CENG 384 - Signals and Systems for Computer Engineers
Spring 2023
Homework 1

Qaryouti, Hashem
e265281@ceng.metu.edu.tr

Mehrabi, Sogol
e254732@ceng.metu.edu.tr

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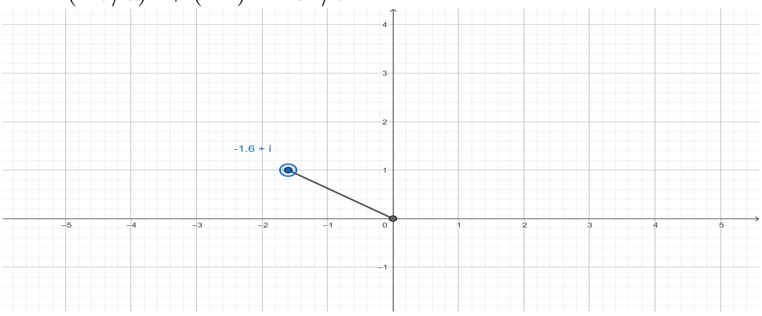
1. (a) 2(x + yj) + 5 = j - (x-yj)
2x+2yj + 5 = j - x + yj
5 - j = -x-2x-2yj+yj
5 - j = -3x - yj
-3x = 5 Therefore X = -5/3
-j = -yj Therefore Y = 1
$Z^2 = (-5/3)^2 + (-1)^2 = 34/9$

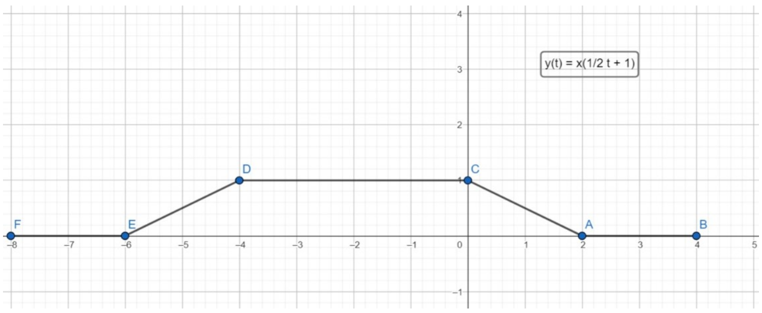


(b) $z^5 = 32j$
$z^5 = r^5[cos(5\theta) + sin(5\theta)j]$
$32j = 32[cos(\pi/2) + sin(\pi/2)j]$
$32[cos(\pi/2) + sin(\pi/2)j] = r^5[cos(5\theta) + sin(5\theta)j]$
$r^5 = 32, r = 2$
$\pi/2 = 5\theta$
$\theta = \pi/10$
$z = 2e^{(}\pi/10j)$

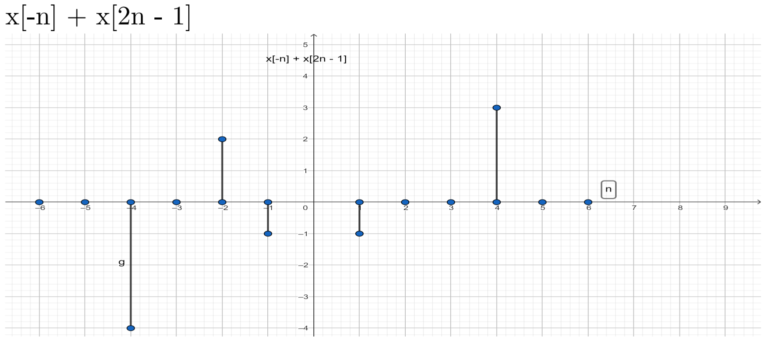
(c) Z = (1+j)(1/2 +j$\sqrt{3}/2)/(j-1)$
$Z = (j+1)(1+j)(1/2 + j\sqrt{3}/2)/(j-1)(j+1)$
$Z = (-1/2)[2j(0.5 + j\sqrt{3}/2)]$
$Z = -j(0.5 + j\sqrt{3}/2)$
$Z = (\sqrt{3}/2 - j0.5)$
$Z = ((\sqrt{3}/2 - j0.5))$
$|Z| = \sqrt{((\sqrt{3}/2)^2 + (-0.5)^2)} = 1$
$\theta = tan^{-}1(-0.5/\sqrt{3}/2) = -30.$

