in the middle (AB=01-11, XY=01-11).

$$Q_A(t+1) = Q_A X' + Q_A Y + Q_B X + Q_A' Q_B' Y'$$



$$Q_B(t+1) = Q'_A Q_B X' + Q'_A Q_B Y + Q'_A Q'_B X$$

Question 8

Q_A	Q_B	Q_C	X	Q_A^+	Q_B^+	Q_C^+	Z	D_A	D_B	D_C
0	0	0	0	0	1	1	0	0	1	1
0	0	0	1	1	0	0	1	1	0	0
0	0	1	0	0	0	1	0	0	0	1
0	0	1	1	1	0	0	1	1	0	0
0	1	0	0	0	1	0	0	0	1	0
0	1	0	1	0	0	0	1	0	0	0
0	1	1	0	0	0	1	0	0	0	1
0	1	1	1	0	1	0	1	0	1	0
1	0	0	0	0	1	0	0	0	1	0
1	0	0	1	0	1	1	0	0	1	1
1	0	1	0	X	X	X	X	X	X	X
1	0	1	1	X	X	X	X	X	X	X
1	1	0	0	X	X	X	X	X	X	X
1	1	0	1	X	X	X	X	X	X	X
1	1	1	0	X	X	X	X	X	X	X
1	1	1	1	X	X	X	X	X	X	X

D	C \ X			
	00	01	11	10
B \ A	00	1	1	0
01	4	5	7	6
11	2	X	X	X
10	3	0	X	X

$$D_A = \bar{C}A\bar{Q}B\bar{X}$$

D	C \ X			
	00	01	11	10
B \ A	00	1	0	0
01	4	5	7	6
11	2	X	X	X
10	3	0	X	X

$$D_B = (\bar{C}C \cdot \bar{X} + C\bar{B}(C \cdot X) + C\bar{B}A)$$

$$D_B = (C\bar{A} + \bar{C}C \cdot \bar{X} + C\bar{B} \cdot C\bar{C} \cdot X)$$

D	C \ X			
	00	01	11	10
B \ A	00	1	0	1
01	4	5	7	6
11	2	X	X	X
10	3	0	X	X

$$D_C = (C\bar{A}X + C\bar{C}\bar{X} + C\bar{A}\bar{C}B\bar{X})$$

Unused States

QA QB QC → Next

1 0 1 → 0 1 1

$D_A = 0, D_B = 1, D_C = 1.$

QA QB QC → Next

1 1 0 →

$D_A = 0, D_B = 1, D_C = X$ (it is not a do not care condition ... it points to the input x
.However, we have to values of x (i.e $x = 0, x = 1$).

For $x = 1$

QA QB QC → Next

1 1 0 → 0 1 1

For $x = 0$

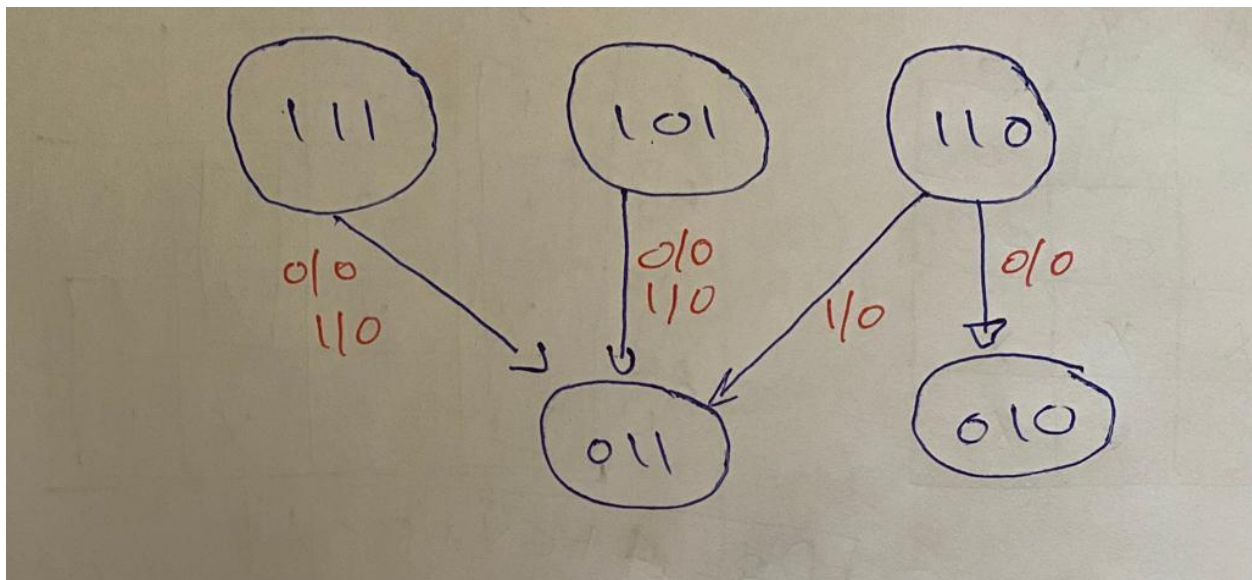
QA QB QC → Next

1 1 0 → 0 1 0

QA QB QC → Next

1 1 1 → 0 1 1

$D_A = 0, D_B = 1, D_C = 1$



Using J-K-F-F

Q_A	Q_B	Q_C	X	Q_A^+	Q_B^+	Q_C^+	Z	J_A	K_A	J_B	K_B	J_C	K_C
0	0	0	0	0	1	1	0	0	X	1	X	1	X
0	0	0	1	1	0	0	1	1	X	0	X	0	X
0	0	1	0	0	0	1	0	0	X	0	X	X	0
0	0	1	1	1	0	0	1	1	X	0	X	X	1
0	1	0	0	0	1	0	0	0	X	X	0	0	X
0	1	0	1	0	0	0	1	0	X	X	1	0	X
0	1	1	0	0	0	1	0	0	X	X	1	X	0
0	1	1	1	0	1	0	1	0	X	X	0	X	1
1	0	0	0	0	1	0	0	X	1	1	X	0	X
1	0	0	1	0	1	1	0	X	1	1	X	1	X
1	0	1	0	X	X	X	X	X	X	X	X	X	X
1	0	1	1	X	X	X	X	X	X	X	X	X	X
1	1	0	0	X	X	X	X	X	X	X	X	X	X
1	1	0	1	X	X	X	X	X	X	X	X	X	X
1	1	1	0	X	X	X	X	X	X	X	X	X	X
1	1	1	1	X	X	X	X	X	X	X	X	X	X

