

CE 211

Digital Systems

Homework # 1

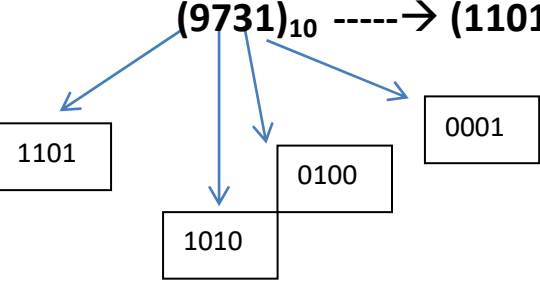
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Question 1

Decimal Digit	5-3-2-1 Code			
	5	3	2	1
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	1	0	0
4	0	1	0	1
5	1	0	0	0
6	1	0	0	1
7	1	0	1	0
8	1	1	0	0
9	1	1	0	1

$(9731)_{10} \rightarrow (1101101001000001)_{5-3-2-1}$



Question 2

$x^2 - 13x + 32 = 0 \rightarrow$ equation 1

Then

$x = 5$ and $x = 4$ are solutions for the equation above

$(x-5)(x-4) = 0 \rightarrow$ in base 10

$x^2 - 5x - 4x - 20 = 0 \rightarrow x^2 - 9x + 20 = 0 \rightarrow$ equation 2

Compare 1 & 2

$(x^2 - 13x + 32)_b = 0$

$(13)_b = (9)_{10}$

$3 \cdot b^0 + 1 \cdot b^1 = 9 \rightarrow 3 + b = 9 \rightarrow b = 6.$

the base of the numbers is 6

also we can find it

$2 \cdot b^0 + 3 \cdot b^1 = 20 \rightarrow 2 + 3b = 20 \rightarrow 3b = 18 \rightarrow b = 6$

Question 3

8 7 6 5 4 3 2 1 0

a. $(100010110)_2 \rightarrow (278)_{10}$

$$M=9, n=0, r=2$$

$$= \sum_{i=-n}^{i=m-1} b^i * (2)^i =$$

$$0*2^0 + 1*2^1 + 1*2^2 + 0*2^3 + 1*2^4 + 0*2^5 + 0*2^6 + 0*2^7 + 1*2^8 =$$

$$0 + 2 + 4 + 0 + 16 + 0 + 0 + 0 + 256 = (278)_{10}$$

7 6 5 4 3 2 1 0

b. $(10101011)_2 \rightarrow (171)_{10}$

$$m=8, n=0, r=2$$

$$= \sum_{i=-n}^{i=m-1} b^i * (2)^i =$$

$$1*2^0 + 1*2^1 + 0*2^2 + 1*2^3 + 0*2^4 + 1*2^5 + 0*2^6 + 1*2^7 =$$

$$1 + 2 + 0 + 8 + 0 + 32 + 0 + 128 = (171)_{10}$$

9 8 7 6 5 4 3 2 1 0

c. $(1011011011)_2 \rightarrow (731)_{10}$

$$m=10, n=0, r=2$$

$$= \sum_{i=-n}^{i=m-1} b^i * (2)^i =$$

$$1*2^0 + 1*2^1 + 0*2^2 + 1*2^3 + 1*2^4 + 0*2^5 + 1*2^6 + 1*2^7 + 1*2^7 + 0*2^8 + 1*2^9 =$$

$$1 + 2 + 0 + 8 + 16 + 0 + 64 + 128 + 512 = (731)_{10}$$

$$d.(10000000000)_2 \rightarrow (1024)_{10}$$

$$m = 11, n = 0, r = 2$$

$$= \sum_{i=-n}^{i=m-1} b^i * (2)^i =$$

$$0*2^0 + 0*2^1 + 0*2^2 + 0*2^3 + 0*2^4 + 0*2^5 + 0*2^6 + 0*2^7 + 0*2^7 + 0*2^8 + 0*2^9 + 1*2^{10} \\ = (1024)_{10}$$


Question 4


a. $(217)_{10} \rightarrow (11011001)_2$


b. $(186)_{10} \rightarrow (10111010)_2$


c. $(265)_{10} \rightarrow (100001001)_2$

d. $(957)_{10} \rightarrow (1110111101)_2$

186 / 2 = 93	0(LSB)	
93/2 = 46	1	
46/2 = 23	0	
23/2 = 11	1	
11/2 = 5	1	
5/2 = 2	1	
2/2 = 1	0	
1/2 = 0	1 (MSB)	

