Microwave Engineering and Antennas

General Introduction

Bart Smolders, Professor

Department of Electrical Engineering

Center for Wireless Technology Eindhoven

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CENTER FOR WIRELESS TECHNOLOGY EINDHOVEN
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Microwave Engineering and Antennas

General

- Master-level course within an Electrical Engineering program
- Trend towards integrated antenna systems
- Ideal combination of passive and active microwave engineering and antenna theory

Microwave Engineering and Antennas

Objectives

- Introduction into microwave engineering, passive and active components

- Introduction into antenna theory and phased-arrays

- Hands-on design of microwave circuits and antennas with state-of-the-art design tools

Microwave Engineering and Antennas

Required Background

- Circuits (Bachelor level)
- Electromagnetics, static and dynamic fields (Bachelor level)

Microwave Engineering and Antennas

Lecturers

Prof. dr. ir. Bart Smolders

Prof. dr. ir. Domine Leenaerts

Microwave Engineering and Antennas

Exercises and Labs

- On-line exercises and quizzes
- Labs using state-of-the-art simulation tools

Microwave Engineering and Antennas

Study material

- Book: ***Modern Antennas and Microwave Circuits***

  by A.B. Smolders, H.J. Visser, U. Johannsen

  on-line available

- Additional material will be provided

Microwave Engineering and Antennas

What we expect from you

- Watch the weblectures each week
- Perform the exercises and quizzes
- Use the discussion forum to seek help

Other requirements

- You need a PC or laptop
- Download open-source software QUCS and CST (student version)

Microwave Engineering and Antennas

Applications

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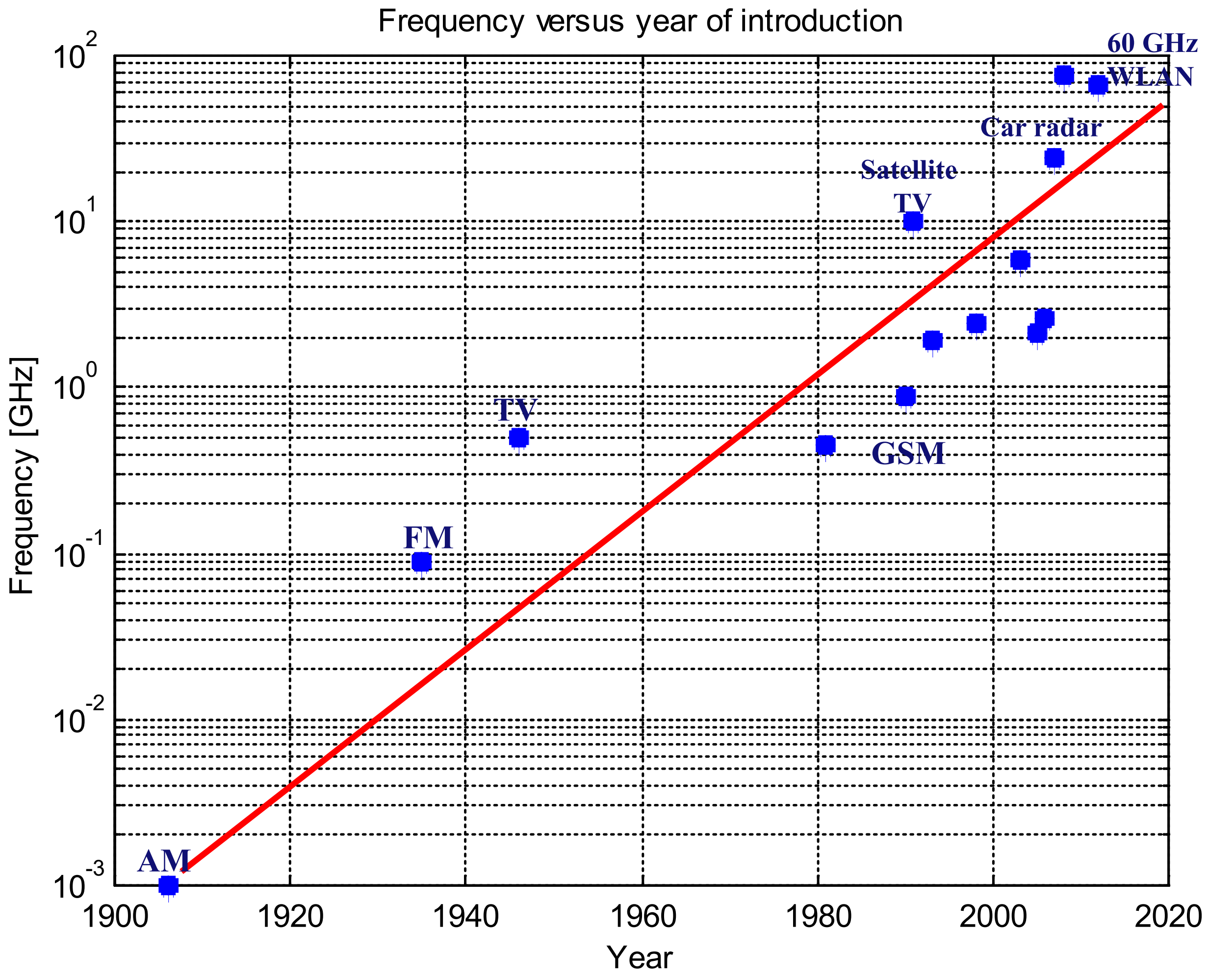
Applications

Objective of this lecture

- Describe trends in wireless applications

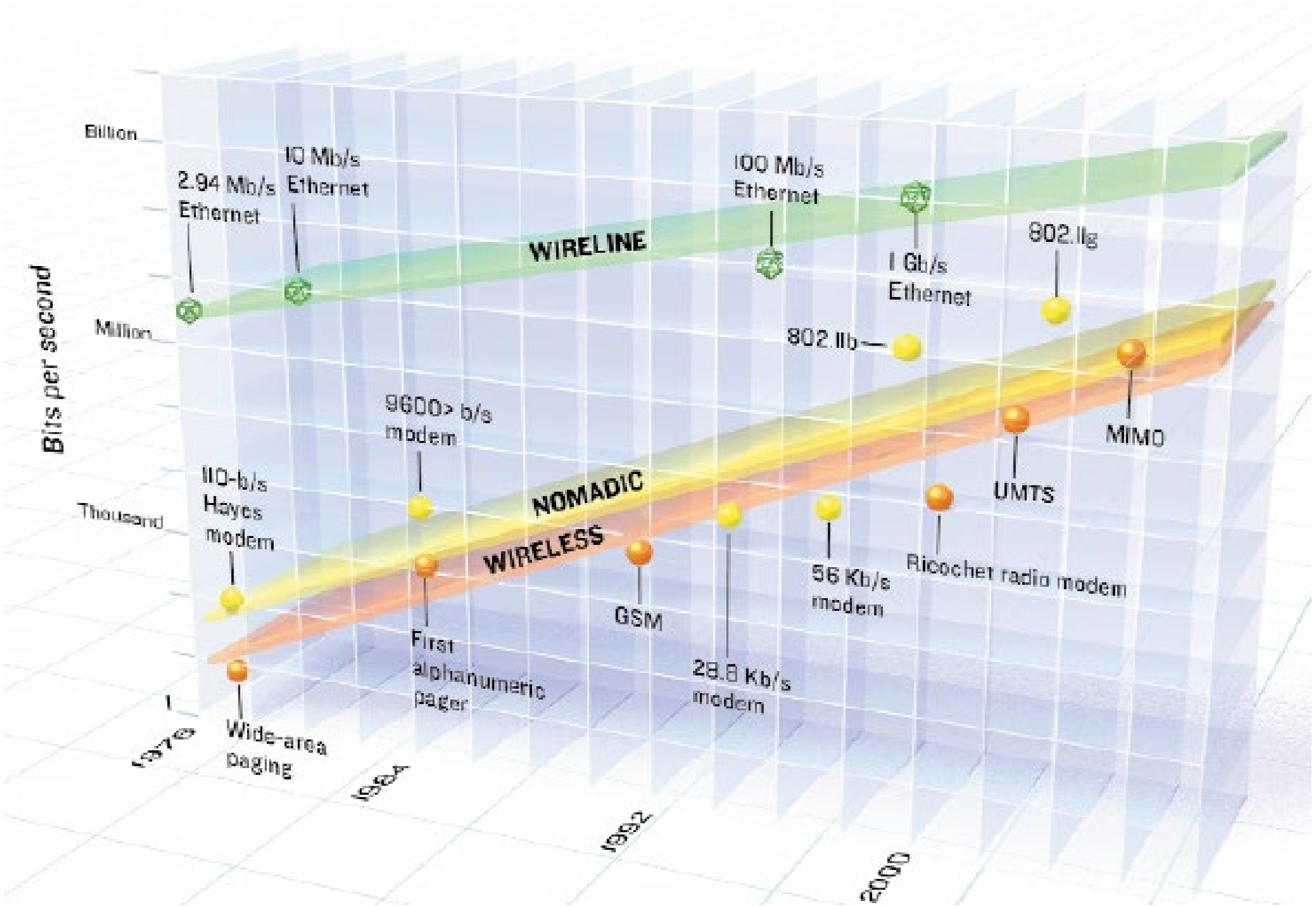
- Introduce examples of applications

Trend 1: increase of operational frequency



- More bandwidth
- New applications

Trend 2: increase of bandwidth and datarate



- Edholm's law
- Required bandwidth/datarate doubles each 18 months

Source: IEEE Spectrum 41 (7): 58–60, 2004

Trend 2: increase of bandwidth and datarate

Evolution of wireless standards

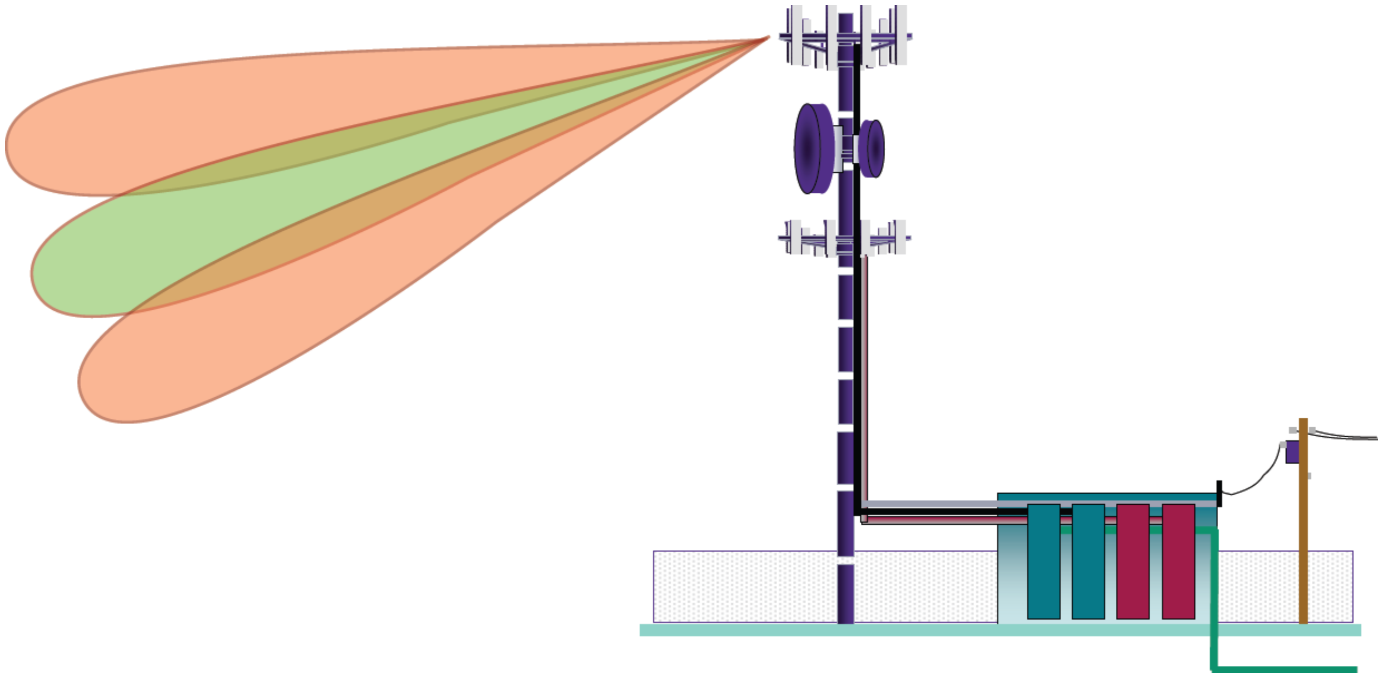
1G	2G	3G	4G	5G	6G
The foundation of mobile telephony	Mobile telephony for everone	The foundation of mobile broadband	Mobile broadband enhanced	Embracing a networked society	Enabling a smart sustainable society
~1980	~1990	~2000	~2010	~2020	~2030
1st GENERATION wireless network	2nd GENERATION wireless network	3rd GENERATION wireless network	4th GENERATION wireless network	5th GENERATION	6th GENERATION wireless network
• Basic Voice service • Analog Based Protocols	• Designed for voice • First digital standards (GSM,CDMA)	• Designed for voice and data • First mobile broadband • Voice through circuit & Data-Packet Switching	• Designed Primarily for Data • IP based protocol • True Mobile broadband.		• Extension to (sub) mm-wave frequencies Real-time cloud computing
2.4 Kbps	*64 Kbps*	*2 Mbps*	*100 Mbps*	*10 Gbps*	*1 Tbps*

Compare with Edholms law: factor 2 more BW in 1.5 year

$2^{(10/1.5)}=101$

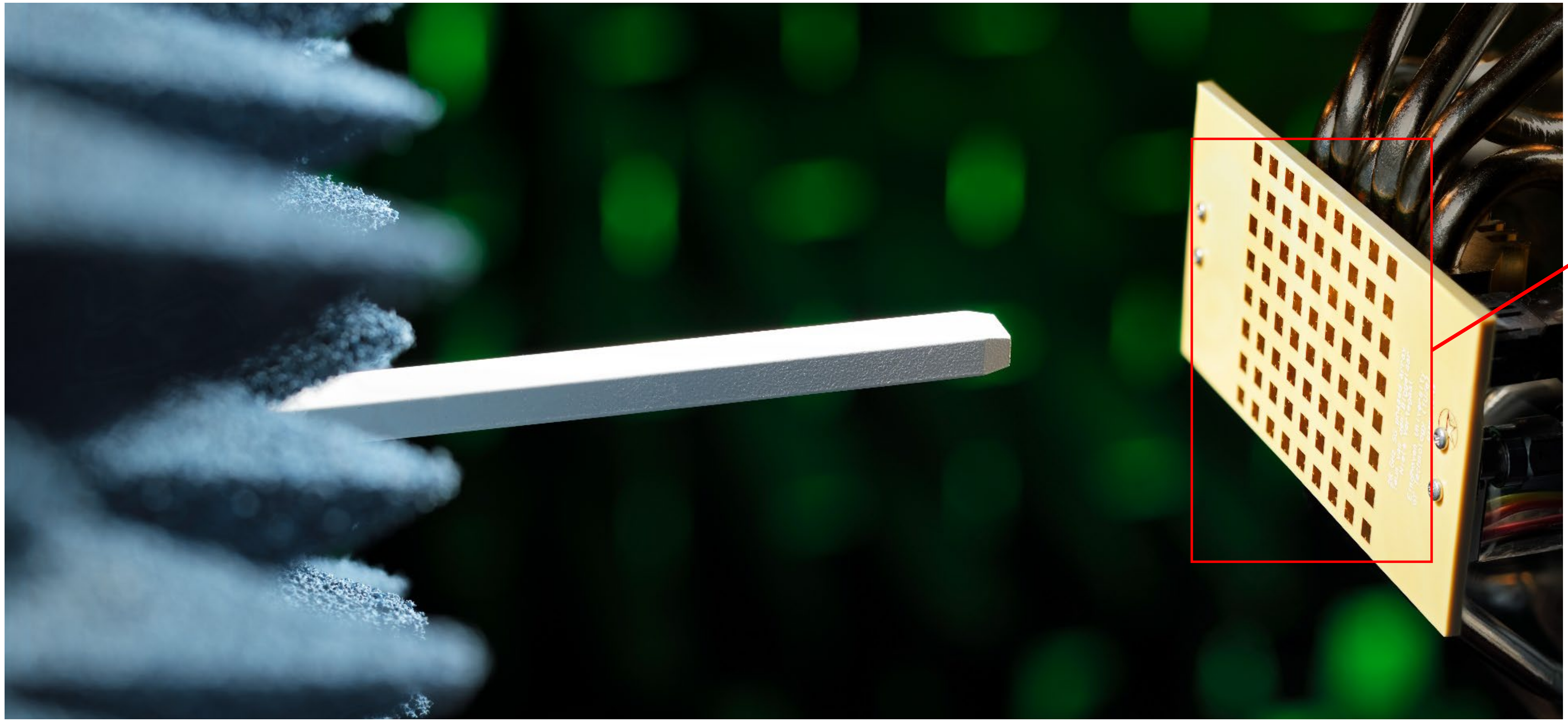
Applications: Wireless communications

Basestations 4G/5G/XG



Applications: Wireless communications

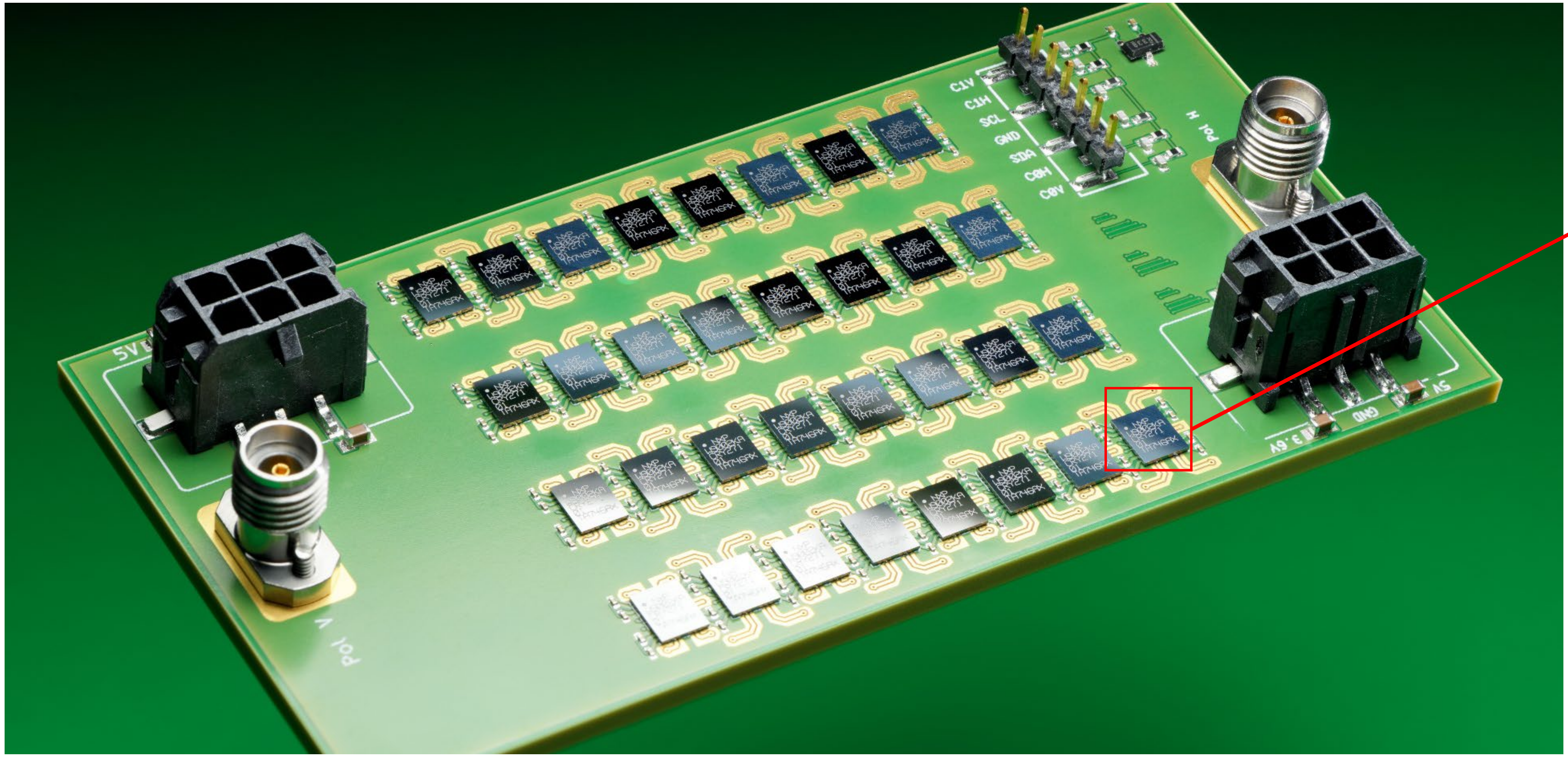
Phased-array for 5G NR Basestations (26.5-29.5 GHz)



8x8 phased array

Applications: Wireless communications

Phased-array for 5G NR Basestations (26.5-29.5 GHz)



Silicon Integrated Circuits

Applications: Wireless communications

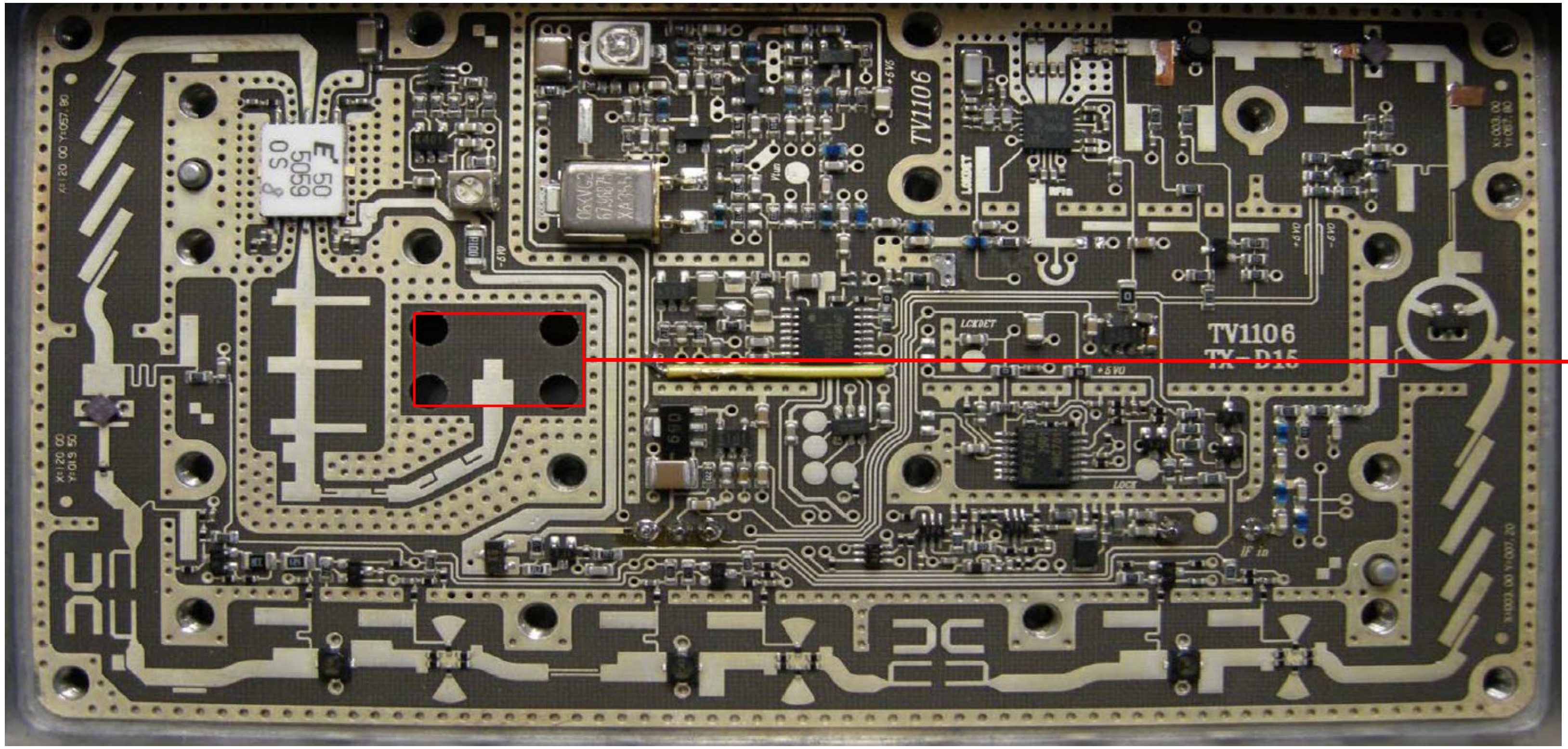
Mobile phone

- Multiple wireless applications
  - WiFi, Bluetooth, 4G/5G, GPS, NFC…
- Large number of antennas
- Large number of integrated circuits (ICs)

Source: https://news.samsung.com/global/wp-content/uploads/2014/04/Galaxy-S5-Tear-Down.jpg

Applications: Wireless communications

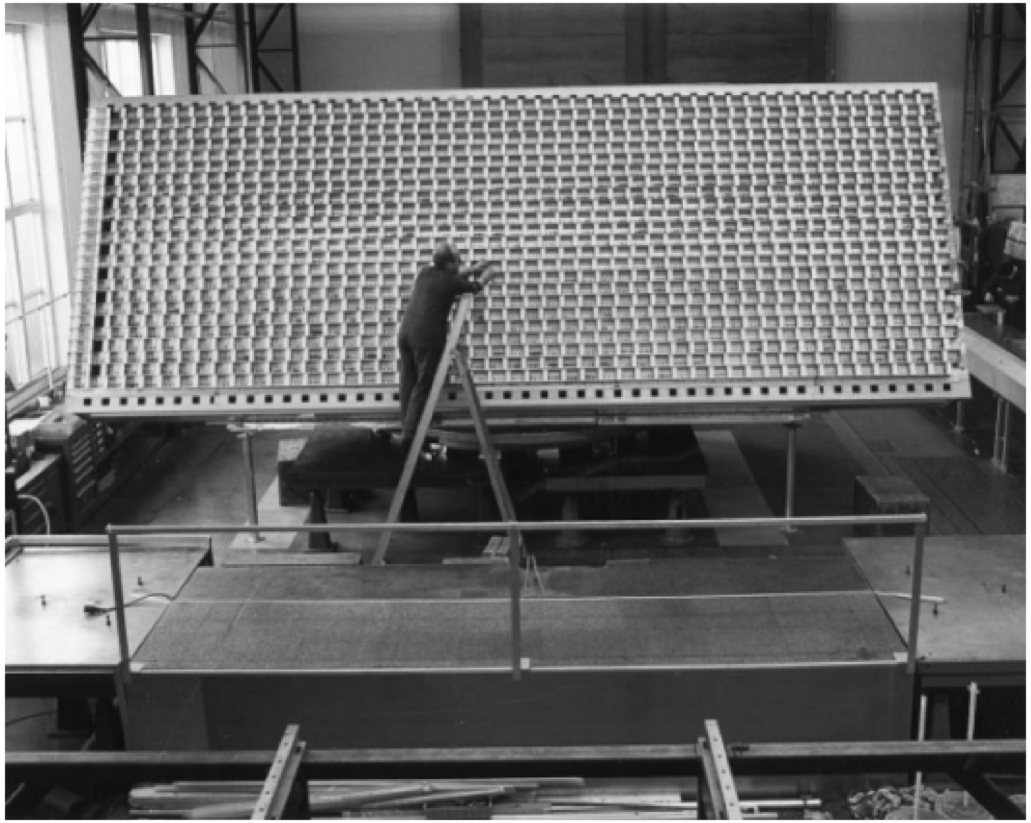
Satellite communications (11-40 GHz)



Antenna input

Applications: Radar

Phased-array radar (L-band)

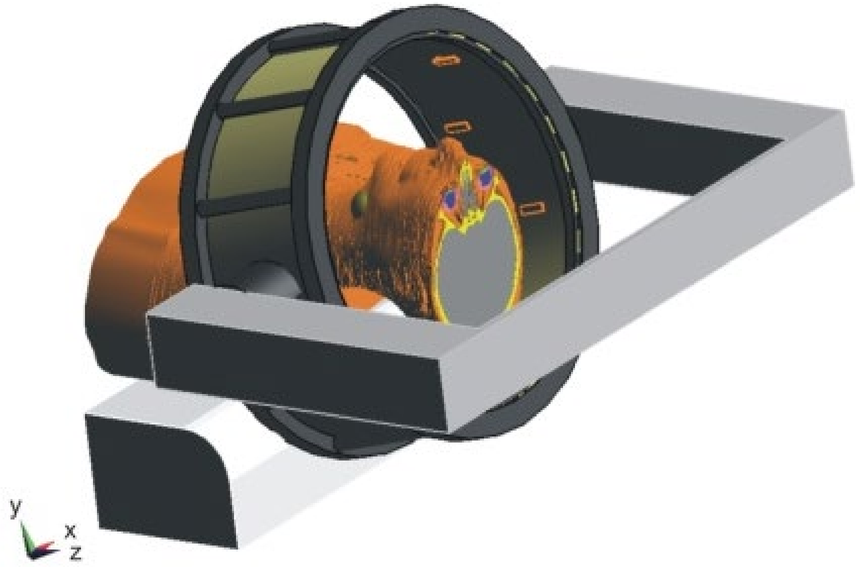


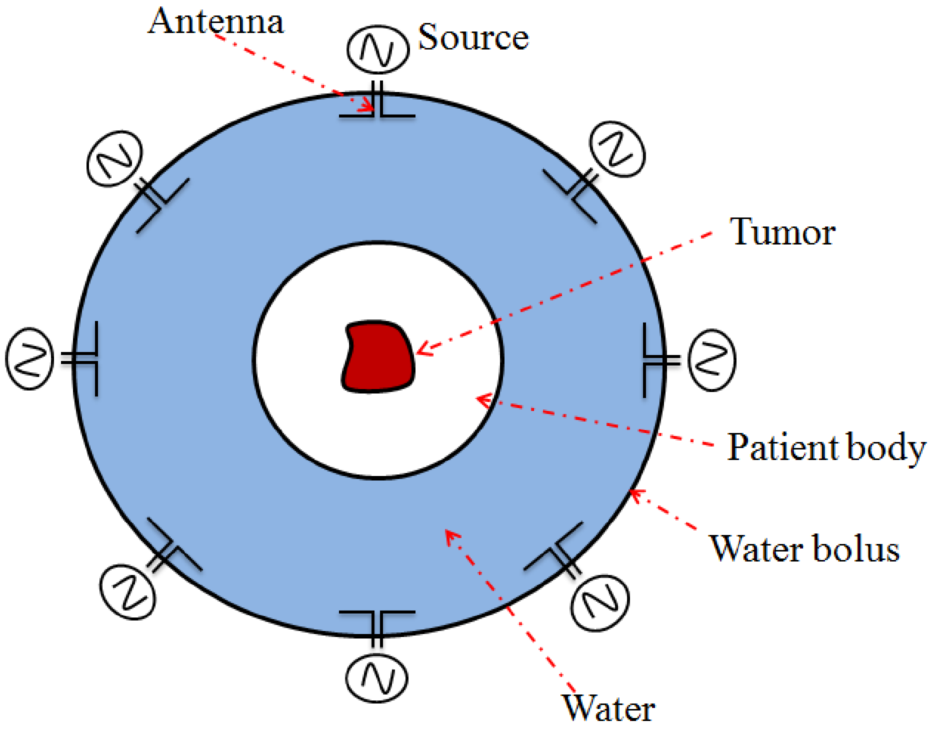
- 1024 antenna elements
- Electronic beamsteering

Source: M.C. Van Beurden, A.B. Smolders, IEEE Trans. AP, 2002, pp.1266-1273

Applications: Medical

Hyperthermia (433 MHz)





Source: https://www6.erasmusmc.nl/radiotherapie/hyperthermie/?lang=en

Applications: Medical

Radio astronomy (10 MHz-500+ GHz)

- Square Kilometer Array
- Multi beams
- Wideband
- Receive only

Source: www.astron.nl

Applications

Summary

- Trends
  - Higher frequencies
  - More bandwidth
- Applications
  - Wireless communications
  - Radar
  - Medical
  - Radio astronomy

Microwave Engineering and Antennas

Time-harmonic signals and fields

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Time-harmonic signals and fields

Objective of this lecture

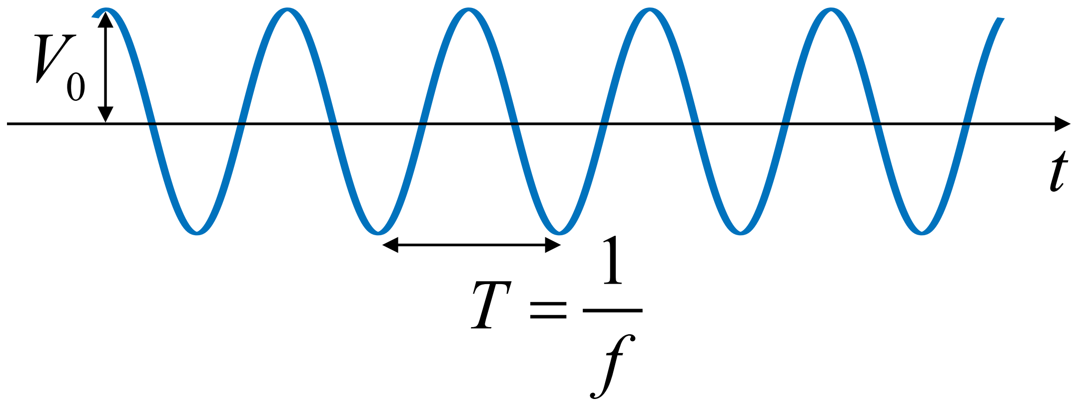
- Signals in the time domain
- Frequency domain and phasors
- Time harmonic fields

Time harmonic signals

Voltage in time domain

$$v(t) = \boxed{V_0} \cos(\omega t + \varphi)$$

$\boxed{\omega = 2\pi f}$, $\quad \varphi \quad$ phase offset



$$v(t) = \mathrm{Re}\left[\boxed{V_0 e^{j\varphi}} e^{j\omega t}\right]$$

$$= \mathrm{Re}\left[\boxed{V} e^{j\omega t}\right]$$

V: Time-independent complex voltage (phasor)

Frequency-domain representation of the voltage

Time harmonic signals

Usage and properties

We will omit the term $e^{j\omega t}$ when working with phasors

Time derivative

$$\frac{\partial v(t)}{\partial t} = \mathrm{Re}\left[\boxed{j\omega V} e^{j\omega t} \right]$$

Time-average power

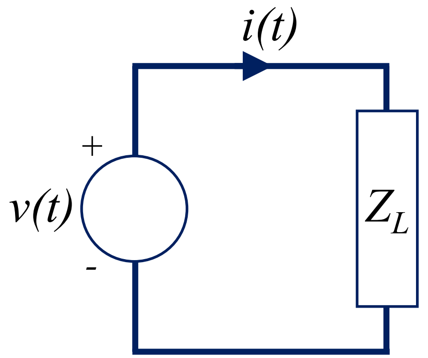
Dissipated power in load

$$p_i(t) = v(t)i(t)$$ Instantaneous power

$$P_{av} = \frac{1}{T}\int_0^T v(t)i(t)dt$$ Time-average power

$$= \frac{1}{T}\int_0^T |V||I|\cos(\omega t + \varphi_1)\cos(\omega t + \varphi_2)dt$$

$$= \boxed{\frac{1}{2}}\mathrm{Re}\left[\boxed{VI^*}\right]$$



$$v(t) = \mathrm{Re}\left[Ve^{j\omega t}\right]$$

$$i(t) = \mathrm{Re}\left[Ie^{j\omega t}\right]$$

$$\boxed{\begin{aligned} V &= |V|e^{j\varphi_1} \\ I &= |I|e^{j\varphi_2} \end{aligned}}$$

Time harmonic Fields

Time domain Electric field vector of a plane wave in free space

$$\vec{\mathrm{E}}\left(\boxed{\vec{r}},t\right)=\vec{E}_0\left(\vec{r}\right)\cos(\omega t-\boxed{k_0}\boxed{r}+\varphi)$$

Position vector

wavenumber

$r=\left|\vec{r}\right|$

$$\vec{\mathrm{E}}\left(\vec{r},t\right)=\mathrm{Re}\left[\boxed{\vec{E}\left(\vec{r}\right)}e^{j\omega t}\right]$$

Time-independent complex Electric field vector (phasor)

$$\boxed{\vec{E}\left(\vec{r}\right)=\vec{E}_0\left(\vec{r}\right)e^{j\varphi}e^{-jk_0 r}}$$

Summary

- In the <u>entire course</u> we will assume time-harmonic signals and fields
- Time domain and frequency domain representation of signals
- Scalar and vector quantities
- Time-average power

Microwave Engineering and Antennas

Hands-On Design

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Microwave Engineering and Antennas – Hands-On Design

Motivation

- Putting theory into practice
- Gaining deeper understanding
- Preparation for a successful microwave engineering and antennas career

Microwave Engineering and Antennas

Objectives

- Applying state-of-the-art design tools
- Designing standard microwave components in practice
- Assessment of various design options to come to the best trade-off

Microwave Engineering and Antennas

Study resources

- Lecture material
- Qucs
- CST Microwave Studio (student version)

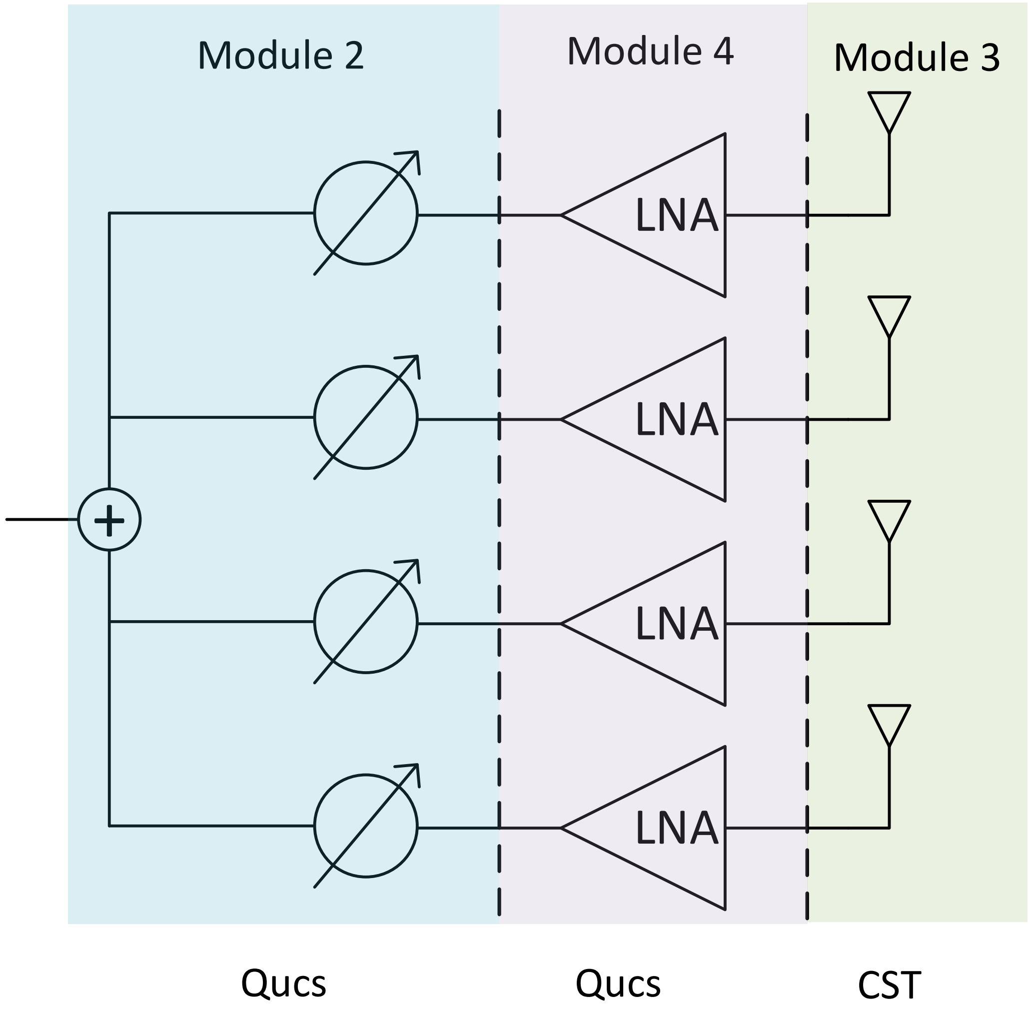
Qucs

- Quite Universal Circuit Simulator (Qucs) is a free-software electronics circuit simulator under GPL.
- You can **download** a copy from http://qucs.sourceforge.net/
- A **short description of mathematical functions** that can be used in Qucs equation (including some standard microwave engineering parameters) can be found here: https://web.mit.edu/qucs_v0.0.19/docs/en/mathfunc.html

CST Microwave Studio (student version)

- CST Studio Suite® is a high-performance **3D EM analysis software** package for designing, analyzing and optimizing electromagnetic components and systems.
- You can download the **student edition** of **CST Studio Suite** from https://www.3ds.com/products-services/simulia/products/cst-studio-suite/student-edition/

Hands-On Design Assignment



4-Element Analogue Beamforming Receiver-Array

- 5.8 GHz ISM band
- ±45˚ beamsteering range
- Main-lobe antenna gain of 9dBi for entire beamsteering range
- System noise figure < 2.7dB

Microwave Engineering and Antennas

What we expect from you

- Work out the assignment (alone or in a small group)
- Use the discussion forum to seek help
- Submit your design report along with the Qucs/CST file(s) of your final design
- Assess the designs of your peers

Other requirements

- You need a PC or laptop
- Download open-source software QUCS and CST (student version)

Microwave Engineering and Antennas

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Transmission lines part 1
The lossless line

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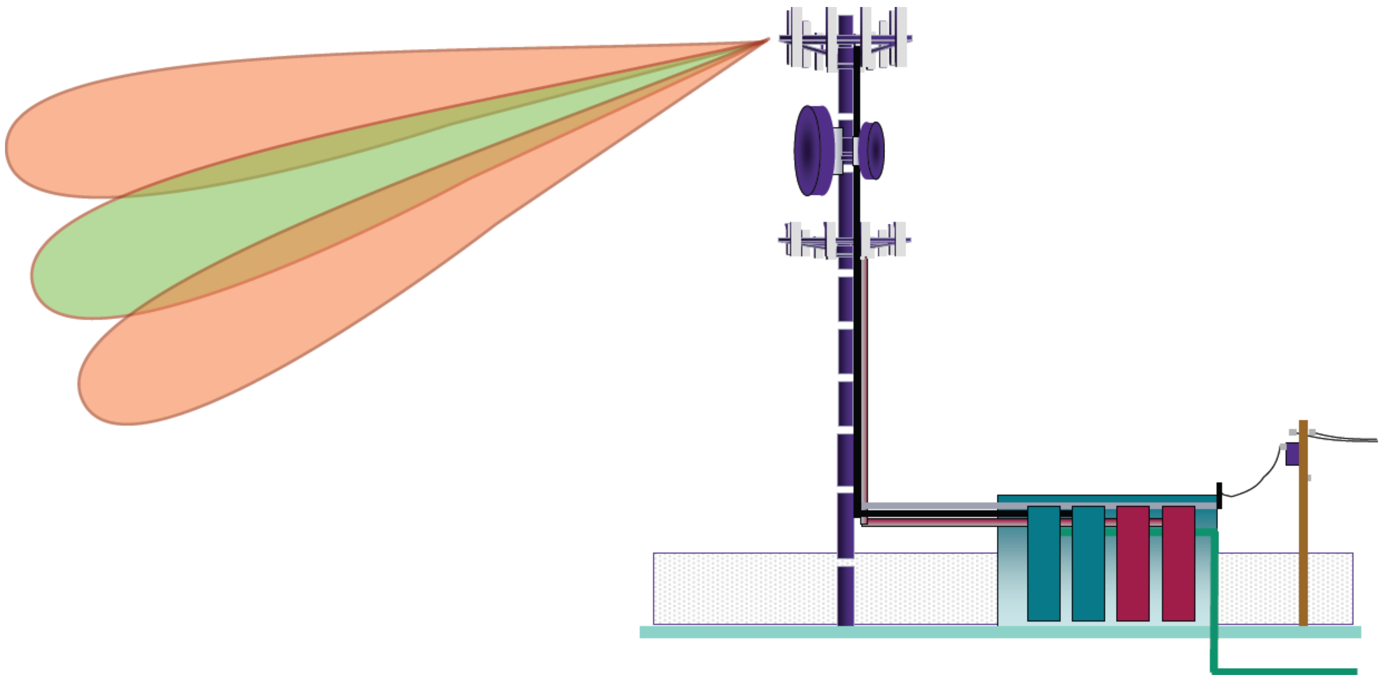
Transmission lines: The lossless line

Objective of this lecture

- Introduce theory of ideal transmission lines
- Apply theory to terminated lossless line.
- Discuss a few examples

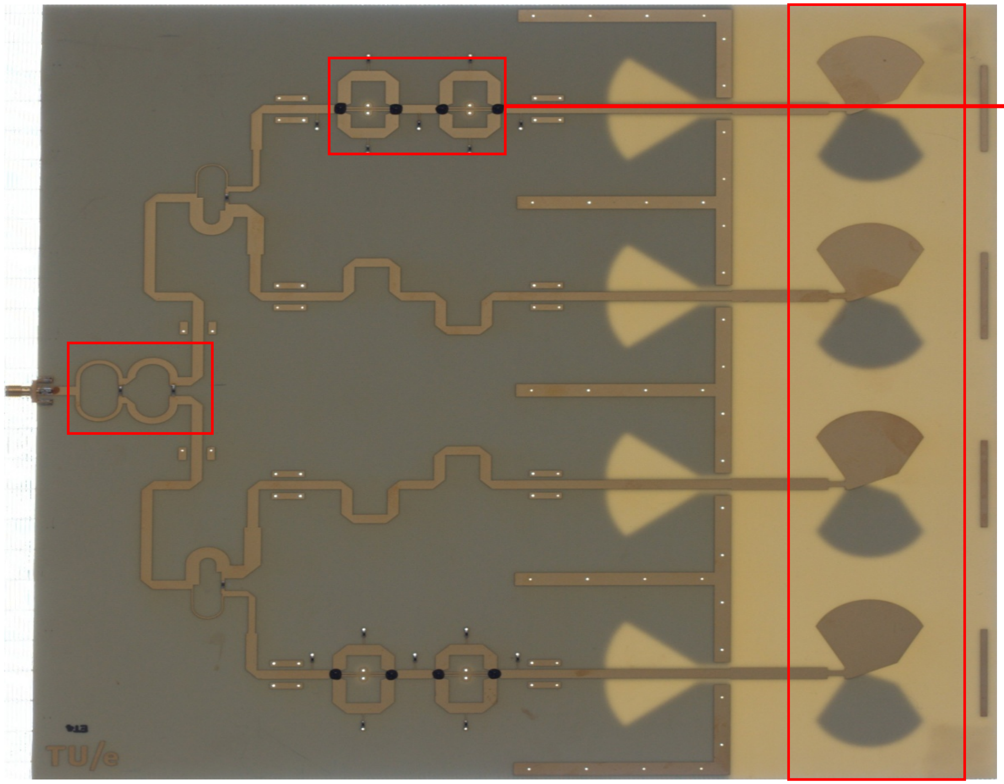
Wireless communications

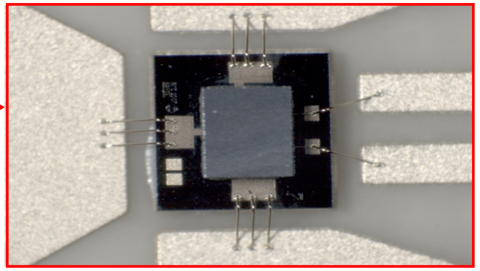
Basestations 4G/5G/XG



Wireless communications

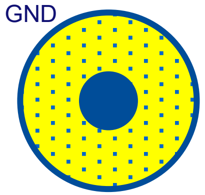
RF Beamformer for Basestation (1.6-2.4 GHz)