(d) Z = j $\exp(-j\pi/2)$
$= j[\cos -\pi/2 + j \sin -\pi/2]$
$= j[0 - j]$
$= -j * j$
$= 1$
$Z = r \exp(j\theta)$
$r = 1, \theta = \tan^{-}(0) = 0$
$Z = 1$

2. y(t) = x(0.5t+ 1)



3. (a) x[-n] + x[2n - 1]



(b) $3\delta(n+7) - 4\delta(n+4) + 2\delta(n+2) - \delta(n+1) - \delta(n-1) + 3\delta(n-4)$

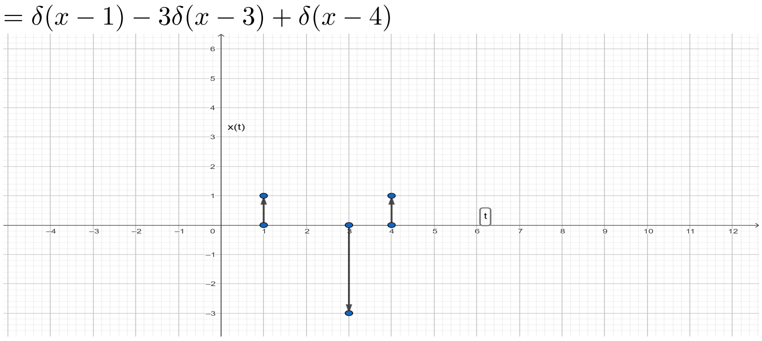
4. (a) w= 2, T fundamental period = $(2\pi/3)$
$x(t) = 5[sin(3t)cos(\pi/4) - cos(3t)sin(\pi/4)]$
$x(t+nT) = 5/\sqrt{2}[sin(3t+3nT) - cos(3t+3nT)]$
$= 5/\sqrt{2}[sin(3t+2\pi n) - cos(3t+2\pi n)]$
$= 5/\sqrt{2}[[sin(3t)cos(2\pi n) + sin(2\pi n)cos(3t)] - [cos(3t)cos(2\pi n) - sin(3t)sin(2\pi n)]]$
$= 5/\sqrt{2}[sin(3t) - cos(3t)]$
$= 5[sin(3t)\sqrt{2} - cos(3t)\sqrt{2}]$
$x(t+nT) = 5sin(3t - \pi/4)$
Since x(t) = x(1/2t + 1), then the signal is periodic.

(b) N = $(2\pi/\omega) * m$
such that m is an integer which makes N an integer since it is discrete signal.
$N1 = (20/13) * m$
we can take m as 13 so N1 = 20
$N2 = (20/7) * m$
we can take m as 7 so N2 = 20
$N = LCM(20, 20) = 20$
The fundamental period is 20.

(c) N = $(2\pi/7) * m$
such that m is an integer which makes N an integer .
There is no integer m that can make N an integer, so the signal is not periodic.

5. (a) x(t) = 0* u(t-1) + 1 (u(t-1) - u(t-3)) -2 (u(t-3) - u(t-4)) - u(t-4)
x(t) = u(t-1) -3 u(t-3) + u(t-4)

(b) dx(t)/dt = $\Sigma(final$value - initialvalue)* $\delta(x - x0)$
$= (1-0)\delta(x-1) + (-2-1)\delta(x-3) + (-1+2)\delta(x-4)$
$= \delta(x-1) - 3\delta(x-3) + \delta(x-4)$



6. (a) y(t) = t * x(2t+ 3)
y(-3) = -3 * x(-3).
y(3) = 3 * x(9).
The system has memory because it depends not only on the present value of the input but also on the future values.

y(3) = 3 * x(9)
The system is not causal because it depends on future values of the input.