217 / 2 = 108	1(LSB)	
108/2 = 54	0	
54/2 = 27	0	
27/2 = 13	1	
13/2 = 6	1	
6/2 = 3	0	
3/2 = 1	1	
1/2 = 0	1(MSB)	

957/2 = 478	1(LSB)	
478/2 = 239	0	
239/2 = 119	1	
119/2 = 59	1	
59/2 = 29	1	
29/2 = 14	1	
14/2 = 7	0	
7/2 = 3	1	
3/2 = 1	1	
1/2 = 0	1 (MSB)	

265/2 = 132	1 (LSB)	
132/2 = 66	0	
66/2 = 33	0	
33/2 = 16	1	
16/2 = 8	0	
8/2 = 4	0	
4/2 = 2	0	
2/2 = 1	0	
1/2 = 0	1 (MSB)	

Question 5

1 0

$$\text{a. } (36)_{16} \rightarrow (54)_{10}$$

$$m = 2, n = 0, r = 16$$

$$= \sum_{i=-n}^{i=m-1} h^i * (16)^i =$$

$$6 * 16^0 + 3 * 16^1 = (54)_{10}$$

3 2 1 0

$$\text{b. } (ABCD)_{16} \rightarrow (43981)_{10}$$

$$m = 3, n = 0, r = 16$$

$$= \sum_{i=-n}^{i=m-1} h^i * (16)^i =$$

$$D * 16^0 + C * 16^1 + B * 16^2 + A * 16^3 = (43981)_{10}$$

1 0

$$\text{c. } (89)_{16} \rightarrow (137)_{10}$$

$$m = 4, n = 0, r = 16$$

$$= \sum_{i=-n}^{i=m-1} h^i * (16)^i =$$

$$9 * 16^0 + 8 * 16^1 = (137)_{10}$$

3 2 1 0

$$\text{d. } (2000)_{16} \rightarrow (8192)_{10}$$

$$m = 2, n = 0, r = 16$$

$$= \sum_{i=-n}^{i=m-1} h^i * (16)^i =$$

$$0 \cdot 16^0 + 0 \cdot 16^1 + 0 \cdot 16^2 + 2 \cdot 16^3 = (8192)_{10}$$

Question 6

a. $(372)_{10} \rightarrow (174)_{16}$

b. $(2313)_{10} \rightarrow (909)_{16}$

c. $(33)_{10} \rightarrow (21)_{16}$

d. $(1024)_{10} \rightarrow (400)_{16}$

$$2313 / 16 = 144 \quad 9$$

$$144 / 16 = 9 \quad 0$$

$$9 / 16 = 0 \quad 9$$

$$372 / 16 = 23 \quad 4 \text{ (LSB)}$$

$$23 / 16 = 1 \quad 7$$

$$1 / 16 = 0 \quad 1 \text{ (MSB)}$$

$$1024 / 16 = 64 \quad 0$$

$$64 / 16 = 4 \quad 0$$

$$4 / 16 = 0 \quad 4$$

$$33 / 16 = 2 \quad 1 \text{ (LSB)}$$

$$2 / 16 = 0 \quad 2 \text{ (MSB)}$$

Question 7

a. $(010101010101)_{6-3-1-1} \rightarrow (444)_{10}$

$$0101 / 0101 / 0101$$

4 / 4 / 4 and concatenate them.

b. $(000110000100)_{6-3-1-1} \rightarrow (163)_{10}$

b. $(000110000100) \rightarrow (163)_{10}$

$$0001 / 1000 / 0100$$

$$1 / 6 / 3$$

c. $(10111) \rightarrow (15)_{10}$

$$0001 / 0111$$

$$1 / 5$$

d. $(1110101) \rightarrow ()_{10}$

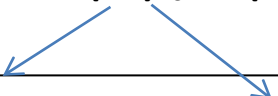
$$0111 / 0101$$

$$5 / 4$$

Question 8

a. $(38)_{10} \rightarrow (1\ 0110\ 1011)_{\text{Excess-3code+even-parity-bit}}$

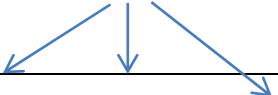
$(38)_{10} \rightarrow (00111000)_{\text{BCD}} \rightarrow (01101011)_{\text{Excess-3code}}$