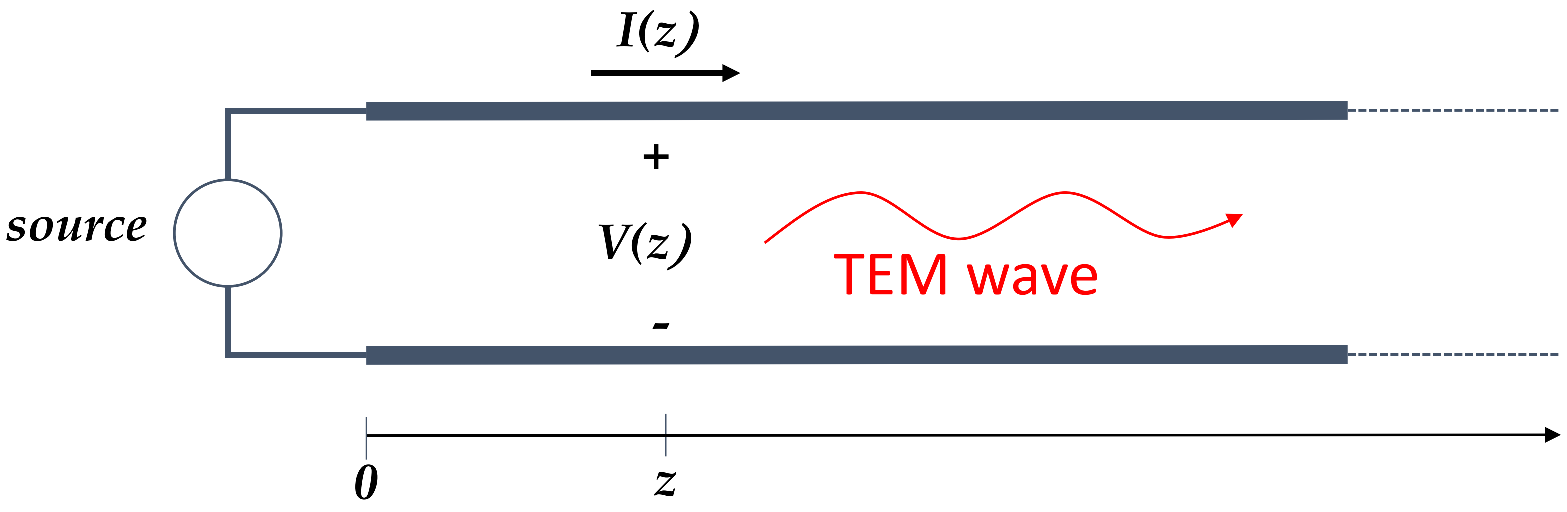
MEMS Switch

Examples of transmission lines (Tlines)



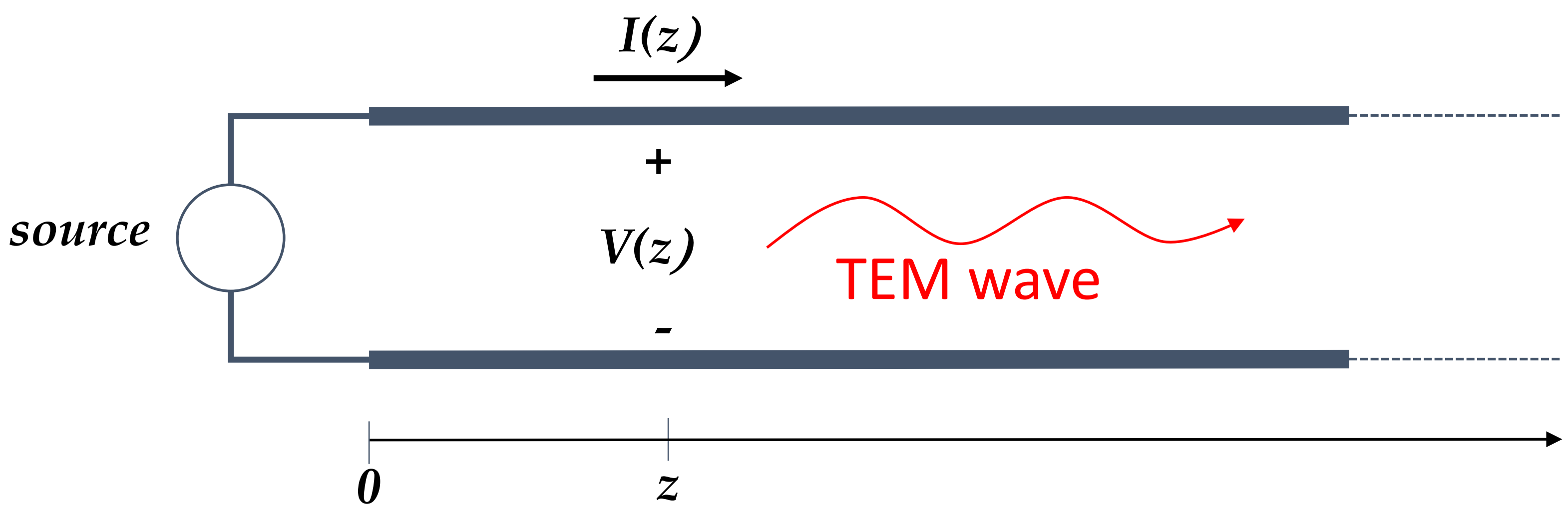
Coaxial cable, TEM mode

Basic Transmission line along z-axis



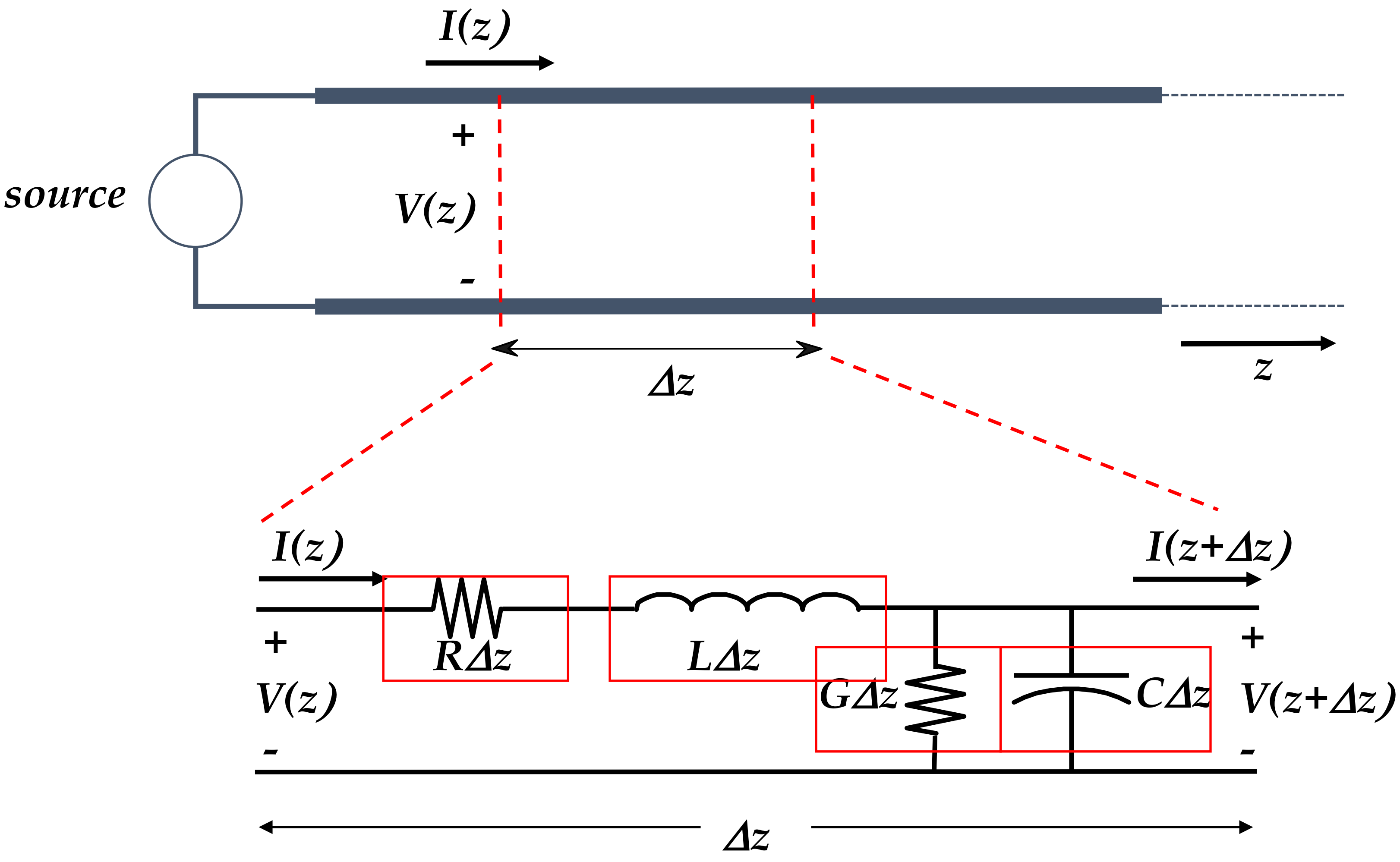
- We will assume Transverse ElectroMagnetic (TEM) waves only.
- <u>Question: How to apply Kirchhoff's law?</u>

Basic Transmission line along z-axis

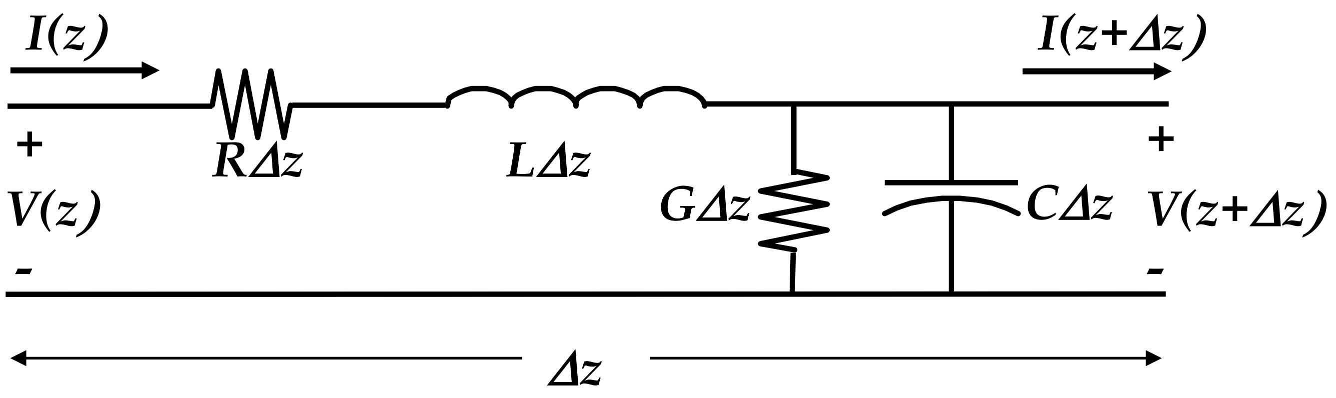


- A short piece of length Δz $(<<\lambda)$ and can be modelled as a lumped-element circuit
- <u>Question: How???</u>

Lumped-element circuit model



Applying circuit theory



Apply Kirchhoff's voltage and current laws:

$$V(z) - \boxed{I(z)R\Delta z} - \boxed{I(z)\,j\omega L\Delta z} - \boxed{V(z+\Delta z)} = 0$$

$$I(z) - \boxed{V(z+\Delta z)G\Delta z} - \boxed{V(z+\Delta z)\,j\omega C\Delta z} - \boxed{I(z+\Delta z)} = 0$$

Telegrapher equation

$$\boxed{V(z)} - I(z)R\Delta z - I(z)\,j\omega L\Delta z - \boxed{V(z+\Delta z)} = 0$$

$$I(z) - V(z+\Delta z)G\Delta z - V(z+\Delta z)\,j\omega C\Delta z - I(z+\Delta z) = 0$$

Divide by Δz and take limit Δz -> 0:

$$\frac{\partial V(z)}{\partial z} = -(R + j\omega L)I(z)$$

$$\frac{\partial I(z)}{\partial z} = -(G + j\omega C)V(z)$$

Solution of the Telegrapher equation

$$\frac{\partial V(z)}{\partial z} = -(R + j\omega L)I(z)$$

$$\frac{\partial I(z)}{\partial z} = -(G + j\omega C)V(z)$$

$$\Downarrow$$

$$V(z) = \boxed{V_0^+ e^{-\gamma z}} + \boxed{V_0^- e^{\gamma z}}$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

Wave propagation on a Tline

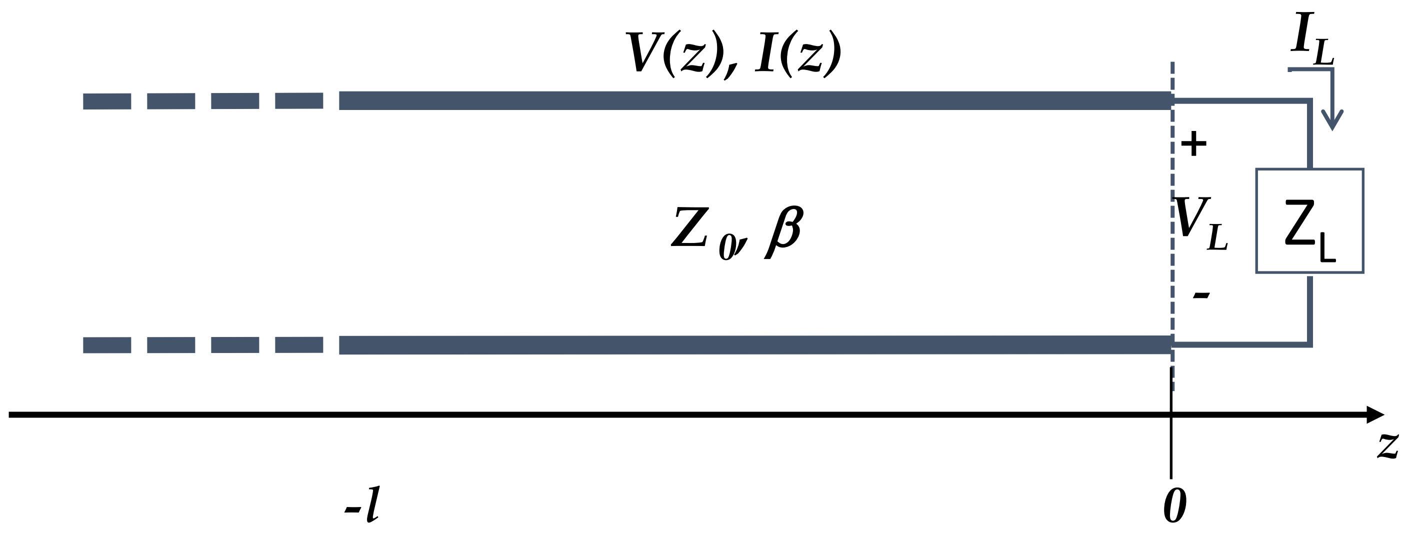
$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$\begin{aligned}\gamma &= \boxed{\alpha} + \boxed{j\beta} \\ &= \sqrt{(R + j\omega L)(G + j\omega C)}\end{aligned}$$

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{R + j\omega L}{\gamma}$$

The terminated lossless Tline (α=0)



$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} \boxed{-} \frac{V_0^-}{Z_0} e^{j\beta z}$$

$$\boxed{Z_L = \frac{V_L}{I_L}} = \frac{V(0)}{I(0)}$$

$$= \boxed{\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} Z_0}$$

The terminated lossless Tline

$$Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} Z_0$$

Rewriting gives:

$$V_0^- = \frac{Z_L - Z_0}{Z_L + Z_0} V_0^+$$

$$\boxed{\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

Reflection coefficient at z=0

Some examples

Short $(Z_L = 0)$

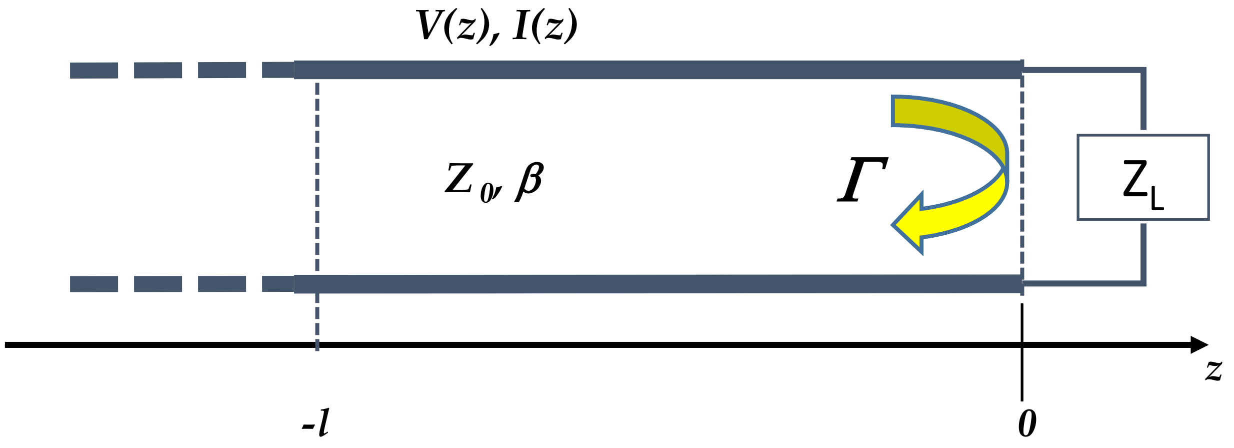
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{0 - Z_0}{0 + Z_0} = -1$$

Open $(Z_L = \infty)$

$$\Gamma = 1$$

Load $(Z_L = Z_0)$

$$\Gamma = 0$$



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The terminated lossless Tline

The voltage along the line at *z=-l* can be written as

$$\left|V(z)\right| = \boxed{\left|V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}\right|} = \boxed{\left|V_0^+\right|\left|1+\Gamma e^{2j\beta z}\right|}$$

$$= \boxed{\left|V_0^+\right|\left|1+\left|\Gamma\right| e^{j(\theta-2\beta l)}\right| \text{ at } z=-l}$$

$$\text{with } \Gamma = \left|\Gamma\right| e^{j\theta}$$

The voltage fluctuates along the line

$$V_{\max} = \left|V_0^+\right|\left|1+\left|\Gamma\right|\right|$$

$$V_{\min} = \left|V_0^+\right|\left|1-\left|\Gamma\right|\right|$$

$$\boxed{VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1+\left|\Gamma\right|}{1-\left|\Gamma\right|}}$$

Summary

- Transmission line is a distributed network
- Telegrapher equation
- Terminated transmission line.
- Reflection coefficient
- Voltage Standing Wave Ratio (VSWR)

Microwave Engineering and Antennas

Transmission lines part 2
The terminated line and lossy lines

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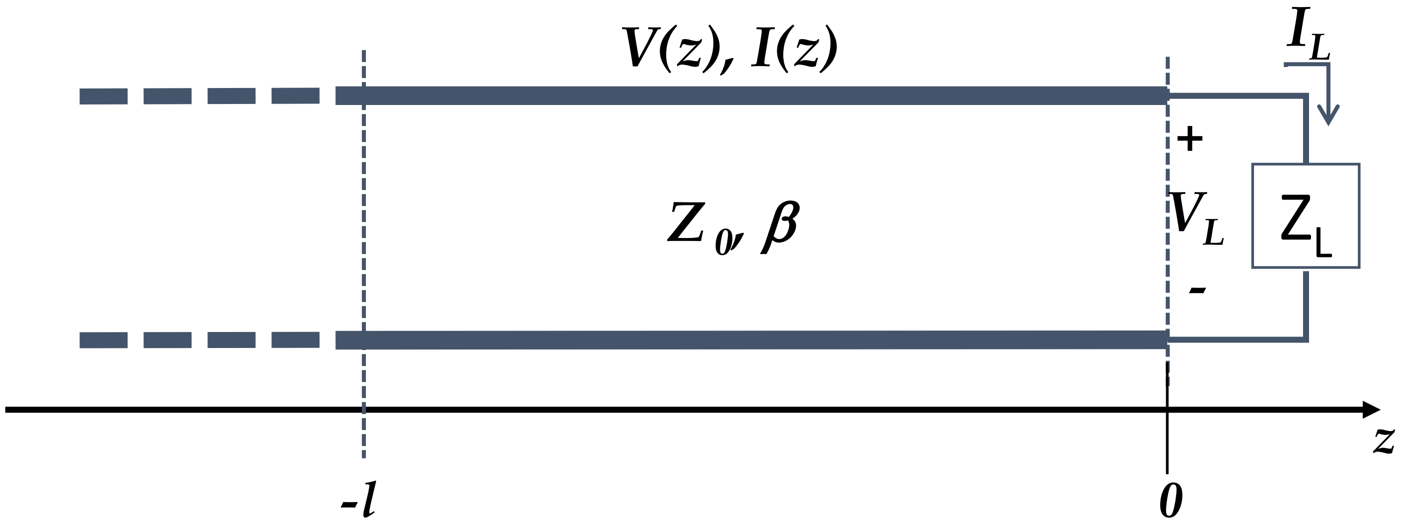
Transmission lines:
The terminated line and lossy lines

Objective of this lecture

- Derive expression for input impedance of terminated line
- Introduce return loss and insertion loss
- Quarter-wave transformer
- Extend theory towards lossy lines

The terminated lossless Tline

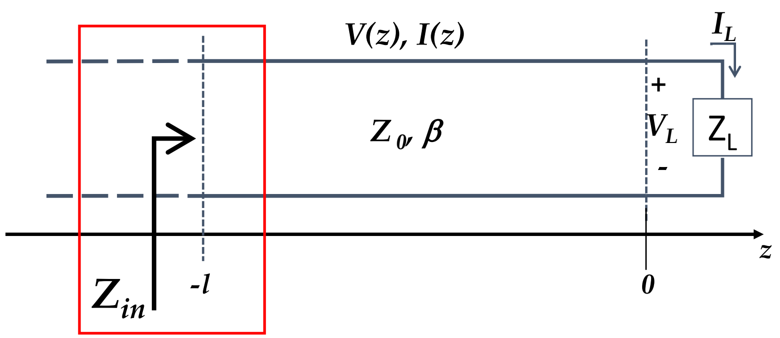


$$V(z) = \boxed{V_0^+ e^{-j\beta z}} + \boxed{V_0^- e^{j\beta z}}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$

$$\Gamma(z=0) = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

The terminated lossless Tline



The input impedance at *z*=-*l* is

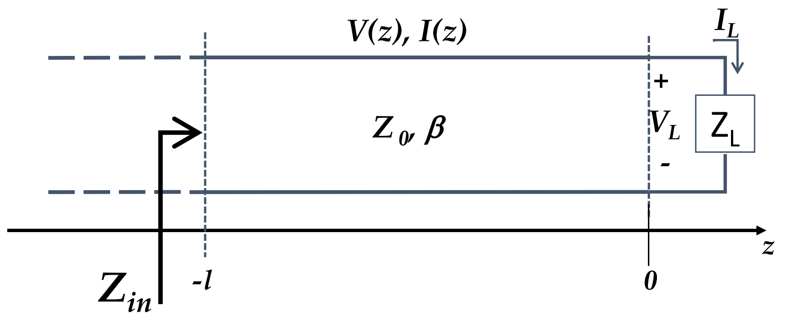
$$Z_{in} = \boxed{\frac{V(z=-l)}{I(z=-l)} = \frac{V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}}{I_0^+ e^{-j\beta z} - I_0^- e^{j\beta z}}} = \boxed{\frac{V_0^+\left[e^{j\beta l} + \Gamma e^{-j\beta l}\right]}{V_0^+\left[e^{j\beta l} - \Gamma e^{-j\beta l}\right]} Z_0}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Reflection coefficient at $z=0$

$$\boxed{Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}}$$

Example



Consider a short-circuited Tline of length *l*:

$$Z_L = 0$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

$$= \boxed{jZ_0 \tan \beta l}$$

This is equivalent to an inductor ($Z_{in}=j\omega L$)

Return loss

The incident (P_{in}) and reflected power (P_{r}) are related to the reflection coefficient Γ

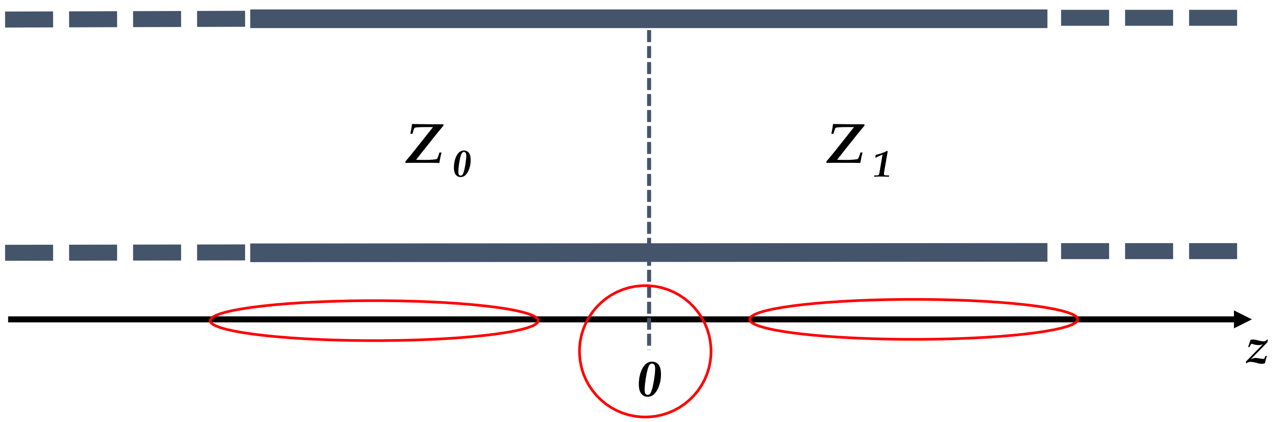
$$\frac{P_r}{P_{in}} = \frac{\left(\boxed{\dfrac{\left|V_0^-\right|^2}{2R_L}}\right)}{\left(\boxed{\dfrac{\left|V_0^+\right|^2}{2R_L}}\right)} = \left|\Gamma\right|^2 \qquad \text{in case of } Z_L = R_L$$

The return loss is now defined as

$$RL = -10\log_{10}\left|\Gamma\right|^2 = -20\log_{10}\left|\Gamma\right| \qquad [\mathrm{dB}]$$

Insertion loss

Consider two Tlines connected to each other



$$z \le 0 \qquad V_0(z) = V_0^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$$

$$z \ge 0 \qquad V_1(z) = V_0^+ T e^{-j\beta z}$$

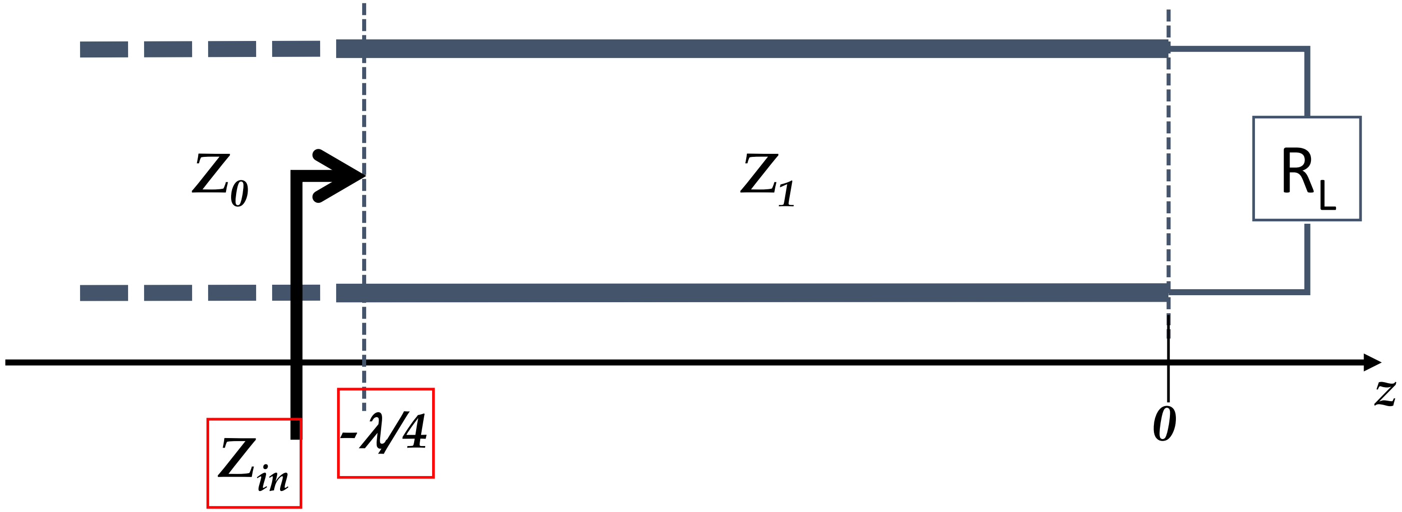
$$z = 0 \qquad T = 1 + \Gamma = 1 + \boxed{\frac{Z_1 - Z_0}{Z_1 + Z_0}}$$

$$= \frac{2Z_1}{Z_1 + Z_0}$$

The insertion loss is now defined as

$$IL = -10\log_{10}\left(\frac{P_1}{P_{in}}\right) = -10\log_{10}\left(\frac{|V_1|^2 / 2Z_1}{|V_0^+|^2 / 2Z_0}\right) = -20\log_{10}|T| + 10\log_{10}\left(\frac{Z_1}{Z_0}\right) \qquad [\mathrm{dB}]$$

The quarter-wave transformer



Input impedance at $z=-\lambda/4$:

$$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan \beta l}{Z_1 + jR_L \tan \beta l}$$

$$\beta l = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z_{in} = \frac{Z_1^2}{R_L}$$

$$\boxed{\Gamma = 0 \text{ if } Z_1 = \sqrt{Z_0 R_L}}$$

Example

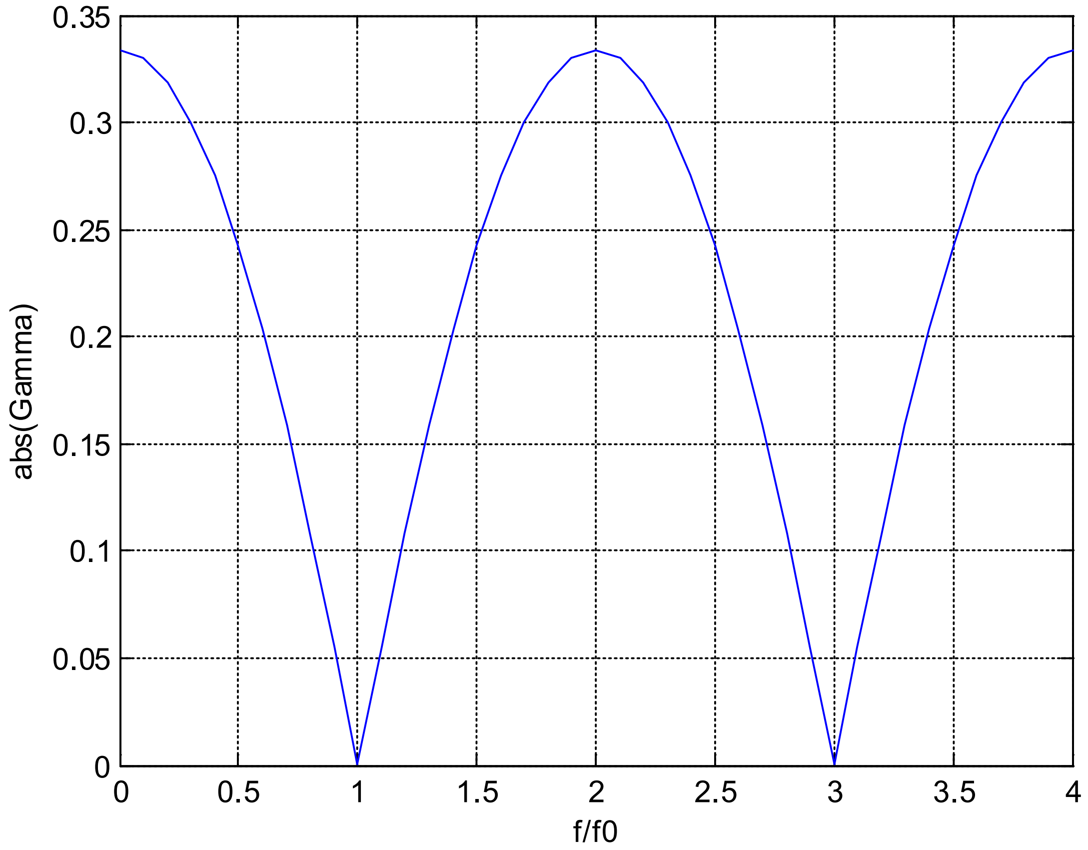
Design a quarter-wave transformer to match a 100 Ohm load to a 50 Ohm transmission line.

$$\Gamma = 0 \text{ if } Z_1 = \sqrt{Z_0 R_L} = 70.71\Omega$$

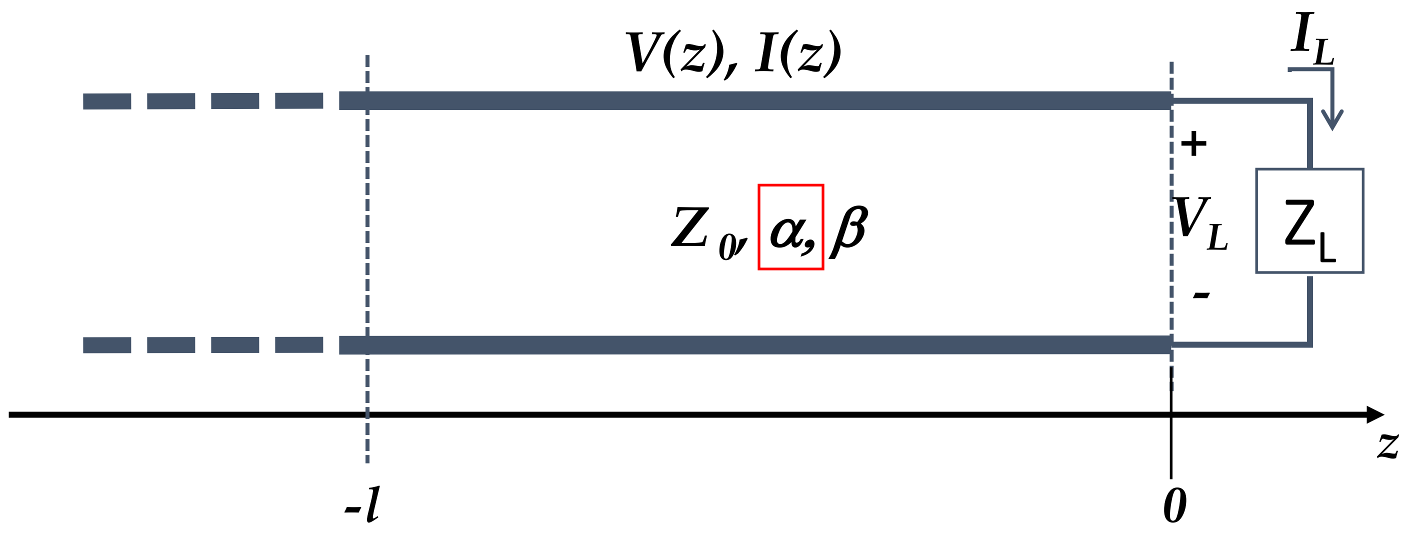
$$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan \beta l}{Z_1 + jR_L \tan \beta l}$$

$$|\Gamma| = \left| \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \right|$$

$$\beta l = \frac{\pi f}{2 f_0}$$



The terminated lossy Tline

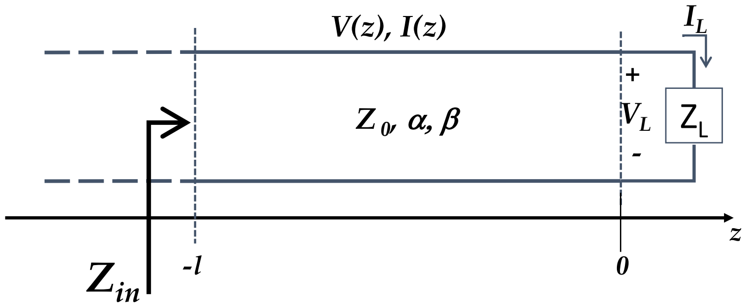


$$V(z) = V_0^+ (e^{-\gamma z} + \Gamma e^{\gamma z})$$

$$\gamma = \alpha + j\beta$$

$$I(z) = \frac{V_0^+}{Z_0} (e^{-\gamma z} - \Gamma e^{\gamma z})$$

The terminated lossy Tline

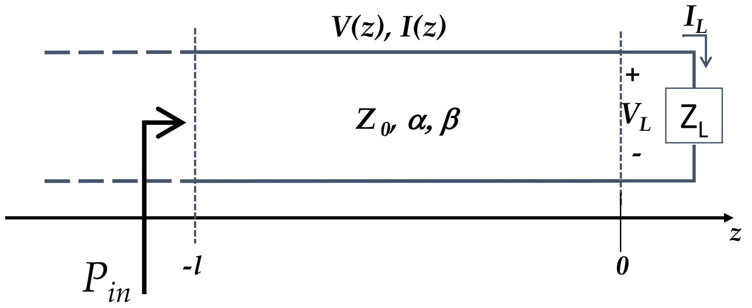


The input impedance at *z*=-*l* on the Tline is:

$$Z_{in} = \frac{V(z=-l)}{I(z=-l)} = \frac{V_0^+\left[e^{\gamma l} + \Gamma e^{-\gamma l}\right]}{V_0^+\left[e^{\gamma l} - \Gamma e^{-\gamma l}\right]} Z_0$$

$$= Z_0 \frac{Z_L + jZ_0 \boxed{\tanh \gamma l}}{Z_0 + jZ_L \tanh \gamma l}$$

The terminated lossy Tline



The power delivered at *z=-l* is:

$$P_{in} = \boxed{\frac{1}{2}}\mathrm{Re}\left(V(-l)I^*(-l)\right) = \frac{\left|V_0^+\right|^2}{2Z_0}\left(e^{2\alpha l} - \left|\Gamma\right|^2 e^{-2\alpha l}\right)$$

The power delivered to the load at *z=0* is:

$$P_L = \frac{1}{2}\mathrm{Re}\left(V(0)I^*(0)\right) = \frac{\left|V_0^+\right|^2}{2Z_0}\left(1 - \left|\Gamma\right|^2\right)$$

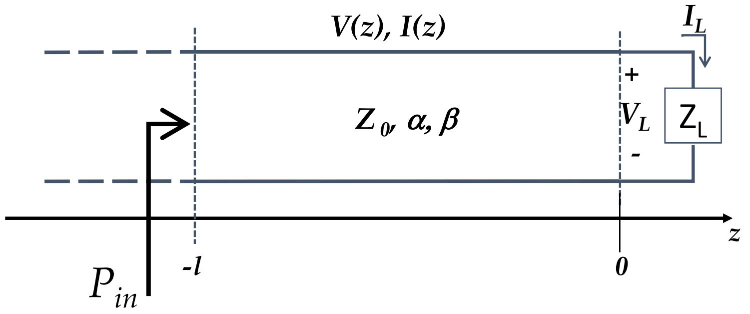
The terminated lossy Tline

The loss in the line is the difference between P_{in} and P_L:

$$P_{loss} = P_{in} - P_L = \frac{|V_0^+|^2}{2Z_0}\left(\boxed{\left[e^{2\alpha l} - 1\right]} + \boxed{|\Gamma|^2\left[1 - e^{-2\alpha l}\right]}\right)$$

Loss of incident wave

Loss of reflected wave



Summary

- The terminated Tline is an impedance tuner
- Special case: Quarter-wave transformer
- Lossy Tline

Microwave Engineering and Antennas

Microwave Networks

Bart Smolders, Professor

Department of Electrical Engineering

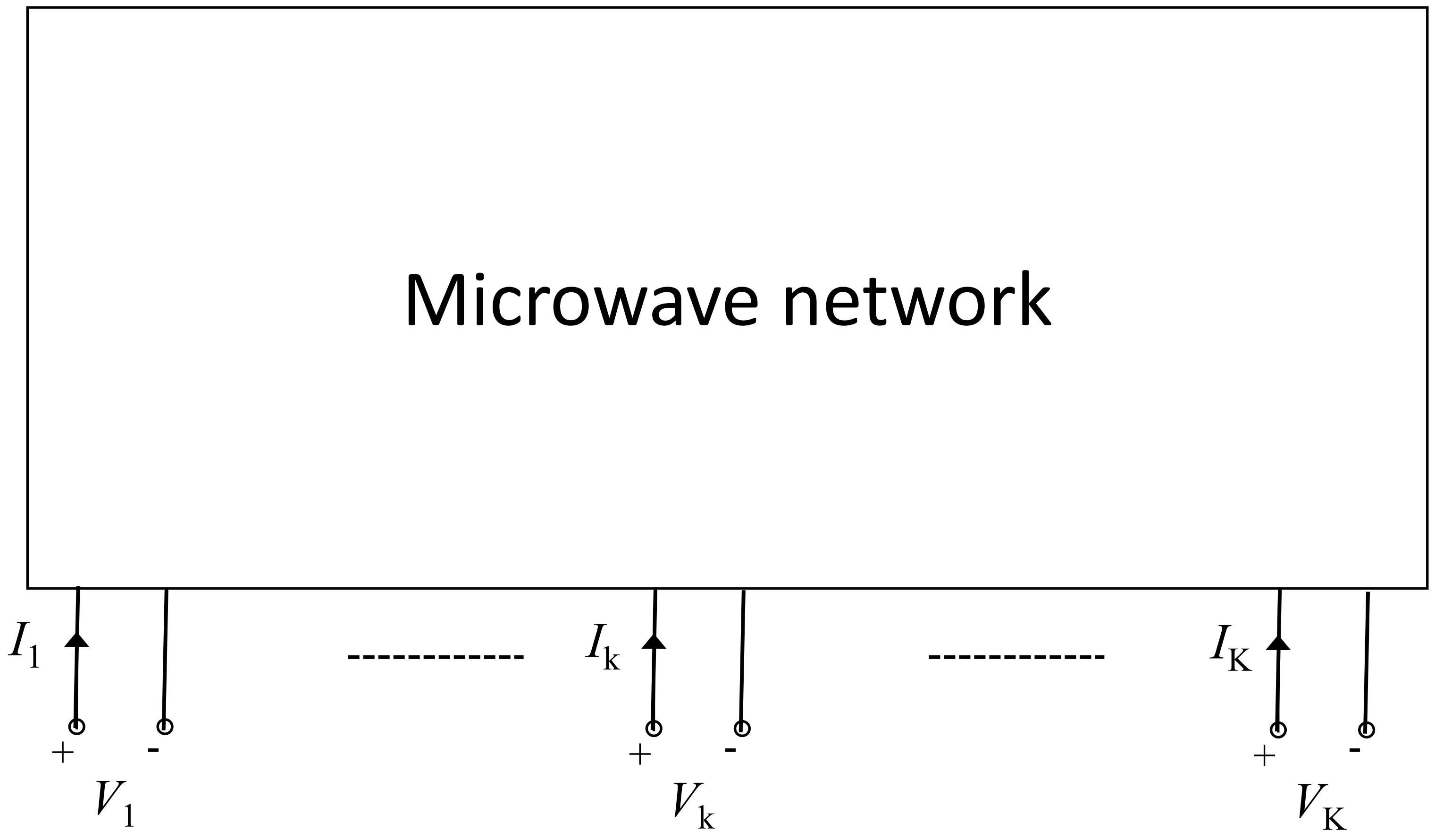
Center for Wireless Technology Eindhoven

Microwave Networks

Objective of this lecture

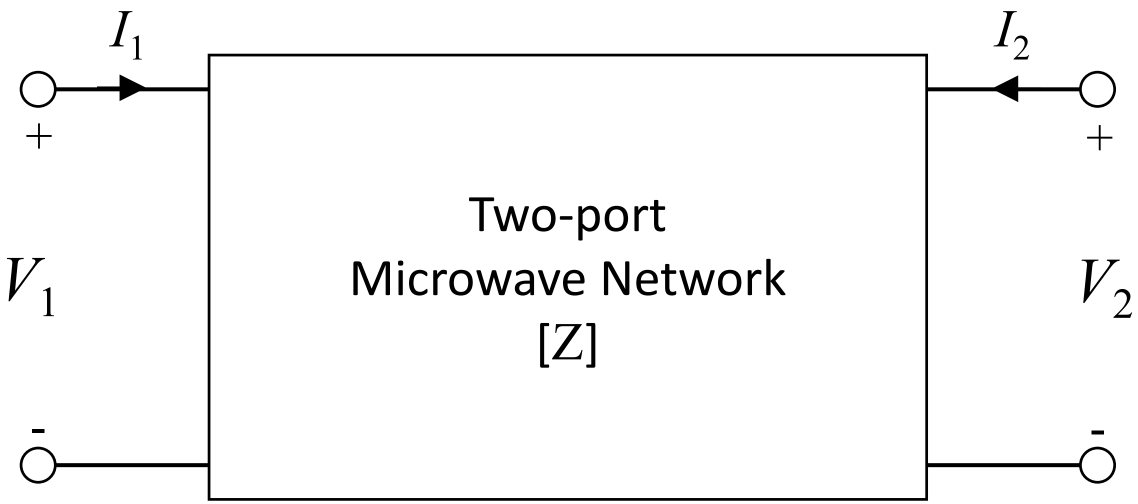
- Introduction of a K-port microwave network
- Introduction of impedance matrix and scattering matrix
- Input and output reflection coefficient of 2-port network
- Discuss example

K-port microwave network



Impedance matrix

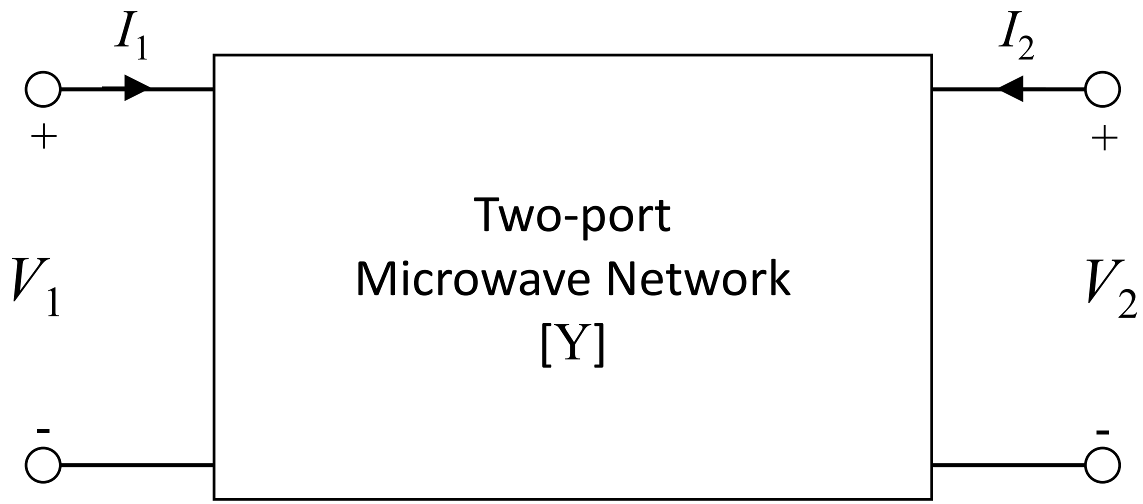
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \boxed{Z_{11}} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_{2=0}} \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_{1=0}} \qquad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_{2=0}} \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_{1=0}}$$

Admittance matrix

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \boxed{Y_{11}} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

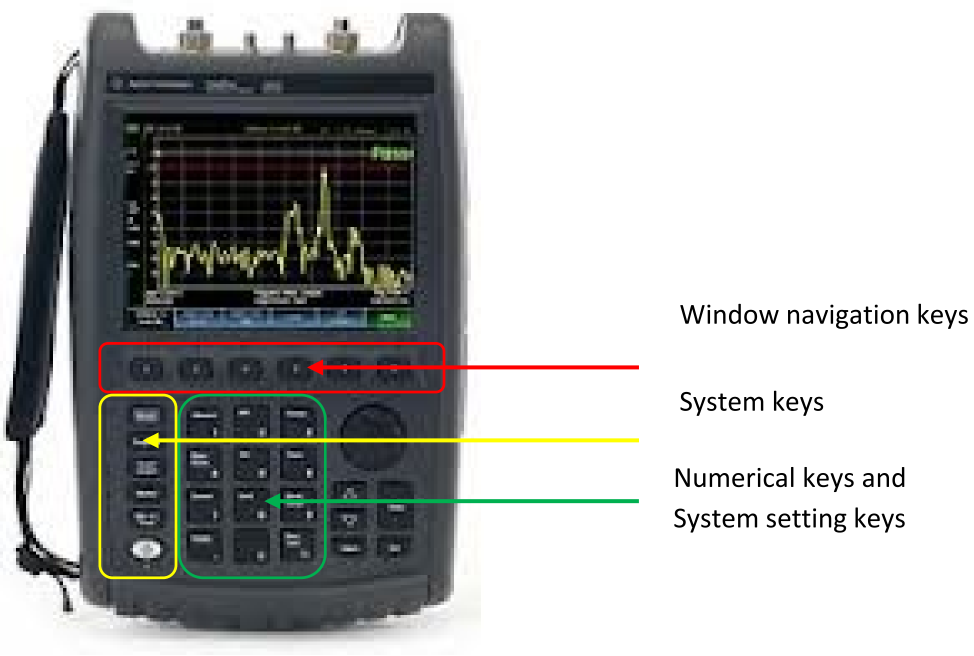


$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_{2=0}} \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_{1=0}} \qquad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_{2=0}} \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_{1=0}}$$

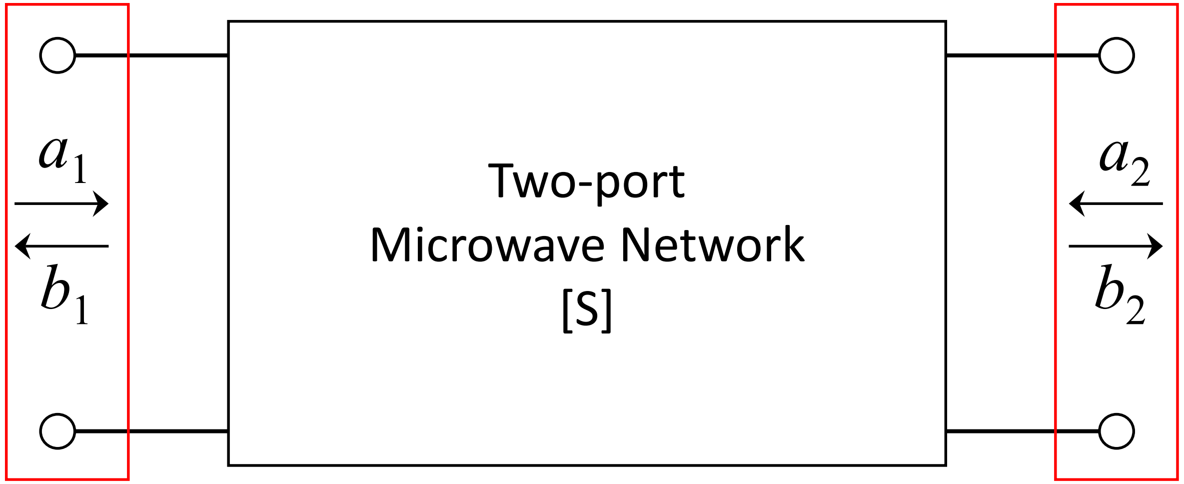
Impedance/Admittance matrix

Problem at microwave frequencies

- We cannot measure voltages and currents directly!
- We can measure the complex amplitude (amplitude and phase) of the incident and reflected waves using a vector network analyser (VNA).