The system is stable since changes in the input do not lead to divergence in the output.

x(2t+3) = y(t) / t
x(3) = y(0) /0
The system is not invertible since distinct inputs do not lead to a certain output (undefined value).

y(t-t0) = (t-t0) * x(2t-2t0+3)
for 2t-t0
y(t) = t * x(2t-t0+3)
The system is variant

for x1(2t+ 3), y1(t) = t * x1(2t+ 3)for x2(2t+3), y2(t) = t * x1(2t+ 3)// for ax1(2t+ 3) + b x2(2t+ 3) y(t) = t (a x1(2t+ 3) + b x2(2t+ 3))
= a t x1(2t+ 3) + b t x2(2t+ 3)
= a y1(t) + b y2(t)
The system is linear because it applies the above equation e.g, it holds the superposition principle.

(b) -The system has memory because it depends on all past inputs.

The system is causal because it depends only on present and past values of the input.

The system is not stable since its amplitude is not bounded and goes to infinity.

checking the superposition property

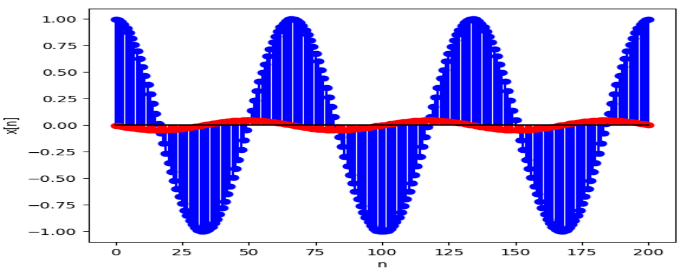
y1[n] $= \Sigma_{k=1}^{\infty} = x1[n-k]$
$y2[n] = \Sigma_{k=1}^{\infty} = x2[n-k]$
$x3[n] = \Sigma_{k=1}^{\infty} = a1x1[n] + a2x2[n]$
$y3[n] = \Sigma_{k=1}^{\infty} = x3[n-k] = a1\Sigma_{k=1}^{\infty} = x1[n-k] + a2\Sigma_{k=1}^{\infty} = x2[n-k] = a1y1[n] + a2y2[n]$

The superposition property holds, so the system is linear.

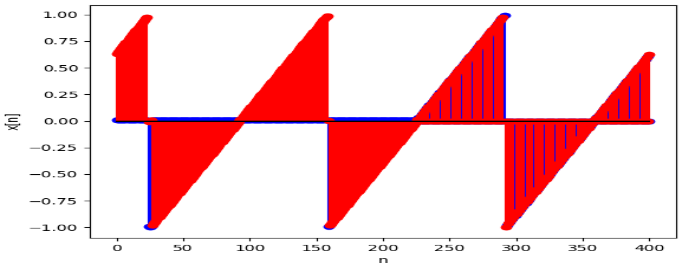
The system is invertible and the inverse system is
x[n] = y[n+1] - y[n]

The system is time invariant since shifting the input by n0 shifts the output by n0, i.e.
y[n - n0] = x[n-n0-k]

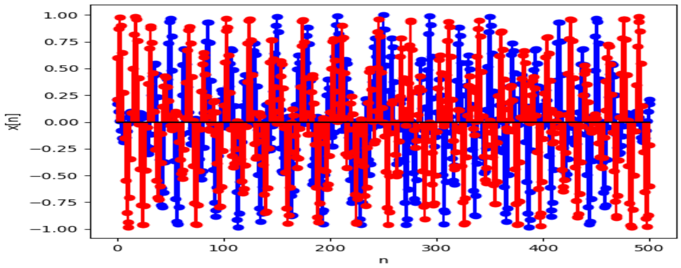
7. (a) The result for sine part a.csv



The result for shifted sawtooth part a.csv



The result for chirp part a.csv

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     import matplotlib.pyplot as plt