0011+0011	1000+0011
0110	1011

b. $(275)_{10} \rightarrow (1\ 0101\ 1010\ 1000)_{\text{Excess-3code+even-parity-bit}}$


$(275)_{10} \rightarrow (001001110101)_{\text{BCD}} \rightarrow (010110101000)_{\text{Excess-3code}}$



0010+0011	0111+0011	0101+0011
0101	1010	1000

c. $(9201)_{10} \rightarrow (1\ 1100\ 0101\ 0011\ 0100)_{\text{Excess-3code+even-parity-bit}}$


$(9201)_{10} \rightarrow (1001001000000001)_{\text{BCD}} \rightarrow (1100010100110100)_{\text{Excess-3code}}$



1001+0011	0010+0011	0000+0011	0001+0011
1100	0101	0011	0100

d. $(51)_{10} \rightarrow (0\ 1000\ 0100)_{\text{Excess-3code+even-parity-bit}}$

$(51)_{10} \rightarrow (01010001)_{\text{BCD}} \rightarrow (10000100)_{\text{Excess-3code}}$



0101+0011	0001+0011
1000	0100

Question 9

a. $(01101)_{2,2S' \text{ complement}} = (+13)_{10}$

+ve

$$R = 2, m = 5, n = 0 \rightarrow \sum_{-n}^{m-1} b_i * 2^i = 1 * 2^0 + 0 * 2^1 + 1 * 2^2 + 1 * 2^3 + 0 * 2^4 = 13$$

b. $(11101)_{2,2S' \text{ complement}} = (-3)_{10}$

-ve

applying 2S' comp $\rightarrow (00011)_{2,2S' \text{ complement}} = (3)$ then add the - sign
 $\rightarrow (-3)_{10}$

c. $(01111011) = (+123)_{10}$

+ve

$$(01111011)_{2,2S' \text{ complement}} \rightarrow (+123)_{10}$$

d. $(11111111) = (-1)_{10}$

-ve

$(11111111)_{2,2S' \text{ complement}} \rightarrow$ applying 2S' comp $\rightarrow (0000001)_{2,2S' \text{ complement}}$
 $\rightarrow (1)_{10}$ then add the - sign $\rightarrow (-1)_{10}$

e. $(01111111) = (+127)_{10}$

+ve

$$(01111111)_{2,2S' \text{ complement}} \rightarrow (+127)_{10}$$

f. $(10000000) = (-128)_{10}$

-ve

$(10000000)_{2,2S' \text{ complement}} \rightarrow$ applying 2S' comp $\rightarrow (10000000)_{2,2S' \text{ complement}}$
 $\rightarrow (1)_{10}$ then add the - sign $\rightarrow (-1)_{10}$

Question 10

a. $(-87) + (256)$

Min #of bits = 10

$$(256)_{10} \rightarrow (0100000000)_2$$

$$(+256)_{10} \rightarrow \text{using 10-bits} \rightarrow (0100000000)_{2,2\text{scomp,signed}}$$

$$(87)_{10} \rightarrow (0001010111)_2$$

$$(+87)_{10} \rightarrow \text{using 10-bits} \rightarrow (0001010111)_{2,2\text{scomp,signed}}$$

$$(-87)_{10} \rightarrow \text{using 10-bits} \rightarrow (1110101001)_{2,2\text{scomp,signed}}$$

Then adding

$$(0100000000)_{2,2\text{scomp,signed}}$$

$$(1110101001)_{2,2\text{scomp,signed}}$$

$$\begin{array}{r} (0100000000)_{2,2\text{scomp,signed}} \\ (1110101001)_{2,2\text{scomp,signed}} \\ \hline (10010101001)_{2,2\text{scomp,signed}} \end{array} \rightarrow (0010101001)_{2,2\text{scomp,signed}} \rightarrow (+169)$$

 **discard**

→ no overflow

a. $(-35) + (65)$

Min #of bits = 8

$$(65)_{10} \rightarrow (01000001)_2$$

$$(+65)_{10} \rightarrow \text{using 8-bits} \rightarrow (01000001)_{2,2\text{scomp,signed}}$$

$$(35)_{10} \rightarrow (00100011)_2$$

$$(+35)_{10} \rightarrow \text{using 10-bits} \rightarrow (00100011)_{2,2\text{scomp,signed}}$$

$$(-35)_{10} \rightarrow \text{using 10-bits} \rightarrow (11011101)_{2,2\text{scomp,signed}}$$