Scattering matrix



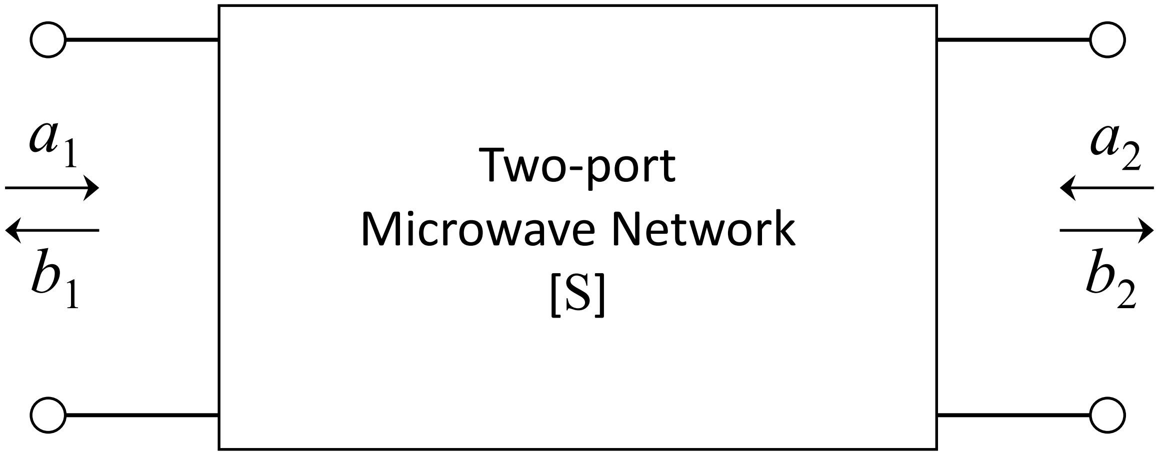
Normalized complex wave amplitudes:

$$a_1 = \frac{V_1^+}{\sqrt{Z_0}} = I_1^+ \sqrt{\boxed{Z_0}} \qquad a_2 = \frac{V_2^+}{\sqrt{Z_0}} = I_2^+ \sqrt{Z_0}$$

$$b_1 = \frac{V_1^-}{\sqrt{Z_0}} = I_1^- \sqrt{Z_0} \qquad b_2 = \frac{V_2^-}{\sqrt{Z_0}} = I_2^- \sqrt{Z_0}$$

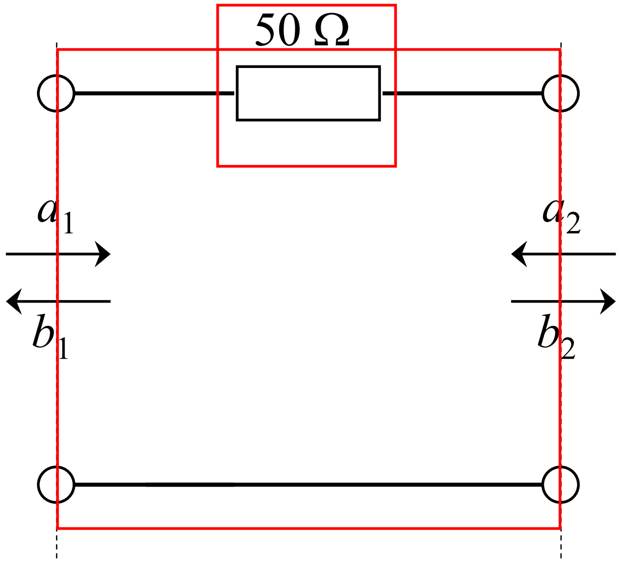
Scattering matrix

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} \boxed{S_{11}} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$



$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} \qquad S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \qquad S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$

Example: 50 Ohm resistor (lumped element)

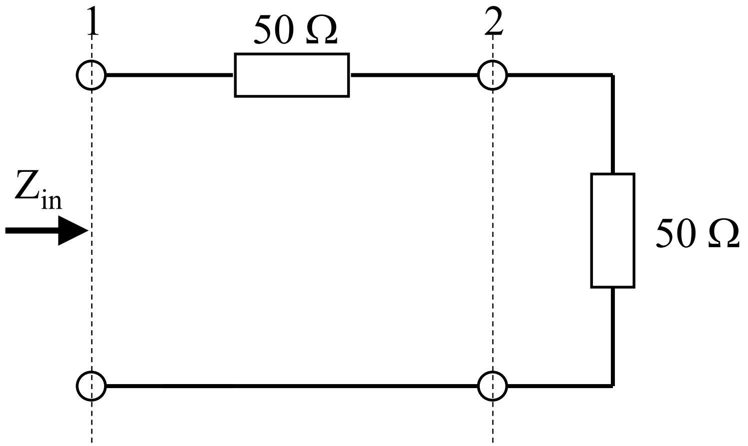


Determine [*S*] matrix of this two-port,

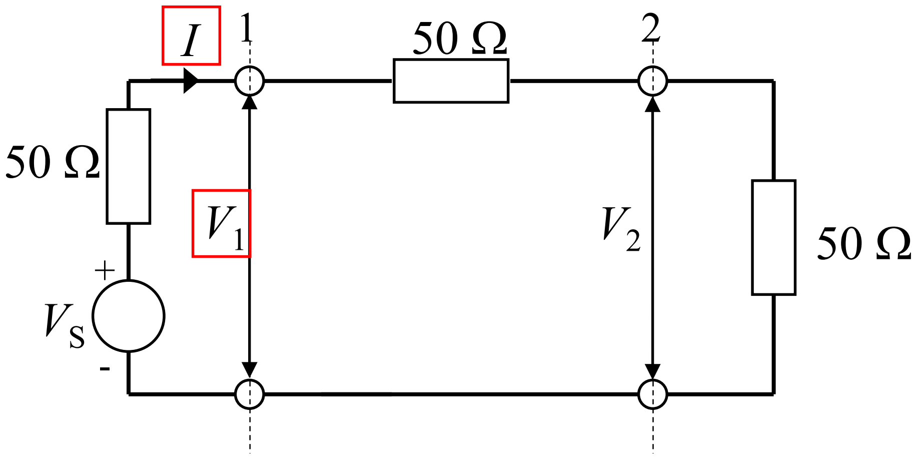
Reference impedance $Z_0 = 50\ \Omega$

Example: 50 Ohm resistor

Corrected version: note errors in video



$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \left.\frac{Z_{in} - Z_0}{Z_{in} + Z_0}\right|_{Z_{in}=100\Omega} = \frac{1}{3}$$



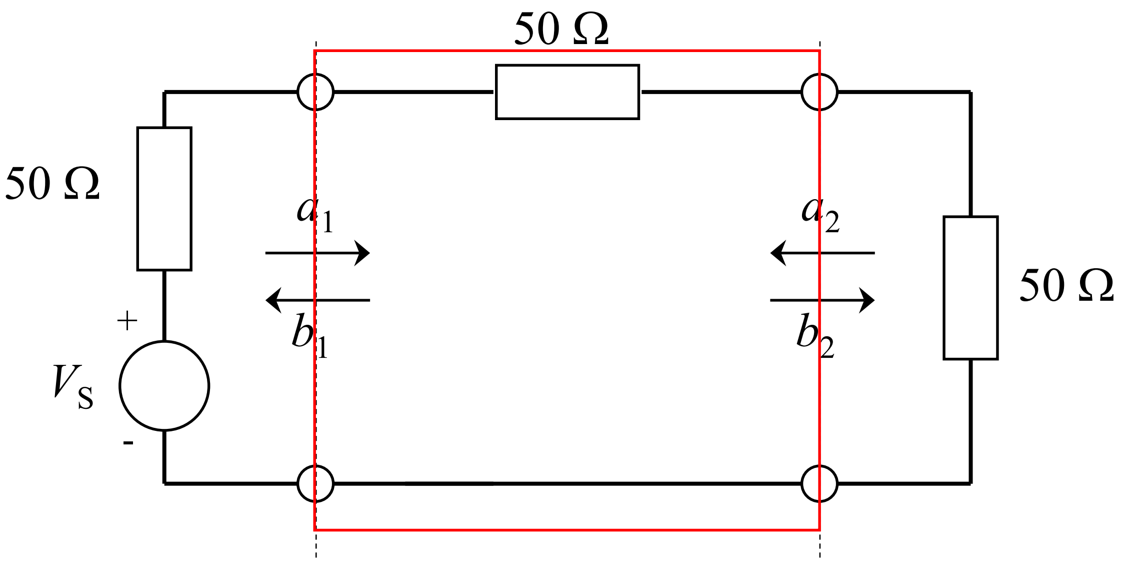
$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2,\Gamma_s=0} = \left.\frac{\frac{1}{\sqrt{Z_0}} V_2^-}{\frac{1}{2\sqrt{Z_0}}(V_1 + Z_0 I)}\right|_{V_2^+=0} = \left.\frac{2V_2^-}{(V_1 + Z_0 I)}\right|_{V_S = V_1 + Z_0 I}$$

$$= \frac{2V_2^-}{V_S} = \frac{2 \cdot 50}{50 + 2 \cdot 50} = \frac{2}{3}$$

$V_2^- = V_2$

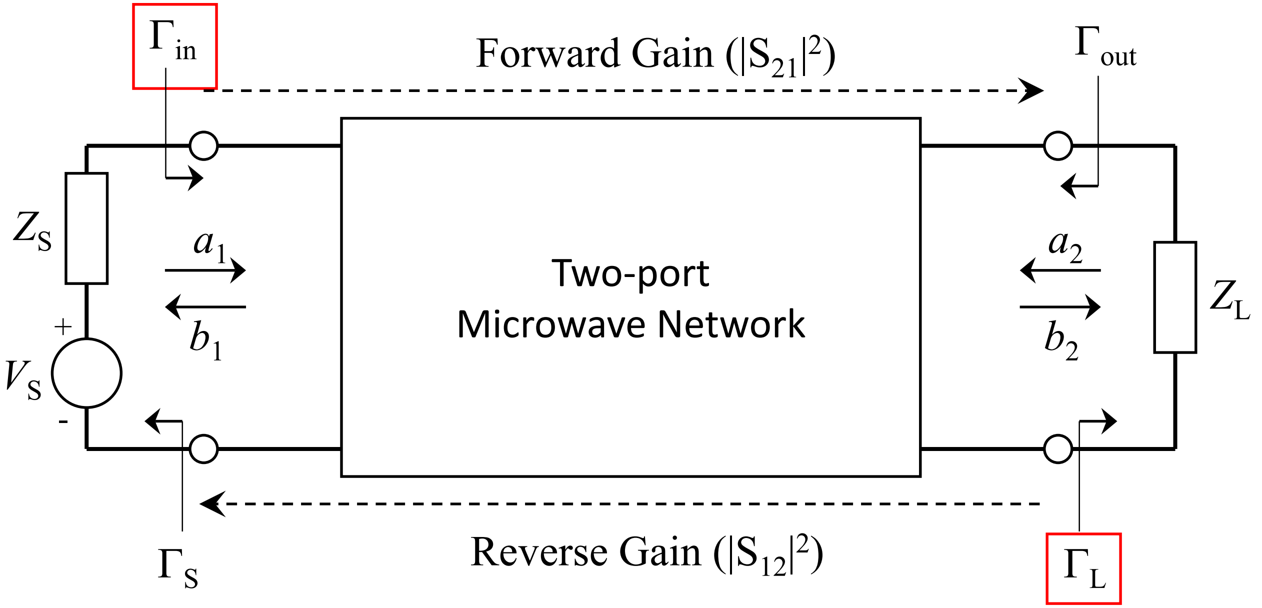
Since $V_2^+ = 0$

Example: 50 Ohm resistor



$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

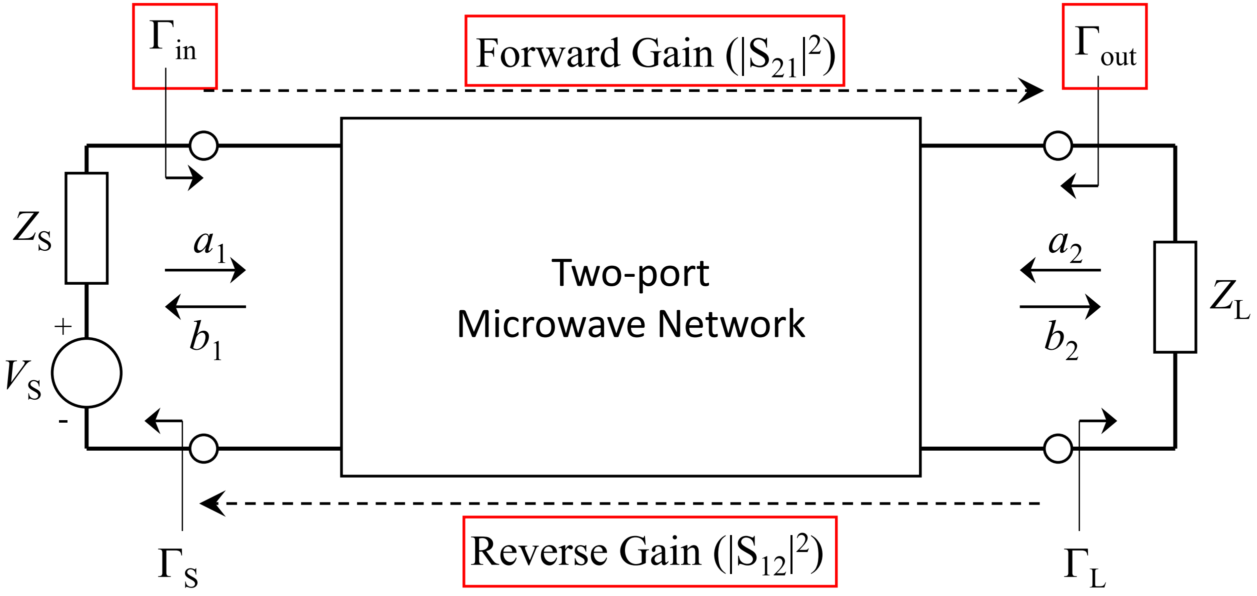
Two-port microwave network



$$\left.\begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2 \\ b_2 &= S_{21}a_1 + S_{22}a_2 \\ \Gamma_L &= \frac{a_2}{b_2} = \frac{Z_L - Z_0}{Z_L + Z_0} \end{aligned}\right\}$$

Three equations provide a relation between b_1 and a_1

Two-port microwave network



$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}}{\frac{1}{\Gamma_L} - S_{22}}, \qquad \Gamma_{in}\big|_{\Gamma_L \to 0} = S_{11}$$

Input Reflection coefficient

$$\Gamma_{out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12}S_{21}}{\frac{1}{\Gamma_S} - S_{11}}, \qquad \Gamma_{out}\big|_{\Gamma_S \to 0} = S_{22}$$

Output Reflection coefficient

Summary

- Microwave networks
- Impedance and Admittance matrix
- Scattering matrix
- Example of a 50 Ohm lumped resistor
- Input/output reflection
- Forward/reverse gain

Microwave Engineering and Antennas

Power Combiners

Bart Smolders, Professor
Department of Electrical Engineering
Center for Wireless Technology Eindhoven

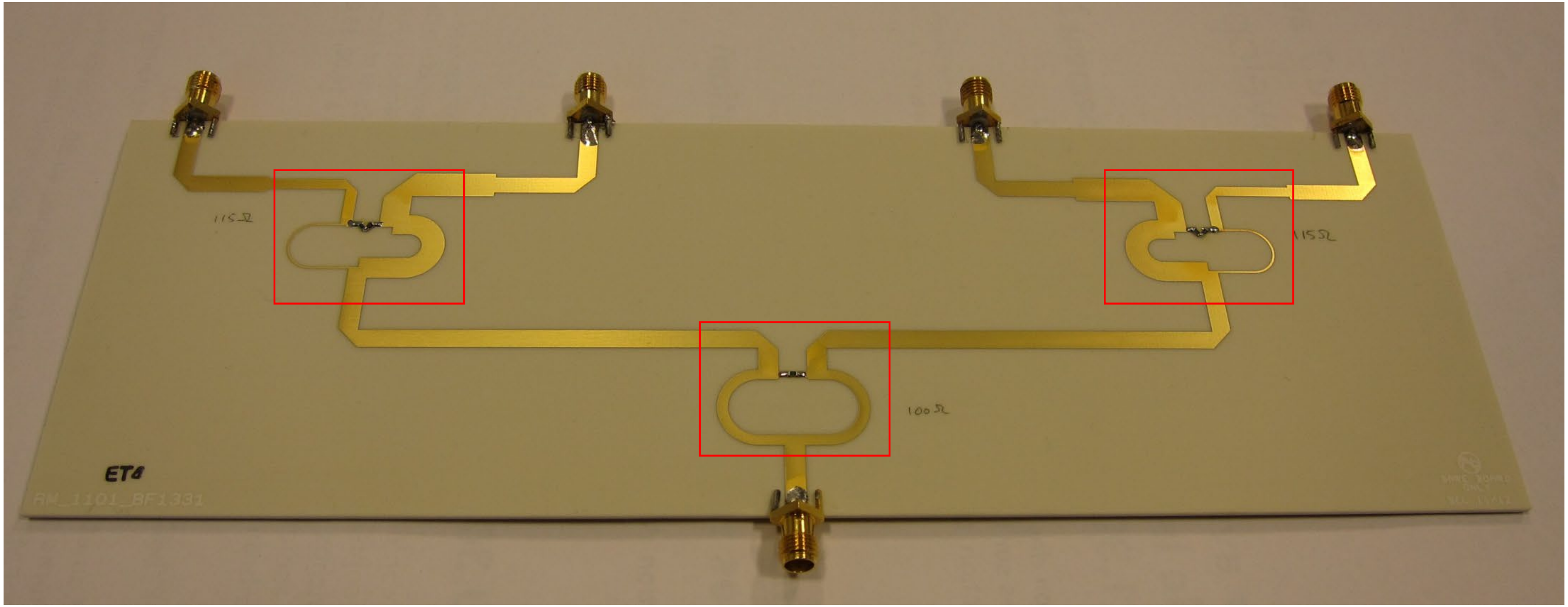
Power Combiners

Objective of this lecture

- General description
- Introduction of various concepts for power combining
- Provide physical insight
- Discuss example

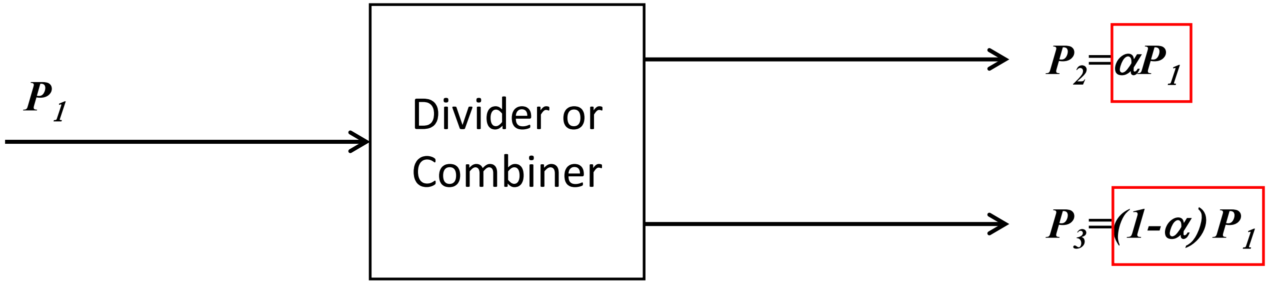
Example of a power combining network

1:4 unequal Wilkinson power combiner using microstrip technology

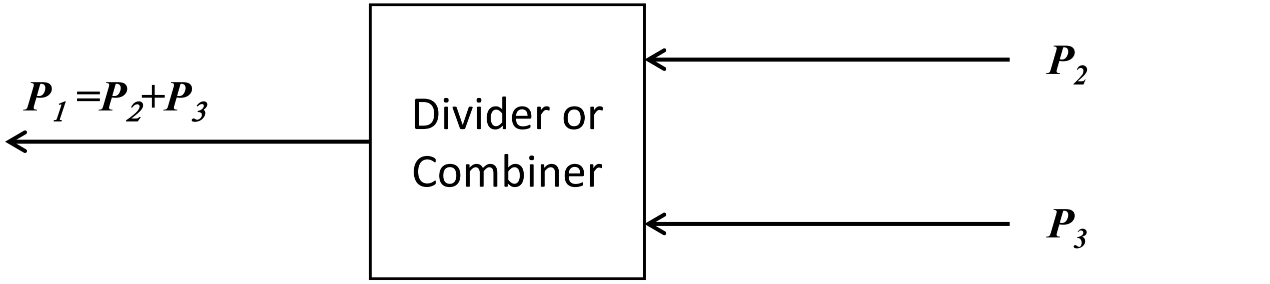


Power Combiners/Dividers

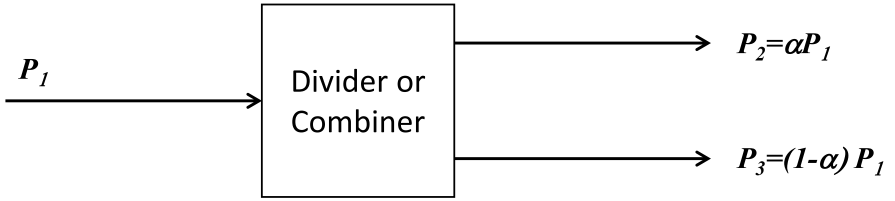
Assymmetrical power divider



Symmetrical power combiner



Power Combiners/Dividers



Three-port network representation

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

When properly matched at all ports: $S_{11}=S_{22}=S_{33}=0$

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

If we require the network to be lossless, [*S*] needs to be unitary

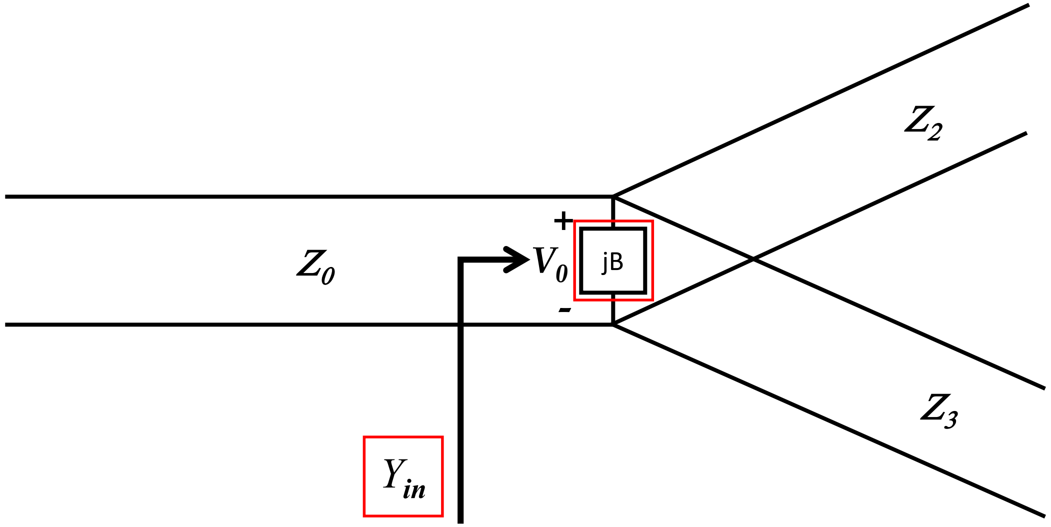
Power Combiners/Dividers

Unitary condition of [*S*]

- Cannot be met when (S_{12}, S_{13}, S_{23}) are all non-zero
- Conclusion: a three-port network cannot be lossless, reciprocal and matched at all ports at the same time.
- We always need to compromise on a requirement

TU/e

Lossless T-junction divider



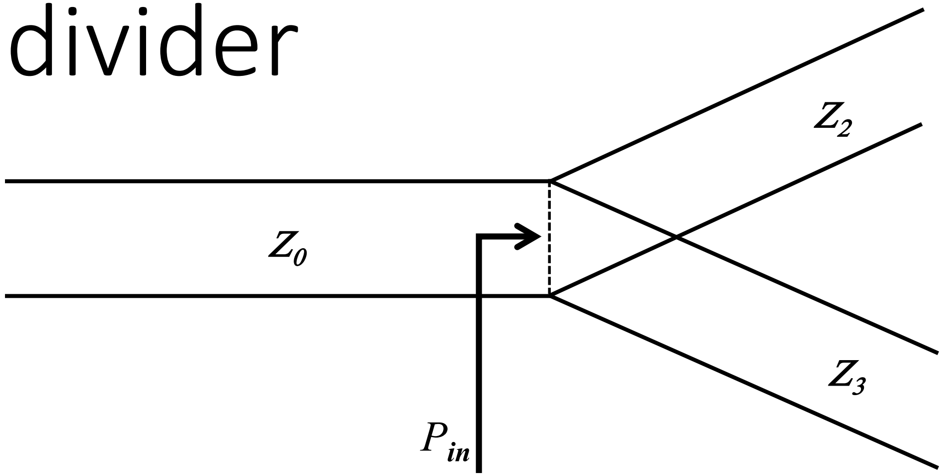
Characteristics

- Junction of three transmission lines
- Susceptance B is related to fringing fields
- For input matching we need:

$$Y_{in} = jB + \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{Z_0} \quad \xrightarrow{B=0} \quad \boxed{Y_{in} = \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{Z_0}}$$

Example of a Lossless T-junction divider

Given a source impedance Z_0=50 Ω,
Find Z_2 and Z_3 for an equal-split divider ratio.



Solution

- The input power delivered to the matched divider is:

$$P_{in} = \frac{V_0^2}{2Z_0}$$

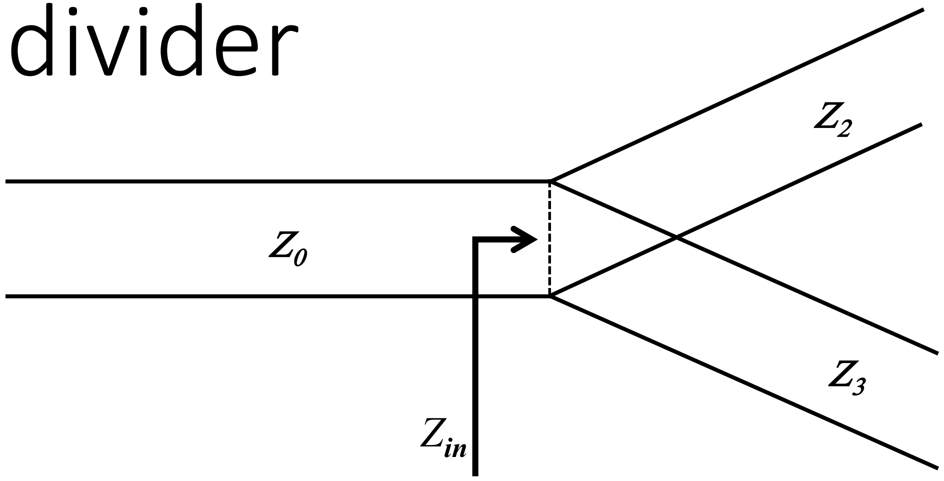
- The output powers are:

$$P_2 = \frac{V_0^2}{2Z_2} = \frac{1}{2}P_{in}$$

$$P_3 = \frac{V_0^2}{2Z_3} = \frac{1}{2}P_{in}$$

$$\boxed{\begin{aligned} Z_2 &= 2Z_0 = 100\Omega \\ Z_3 &= 2Z_0 = 100\Omega \end{aligned}}$$

Example of a Lossless T-junction divider



Solution

- The input impedance is now matched:

$$Z_{in} = 100 \,||\, 100 = 50\Omega$$

- The output are **not** matched:

$$\Gamma_2 = \frac{Z_{out,2} - Z_2}{Z_{out,2} + Z_2} = \frac{33 - 100}{33 + 100} = -0.5$$

$$\Gamma_3 = \frac{Z_{out,3} - Z_3}{Z_{out,3} + Z_3} = \frac{33 - 100}{33 + 100} = -0.5$$

Resistive divider

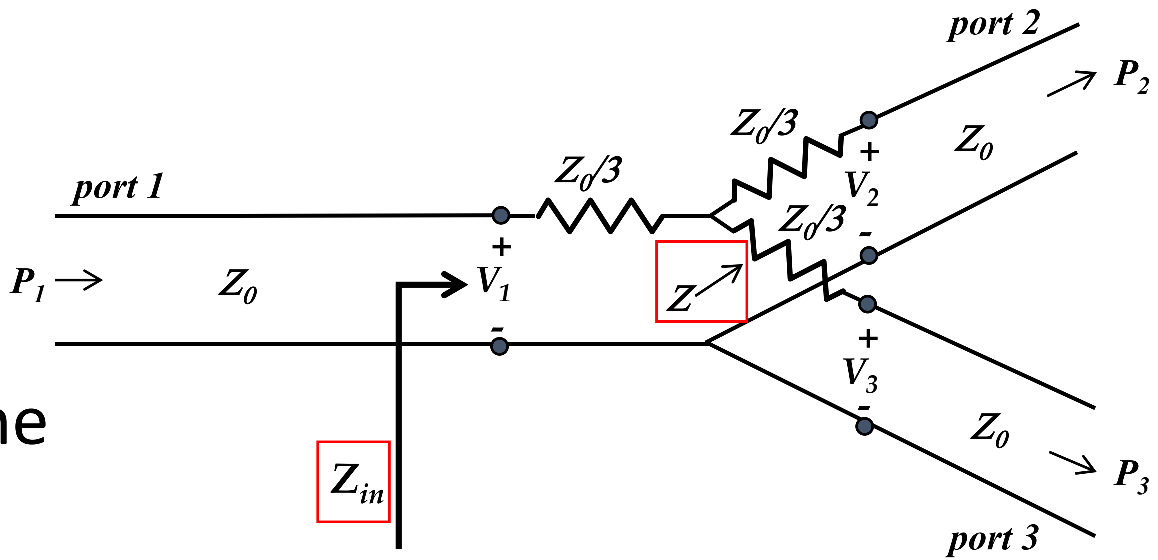
Characteristics

- The impedance Z (looking into the $Z_0/3$ resistor into port 2) is:

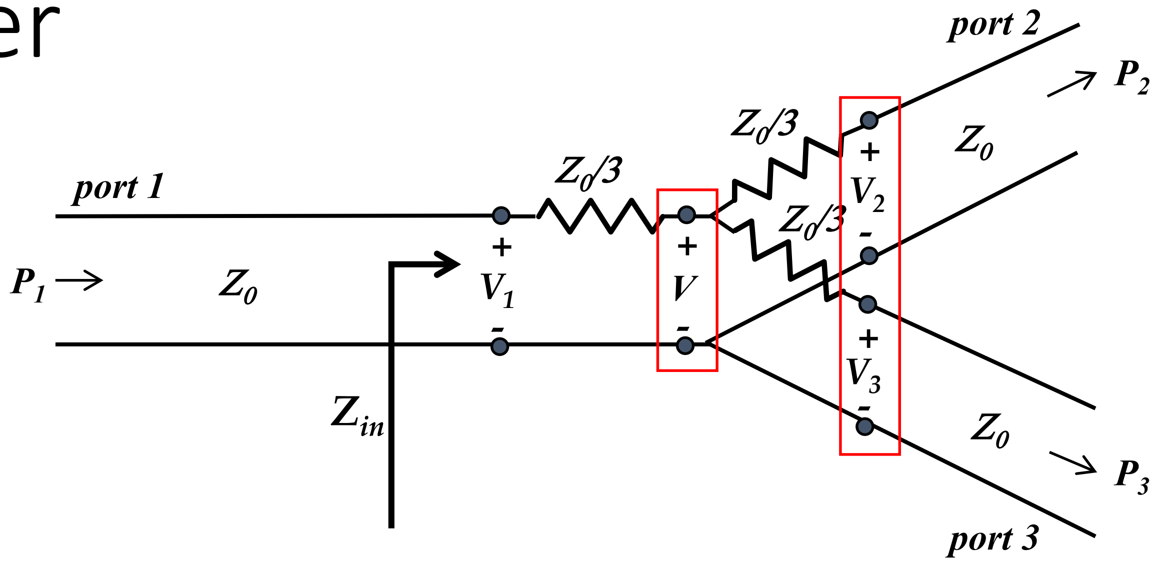
$$Z = \frac{Z_0}{3} + Z_0 = \frac{4Z_0}{3}$$

- The input impedance Z_{in} is now:

$$Z_{in} = \frac{Z_0}{3} + \frac{2Z_0}{3} = Z_0$$



Resistive divider/combiner



Derivation of [S]

- Voltage at central node:

$$V = V_1 \frac{2Z_0/3}{Z_0/3 + 2Z_0/3} = \frac{2V_1}{3}$$

- Output voltages become

$$V_2 = V_3 = V \frac{Z_0}{Z_0 + Z_0/3} = \frac{3V}{4} = \frac{1}{2} V_1$$

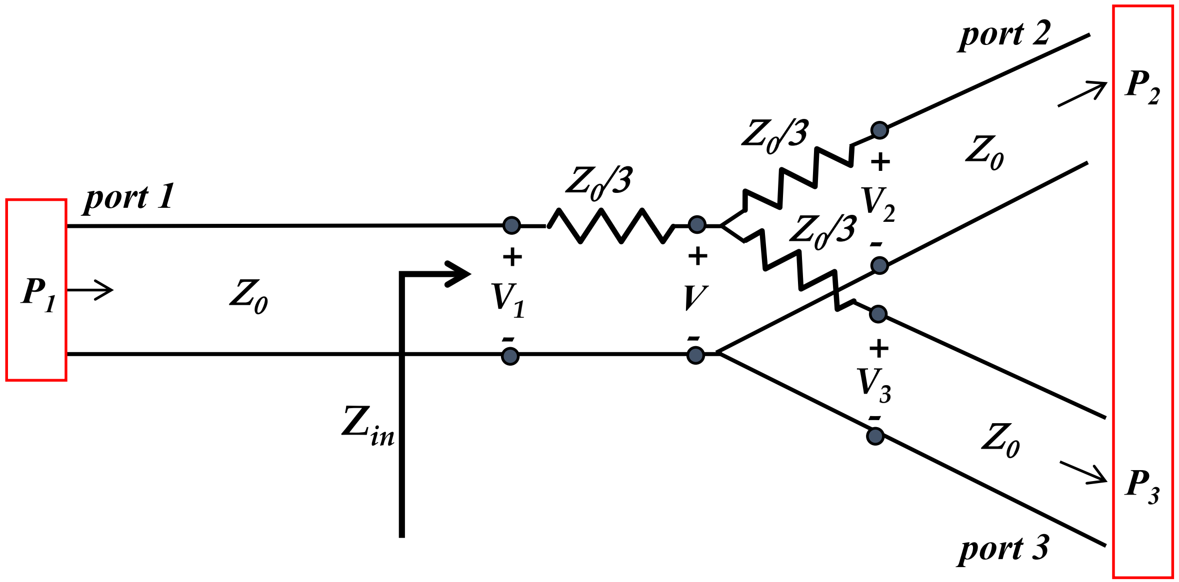
- Scattering matrix

$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Power loss

Matching is excellent.

Resistive divider



Power loss

- Input power

$$P_{in} = P_1 = \frac{V_1^2}{2Z_0}$$

- Output power

$$P_{out} = P_2 = P_3 = \frac{(1/2V_1)^2}{2Z_0} = \frac{V_1^2}{8Z_0} = \frac{1}{4}P_{in}$$

So 50% of the input power is dissipated in the resistors.

Therefore, resistive dividers are only interesting is a limited number of applications, e.g. in a compact on-chip (IC) design.

Summary

- Power combiners/dividers cannot be lossless, have perfect matching, be reciprocal and have perfect isolation at the same time.
- We discussed T-junction and resistive combiners/dividers
- Next web-lecture we will introduce an almost perfect component, the Wilkinson power combiner/divider.