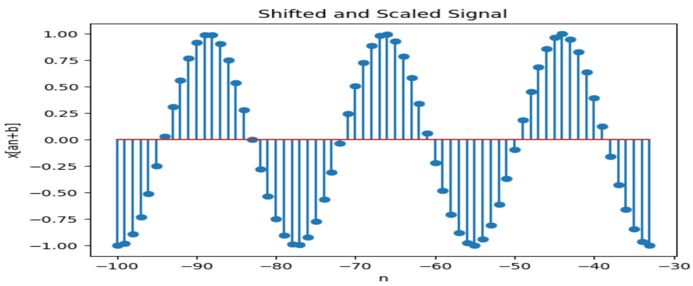
def even_odd_parts(x, si):
    even = []
    odd = []
    l = len(x)
    for i in range(si, si + l):
        if i % 2 == 0:
            even_val = (x[i] + x[-i]) / 2 if i <= l - i else x[i]
            even.append(even_val)
            odd_val = (x[i] - x[-i]) / 2 if i <= l - i else 0
            odd.append(odd_val)
        else:
            even_val = (x[i] + x[-i]) / 2 if i <= l - i else 0
            even.append(even_val)
            odd_val = (x[i] - x[-i]) / 2 if i <= l - i else x[i]
            odd.append(odd_val)
    plt.stem(even, linefmt='b-', markerfmt='bo', basefmt='k-')
    plt.stem(odd, linefmt='r-', markerfmt='ro', basefmt='k-')
    plt.xlabel('n')
    plt.ylabel('x[n]')
    plt.show()
filename = 'D:\Computer Engineering\GJU Years\Years\Third Year\Exchange Semester at METU\Signals and Systems\Homeworks\Homework 1/chirp_part_a.csv'
with open(filename, 'r') as f:
    data = f.read().split(',')
si = int(data[0])
x = [float(val) for val in data[1:]]

even_odd_parts(x, si)

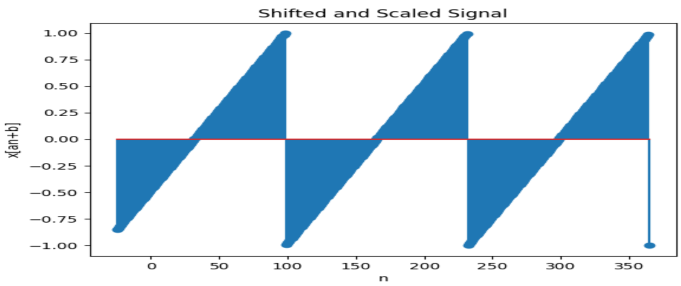



```

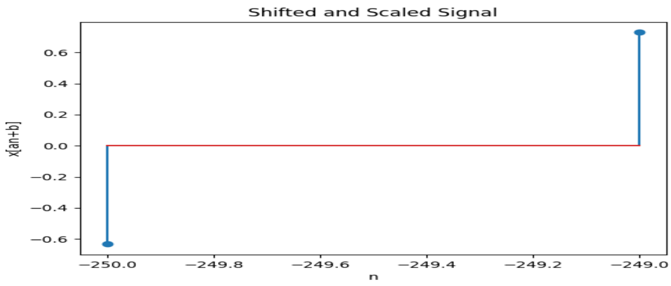
(b) The result for sine part b.csv



The result for shifted sawtooth part b.csv



The result for chirp part b.csv




```

     import matplotlib.pyplot as plt

def shifted_scaled(x, si, a, b):
    l = len(x)
    n = [i for i in range(si, si + 1)]
    new_signal = []
    for i in n:
        ind = int(a*(i-si)+b)
        if 0 <= ind < l:
            new_signal.append(x[ind])

    m = len(new_signal)
    plt.stem(n[:m], new_signal)
    plt.title('Shifted and Scaled Signal')
    plt.xlabel('n')
    plt.ylabel('x[an+b]')
    plt.show()

filename = 'D:\Computer Engineering\GJU Years\Years\Third Year\Exchange Semester at METU\Signals and Systems\Homeworks\Homework 1/chirp_part_b.csv'
with open(filename, 'r') as f:
    data = f.read().split(',')
si = int(data[0])
a = int(data[1])
b = int(data[2])
x = [float(val) for val in data[3:]]

shifted_scaled(x, si, a, b)



```