Then adding

$$(01000001)_{2,2\text{scomp,signed}}$$

$$(11011101)_{2,2\text{scomp,signed}}$$

$$\begin{array}{r} (01000001)_{2,2\text{scomp,signed}} \\ (11011101)_{2,2\text{scomp,signed}} \\ \hline (100011110)_{2,2\text{scomp,signed}} \end{array} \rightarrow (00011101)_{2,2\text{scomp,signed}} \rightarrow (+30)_{10} \rightarrow \text{no overflow}$$

 **discard**

c. (+490) + (+22)

Min #of bits = 10

$(490)_{10} \rightarrow (0111101010)_2$

$(+490)_{10} \rightarrow \text{using 10-bits} \rightarrow (0111101010)_{2,2scomp}$

$(22)_{10} \rightarrow (10110)_2$

$(+22)_{10} \rightarrow \text{using 10-bits} \rightarrow (0000010110)_{2,2scomp,signed}$

Then adding

$(0111101010)_{2,2scomp,signed}$

$(0000010110)_{2,2scomp,signed}$

$(1000000000)_{2,2scomp,signed} \rightarrow \text{overflow}$

The min #of bits to avoid overflow condition is 11-bits

Using 11-bits

$(00111101010)_{2,2scomp}$

$(00000010110)_{2,2scomp}$

$(01000000000)_{2,2scomp}$

$\rightarrow (+512)_{10}$

d. (-255) + (-230)

Min #of bits = 9

$(255)_{10} \rightarrow (011111111)_2$

$(+255)_{10} \rightarrow \text{using 9-bits} \rightarrow (011111111)_{2,2scomp}$

$(-255)_{10} \rightarrow \text{using 9-bits} \rightarrow (100000001)_{2,2scomp}$

$(230)_{10} \rightarrow (011100110)_2$

$(+230)_{10} \rightarrow \text{using 9-bits} \rightarrow (011100110)_{2,2scomp}$

$(-230)_{10} \rightarrow \text{using 9-bits} \rightarrow (100011010)_{2,2scomp}$

Then adding

$(100000001)_{2,2scomp,signed}$

$(100011010)_{2,2scomp,signed}$

$(1000011011)_{2,2scomp,signed} \rightarrow (000011011) \rightarrow \text{overflow}$

discard

The min #of bits to avoid overflow condition is 10-bits

Using 10-bits

$(1100000001)_{2,2scomp}$

$(1100011010)_{2,2scomp}$

$(11000011011)_{2,2scomp}$

discard

$(-485) = (1000011011)_{2,2scomp}$

Check by applying 2S'comp

(0111100101)

e. (-129) + (128)

Min #of bits = 9

$(128)_{10} \rightarrow (010000000)_2$

$(+128)_{10} \rightarrow \text{using 9-bits} \rightarrow (010000000)_{2,2\text{scomp,signed}}$

$(129)_{10} \rightarrow (010000001)_2$

$(+129)_{10} \rightarrow \text{using 9-bits} \rightarrow (010000001)_{2,2\text{scomp,signed}}$

$(-129)_{10} \rightarrow \text{using 9-bits} \rightarrow (101111111)_{2,2\text{scomp,signed}}$

Then adding

$(010000000)_{2,2\text{scomp,signed}}$

$(101111111)_{2,2\text{scomp,signed}}$

$(111111111)_{2,2\text{scomp,signed}} \rightarrow (-1)_{10}$

\rightarrow no overflow

f. (+986) + (+123)

Min #of bits = 11

$(986)_{10} \rightarrow (01111011010)_2$

$(+986)_{10} \rightarrow \text{using 11-bits} \rightarrow (01111011010)_{2,2\text{scomp}}$

$(123)_{10} \rightarrow (00001111011)_2$

$(+123)_{10} \rightarrow \text{using 11-bits} \rightarrow (00001111011)_{2,2\text{scomp}}$

Then adding

$(01111011010)_{2,2\text{scomp,signed}}$

$(00001111011)_{2,2\text{scomp,signed}}$

$(10001010101)_{22,2\text{scomp,signed}} \rightarrow \text{overflow}$

The min #of bits to avoid overflow condition is 12-bits

Using 12-bits

$(001111011010)_{2,2\text{scomp}}$

$(000001111011)_{2,2\text{scomp}}$

$(010001010101)_{2,2\text{scomp}}$

$\rightarrow (+1109)_{10}$