Microwave Engineering and Antennas

Wilkinson Power Combiner/Divider

Bart Smolders, Professor

Department of Electrical Engineering

Center for Wireless Technology Eindhoven

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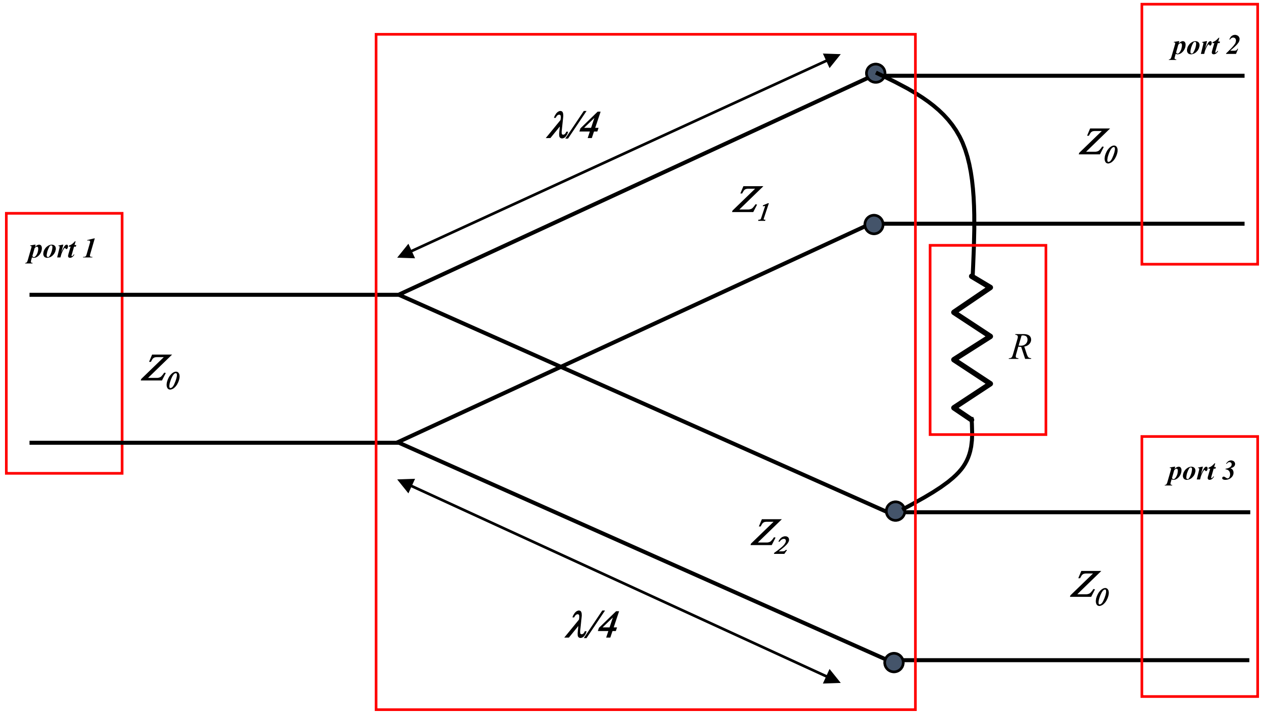
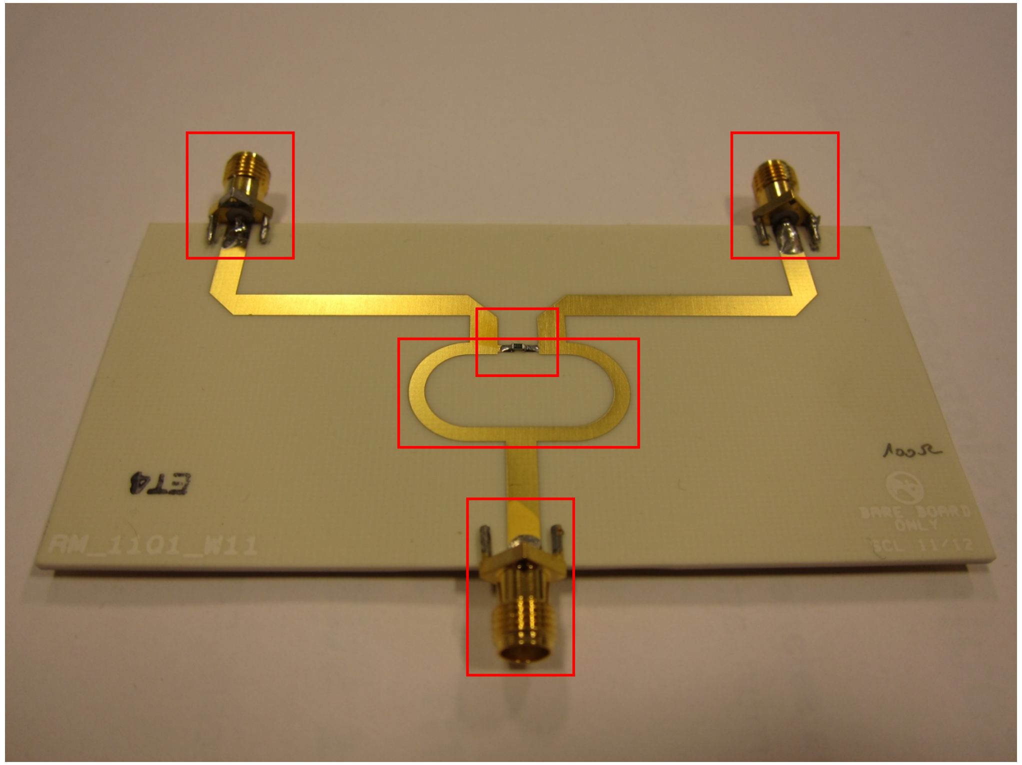
Wilkinson power combiner/divider

Objective of this lecture

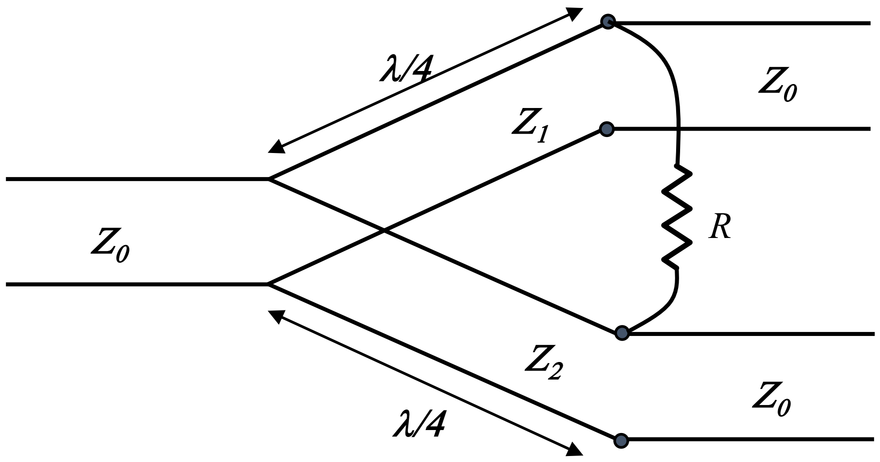
- Introduction of the Wilkinson power combiner
- Even-odd mode analysis
- Derive scattering matrix

Wilkinson Power Combiners/Dividers





Wilkinson Power Combiners/Dividers



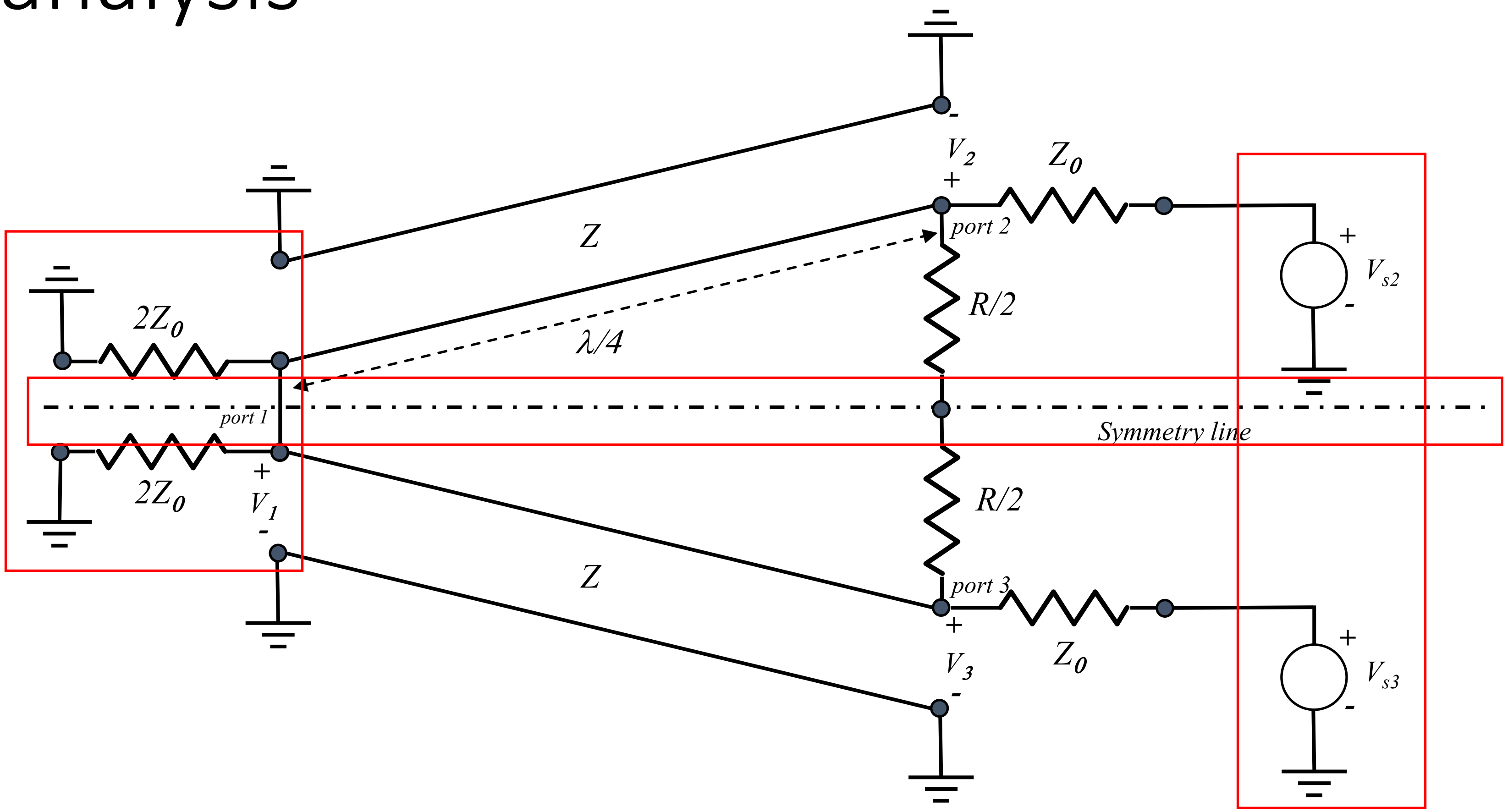
Advantages

- Good matching at all ports
- Low loss
- High isolation between port 2 and port 3
- Various divider-ratios possible

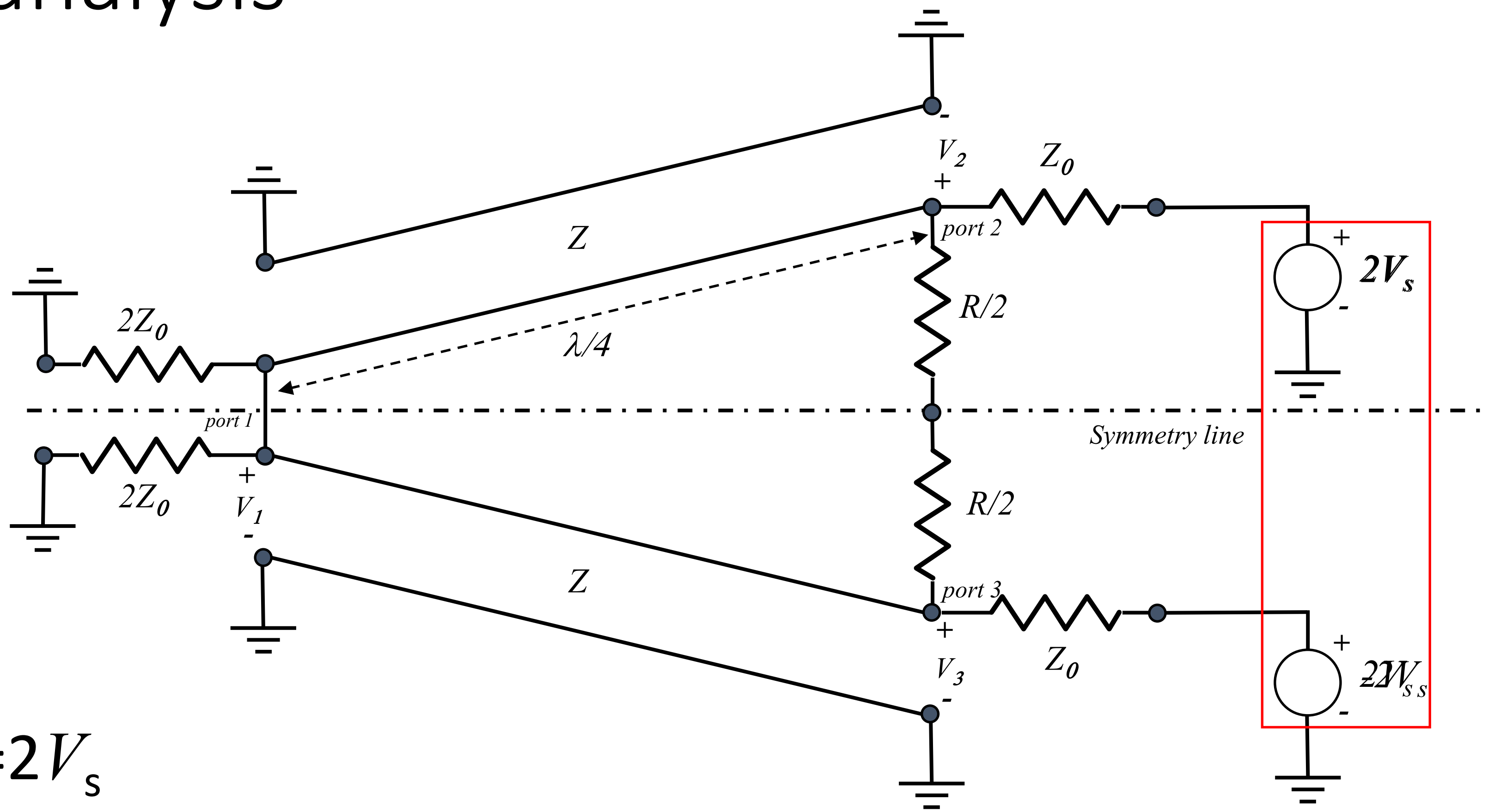
Modelling approach

- Consider a combiner
- Consider only the symmetrical equal-split case: Z_1=Z_2=Z
- Use odd-even mode technique
- Resistor $R = 2Z_0$

Even-odd mode analysis



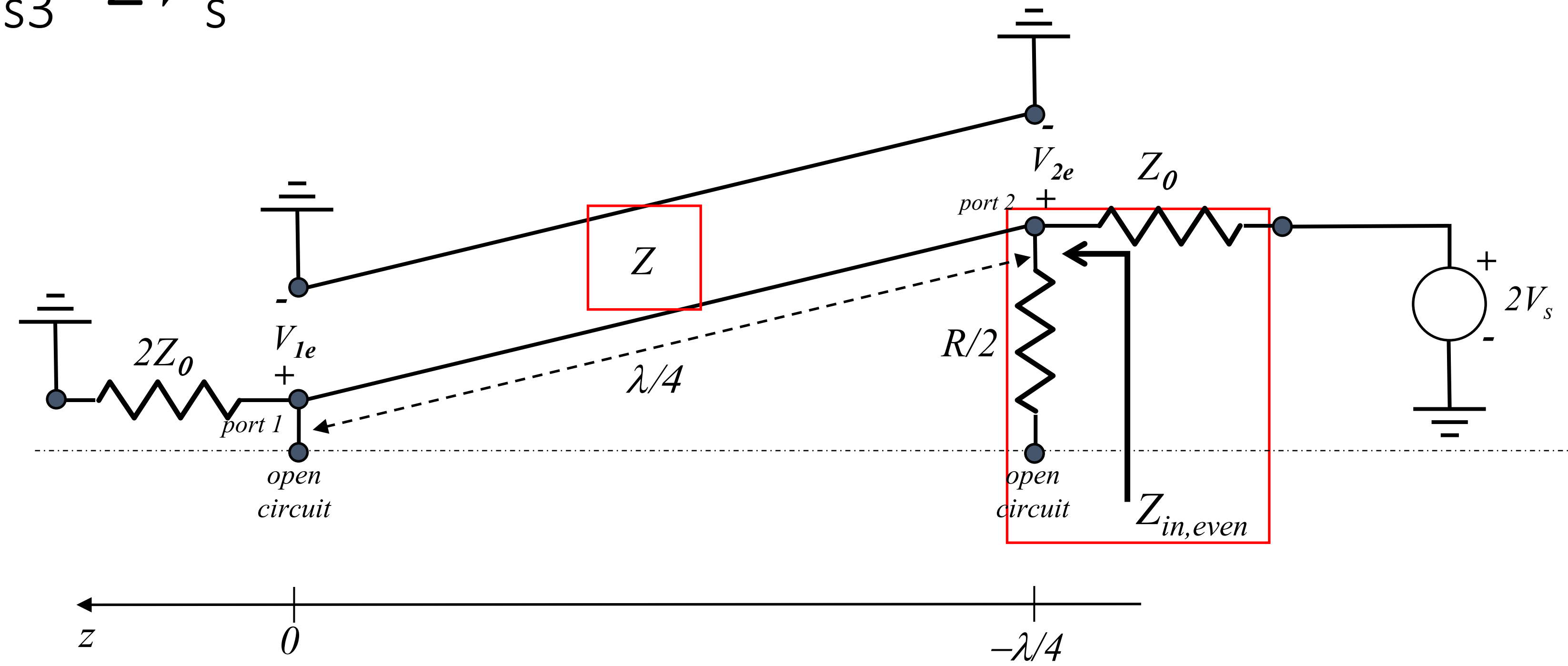
Even-odd mode analysis



Even-odd mode

- Even-mode: $V_{s3}=V_{s2}=2V_s$
- Odd-mode: $V_{s3}=-V_{s2}=-2V_s$

Even mode V_{s2}=V_{s3}=2V_s



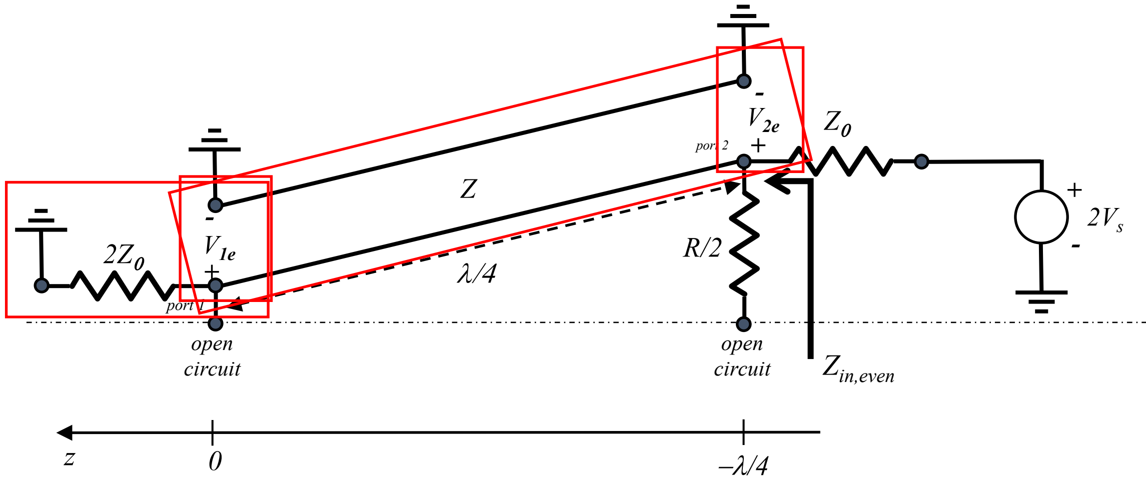
Impedance looking into port 2:

$$Z_{in,even} = \frac{Z^2}{2Z_0} = \frac{\left(\sqrt{2}Z_0\right)^2}{2Z_0} = Z_0 \text{ with choice } \boxed{Z = \sqrt{2}Z_0}$$

Even mode

Voltage along the $\lambda/4$ Tline:

$$V(z) = V_0^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$$



Port voltages are now:

$$V_{2e} = V\left(z = -\frac{\lambda}{4}\right) = jV_0^+(1-\Gamma) = V_s$$

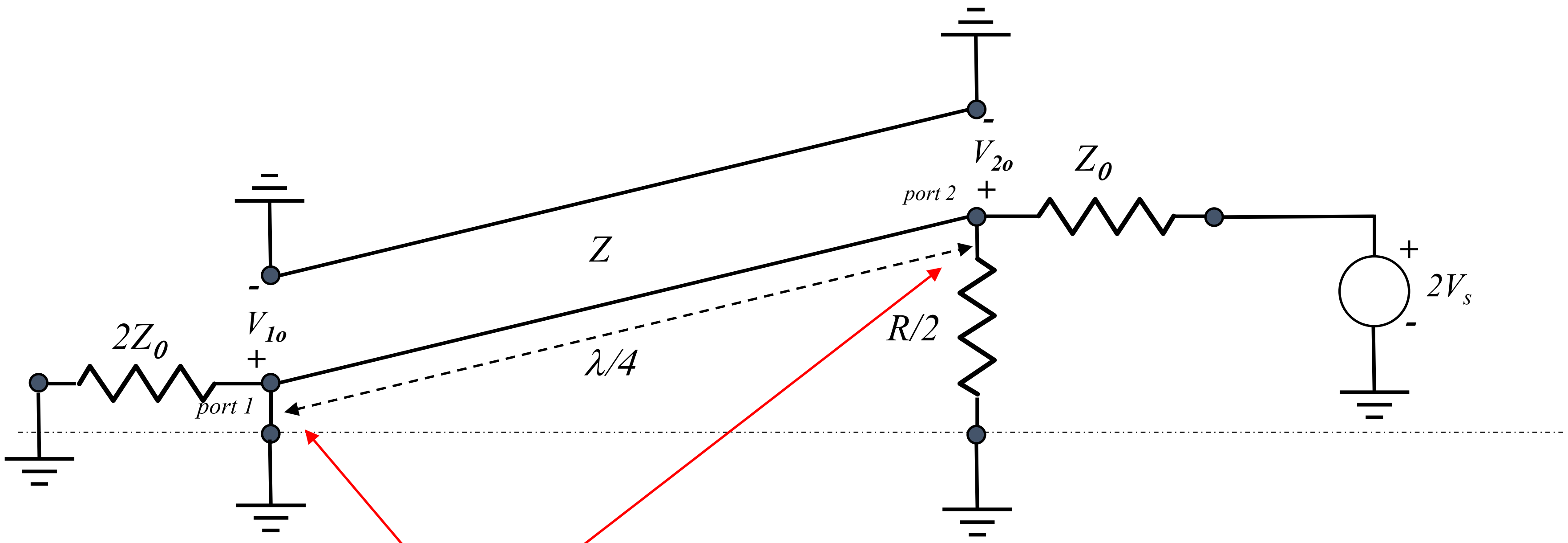
$$V_{1e} = V(z=0) = V_0^+(1+\Gamma) = jV_s \frac{\Gamma+1}{\Gamma-1}$$

With Γ the reflection coefficient looking at port 1 into the load $2Z_0$

$$\Gamma = \frac{2Z_0 - \sqrt{2}Z_0}{2Z_0 + \sqrt{2}Z_0} = \frac{2-\sqrt{2}}{2+\sqrt{2}}$$

$$\boxed{V_{1e} = -jV_s\sqrt{2}}$$

Odd mode V_{s2}=-V_{s3}=2V_s

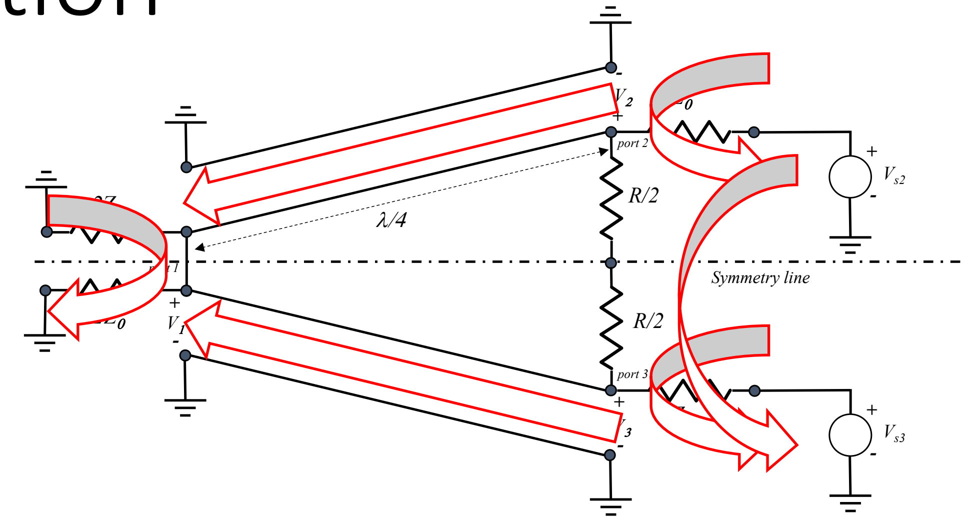


λ/4 Tline transforms a short into an open: $Z_{in} = \dfrac{Z^2}{Z_{load}}$

$V_{2o} = V_s$ $\quad$ $R = 2Z_0$

$V_{1o} = 0$

Total: Even+odd using superposition



- Scattering parameters

$$S_{22} = S_{33} = 0$$

$$S_{12} = S_{21} = \frac{V_1^e + V_1^o}{V_2^e + V_2^o} = -j/\sqrt{2}$$

$$S_{13} = S_{31} = -j/\sqrt{2}$$

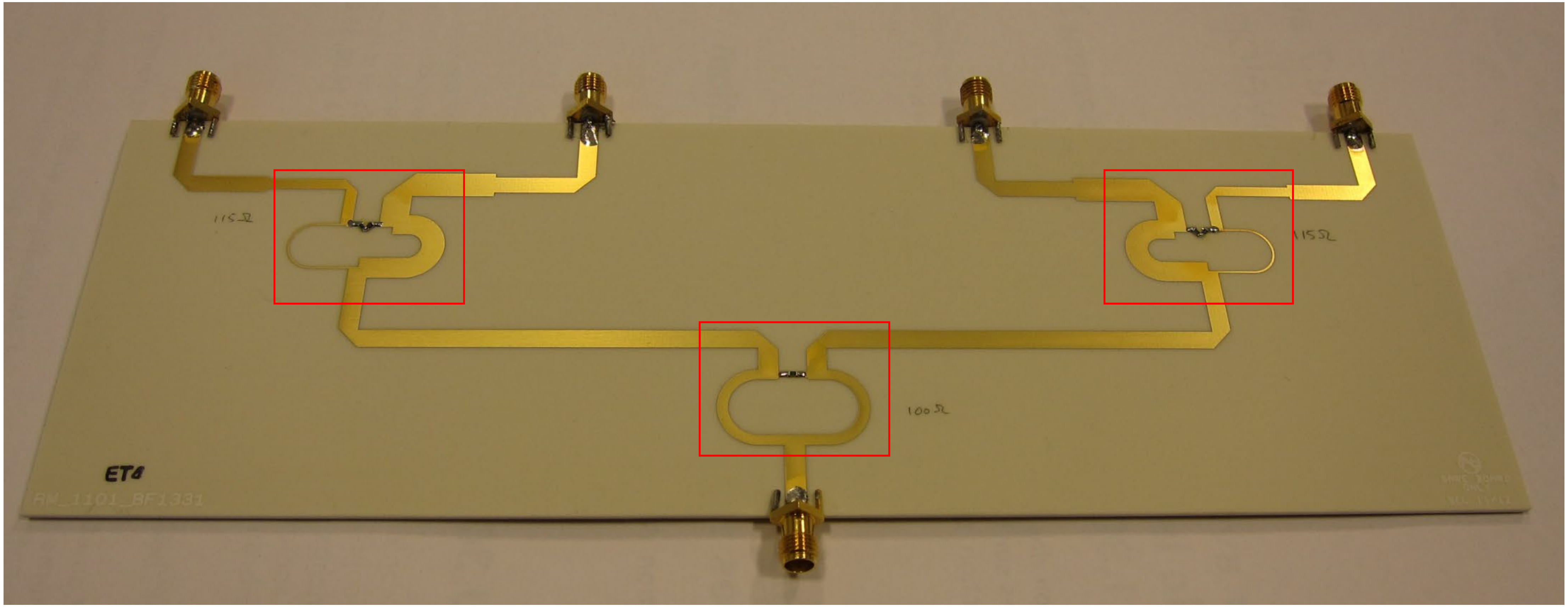
$$S_{23} = S_{32} = 0$$

- Furthermore, it can be shown that with $Z = \sqrt{2}Z_0$

$$S_{11} = 0$$

Example of a power combining network

1:4 Wilkinson power combiner using microstrip technology



Summary

- The ideal Wilkinson power combiner behaves as the almost “perfect” combiner
  - Lossless combiner
  - All ports are matched
  - Input ports are isolated
  - Can be used as divider or combiner
- In practice, losses will occur (typ. 1 dB) and the isolation will be limited (typ. -20 dB).
- Practical designs will also be frequency dependent (typ. 20% bandwidth).

Microwave Engineering and Antennas

Hands-On Design: Beamsteering Network

Ulf Johannsen, Assistant Professor

Department of Electrical Engineering

Center for Wireless Technology Eindhoven

CWTe
CENTER FOR WIRELESS TECHNOLOGY EINDHOVEN
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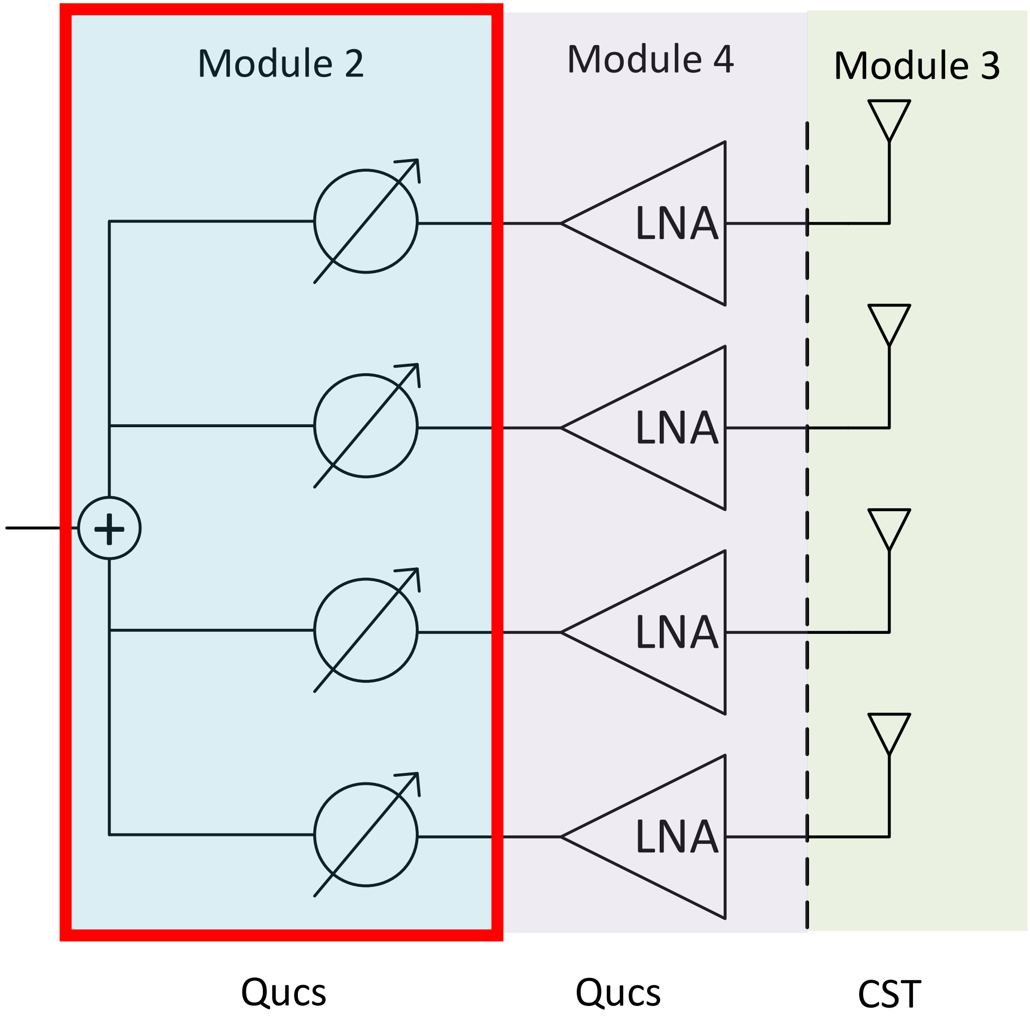
Hands-On Design Assignment



4-Element Analogue Beamforming Receiver-Array

- 5.8 GHz ISM band
- ±45˚ beamsteering range
- Main-lobe antenna gain of 9dBi for entire beamsteering range
- System noise figure < 2.7dB

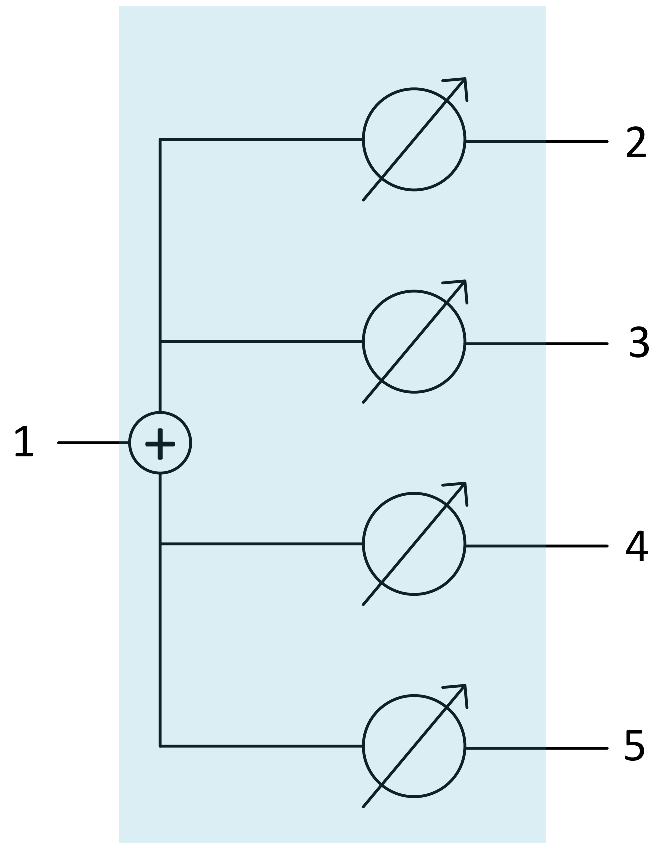
Qucs

- Quite Universal Circuit Simulator (Qucs) is a free-software electronics circuit simulator under GPL.
- You can **download** a copy from http://qucs.sourceforge.net/
- A **short description of mathematical functions** that can be used in Qucs equation (including some standard microwave engineering parameters) can be found here: https://web.mit.edu/qucs_v0.0.19/docs/en/mathfunc.html

Task and Design Requirements

- Design a 4:1 power combiner network
- Design a passive phase shifter
- The overall design, shown on the right, must fulfil the following requirements:

ID	Parameter	Requirement
1	Operating bandwidth	5.725 – 5.875 GHz
2	Port match (within operating bandwidth)	$S_{ii} < -10dB$, for $i = 1, 2, 3, 4, 5$
3	Insertion loss	$IL < 13dB$
 4 5	Phase shifter: 1. Range 2. Resolution	 1. $0° \leq \varphi \leq 354°$ 2. $\Delta\varphi \leq 6°$



Rules and Tips

- You **may use the resistor, capacitor and inductor models from the Qucs library**.
- For the power combiner, you may want to study the **Wilkinson power combiner**.
- Show your **genuine attempt** to design what was asked of you. For a genuine attempt you
  - Have **all design choices** as long as you design the components yourself in Qucs or CST (student version). Commercial **off-the-shelf solutions are not permitted**.
  - Must use a **switched transmission line phase shifter**. The required phase shift is then achieved by a time delay. You must use the SP4T **switch model provided** on Coursera.
  - Must upload **<u>all</u> necessary design files** and a **design report** that describes and explains the design.

Rubric for peer review

Excellent (6 pts)	Very good (5 pts)	Good (4 pts)	Fair (3 pts)	Sufficient (2)	Insufficient (0 pts)
• The report describes **all** design aspects in detail, such that the reader/reviewer can implement the design in Qucs her-/himself. • **All** design aspects are explained clearly and correctly. • The design achieves the specified requirements **without exception**.	• The report describes almost all design aspects in detail. Only **minor details are missing** such that the reviewer **can implement the design** in Qucs her-/himself. • Most design aspects are explained clearly and correctly. **Only minor inaccuracies** are present. The understanding is not significantly affected by this. • The design achieves the specified requirements with **one exception**.	• The report describes most design aspects in detail. **A few details are missing** such that the reviewer may not be able to implement **some minor details** of the design in Qucs her-/himself. • There are **only a few inaccuracies/inconsistencies** in the explanation of the design aspects. The understanding of a few aspects of the design is slightly affected. • The design achieves the specified requirements with **two exceptions**.	• The report describes most design aspects in detail. **Some details are missing** such that the reviewer may **not** be able to implement **every aspect of the design correctly** in Qucs her-/himself. • There are **some inaccuracies/inconsistencies** in the explanation of the design aspects. Not every aspect of the design is entirely clear. • The design achieves the specified requirements with **three exceptions**.	• The report describes most design aspects in detail. **Several details are missing** such that the reviewer **struggles** to implement **the design correctly** in Qucs her-/himself. • There are **several inaccuracies/inconsistencies or some mistakes** in the explanation of the design aspects. About half of the design aspects are not entirely clear. • The design achieves the specified requirements with **four exceptions**.	• The report provides insufficient detail such that the reader/**reviewer cannot implement the design** in Qucs her-/himself. • There are **major design aspects missing** or are **mostly incorrectly** explained. • The **design does not achieve any of the requirements.**

Microwave Engineering and Antennas

Smith chart

Bart Smolders, Professor
Department of Electrical Engineering
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Smith chart

Objective of this lecture

- History and use of Smith Chart
- Visualization of the reflection coefficient and impedance
- Show some examples

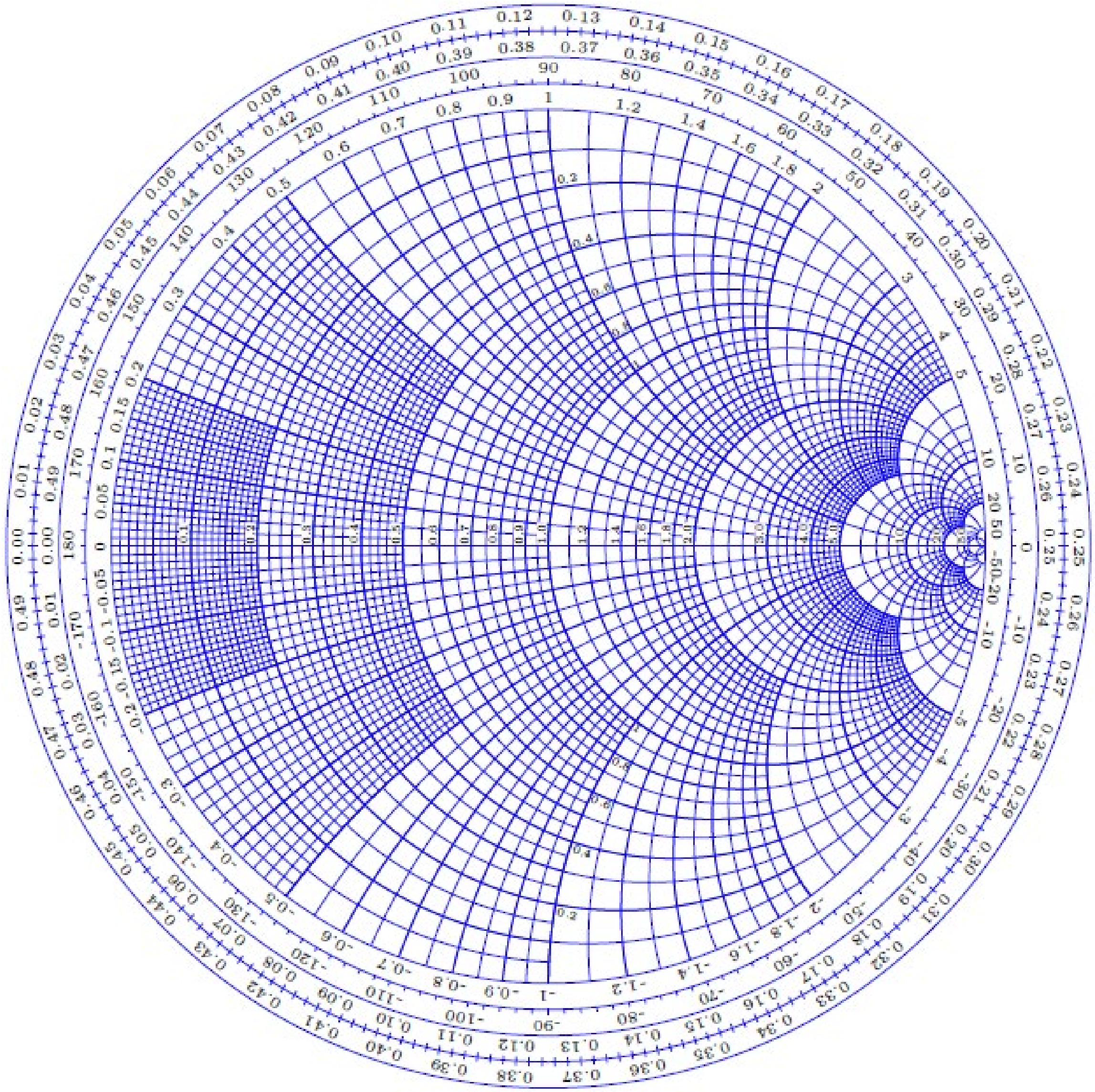
Smith chart

History

- Originates from 1939
- Invented by Philip Smith

Use

- Graphical representation of
  - Reflection coefficient
  - Complex impedance
- Matching and tuning



Smith chart

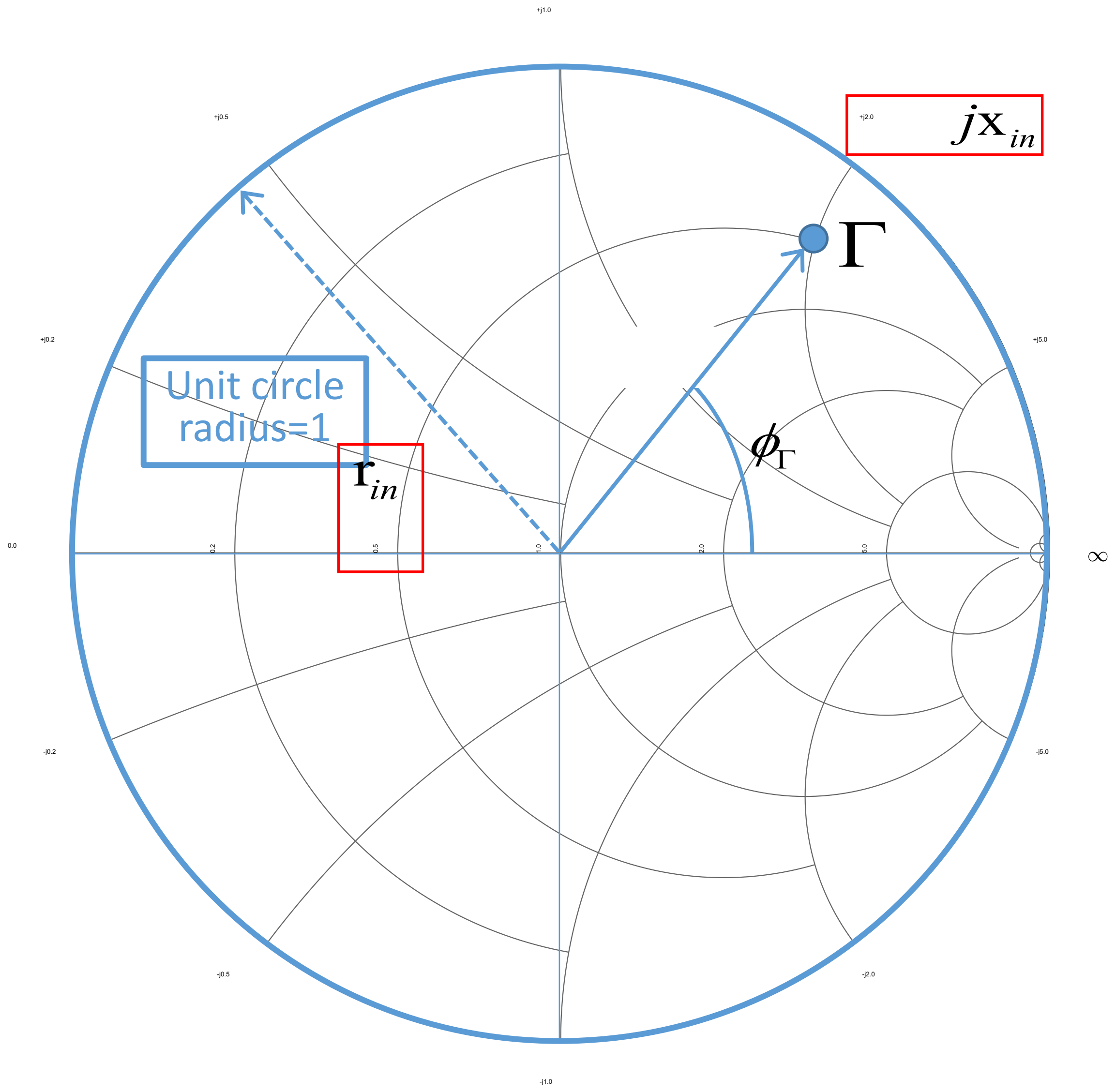
Complex reflection coefficient

$$\Gamma = \frac{V_0^-}{V_0^+} = \Gamma_r + j\Gamma_i = |\Gamma| e^{j\phi_\Gamma}$$

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{z_{in} - 1}{z_{in} + 1}$$

$$z_{in} = \frac{Z_{in}}{Z_0} = \boxed{\mathrm{r}_{in}} + \boxed{j\mathrm{x}_{in}}$$ Normalized input impedance

$$Z_0 = 50\ \Omega$$



Smith chart

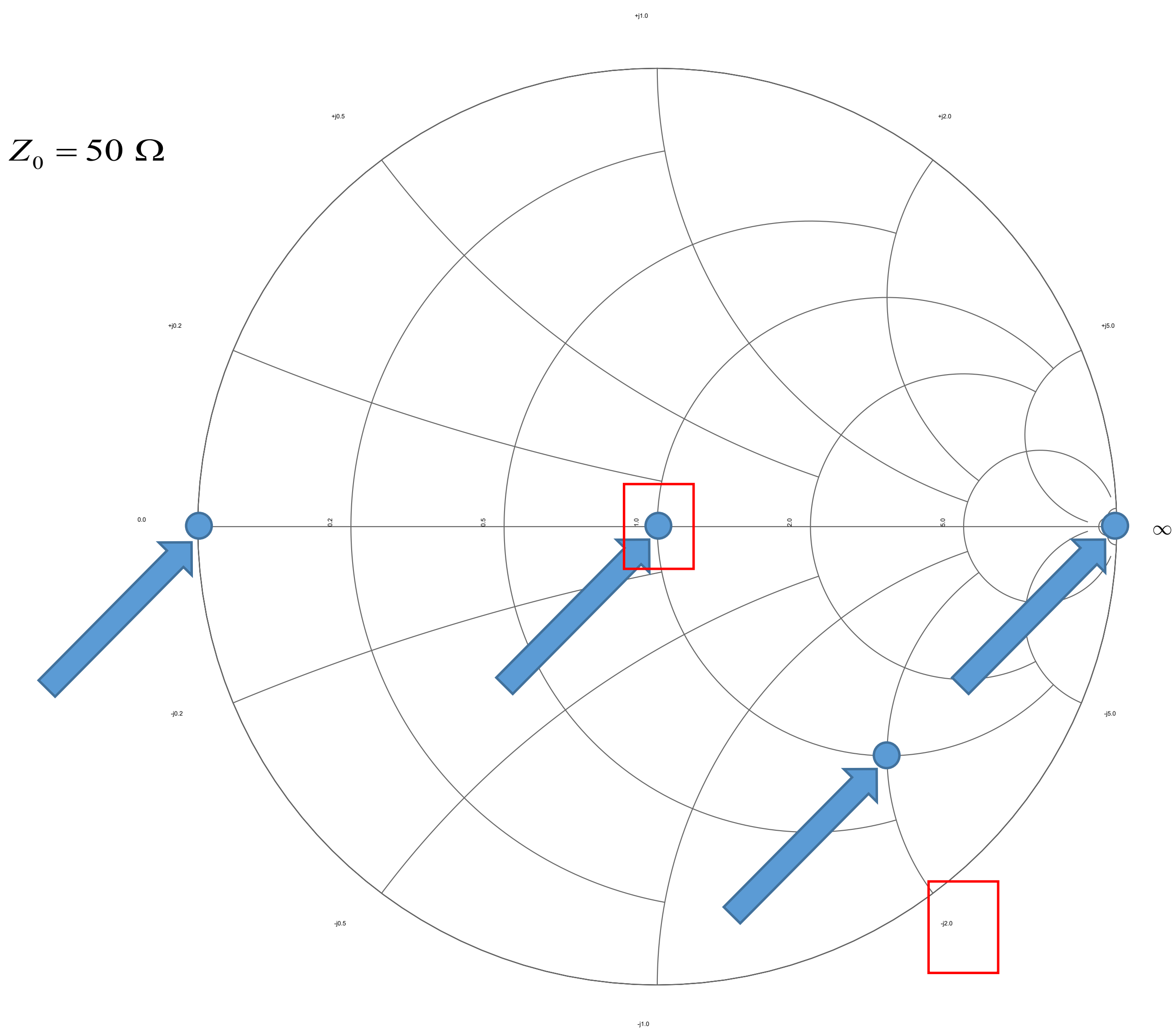
Some examples

$$Z_{in} = 0\ \Omega$$

$$Z_{in} = \infty\ \Omega$$

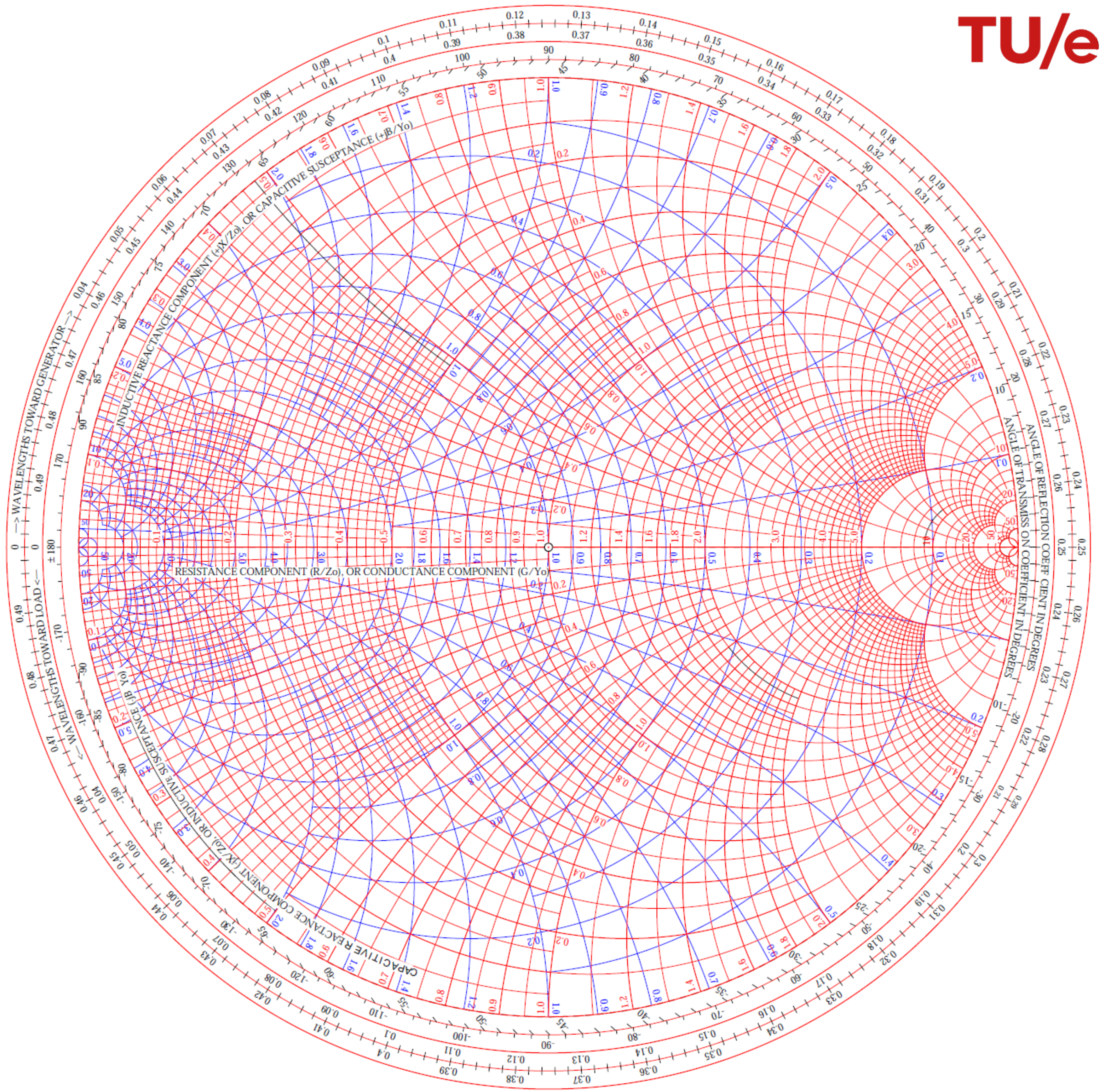
$$Z_{in} = 50\ \Omega$$

$$Z_{in} = \boxed{50} - \boxed{j100}\ \Omega$$



Smith chart

Combined impedance and admittance chart



Summary

- Smith chart is useful tool to visualize Γ and Z_{in}
- Used in CAD tools and measurement equipment
- We will use it for the design of matching circuits

Microwave Engineering and Antennas

Impedance matching with lumped elements

Bart Smolders, Professor

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Impedance matching with lumped elements

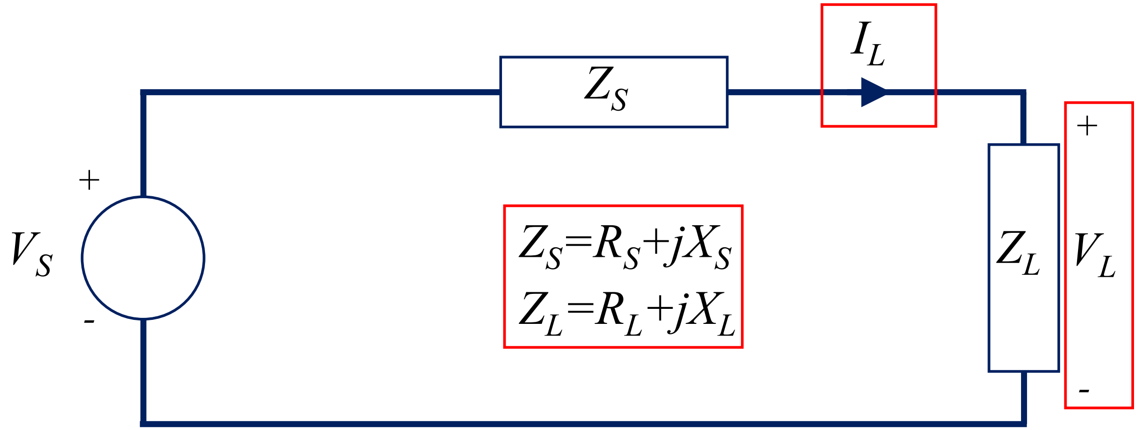
Objective of this lecture

- Explain need for impedance matching
- LC matching
- Show example using the Smith chart

Power matching

Time average power delivered to load

$$\left.\begin{aligned}
P_L &= \frac{1}{2}\mathrm{Re}\left\{V_L I_L^*\right\} \\
&\boxed{V_L = \frac{V_S Z_L}{Z_S + Z_L}} \\
&\boxed{I_L = \frac{V_S}{Z_S + Z_L}}
\end{aligned}\right\}$$

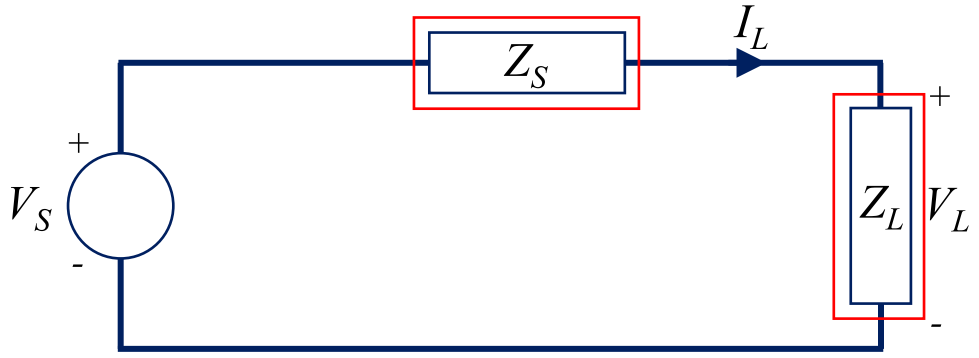


$$\Rightarrow P_L = \frac{1}{2}\mathrm{Re}\left\{\frac{|V_S|^2 Z_L}{|Z_S + Z_L|^2}\right\} = \frac{1}{2}\frac{|V_S|^2 R_L}{\boxed{(R_S + R_L)^2 + (X_S + X_L)^2}}$$

Power matching

Time average power delivered to load

$$P_L = \frac{1}{2}\frac{|V_S|^2 R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}$$

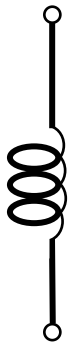


Maximum power is delivered when

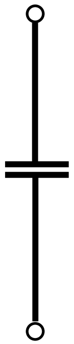
$$\frac{\partial P_L}{\partial R_L} = \frac{\partial P_L}{\partial X_L} = 0 \quad \Rightarrow \quad \begin{array}{l} R_L = R_S \\ X_L = -X_S \end{array} \quad \Rightarrow \quad \boxed{Z_L = Z_S^*}$$

Matching circuit

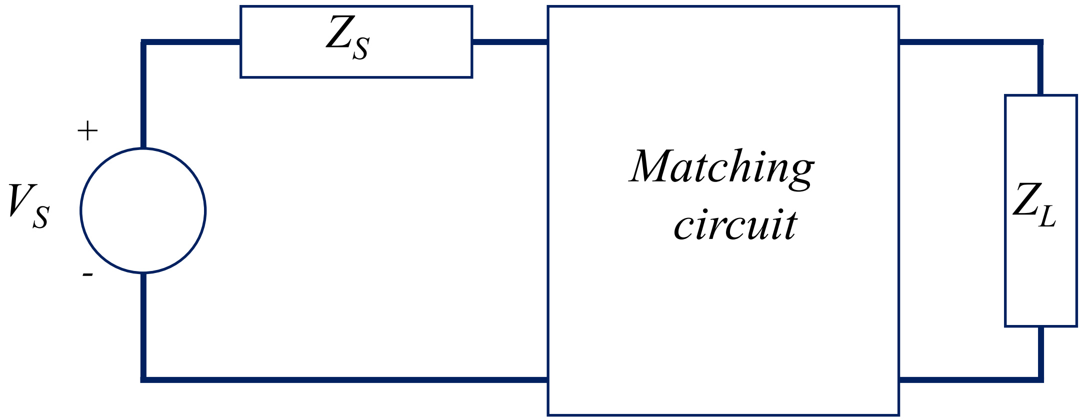
Lumped matching using L and C



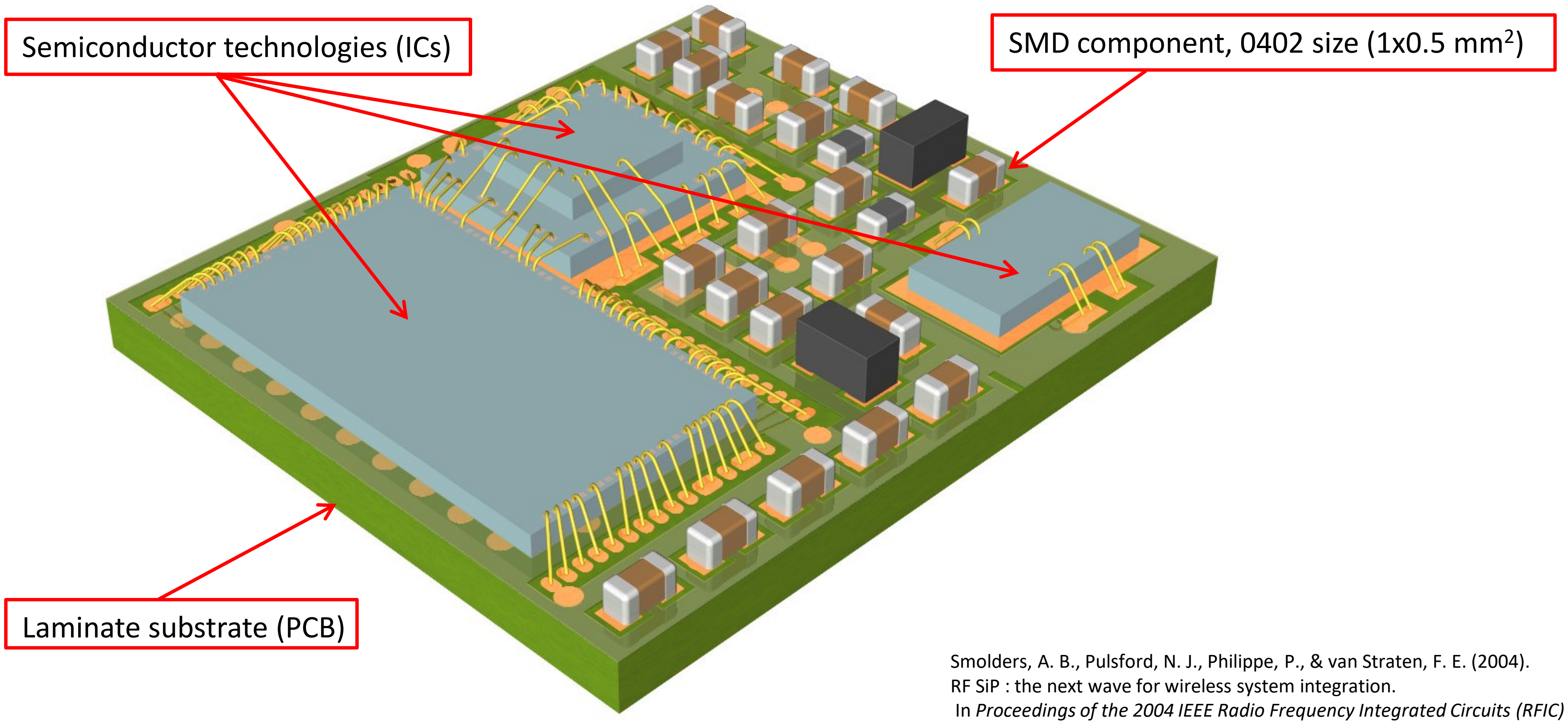
$$Z = j\omega L$$



$$Z = \frac{1}{j\omega C}$$



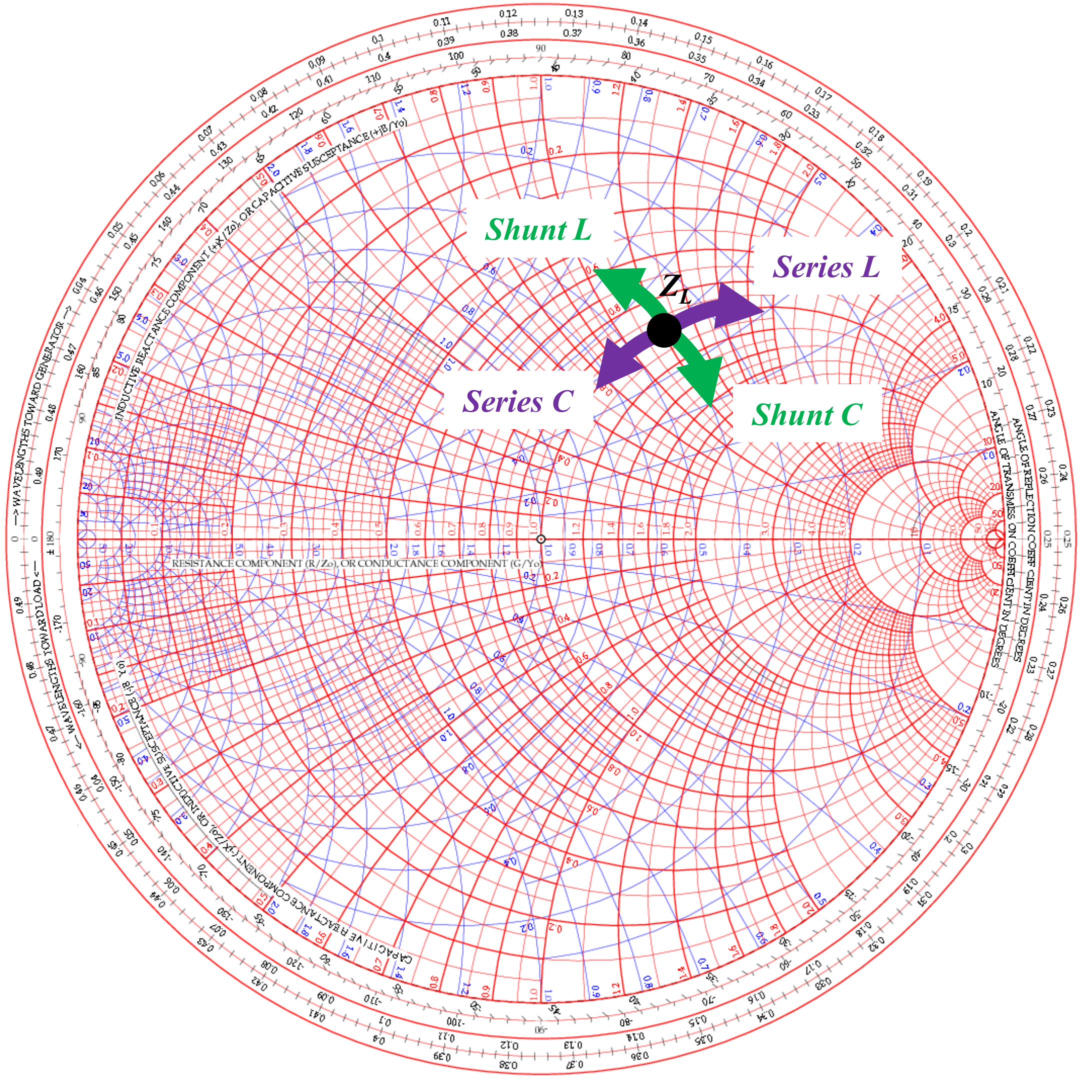
When can we use lumped element matching?

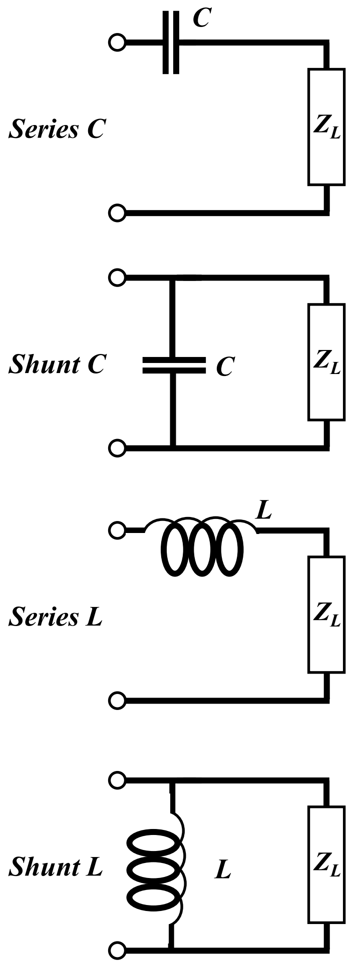


Smolders, A. B., Pulsford, N. J., Philippe, P., & van Straten, F. E. (2004).
RF SiP : the next wave for wireless system integration.
In *Proceedings of the 2004 IEEE Radio Frequency Integrated Circuits (RFIC) Symposium*, pp. 233-236, 2004

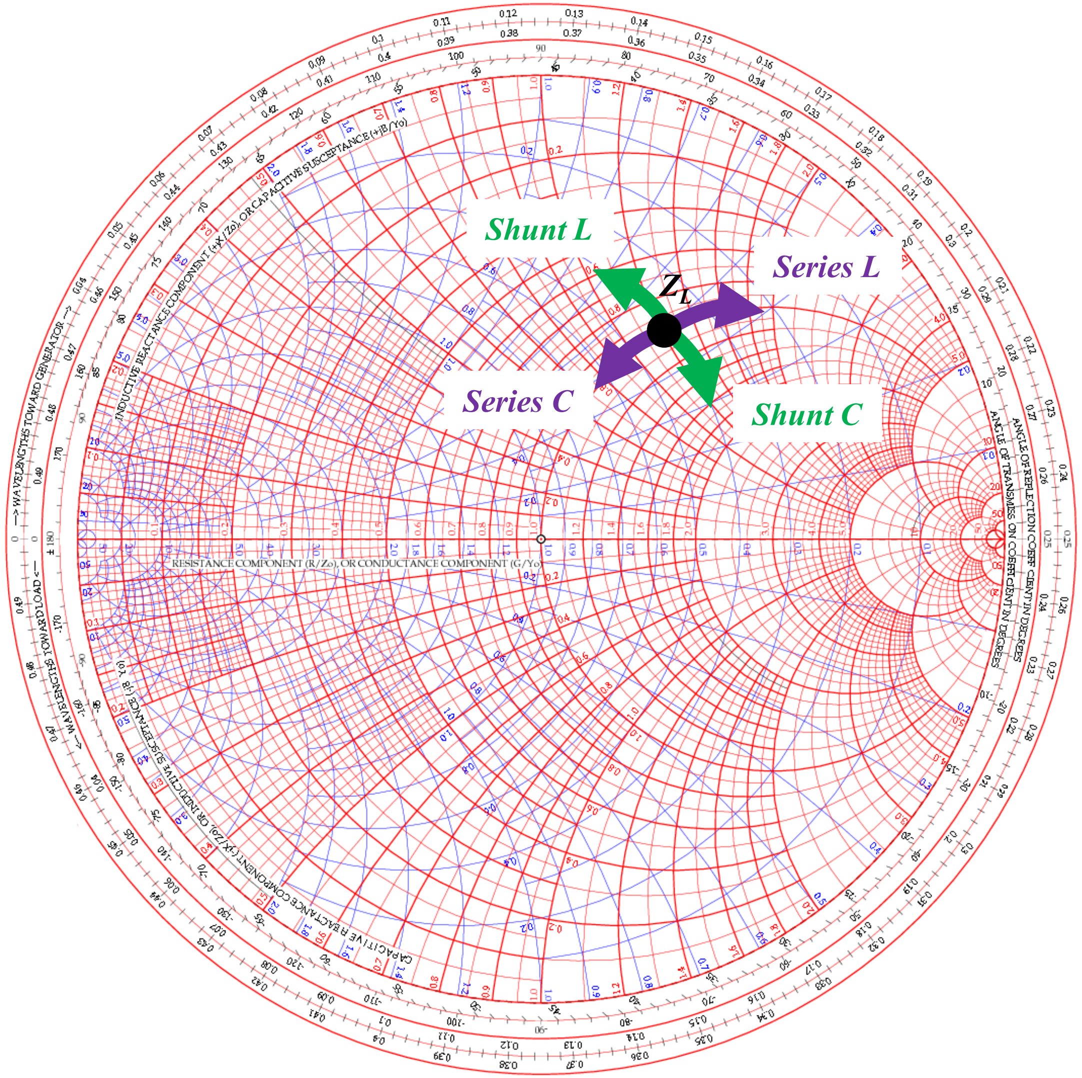
Wavelength must be much larger than size component

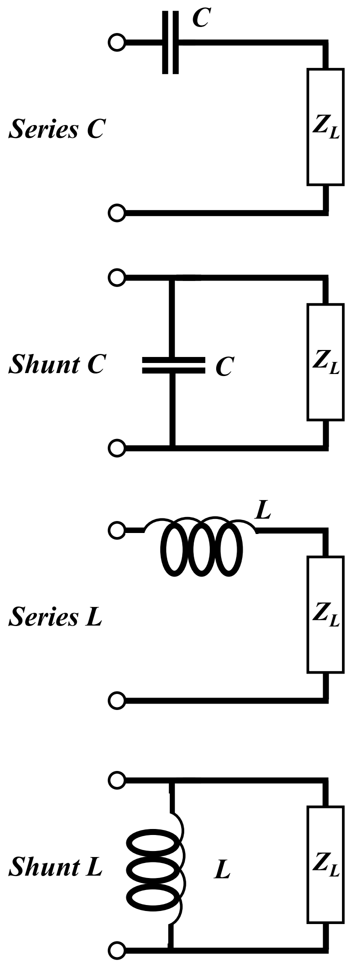
LC-matching using Smith chart



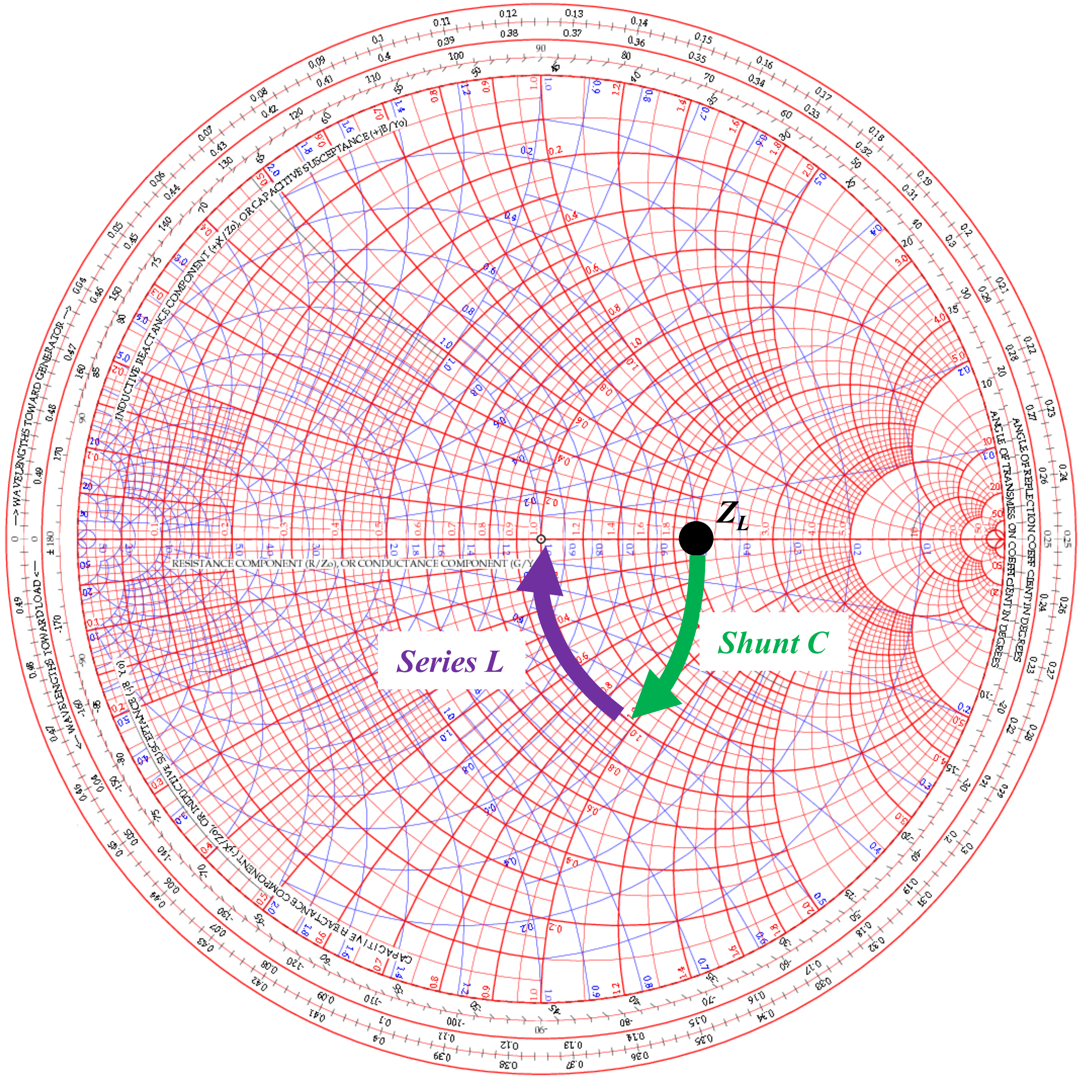


LC-matching using Smith chart

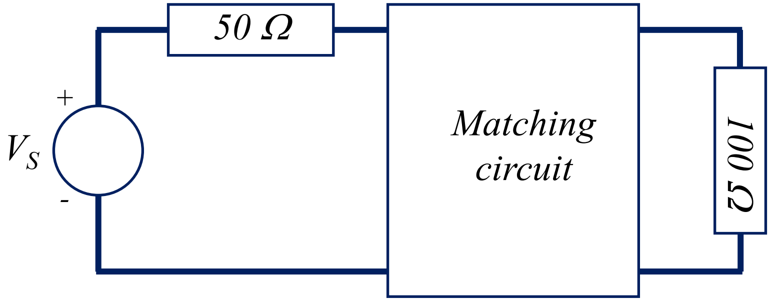




Example LC-matching



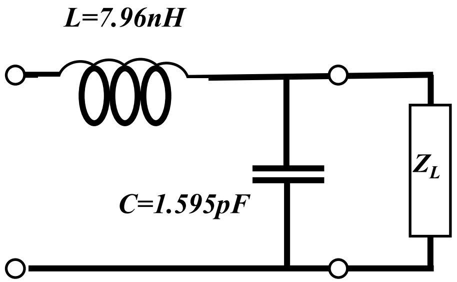
Design matching circuit to match a 100 Ohm load to a 50 Ohm source at f=1 GHz



Step 1: Add Shunt C

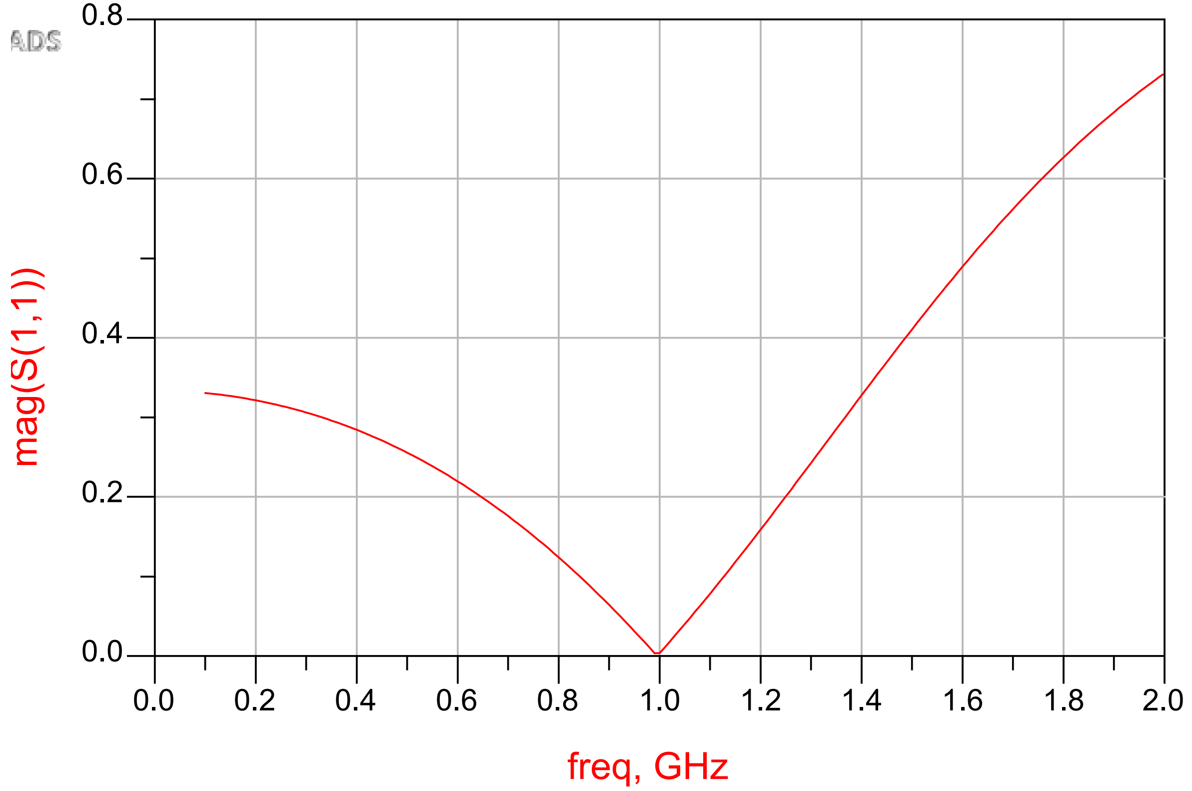
Step 2: Add Series L

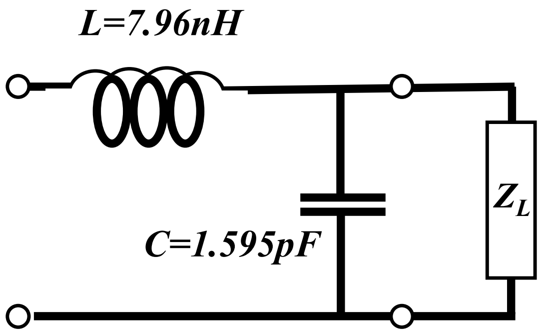
Realized matching circuit:



Example LC-matching

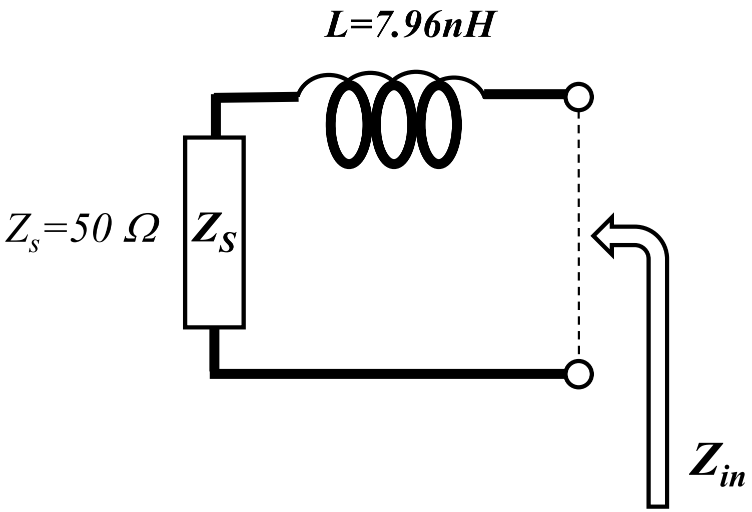
Frequency response realized matching circuit

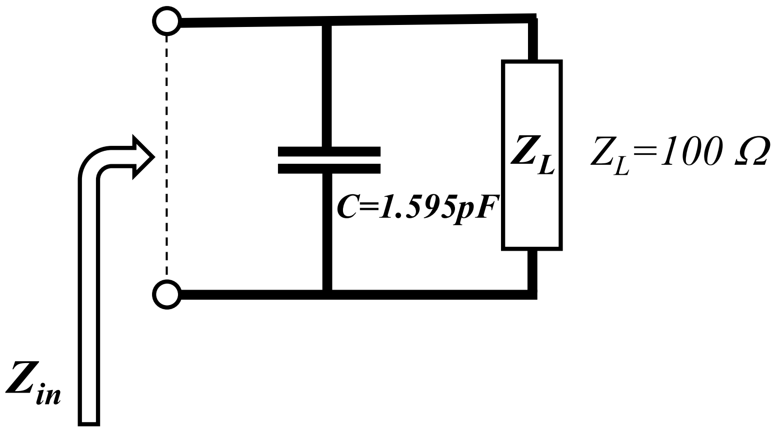




Example LC-matching

What about conjugate matching?





$Z_{in}=50 + j50\ \Omega$

$Z_{in}=50 - j50\ \Omega$

Summary

- Lumped-element matching when size components << λ
- Smith chart is a useful tool for design of matching circuits
- Example LC matching
- Next web lecture we will extend concept to matching with transmission lines.

Microwave Engineering and Antennas

Impedance matching with distributed elements

Bart Smolders, Professor

Department of Electrical Engineering

Center for Wireless Technology Eindhoven

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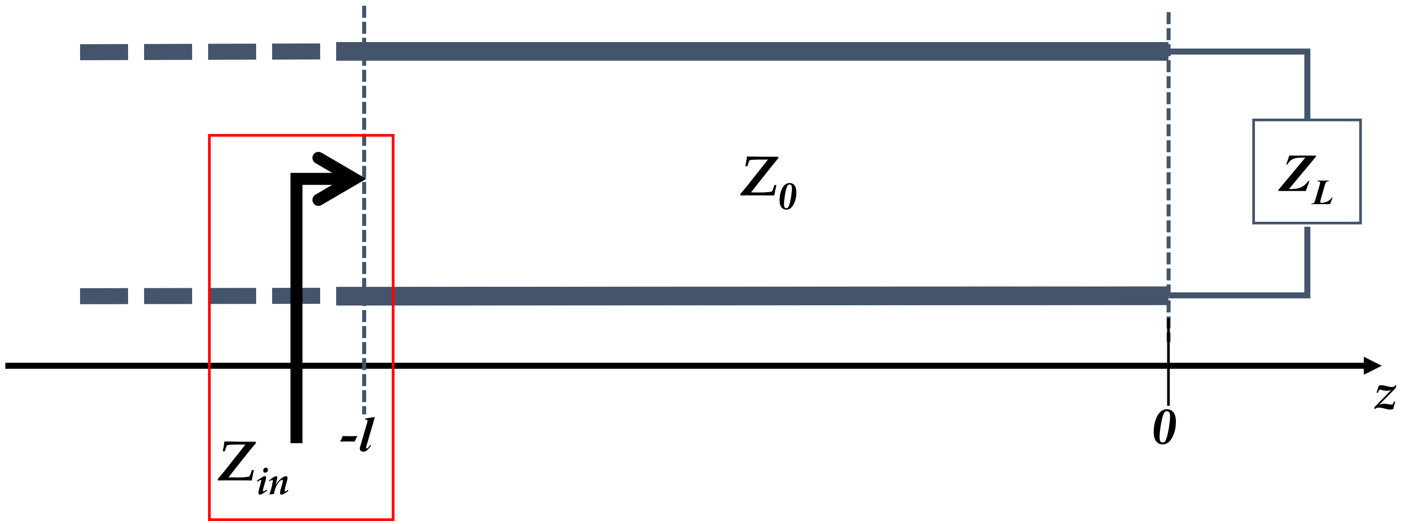
Impedance matching with distributed elements

Objective of this lecture

- Richards transformation
- Shunt-stub matching
- Example

The terminated transmission line



Input impedance at z=-l:

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

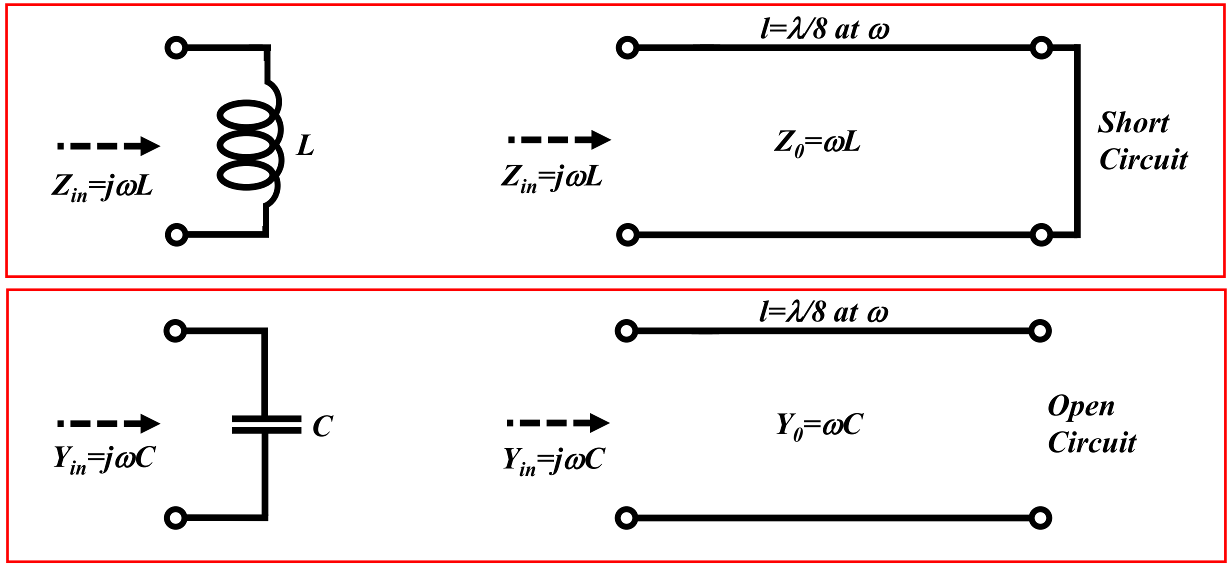
Impedance tuner

Richards' transformation

Special cases are short circuit and open terminations:

$$Z_{in}(Z_L = 0) = jZ_0 \tan\beta l = j\Omega Z_0 = \boxed{j\omega L}$$

$$Z_{in}(Z_L = \infty) = \frac{Z_0}{j\tan\beta l} = \frac{Z_0}{j\Omega} = \boxed{\frac{1}{j\omega C}}$$

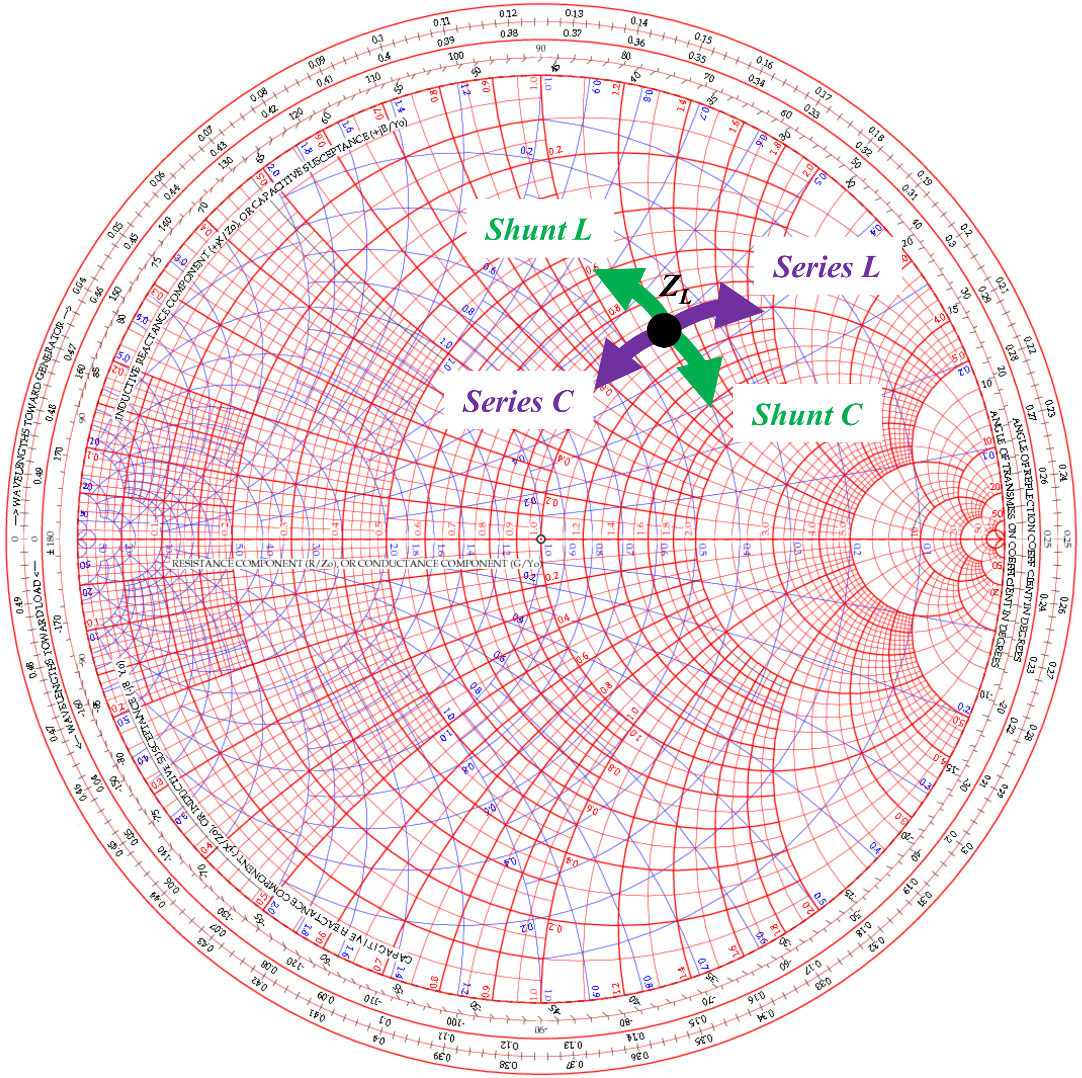


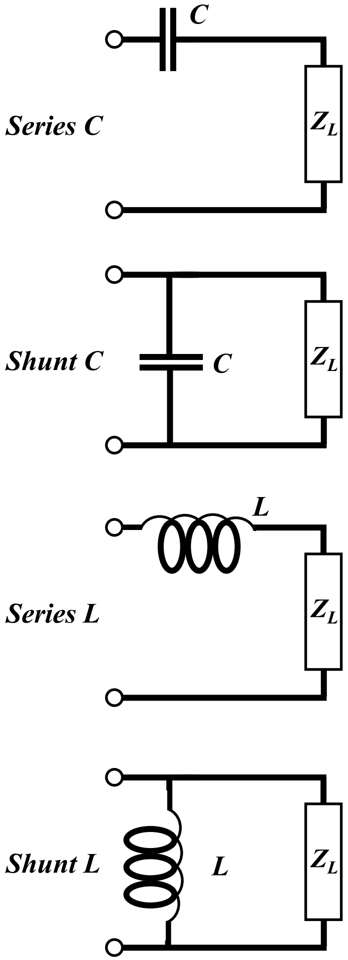
$$\beta = \frac{2\pi}{\lambda}, \text{ with } l = \lambda/8 \text{ we get:}$$

$$\Omega = \tan\beta l = \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{8}\right)$$

$$= \tan\left(\frac{\pi}{4}\right) = 1$$

Equivalent to LC-matching using Smith chart





Single shunt-stub matching

Terminated line

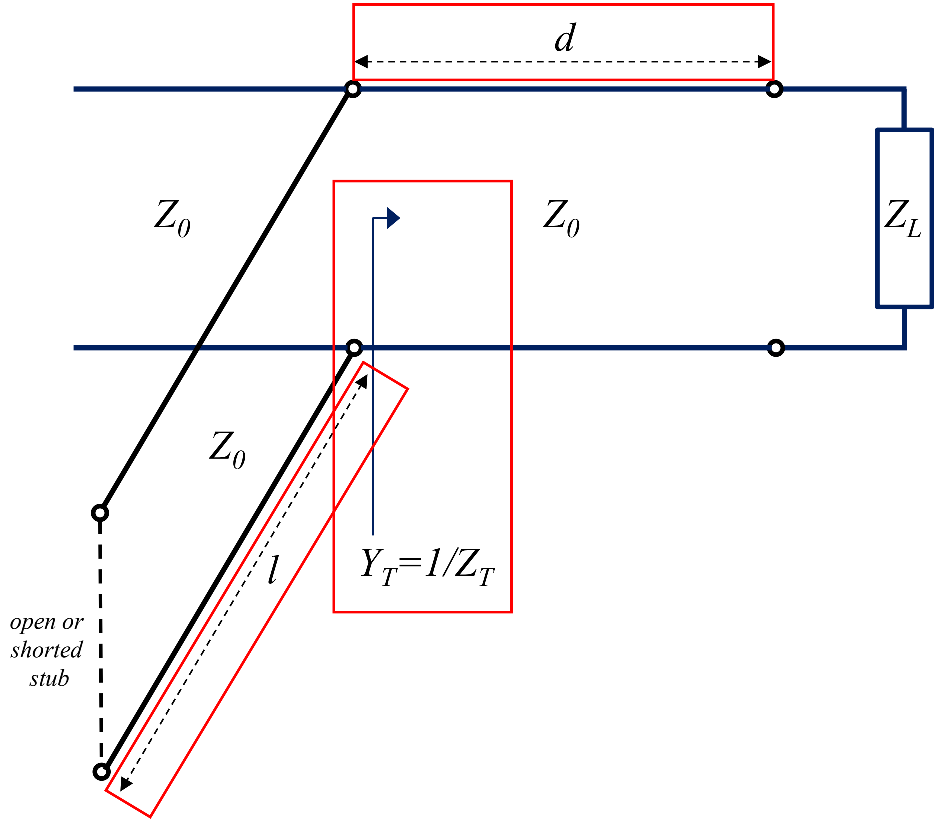
$$Y_T = \frac{1}{Z_T} = \boxed{G_T} + \boxed{jB_T}$$

$$Z_T = Z_0 \frac{Z_L + jZ_0 \tan \beta d}{Z_0 + jZ_L \tan \beta d}$$

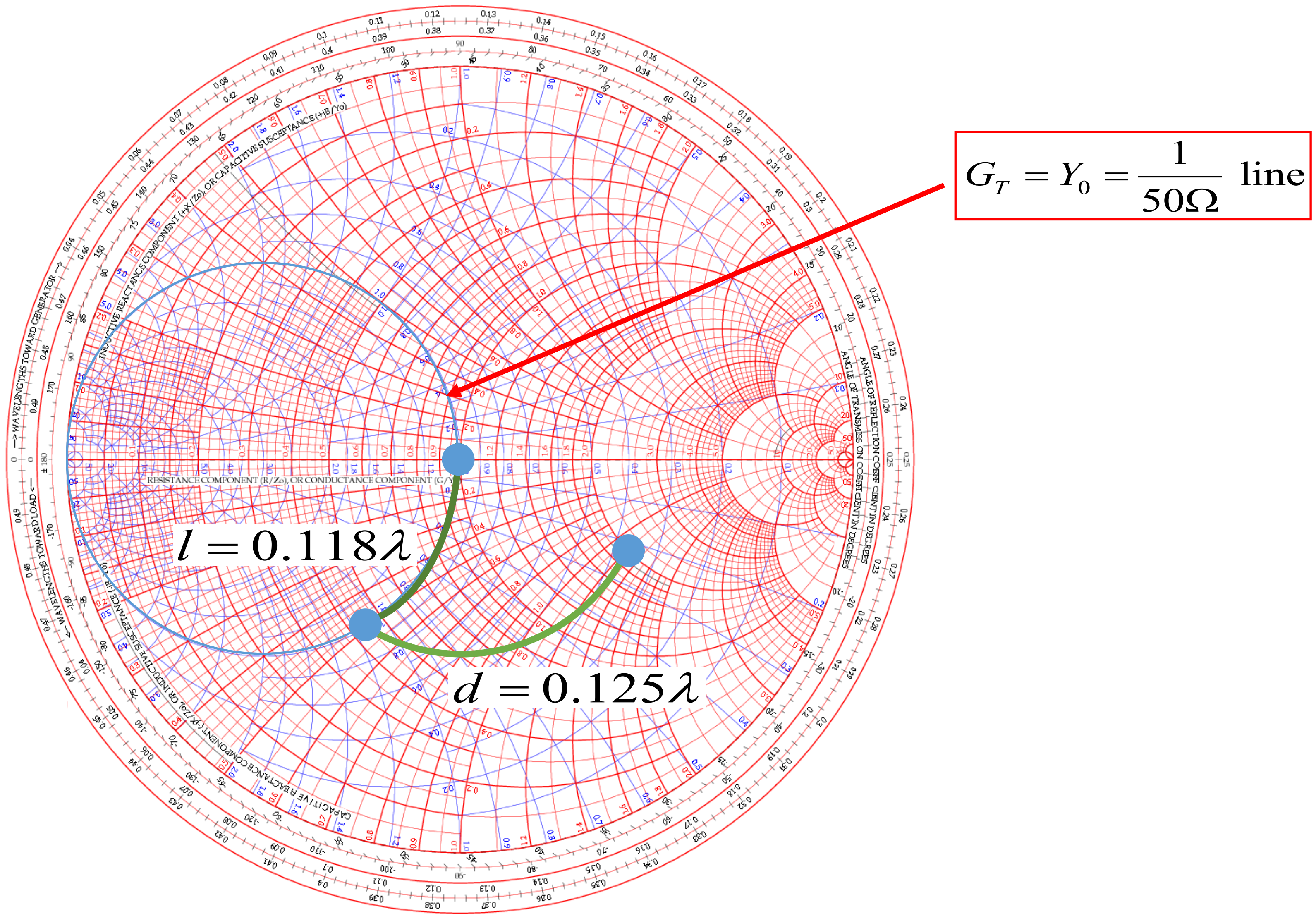
Matching with stub

$$G_T = Y_0 = \frac{1}{Z_0}$$ ➡ determine d

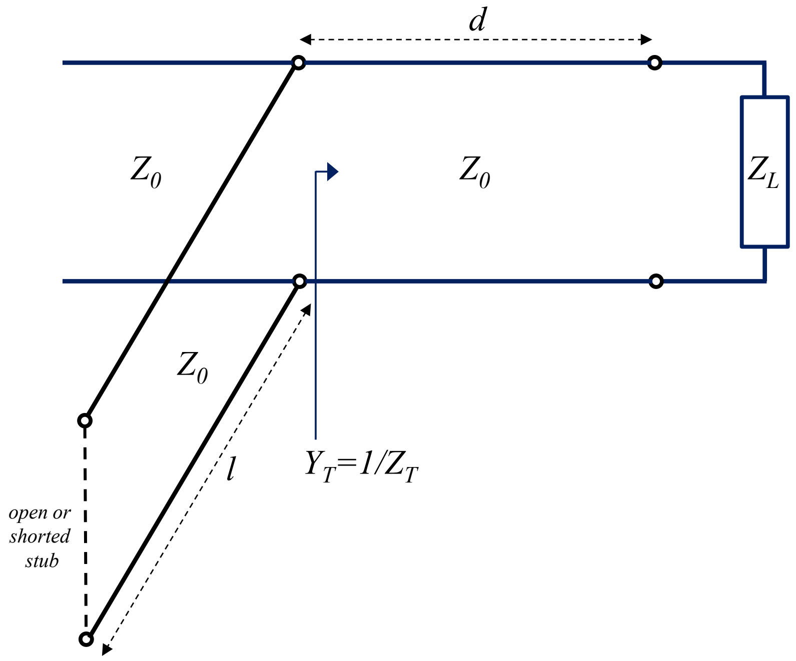
$$Y_s = -jB_T$$ ➡ determine l



Example single shunt-stub matching

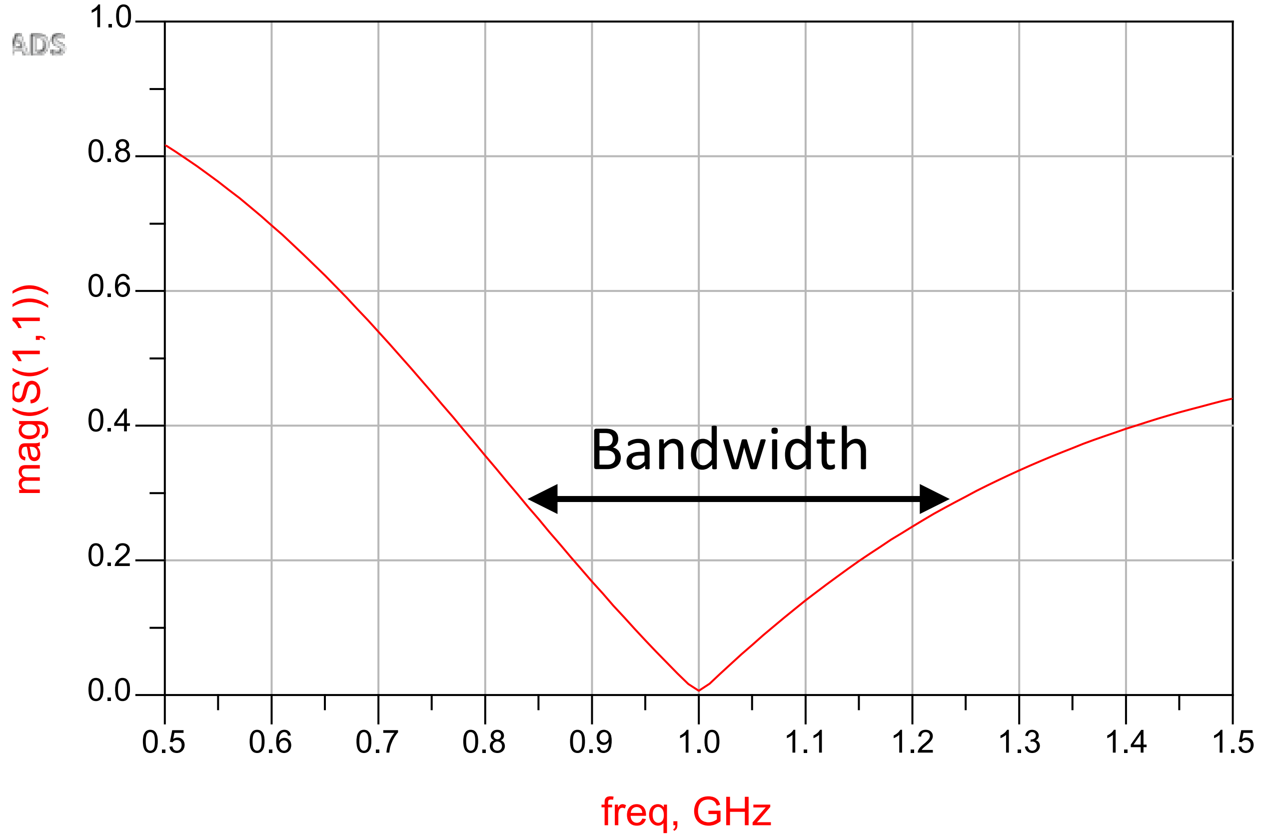


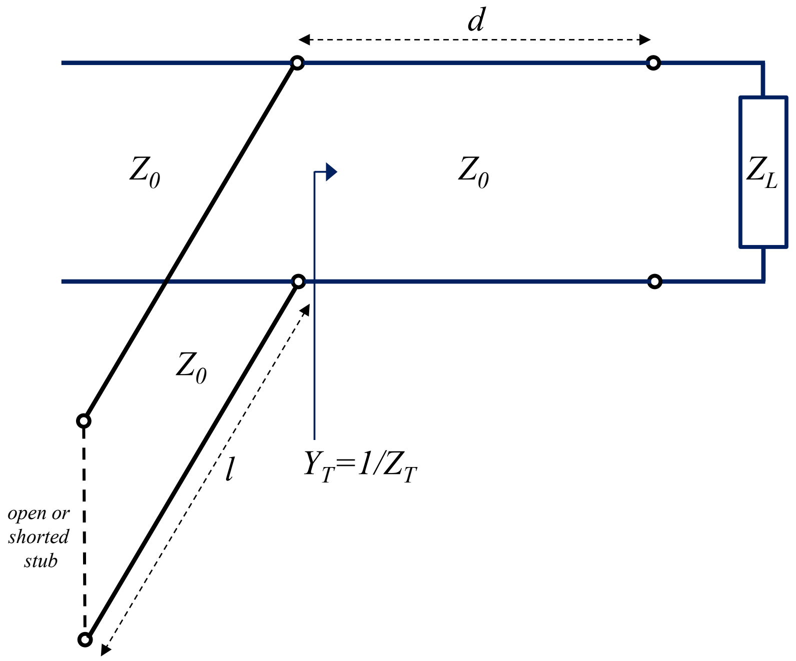
For a load impedance Z_L=100-j60 Ω, design a single-stub shunt matching network to match a Z_0=50 Ω line.



Frequency response

Design frequency 1 GHz





$Z_L = 100 - j60 \ \ \Omega$

$Z_S = 50 \ \ \Omega$

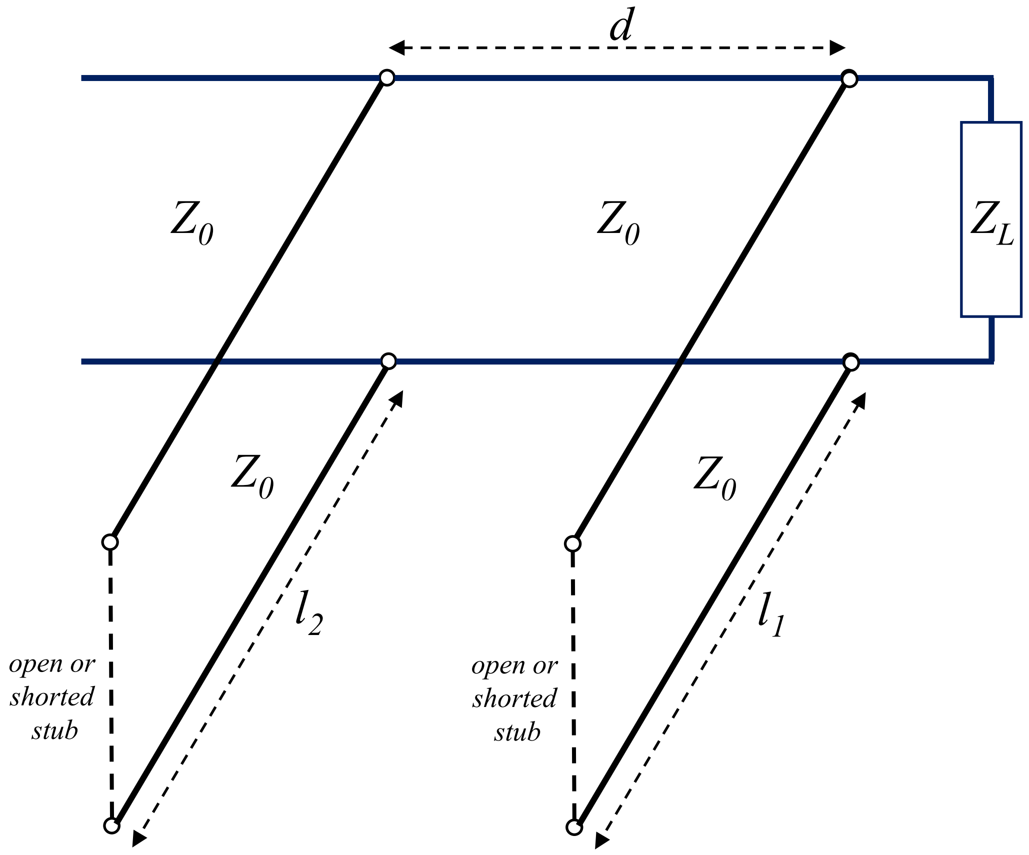
$d = 0.125\lambda$

$l = 0.118\lambda$

Double shunt-stub matching

Features

- Fixed line between stubs
- Easier to implement
- Limited in matching range



Summary

- Transmission line equivalence of LC components
- We can add additional line sections
- Shunt-stub matching
- Smith chart is useful tool

Microwave Engineering and Antennas

Microwave Filters

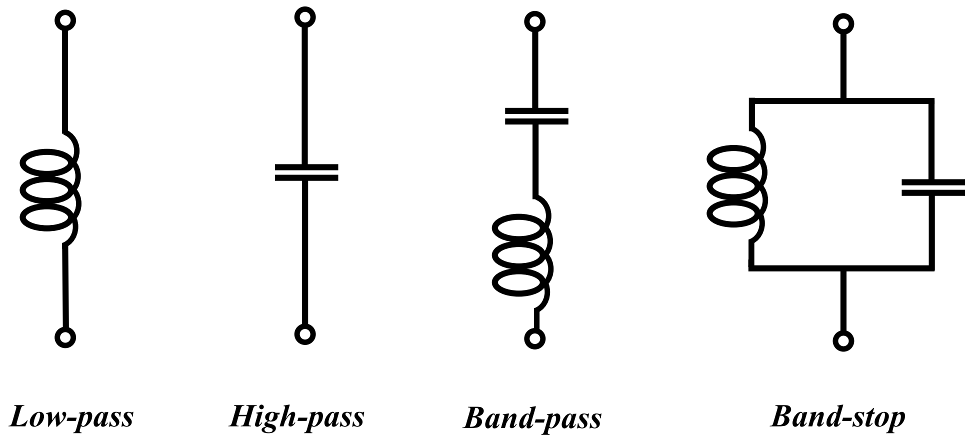
Bart Smolders, Professor
Department of Electrical Engineering
Center for Wireless Technology Eindhoven

Microwave filters

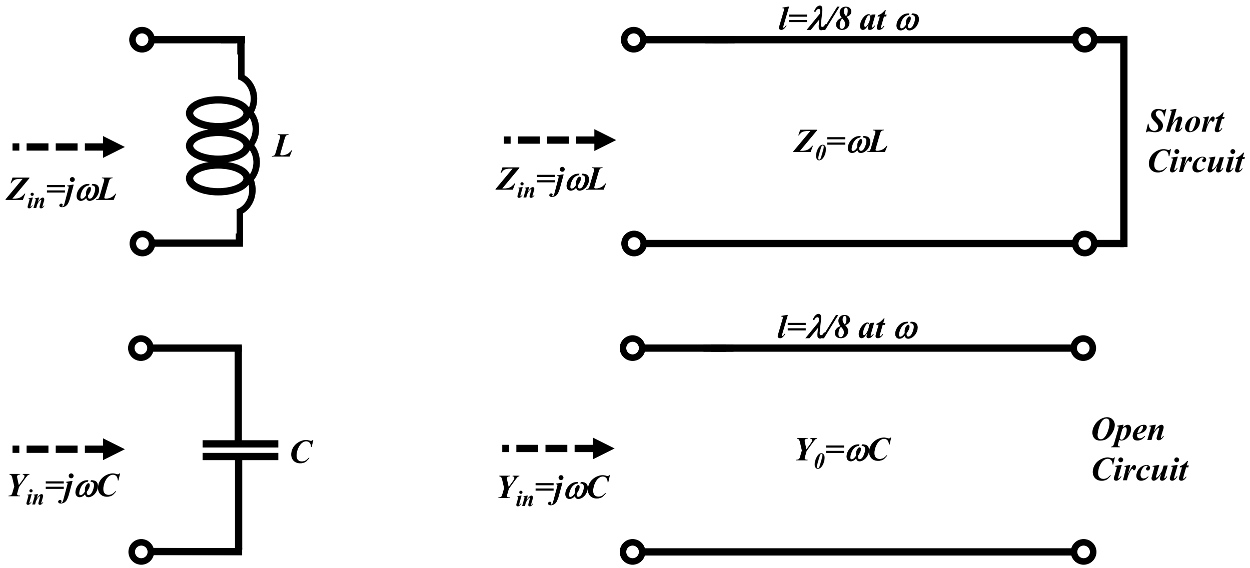
Objective of this lecture

- Lumped-element filters
- Kuroda's identities
- Low-pass filters
- Coupled-line bandpass filters

Lumped-element filter types

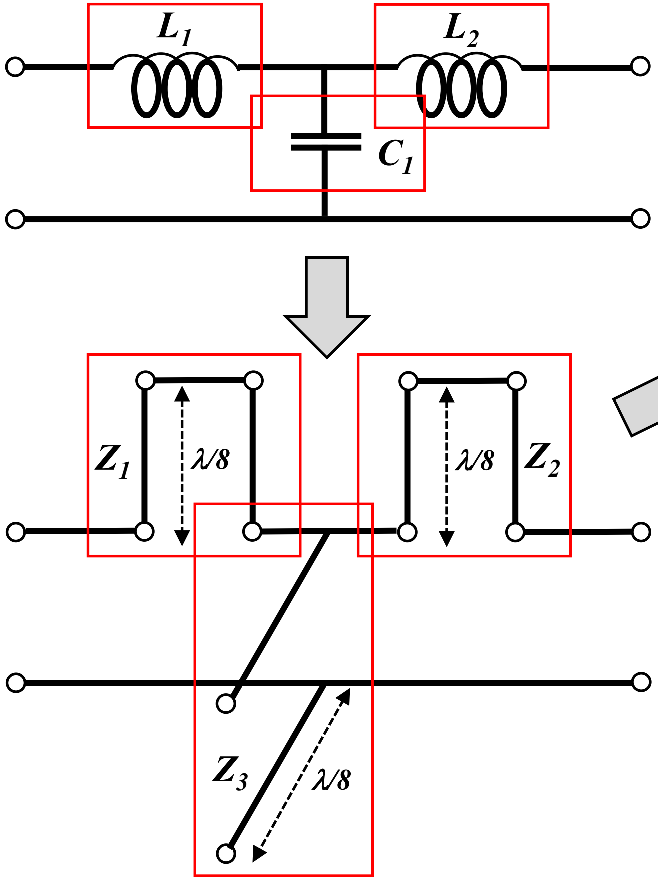


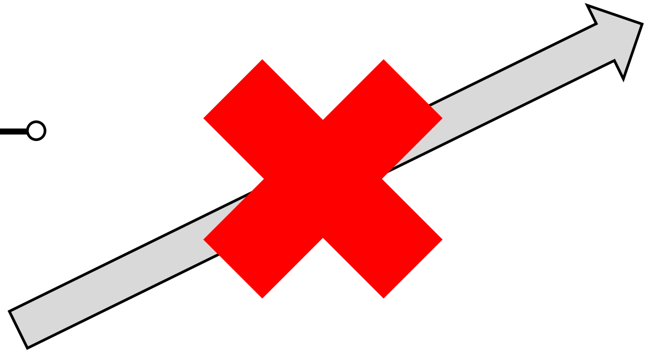
Richards' transformation:

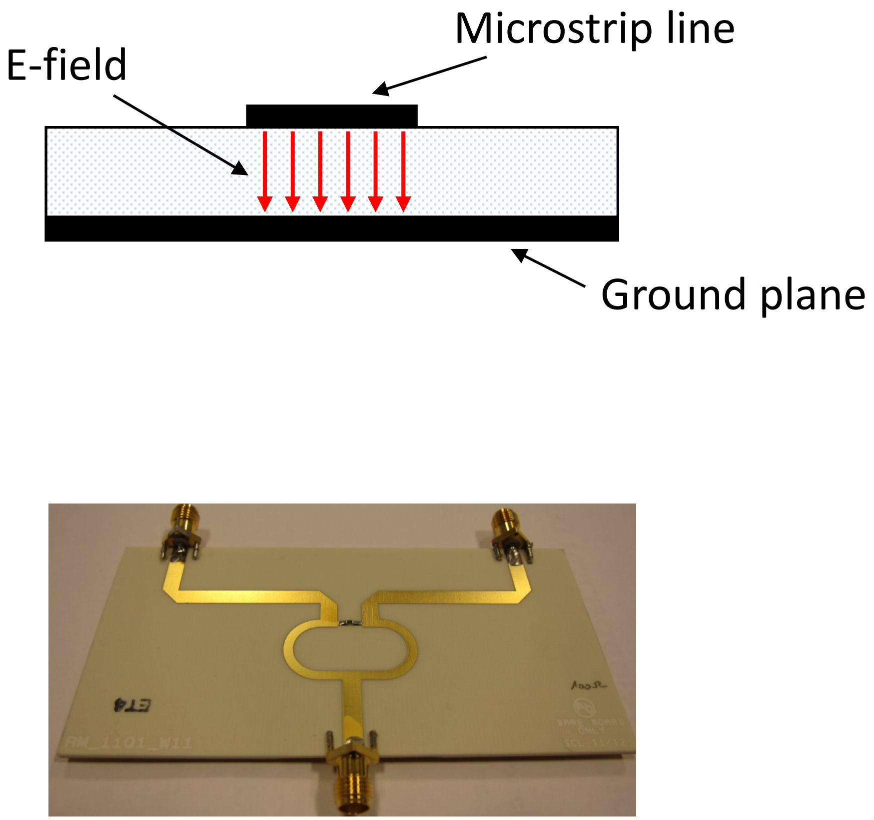


Problem in conversion to Tlines

Low-pass prototype filter

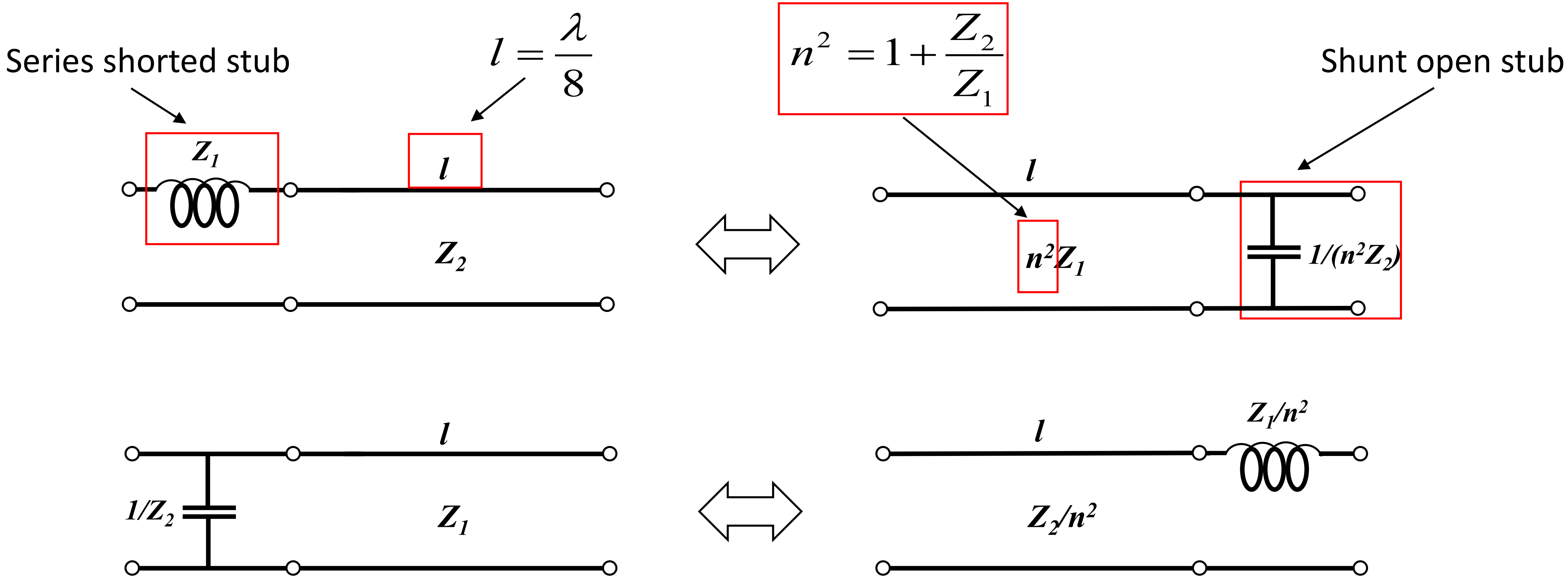






Kuroda’s identities

Transforms series stubs into parallel stubs

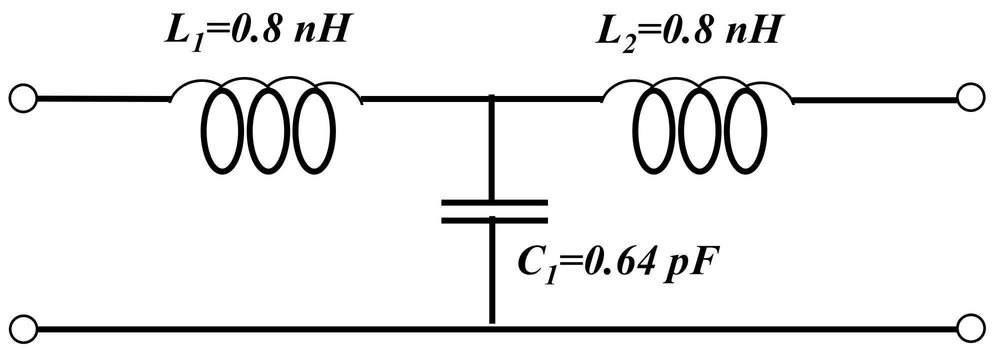


Design of a microstrip low-pass filter

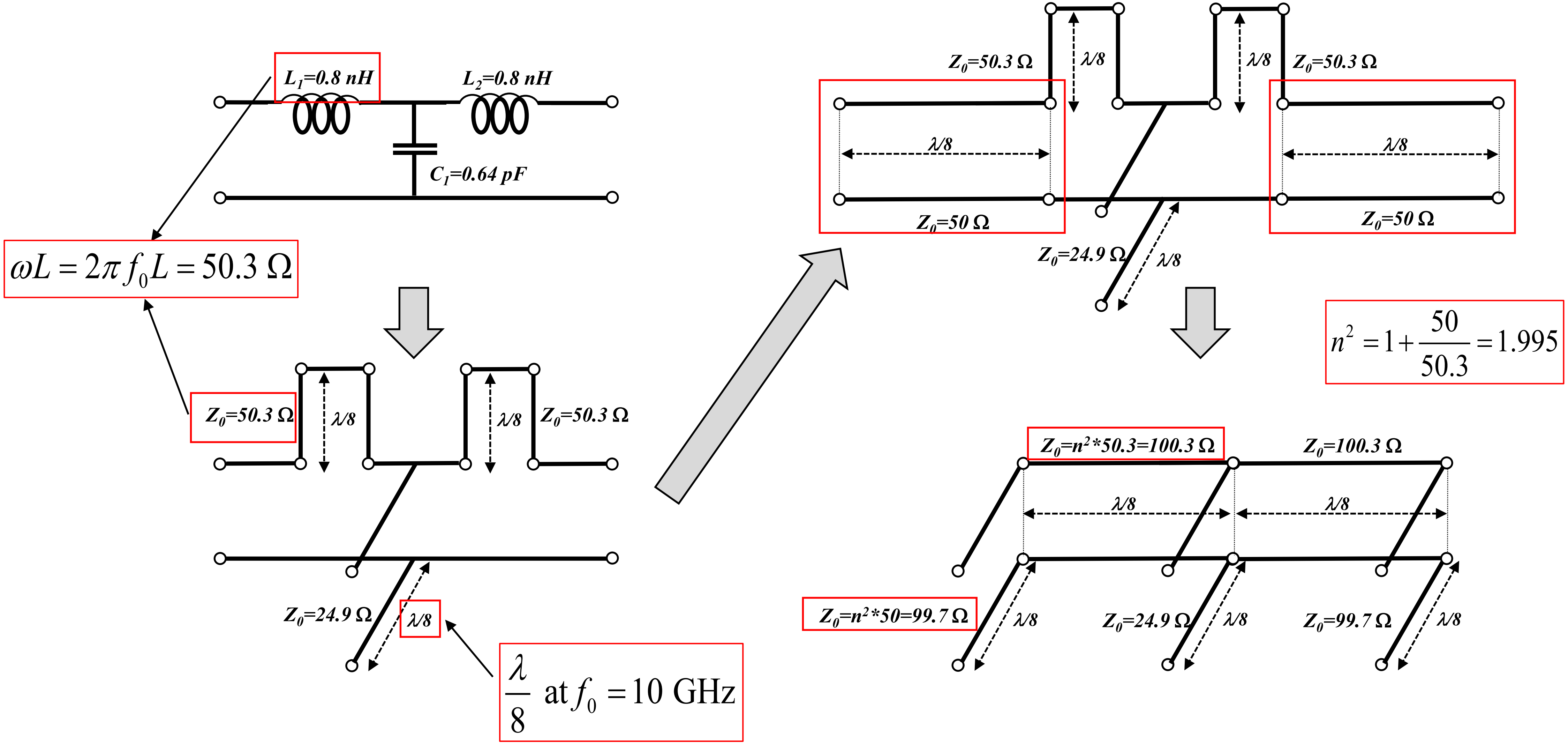
Specifications

- Maximally-flat
- Third order, N=3
- $f_{-3\mathrm{dB}} = f_0$ = 10 GHz
- Reference impedance Z_0=50 Ω
- Input/output ports 50 Ω

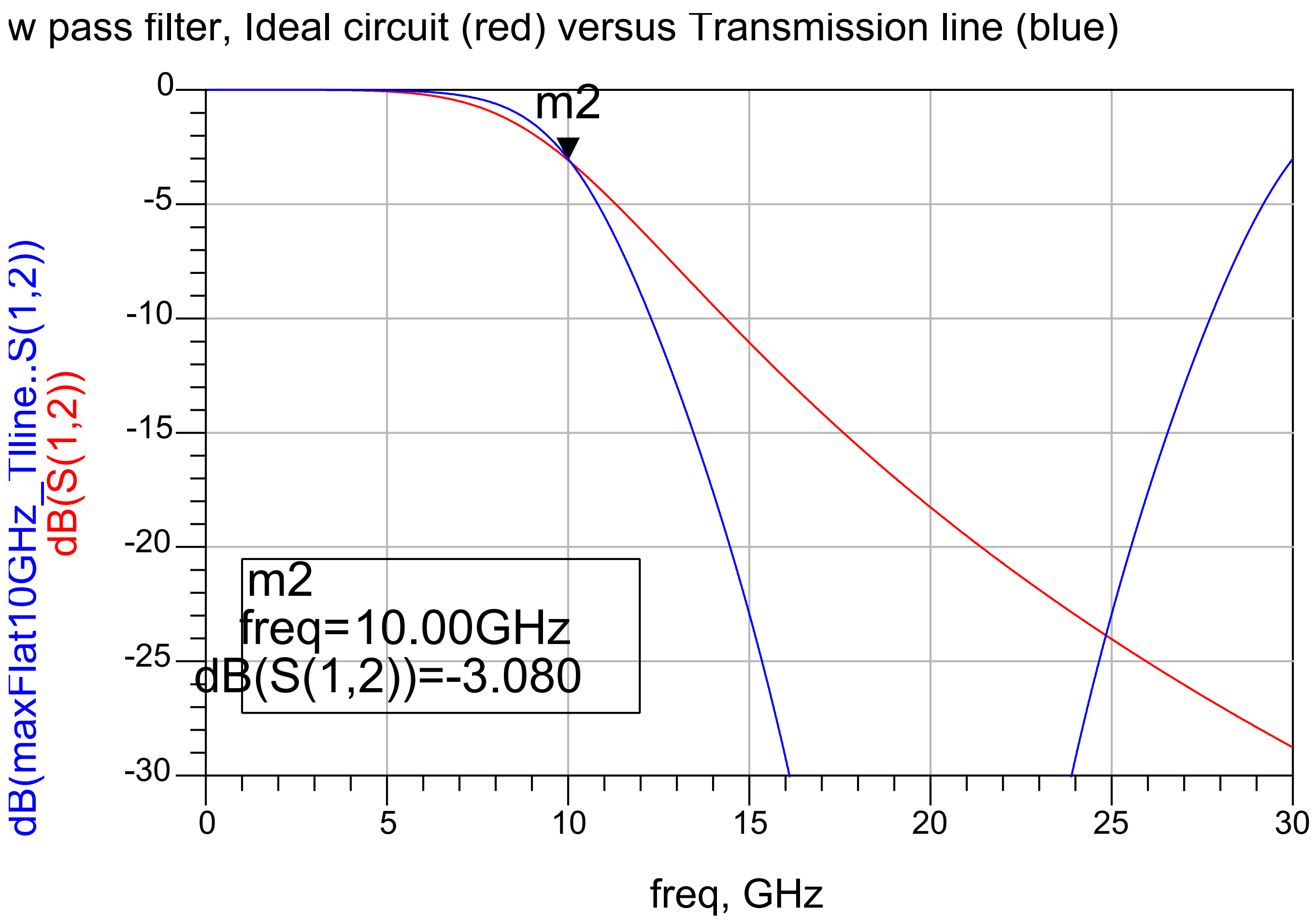
Step 1: Lumped-element prototype



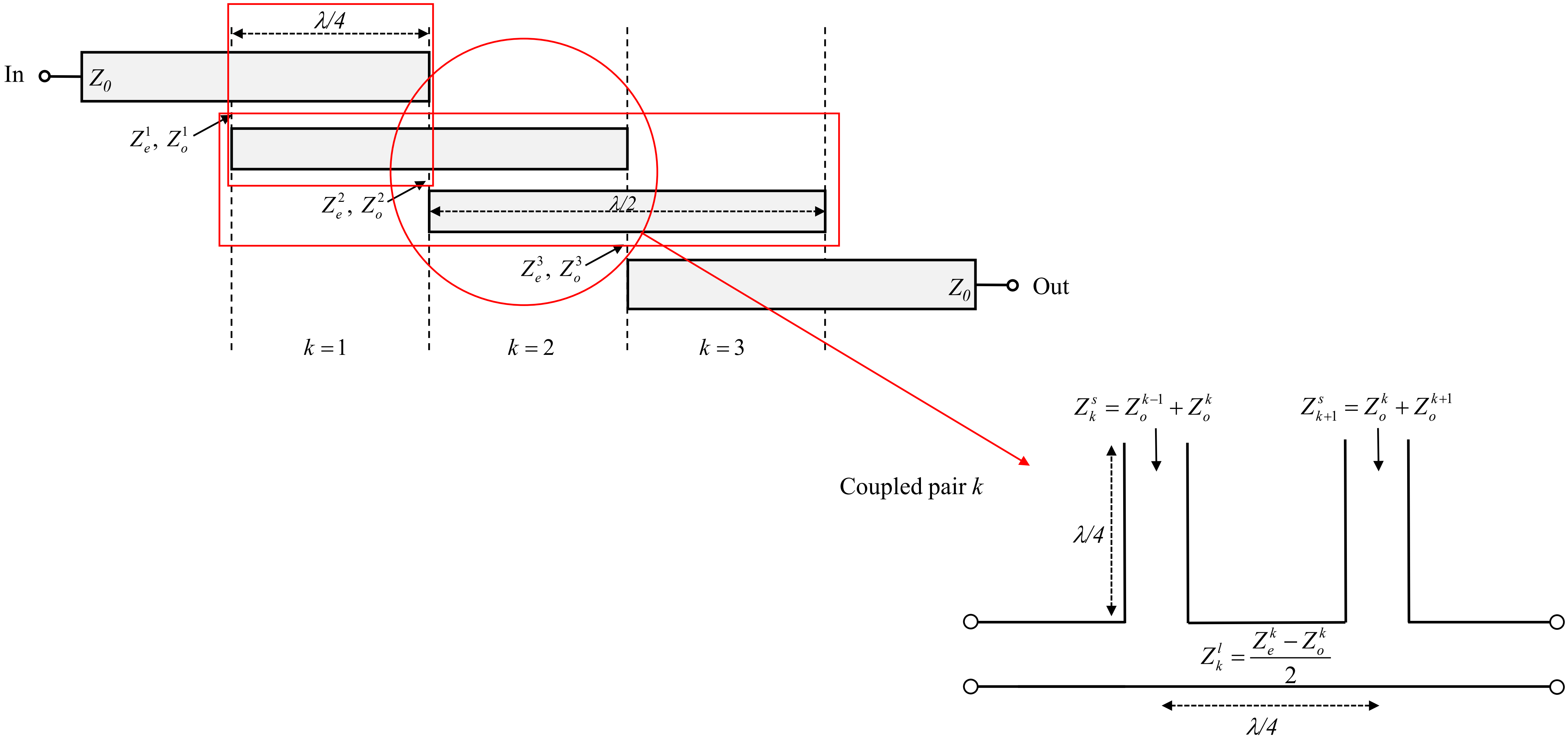
Design of a microstrip low-pass filter



Frequency response low-pass filter



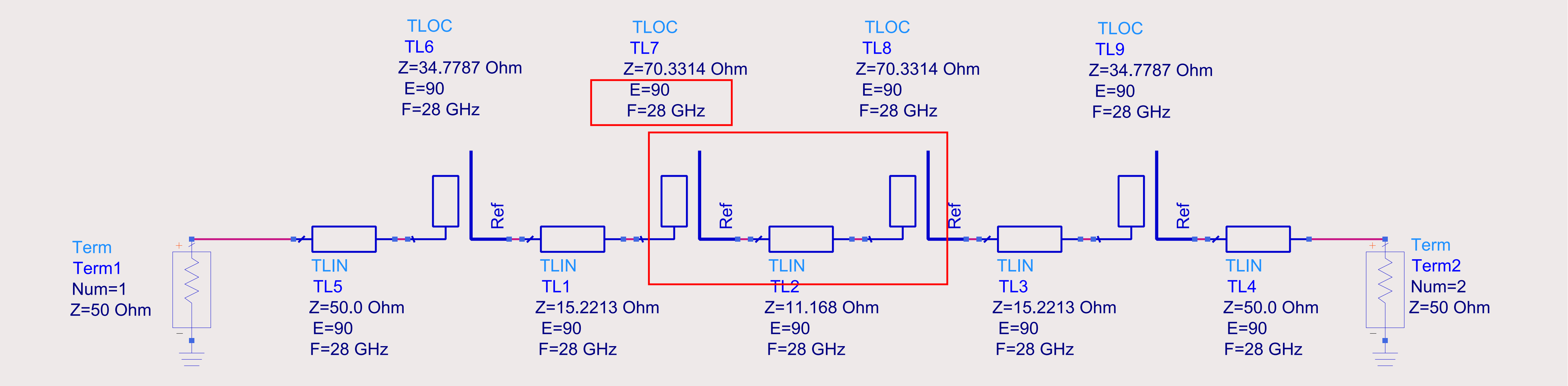
Coupled-line filter



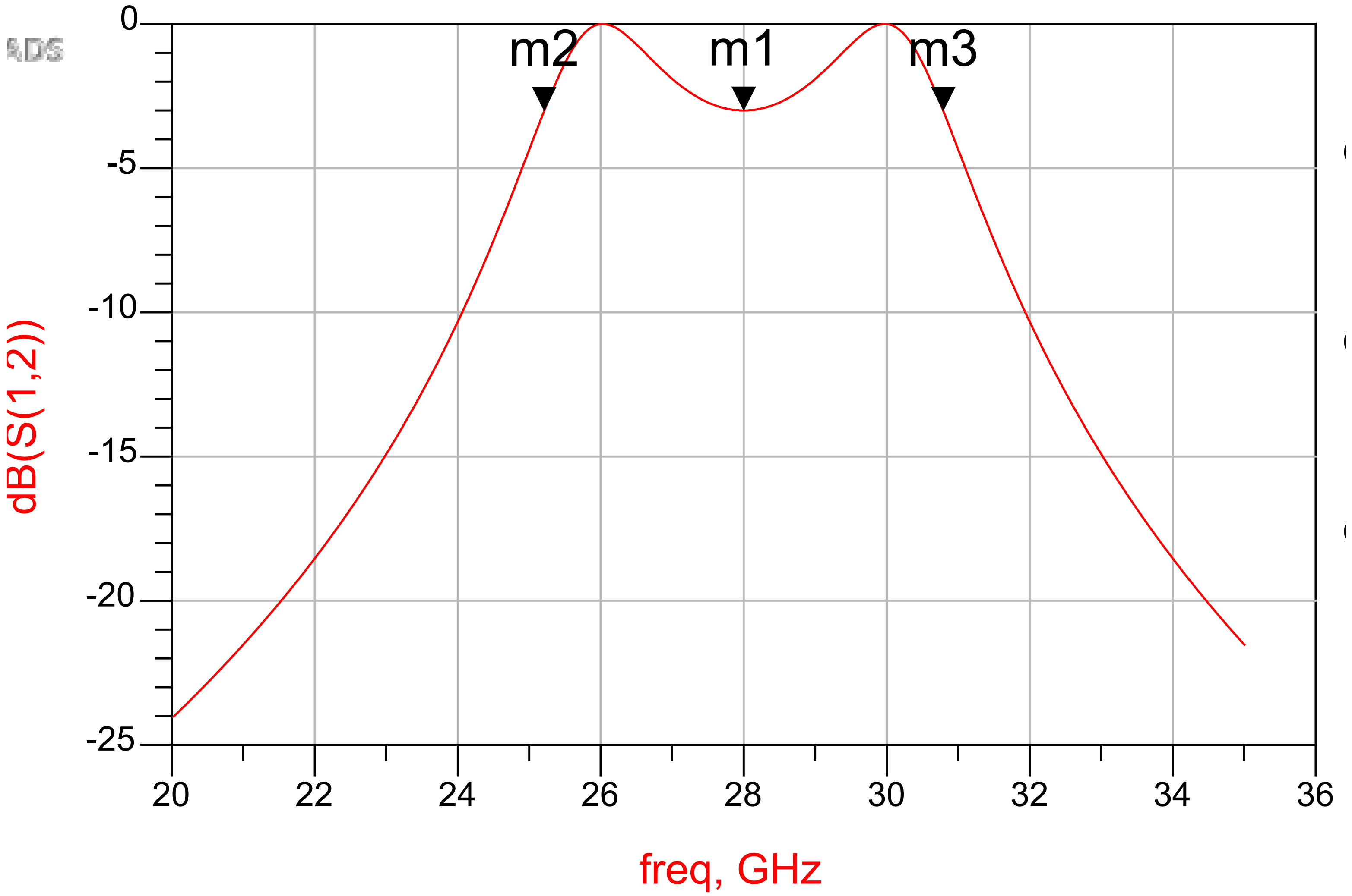
Coupled-line filter, example

Specifications

- Chebyshev equal ripple, 3dB in passband
- Two resonators, $N=2$
- f_0= 28 GHz, 20% bandwidth



Coupled-line filter, example



Summary

- Start with low-pass lumped element prototype
- Use Richard’s transformation and Kuroda’s identities.
- Discussed examples of low-pass and band-pass filter.
- Check the book for design equations

Microwave Engineering and Antennas

Introduction of Antennas

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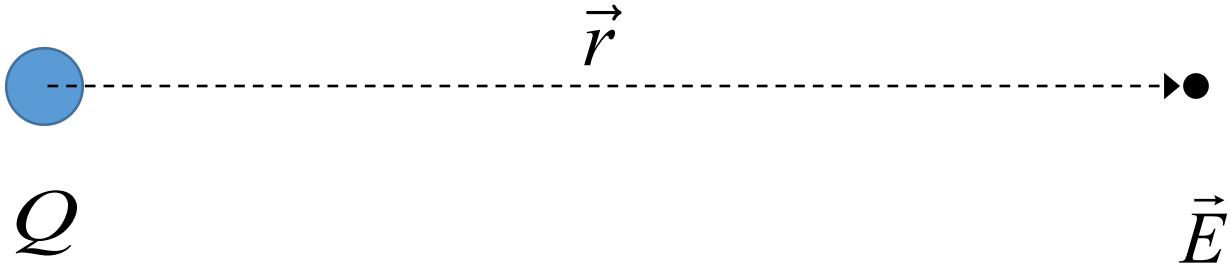
Introduction of Antennas

Topics of this lecture

- Electromagnetic radiation
- History of antennas
- Examples of antennas
- Coordinate system
- Field regions

Electromagnetic radiation

Electric field due to a <u>static</u> charge

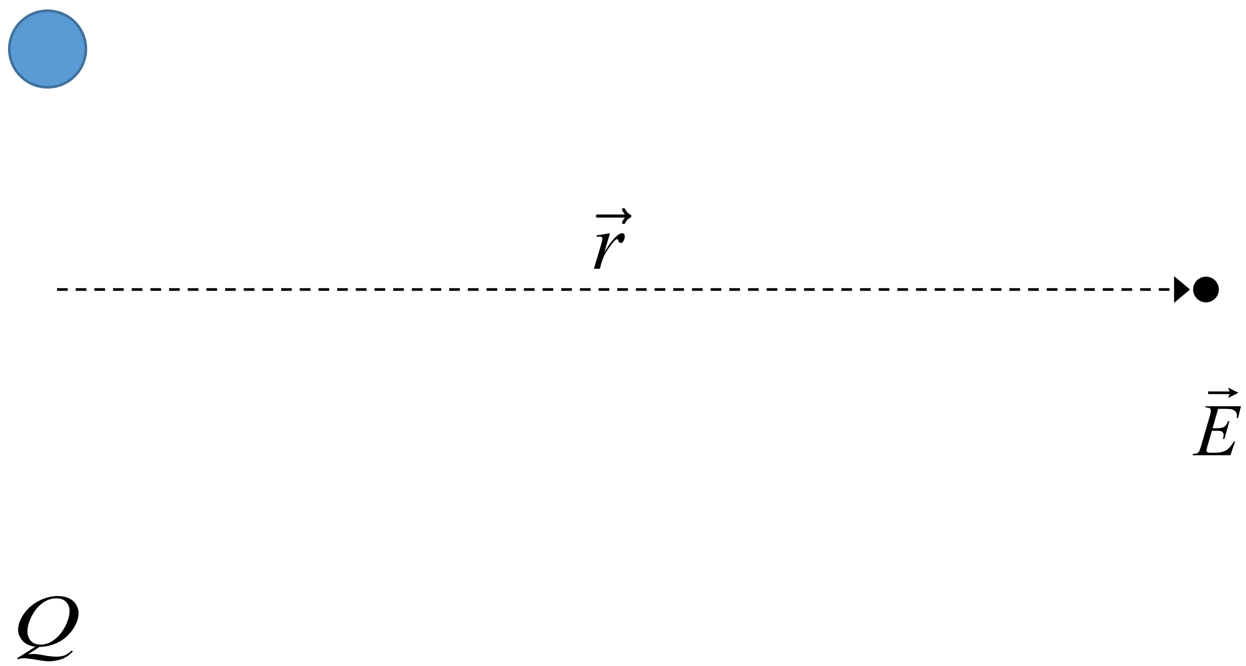


$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 \boxed{r^2}} \boxed{\vec{u}_r}$$

Unit vector

Electromagnetic radiation

Electric field due to an accelerating charge

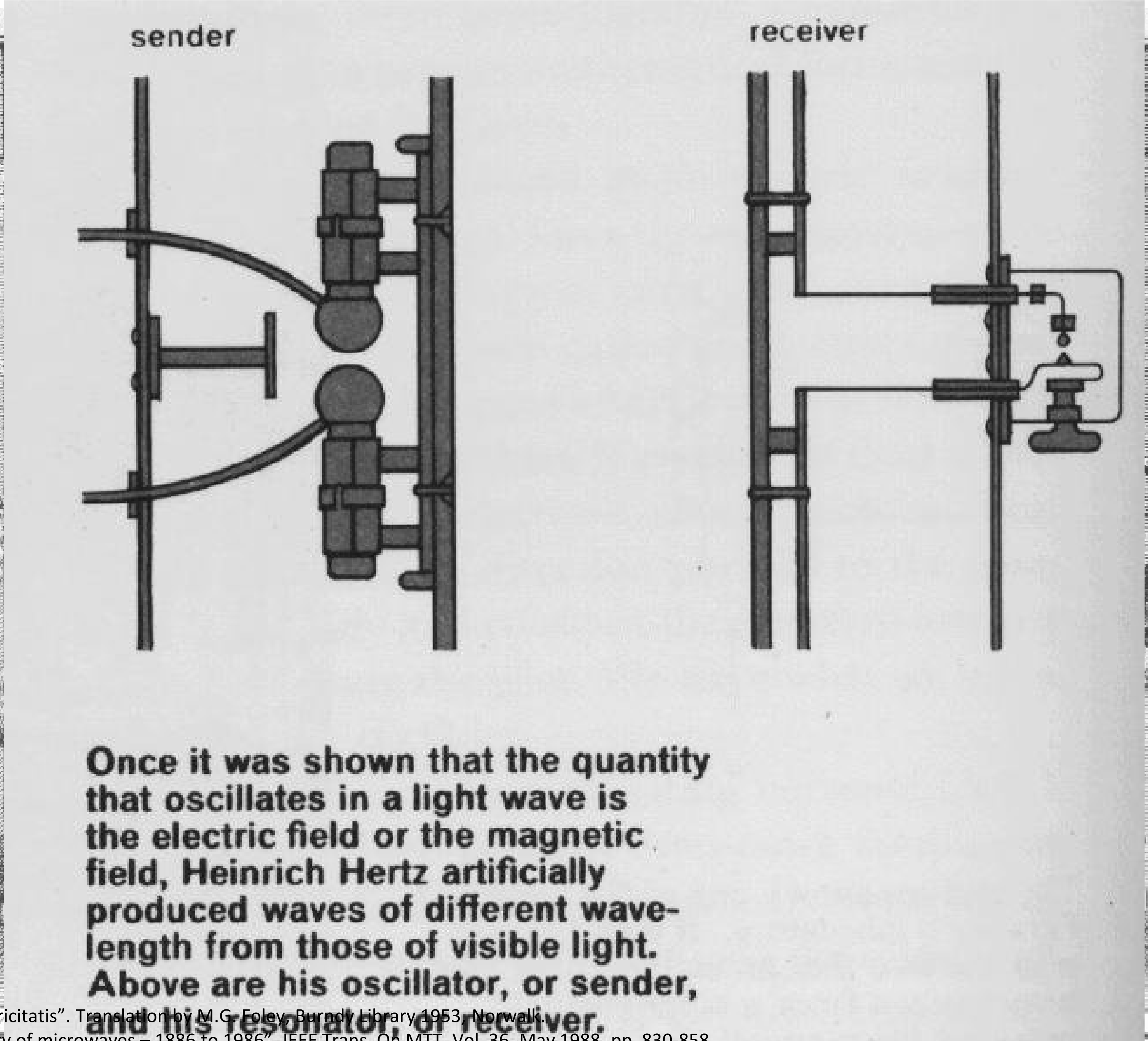


$$\vec{E} = \frac{\vec{E}_0}{4\pi\varepsilon_0 \boxed{r}}$$

$1/r$ dependence far away from the antenna

Hi

Tin



tlantic

Marconi)

[1] Liuigi Galvani, "Viribus electricitatis". Translation by M.G. Foley, Burndy Library 1953, Norwalk.
[2] J.H. Bryant, "The first century of microwaves – 1886 to 1986". IEEE Trans. On MTT, Vol. 36, May 1988, pp. 830-858.

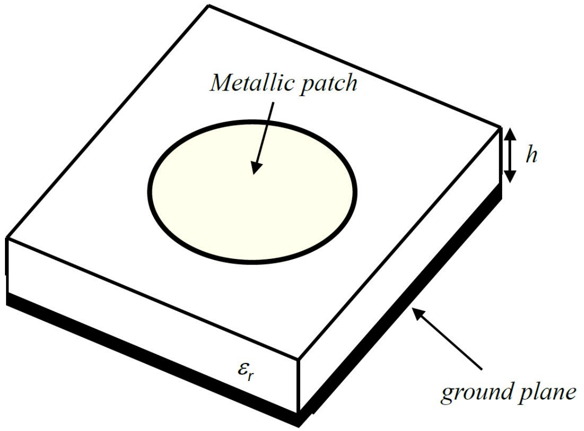
Examples of antennas

Reflector antenna (Westerbork Synthesis Radio Telescope)

Wire antenna

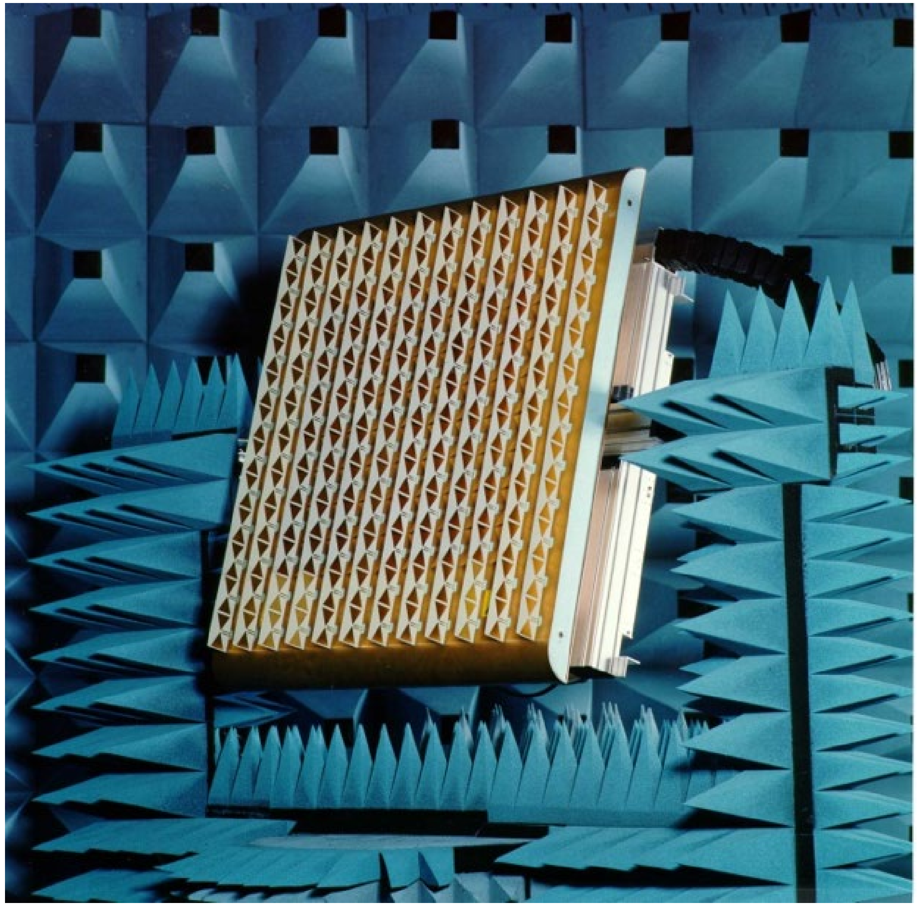
Horn antenna

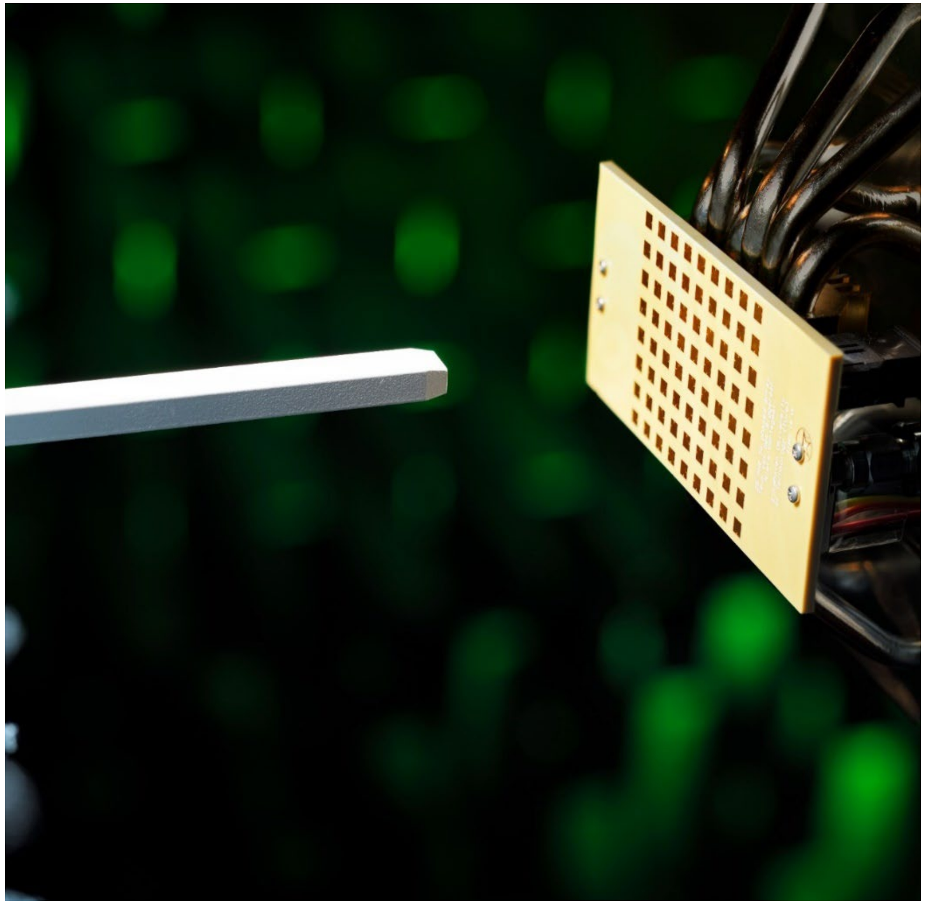
Microstrip antenna



Examples of antennas

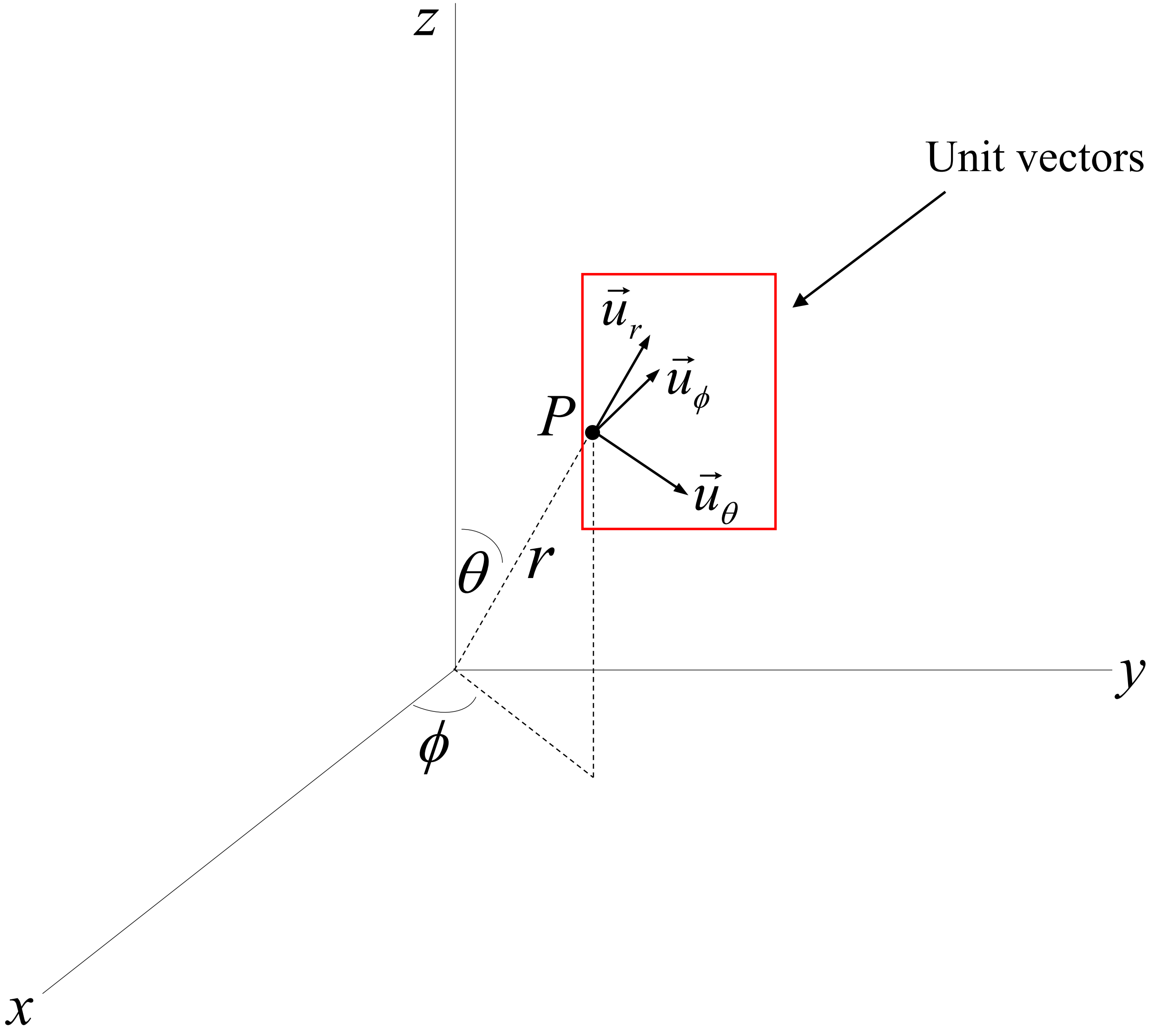
Phased-arrays





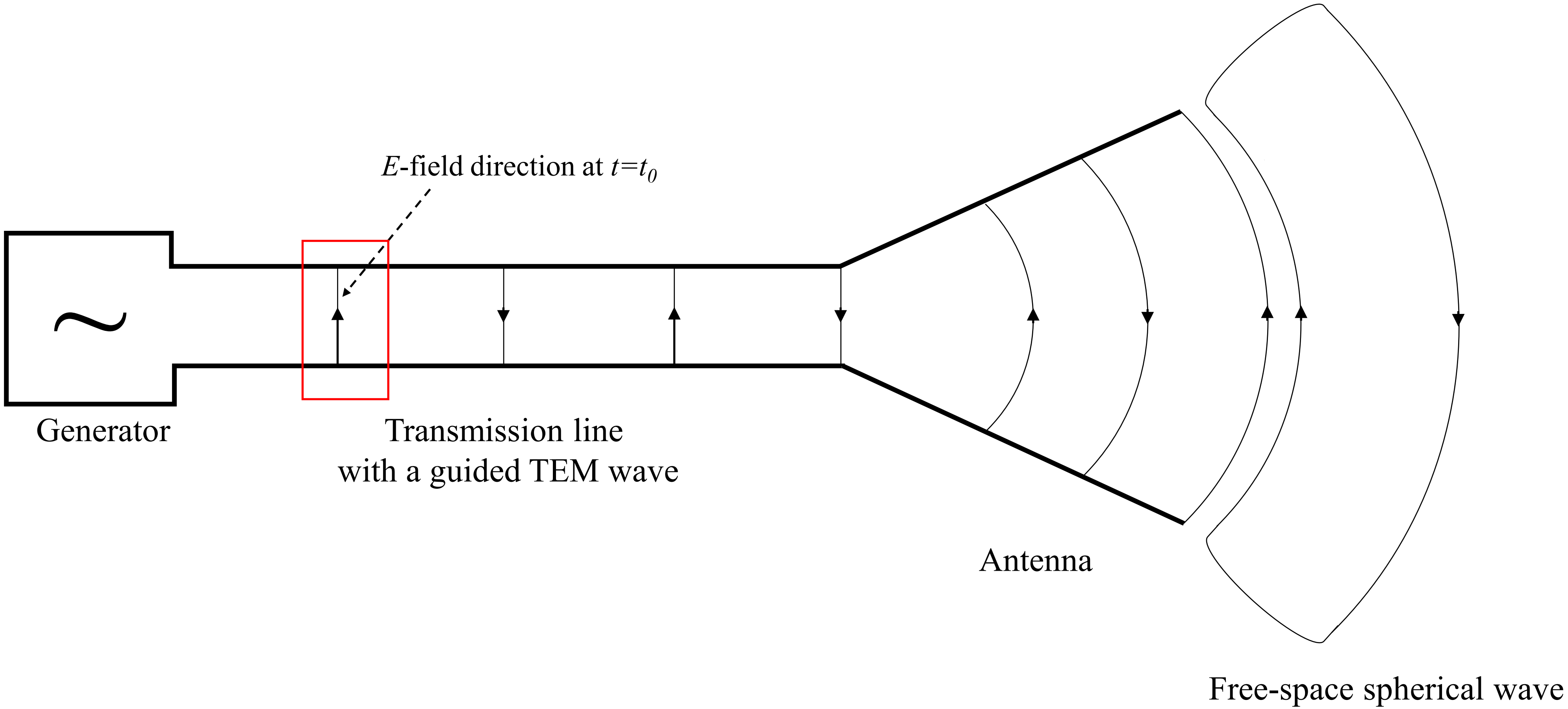
Coordinate system

Spherical coordinates

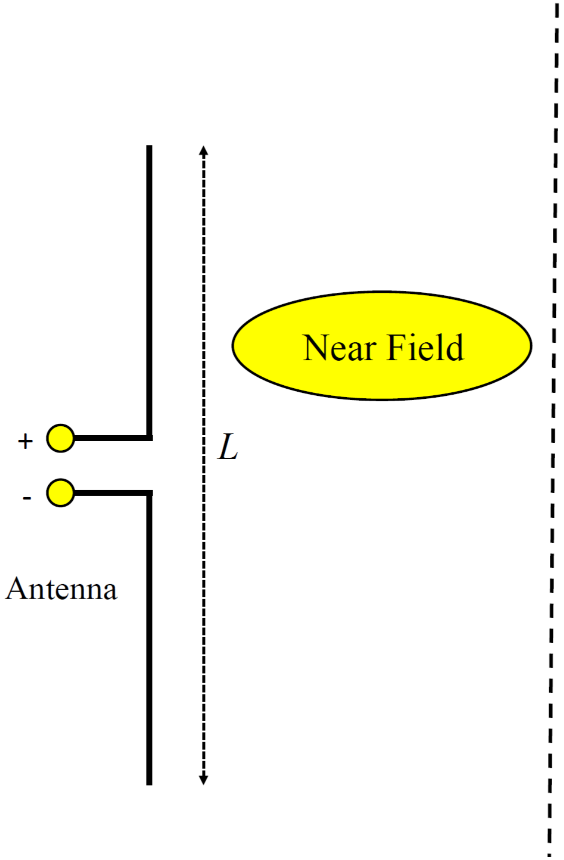


Field regions

Creation of waves, situation at $t\!=\!t_0$



Field regions



Summary

- Acceleration of charges is the source of electromagnetic radiation
- Several antenna types exist:
  - Wire antennas
  - Aperture antennas
  - Phased-arrays
- Field regions

Microwave Engineering and Antennas

Antenna Parameters

Bart Smolders, Professor
Department of Electrical Engineering
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Antenna parameters

Topics of this lecture

- Pointing vector in the far-field
- Radiation pattern
- Directivity and Antenna Gain
- Effective antenna aperture

Far field properties antennas

At large distance from antenna

$$\vec{E} = E_\theta \vec{u}_\theta + E_\phi \vec{u}_\phi$$

$$E_\theta(\vec{r}) = E_\theta(\theta,\phi)\frac{e^{\boxed{-jk_0 r}}}{\boxed{r}}$$

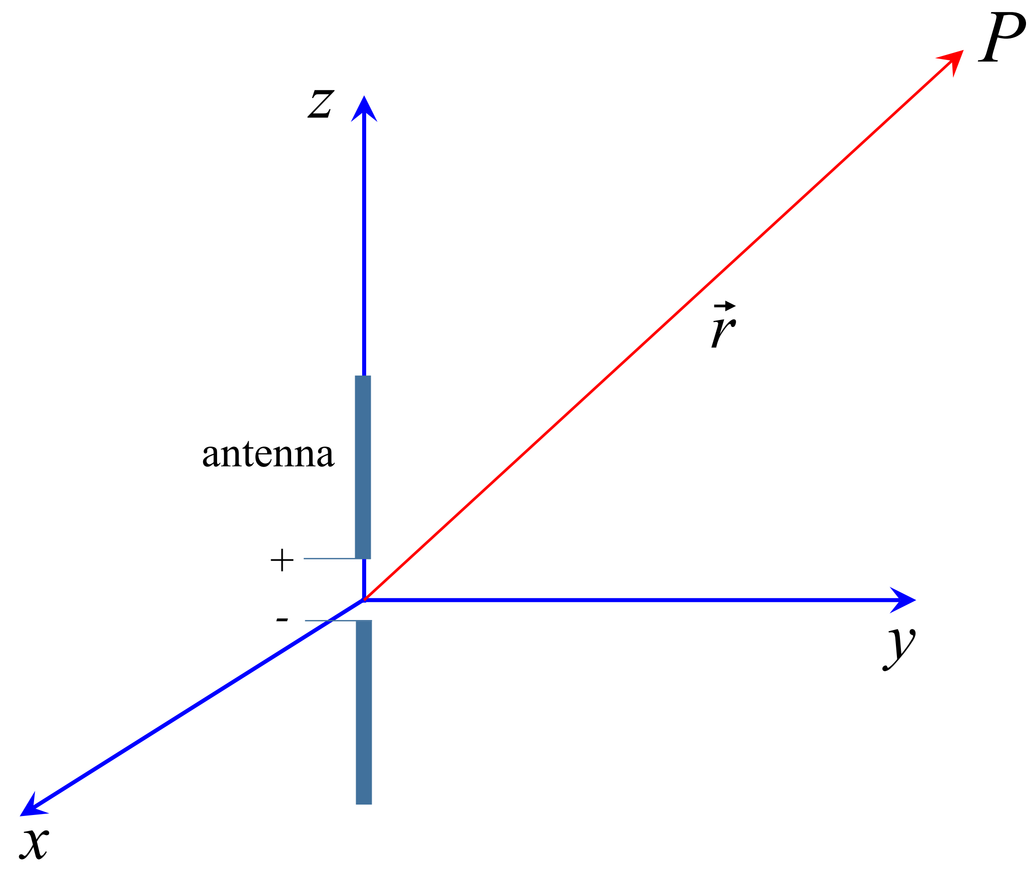
Phase term $k_0 = \dfrac{2\pi}{\lambda_0}$

$$E_\phi(\vec{r}) = E_\phi(\theta,\phi)\frac{e^{-jk_0 r}}{r}$$

1/r dependence

$$\vec{H} = \frac{1}{\boxed{Z_0}}\vec{u}_r \times \vec{E}(\vec{r})$$

free-space impedance =377 Ω



Far field properties antennas

Time-average Poynting vector

$$\vec{S}_p = \frac{1}{T}\int_0^T \vec{S}_p(\vec{r},t)dt = \frac{1}{T}\int_0^T \boxed{\vec{\mathrm{E}}(\vec{r},t)} \times \vec{\mathrm{H}}(\vec{r},t)dt$$

Time-domain Electric field vector

$$= \frac{1}{2}\mathrm{Re}\left[\boxed{\vec{E}(\vec{r})} \times \vec{H}^*(\vec{r})\right]$$

Frequency-domain Electric field vector

Substituting

$$\vec{H} = \frac{1}{Z_0}\vec{u}_r \times \vec{E}(\vec{r})$$

$$\vec{S}_p = \frac{1}{2Z_0}\left|\vec{E}(\vec{r})\right|^2 \boxed{\vec{u}_r} \quad \left[\mathrm{W/m^2}\right]$$

Power flow in the r-direction

Radiation pattern

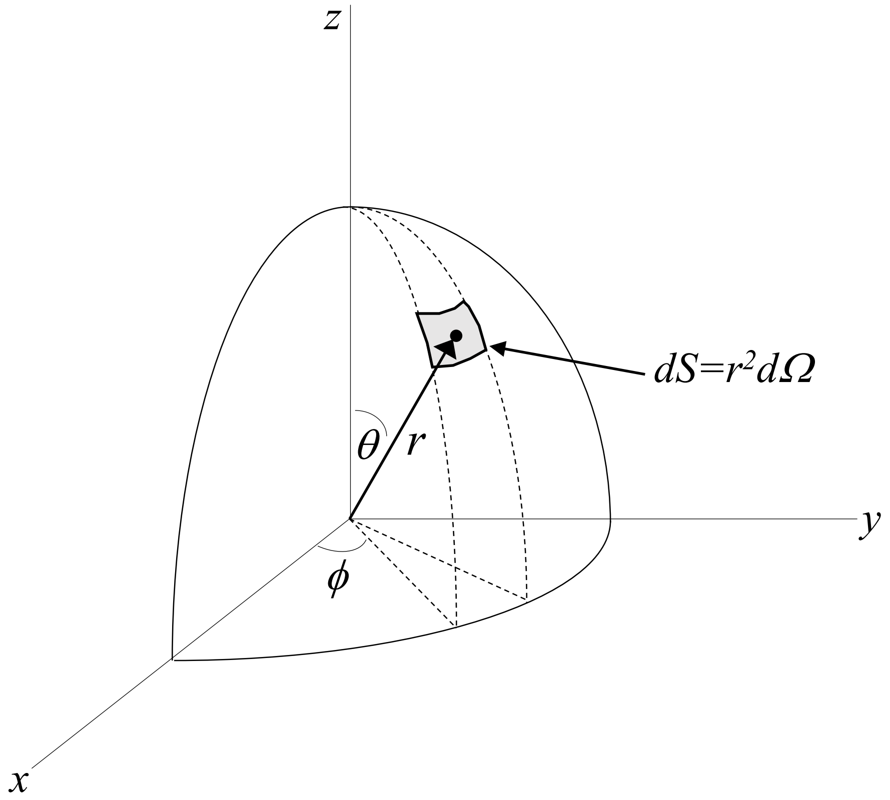
Radiated power per element of solid angle:

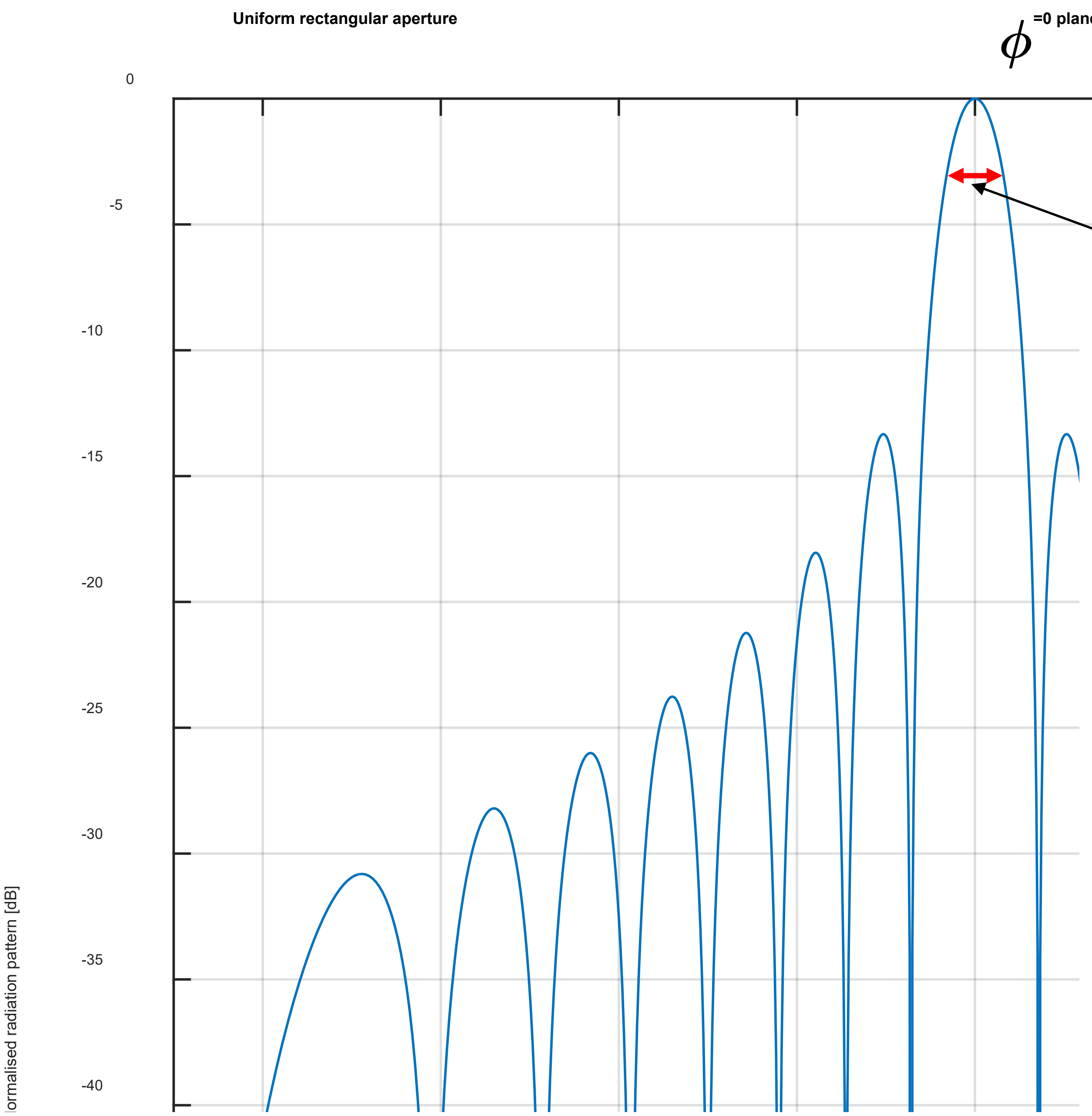
$$P(\theta,\phi)=\left|r^2\vec{S}_p(\vec{r})\right|$$

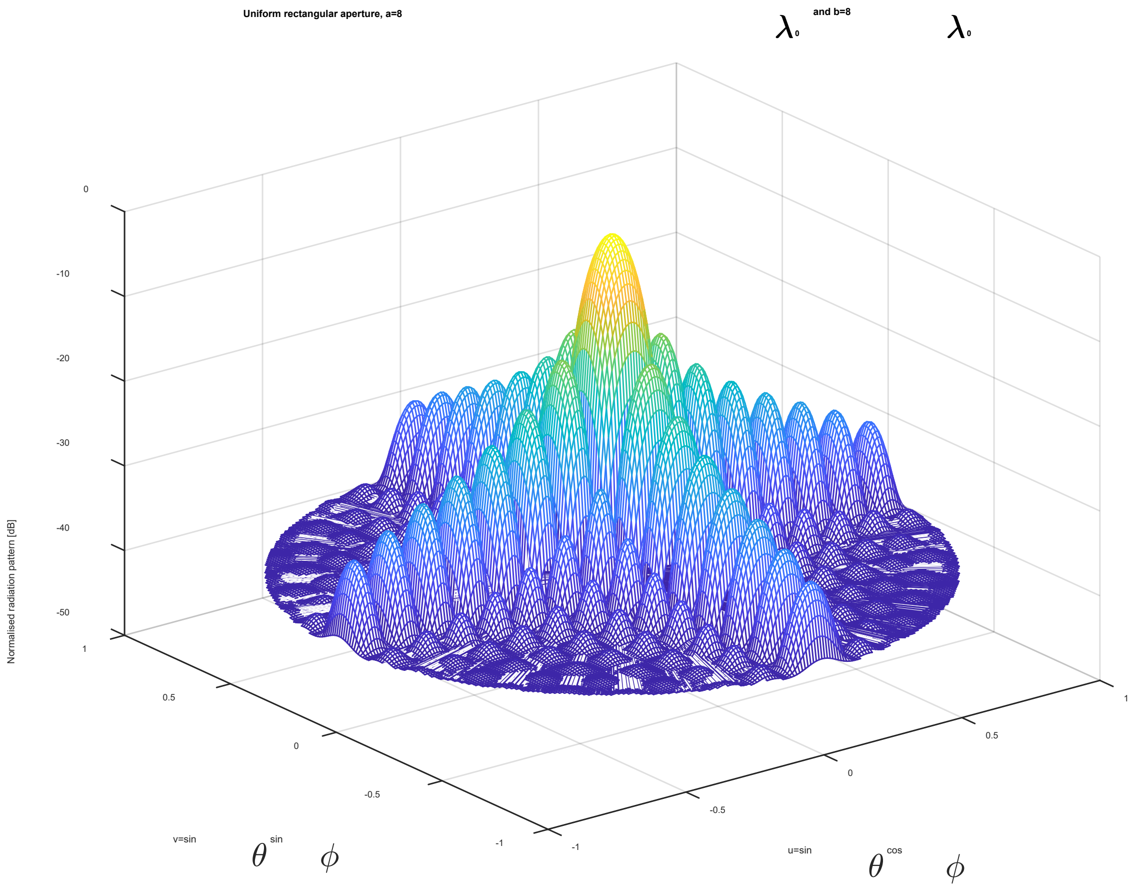
Normalized radiation pattern

$$F(\theta,\phi)=\frac{P(\theta,\phi)}{\boxed{P_{\max}}}$$

Maximum value of $P(\theta,\phi)$




Uniform rectangular aperture
=0 plane, a=8
and b=8
0
-5
-10
-15
-20
-25
-30
-35
-40
ormalised radiation pattern [dB]
Half-power beamwidth (HPBW)


Uniform rectangular aperture, a=8
and b=8
Normalised radiation pattern [dB]
0
-10
-20
-30
-40
-50
1
0.5
0
-0.5
-1
v=sin
sin
u=sin
cos

Directivity and Antenna Gain

Directivity function

$$D(\theta,\phi) = \frac{P(\theta,\phi)}{\boxed{P_t}\,\boxed{/\,4\pi}}$$

Total radiated (transmitted) power

Radiated power density of an ideal isotropic radiator

Directivity

$$\boxed{D} = \max\left[D(\theta,\phi)\right]$$

Expressed in dBi

Antenna Gain

$$\boxed{G} = \eta D \quad \text{with } \eta = \frac{P_t}{P_{in}} \text{ the radiation efficiency}$$

Effective antenna aperture

Assuming receiving antenna is matched to the receiver electronics:

$$A_e = G\frac{\boxed{\lambda_0}^2}{4\pi}$$

Free-space wavelength $\lambda_0 = c/f$, with c speed of light

Summary

- Introduction of Poynting vector
- Graphical representation of radiated power using radiation pattern
- Directivity and Antenna Gain [dBi]
- Effective antenna aperture

Microwave Engineering and Antennas

Link budget: Radio and Radar equation

Bart Smolders, Professor

Department of Electrical Engineering

Center for Wireless Technology Eindhoven

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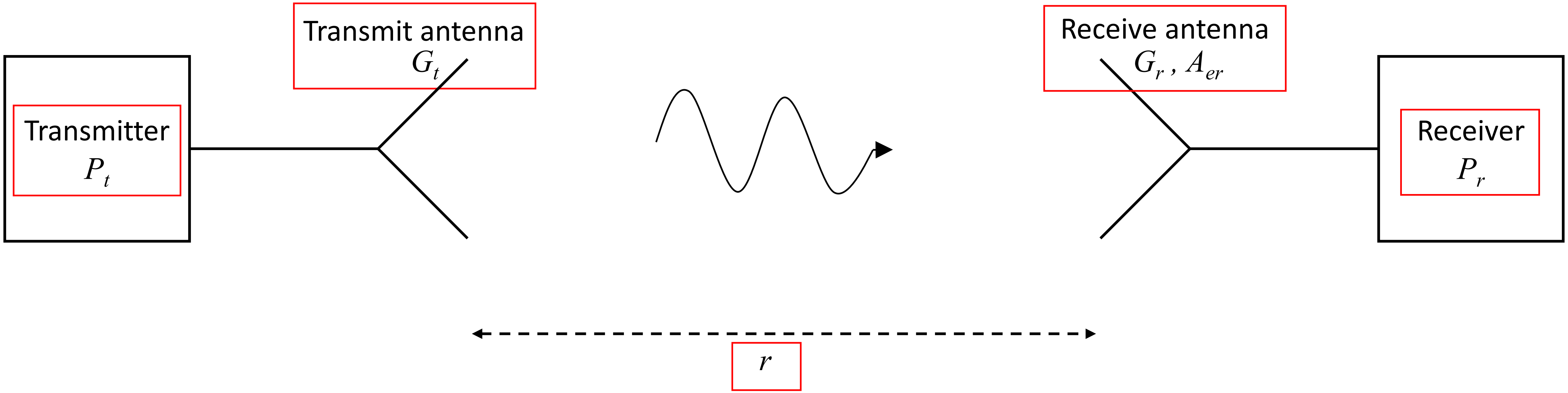
Link budget

Objective of this lecture

- Introduce the Radio equation
- Show an example, Bluetooth
- Derive the Radar equation

Radio equation

Wireless communication system



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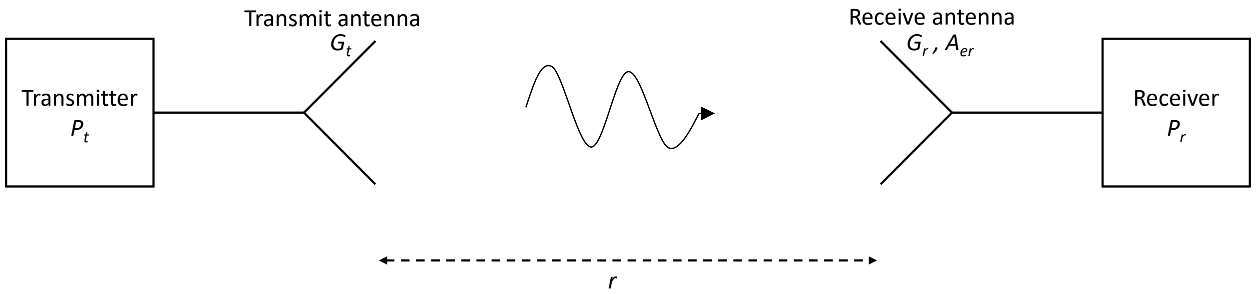
Radio equation



Power density at RX

$$S_r = \frac{P_t G_t}{\boxed{4\pi r^2}} \left[\frac{W}{m^2}\right]$$

Due to spherical expansion of the transmitted wave

Received power

$$P_r = \frac{P_t G_t}{4\pi r^2} A_{er} = \frac{P_t G_t G_r \lambda_0^2}{(4\pi)^2 r^2}$$

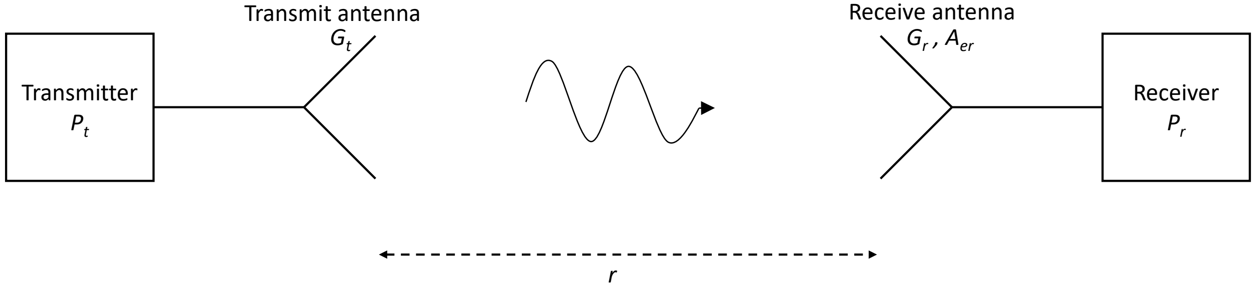
Free-space path loss

Maximum range

$$r_{\max} = \sqrt{\frac{P_t G_t G_r \lambda_0^2}{(4\pi)^2 \boxed{P_{r,\min}}}}$$

Receiver sensitivity

Example Bluetooth



Specifications

- Frequency 2.4-2.48 GHz
- Omni directional antennas $G_t = G_r = 1$
- Output power P_t=1 mW (0 dBm)
- Receiver sensitivity $P_{r,\min}$=10^{-7} mW (-70 dBm)

Maximum range

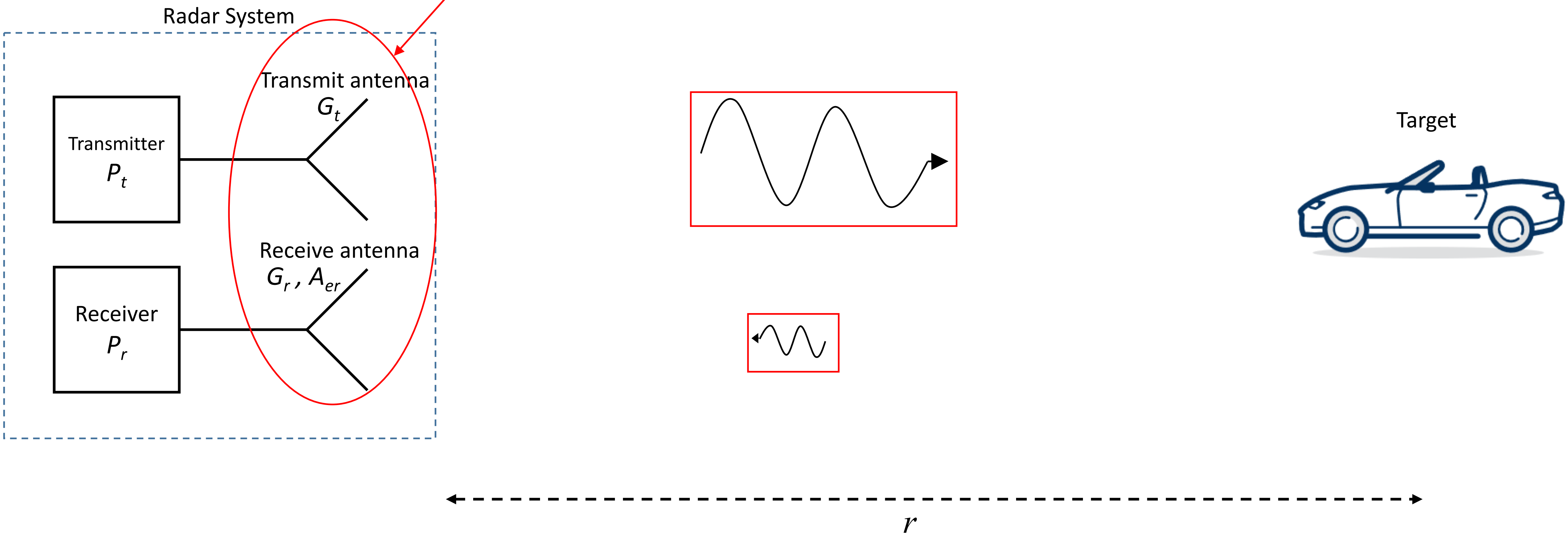
Corrected

$$r_{\max} = \sqrt{\frac{P_t G_t G_r \boxed{\lambda_0^2}}{(4\pi)^2 P_{r,\min}}} \approx 30 \text{ m} \qquad \lambda_0 = \frac{c}{f_0} = \frac{3 \cdot 10^8}{2.45 \cdot 10^9} = 12.3 \text{ cm}$$

Radar equation

Radar system

Often the Transmit and Receive antennas are combined into single antenna ($G_t = G_r = G$)



Radar equation

Power density at RX

$$S_r = \frac{P_t G_t}{4\pi r^2} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$

Radar cross section target

$$\sigma \quad [\mathrm{m}^2]$$

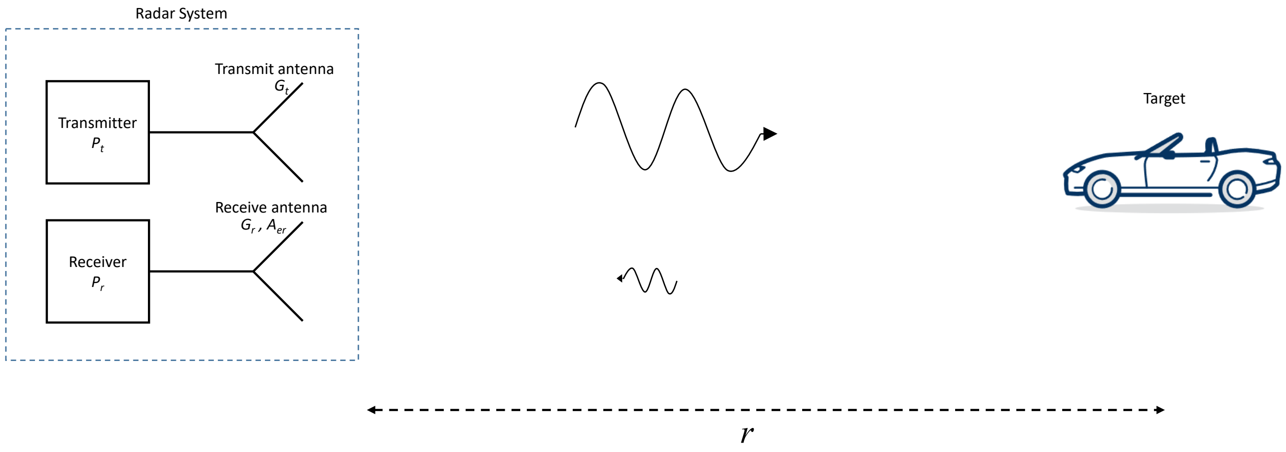
Received power

$$P_r = S_r \frac{\sigma}{4\pi r^2} A_{er} = \frac{P_t G_t G_r \sigma \lambda_0^2}{(4\pi)^3 r^4}$$

Range

$$r_{\max} = \left(\frac{P_t G_t G_r \sigma \lambda_0^2}{(4\pi)^3 \boxed{P_{r,\min}}}\right)^{\frac{1}{4}}$$

Receiver sensitivity



Summary

- Introduction of the radio and radar equations
- Determine the maximum range of a system under ideal conditions
- A more detailed model should include the propagation channel as well

Microwave Engineering and Antennas

Antenna impedance: Circuit representation of antennas

Bart Smolders, Professor

Department of Electrical Engineering

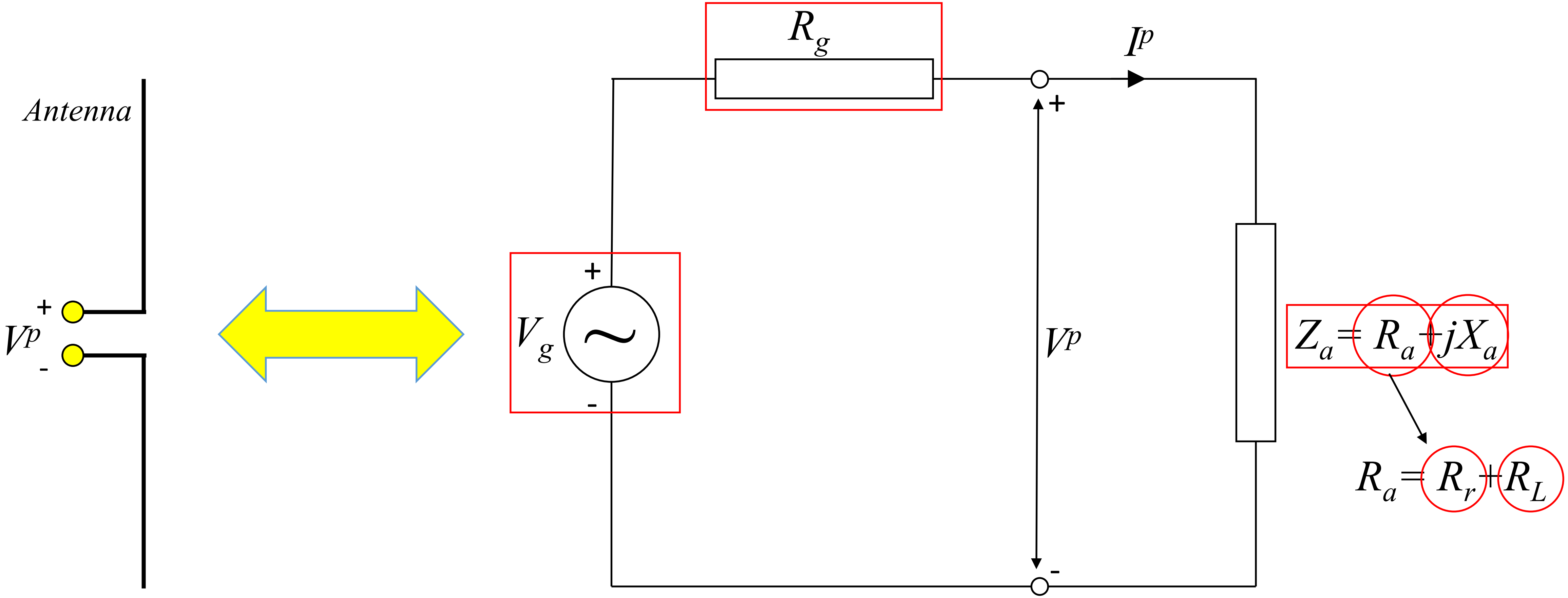
Center for Wireless Technology Eindhoven

Antenna impedance

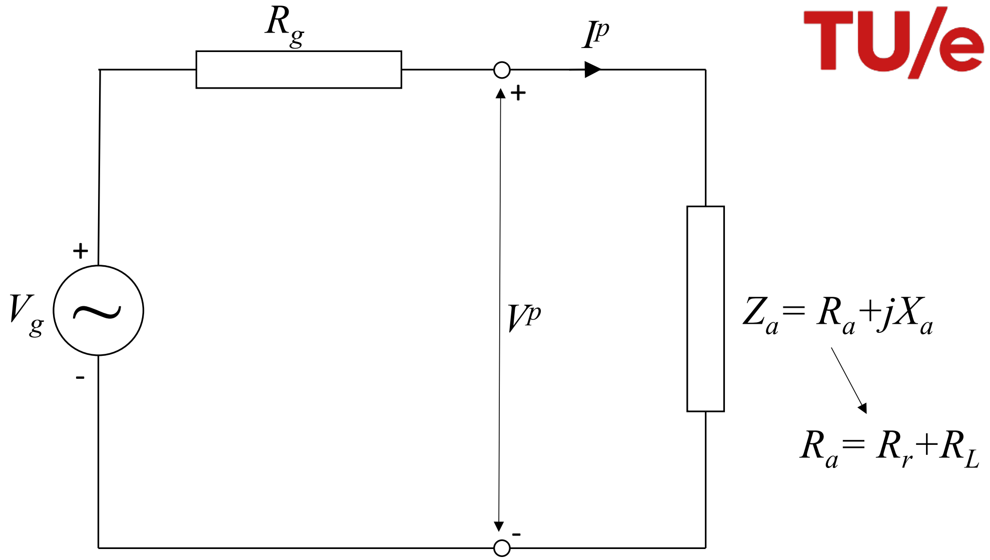
Objective of this lecture

- Introduce equivalent circuit of antenna
- Derive expression for total radiated power
- Determine input reflection coefficient

Equivalent circuit



Equivalent circuit



Radiation resistance

$$R_r = \frac{P_t}{\frac{1}{2}\left|I^p\right|^2}$$

Total radiated power

Time-average Pointing vector

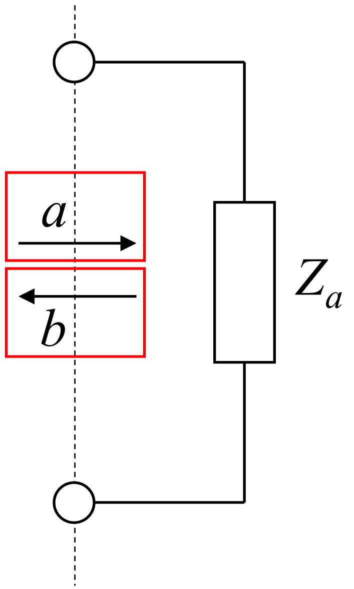
$$P_t = \iint \boxed{\vec{S}_p(\vec{r})} \cdot \vec{u}_r r^2 d\Omega$$

Far-field components

$$= \frac{1}{2} \boxed{Z_0^{-1}} \int_0^{2\pi} \int_0^{\pi} \left[\left| \boxed{E_\theta(\theta, \phi)} \right|^2 + \left| \boxed{E_\phi(\theta, \phi)} \right|^2 \right] \sin\theta d\theta d\phi$$

free-space impedance (=377 Ω)

Input reflection coefficient



$$\Gamma = \frac{b}{a}$$

$$= \frac{Z_a - \boxed{Z_0^t}}{Z_a + Z_0^t}$$

Characteristic impedance of connected transmission line

Summary

- Introduction of a circuit model
- Related radiation resistance to the radiated electromagnetic waves
- Determined the input reflection coefficient when the antenna is connected to a transmission line.

Microwave Engineering and Antennas

Phased Arrays: Introduction of Linear Arrays

Bart Smolders, Professor
Department of Electrical Engineering
Center for Wireless Technology Eindhoven

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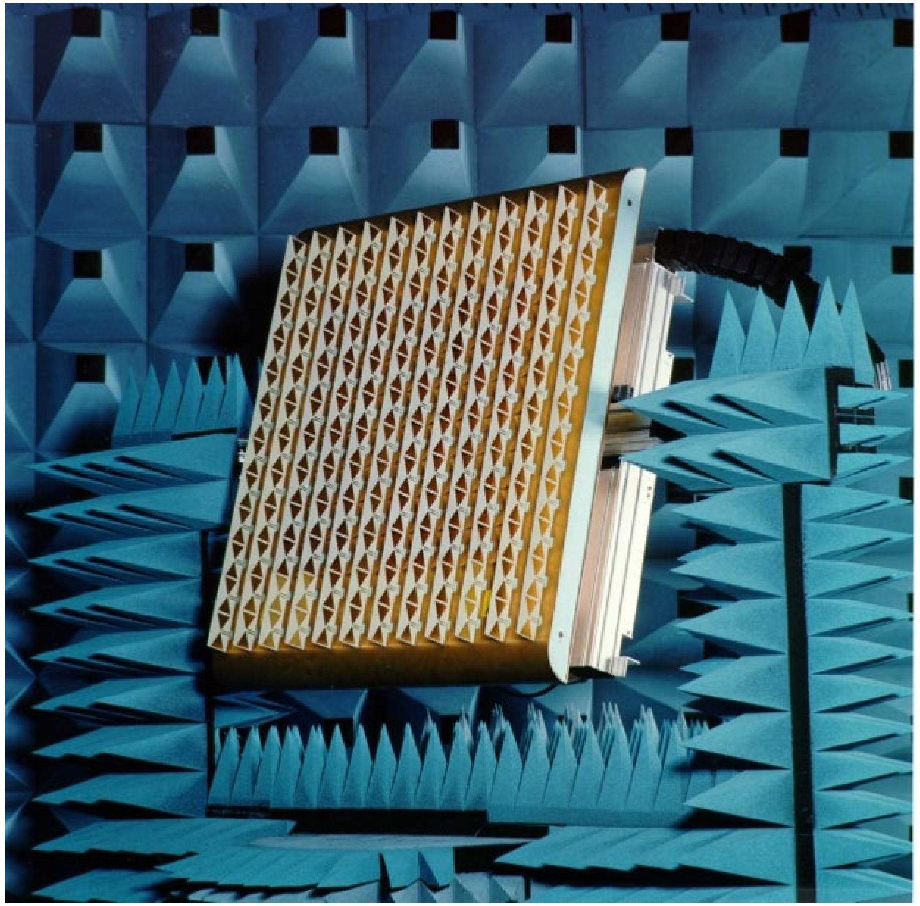
Phased Arrays

Objective of this lecture

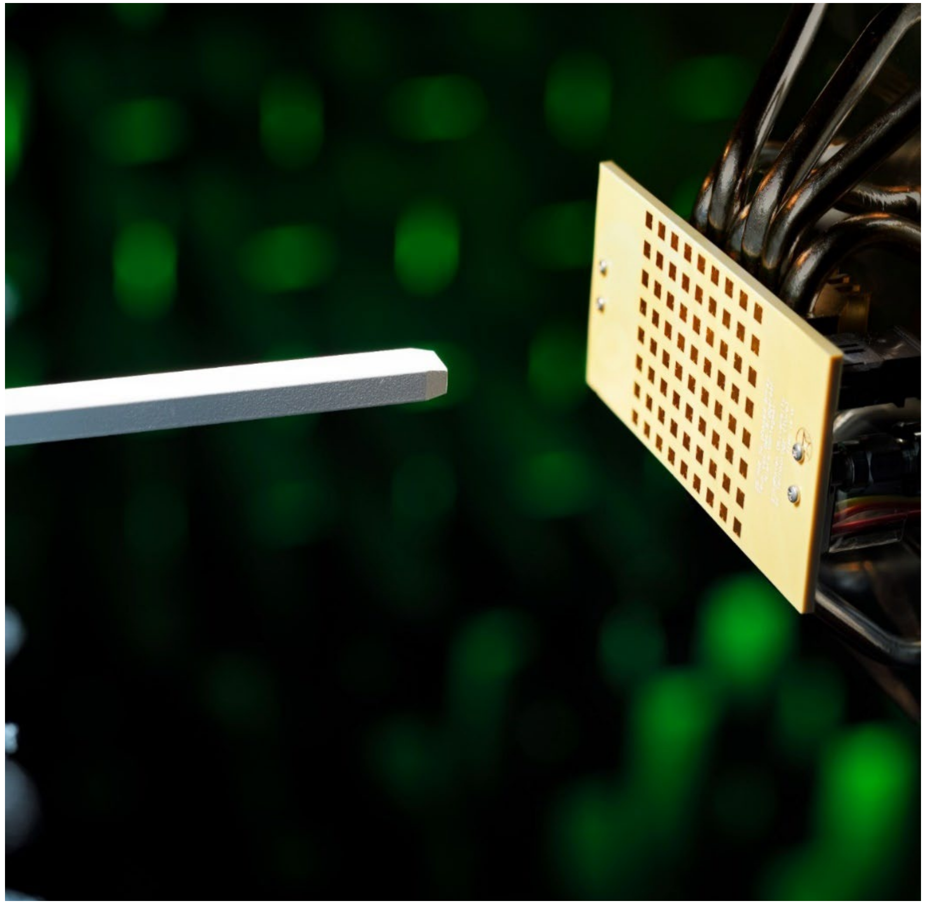
- Introduce the concept of phased arrays.
- Derive expression for the radiation pattern of linear arrays.
- Introduction of the array factor.

Applications of phased arrays

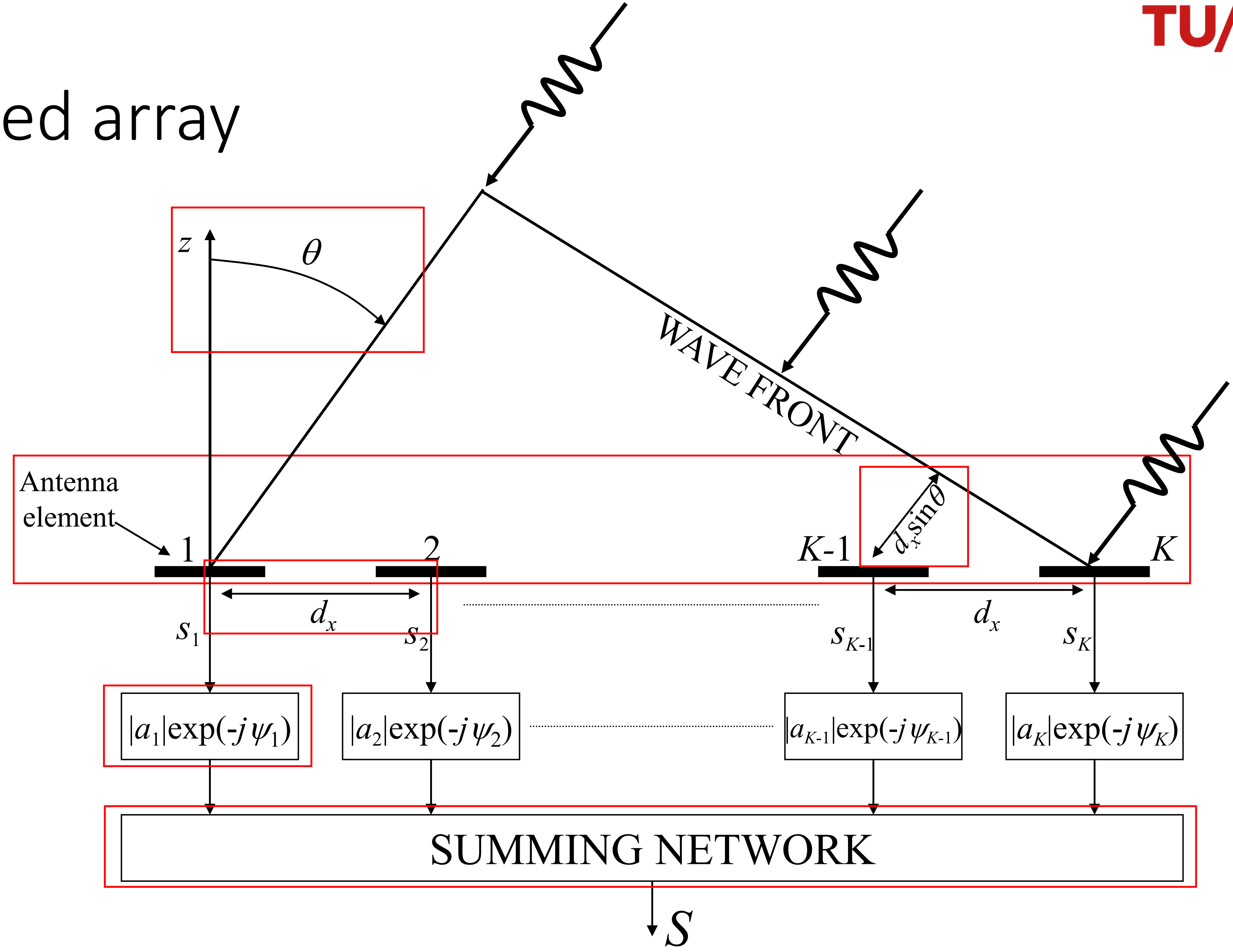
Radio astronomy



Mm-wave 5G basestations



Linear phased array



Linear array of isotropic antennas

Received signal s_k

check $k = K-1$: $jk_0 d_x \sin\theta$

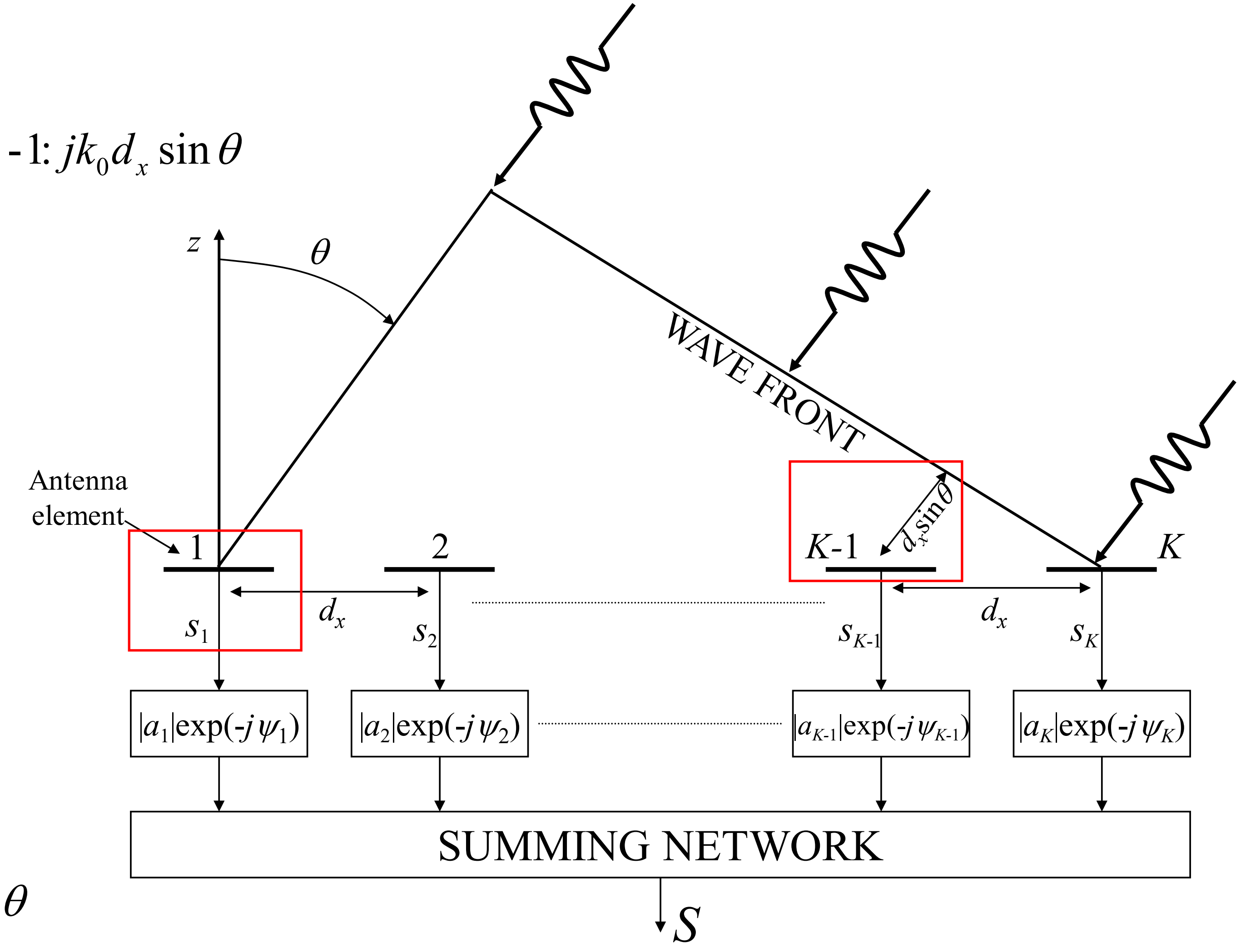
$$s_k = e^{\boxed{jk_0 (k-K) d_x \sin\theta}}$$

$$k_0 = \frac{2\pi}{\lambda_0}$$

Transfer phase reference to element k=1

$$s'_k = e^{\boxed{jk_0 (k-1) d_x \sin\theta}}$$

substracted phase $jk_0 (1-K) d_x \sin\theta$



Linear array of isotropic antennas

Apply amplitude weighting and phase shifting and combine all signals:

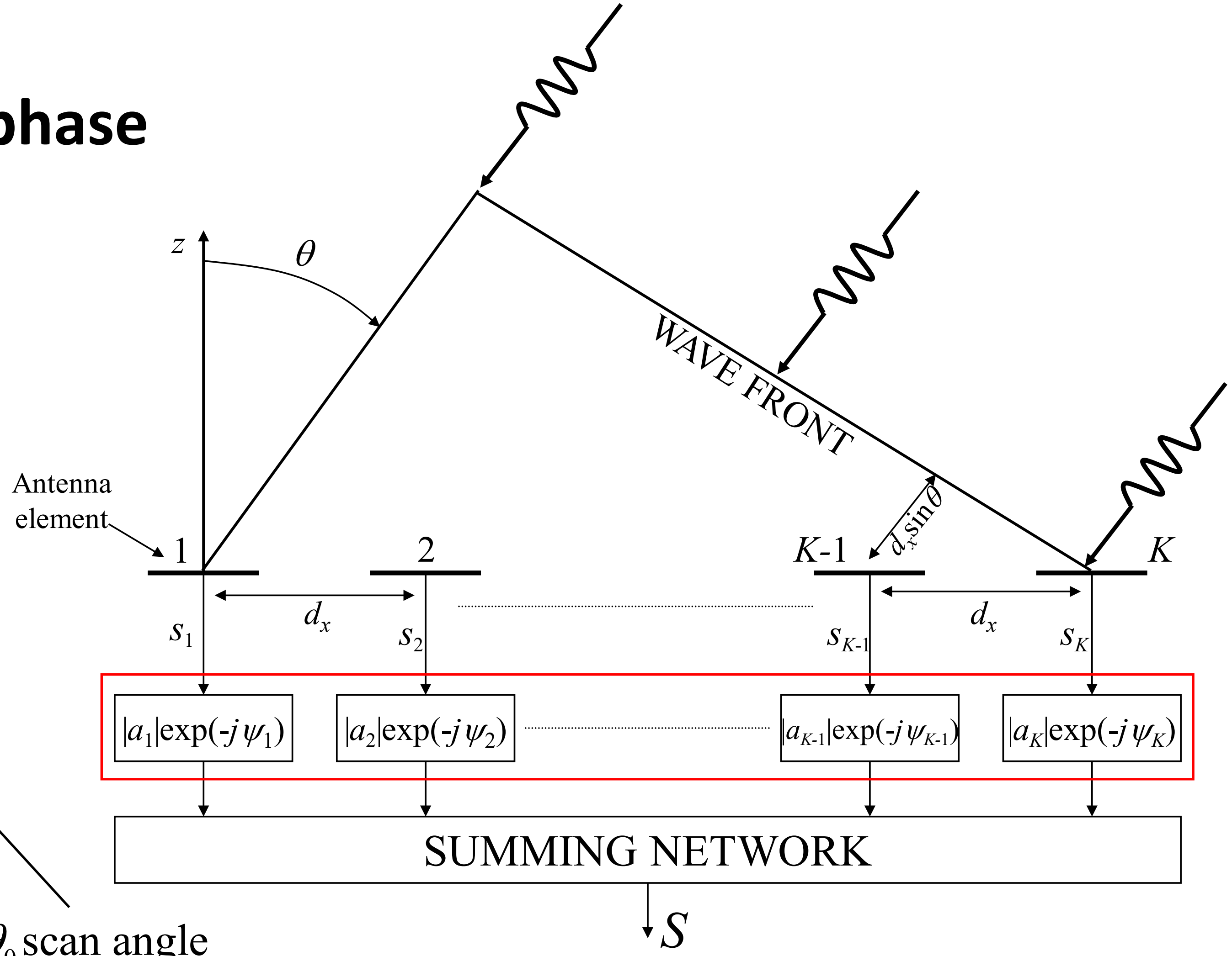
$$S(\theta) = \sum_{k=1}^{K} \boxed{\left|a_k\right| e^{-j\psi_k}} s'_k$$

Complex multiplication by electronics

$$= \sum_{k=1}^{K} \left|a_k\right| e^{-j\psi_k} e^{jk_0 (k-1) d_x \sin\theta}$$

$$= \sum_{k=1}^{K} \left|a_k\right| e^{\boxed{jk_0 (k-1) d_x (\sin\theta - \sin\boxed{\theta_0})}}$$

with $\psi_k = k_0 (k-1) d_x \sin\theta_0$ scan angle



Example radiation pattern linear array

Radiation pattern is obtained from

$$F(\theta) = \frac{|S(\theta)|^2}{|S(\theta_0)|^2} \Rightarrow \text{Use dB scale: } 10\log_{10}\left[F(\theta)\right]$$

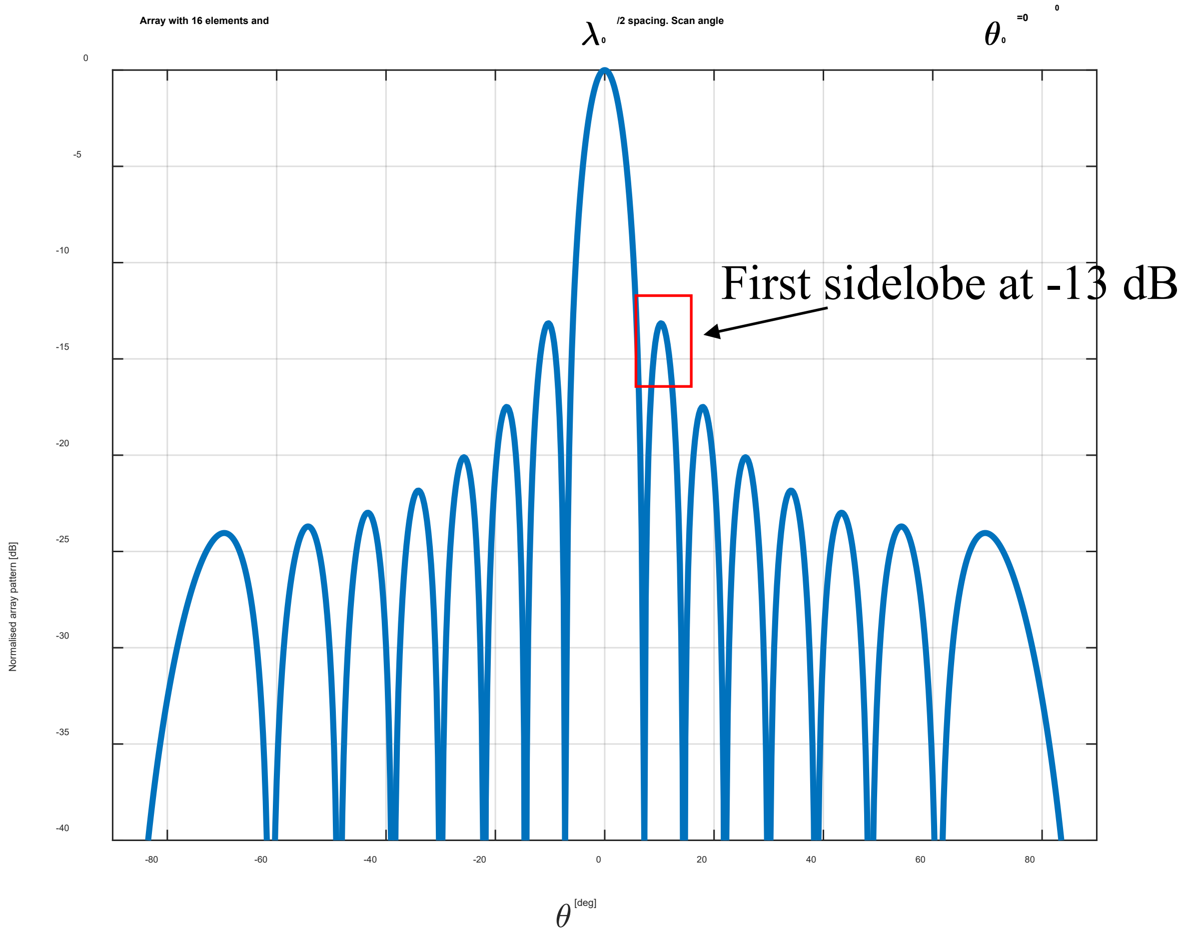
Example

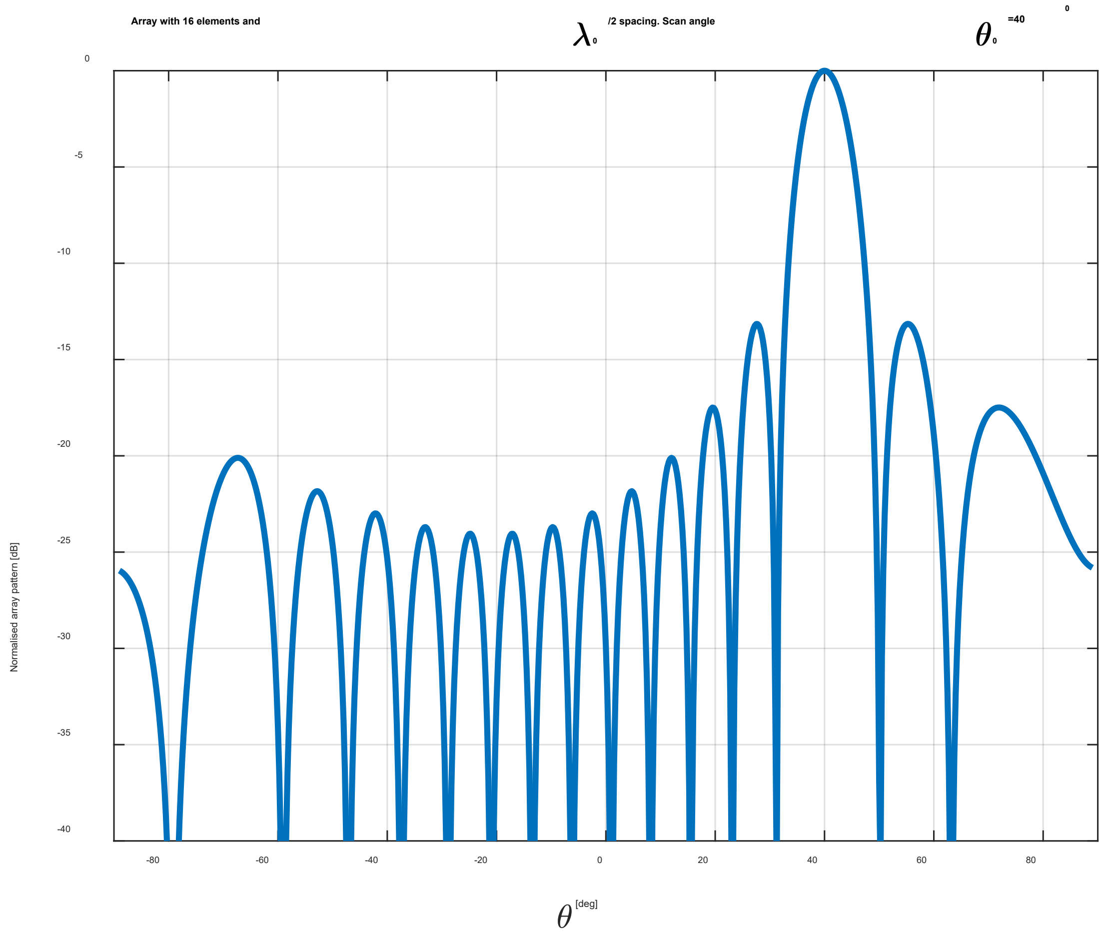
$K = 16$ with spacing $d_x = \dfrac{\lambda_0}{2}$,

Uniform amplitude weighting: $|a_k| = 1$,

$\theta_0 = 0^0$ and $\theta_0 = 40^0$.

Example radiation pattern linear array





Relation to DFT/FFT

Array factor

$$S(u) = \sum_{k=1}^{K} |a_k| e^{\boxed{jk_0(k-1)d_x(u-u_0)}} \quad \text{with } u = \sin\theta \text{ and } u_0 = \sin\theta_0$$

Discrete Fourier Transformation (DFT)

$$F(n) = \sum_{k=1}^{K} f(k) e^{j\boxed{\frac{2\pi(k-1)(n-1)}{K}}} \quad \text{for } 1 \le n \le K$$

$$u - u_0 = \frac{\lambda_0(n-1)}{Kd_x} \quad \Rightarrow \quad \text{FFT if } K \text{ is power of 2}$$

Summary

- Array factor of a linear array was derived.
- Relation with DFT/FFT was shown.
- We only considered isotropic radiators.
- To extend to real antennas we need to introduce **vector-based antenna theory**.
- This is the topic of the next lectures.
- More details on arrays can be found in another course at our University.

Microwave Engineering and Antennas

Hands-On Design: Antenna

Ulf Johannsen, Assistant Professor

Department of Electrical Engineering

Center for Wireless Technology Eindhoven

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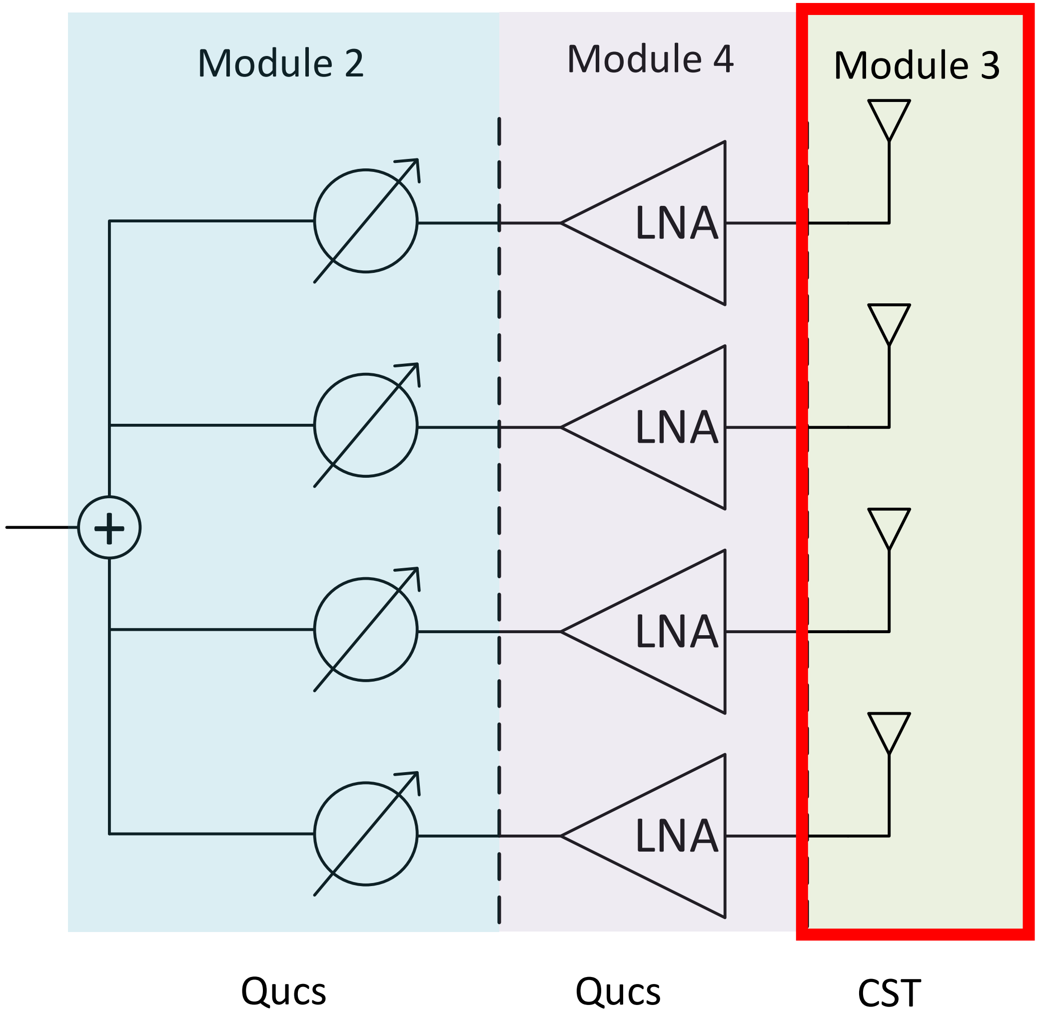
Hands-On Design Assignment



4-Element Analogue Beamforming Receiver-Array

- 5.8 GHz ISM band
- ±45˚ beamsteering range
- Main-lobe antenna gain of 9dBi for entire beamsteering range
- System noise figure < 2.7dB

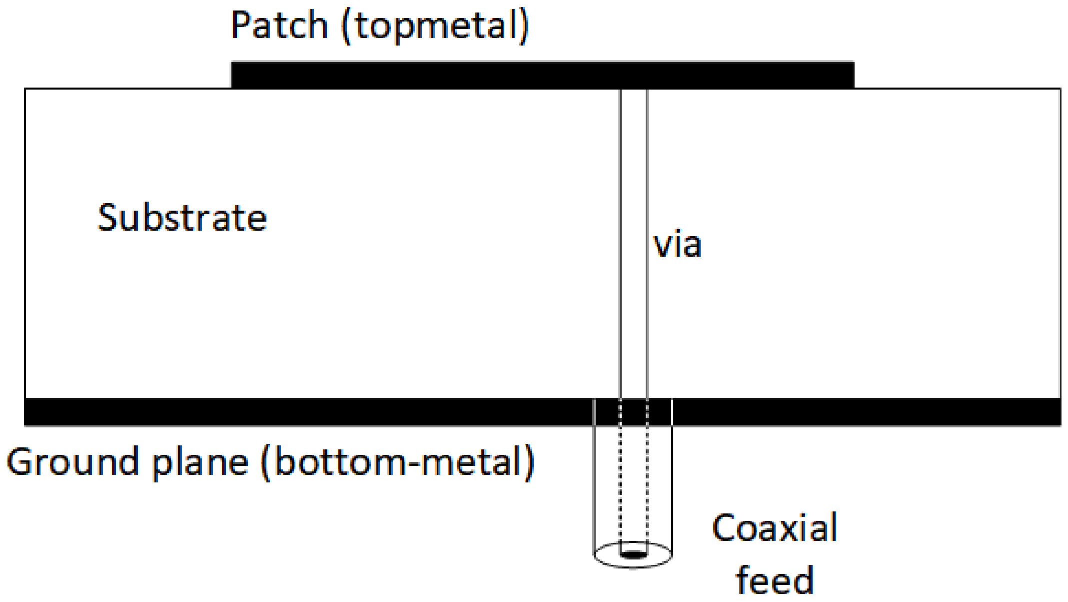
CST Microwave Studio (student version)

- CST Studio Suite® is a high-performance **3D EM analysis software** package for designing, analyzing and optimizing electromagnetic components and systems.
- You can download the **student edition** of **CST Studio Suite** from https://www.3ds.com/products-services/simulia/products/cst-studio-suite/student-edition/

Task and Design Requirements

- Design a via-fed patch antenna that can be used in a 4-element array
- The overall design, shown on the bottom right, must fulfil the following requirements:

ID	Parameter	Requirement
1	Operating bandwidth	5.725 – 5.875 GHz
2	Port match	$S_{11} < -10dB$ @ $Z_0 = 50\Omega$
3	Radiation efficiency	$\eta_{rad} > 80\%$
4	Antenna gain	6dBi
5	Simulation setup	Make use of a symmetry plane in CST



Rules and Tips

- You might want to study some **literature on patch antennas**.
- **Vary some antenna dimensions** (including substrate thickness) to find out how your design changes.
- Show your **genuine attempt** to design what was asked of you. For a genuine attempt you
    - have **all design choices** as long as you design the components yourself in CST (student version).
    - must upload **<u>all</u> necessary design files** and a **design report** that describes and explains the design.
    - **must use RO4350B as substrate** from Rogers Corporation as PCB for your design: https://www.rogerscorp.com/Advanced%20Connectivity%20Solutions/RO4000%20Series%20Laminates/RO4350B%20Laminates

Rubric for peer review

Excellent (6 pts)	Very good (5 pts)	Good (4 pts)	Fair (3 pts)	Sufficient (2)	Insufficient (0 pts)
• The report describes **<u>all</u>** design aspects in detail, such that the reader/reviewer can implement the design in CST her-/himself. • **<u>All</u>** design aspects are explained clearly and correctly. • The design achieves the specified requirements **<u>without exception</u>**.	• The report describes almost all design aspects in detail. Only **<u>minor details are missing</u>** such that the reviewer **<u>can implement the design</u>** in CST her-/himself. • Most design aspects are explained clearly and correctly. **<u>Only minor inaccuracies</u>** are present. The understanding is not significantly affected by this. • The design achieves the specified requirements with **<u>one exception</u>**.	• The report describes most design aspects in detail. **<u>A few details are missing</u>** such that the reviewer may not be able to implement **<u>some minor details</u>** of the design in CST her-/himself. • There are **<u>only a few inaccuracies/inconsistencies</u>** in the explanation of the design aspects. The understanding of a few aspects of the design is slightly affected. • The design achieves the specified requirements with **<u>two exceptions</u>**.	• The report describes most design aspects in detail. **<u>Some details are missing</u>** such that the reviewer may **<u>not</u>** be able to implement **<u>every aspect of the design correctly</u>** in CST her-/himself. • There are **<u>some inaccuracies/inconsistencies</u>** in the explanation of the design aspects. Not every aspect of the design is entirely clear. • The design achieves the specified requirements with **<u>three exceptions</u>**.	• The report describes most design aspects in detail. **<u>Several details are missing</u>** such that the reviewer **<u>struggles</u>** to implement **<u>the design correctly</u>** in CST her-/himself. • There are **<u>several inaccuracies/inconsistencies or some mistakes</u>** in the explanation of the design aspects. About half of the design aspects are not entirely clear. • The design achieves the specified requirements with **<u>four exceptions</u>**.	• The report provides insufficient detail such that the reader/**<u>reviewer cannot implement the design</u>** in CST her-/himself. • There are **<u>major design aspects missing</u>** or are **<u>mostly incorrectly</u>** explained. • The **<u>design does not achieve any of the requirements.</u>**

Microwave Engineering and Antennas

A quick-guide to CST

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CWTe
CENTER FOR WIRELESS TECHNOLOGY EINDHOVEN
TU/e

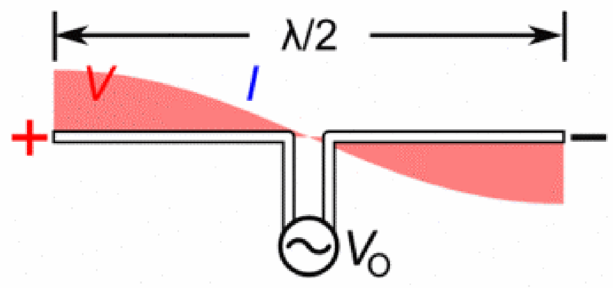
Content

- CST Workflow
  - Boundary definition
  - Solvers
  - Mesh
  - Ports
  - Analysis of results

Why simulate antennas?

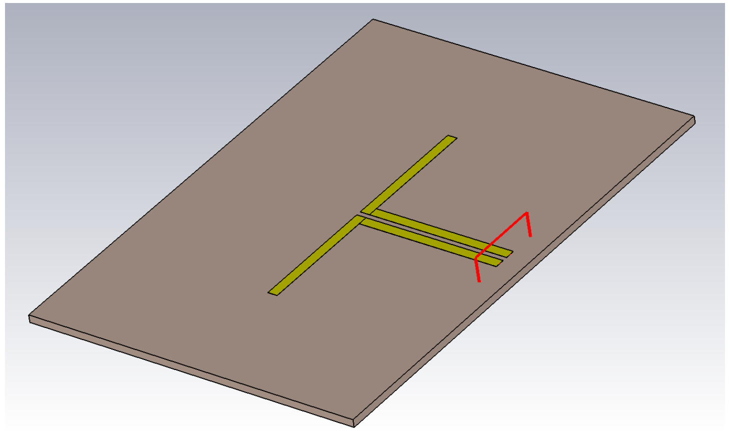
- CST: numerical EM-simulation of antennas
- Bridge gap between theory and measurements
- Design & optimize antennas

Theory

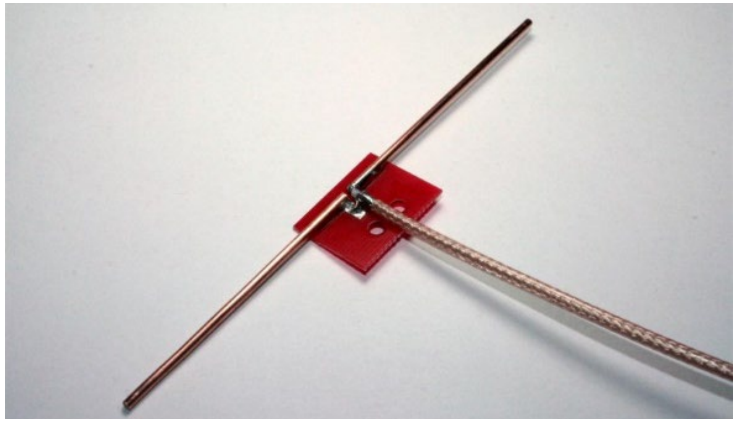


https://en.wikipedia.org/wiki/Dipole_antenna

Simulation



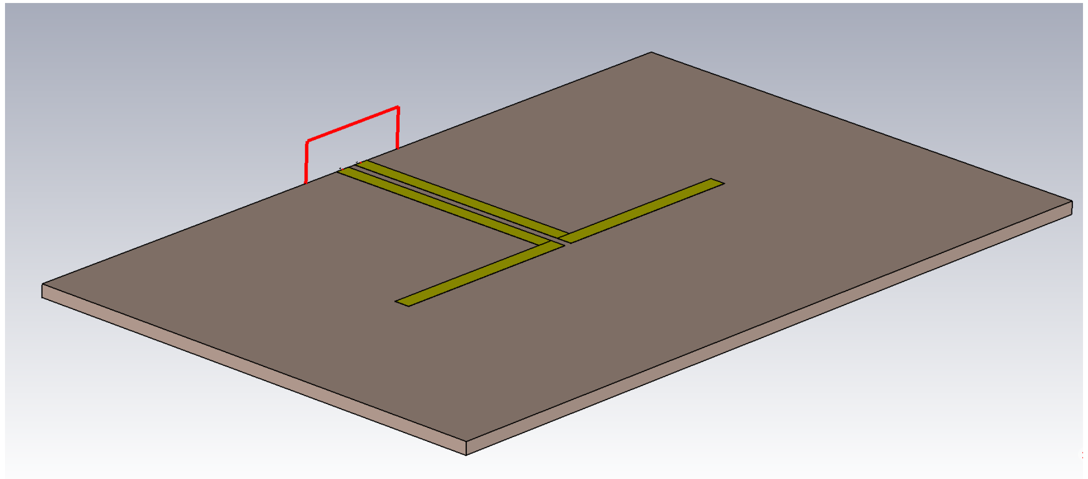
Measurement

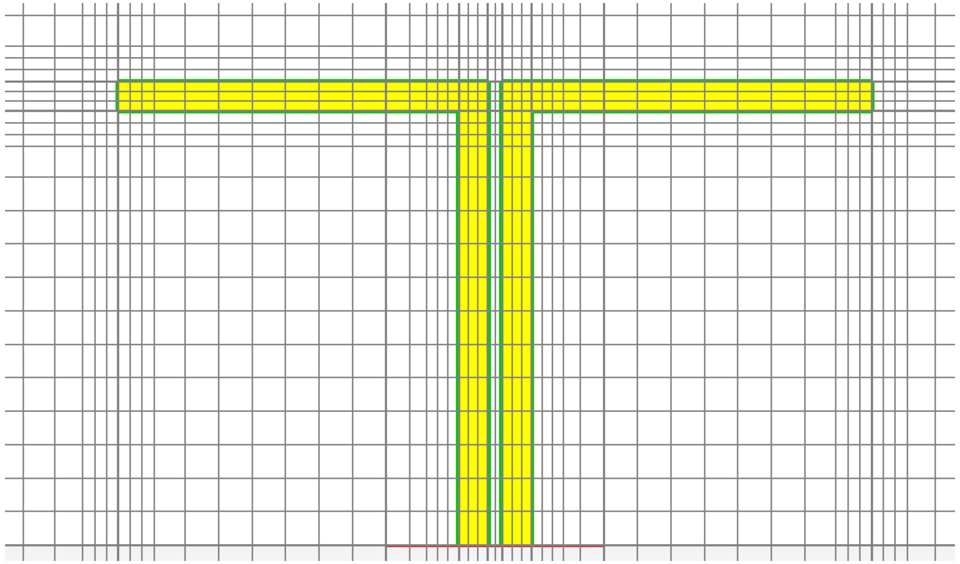


http://www.flytron.com/antennas-filters/100-fine-tuned-dipole-antenna.html

CST Workflow

1. Build 3D model
2. Define ports and boundaries
3. Choose solver
4. Mesh is generated
5. Maxwell equations are solved at mesh points until steady state criterion is met
6. Analyze results: S-parameters, input impedance, radiation pattern, efficiency, fields





Boundaries

High Frequency

Electric: Operates like a perfect electric conductor: all tangential electric fields and normal magnetic fluxes are set to zero.

Magnetic: Operates like a perfect magnetic conductor: all tangential magnetic fields and normal electric fluxes are set to zero.

Open (PML): The open boundary extends the touching geometry virtually to infinity by using a perfectly matched layer (PML) boundary. Waves can pass this boundary with minimal reflections. Note that in case of a unit cell simulation with the general purpose frequency domain solver open boundaries are realized by a Floquet port.

Open (add space): Same as Open (PML), but adds some extra space for farfield calculation. This option is recommended for antenna problems. **PIC solver:** Besides the waves that can pass this boundary with minimal reflections, additionally charged particles can be absorbed by the PML. Only available for PIC on CPU.

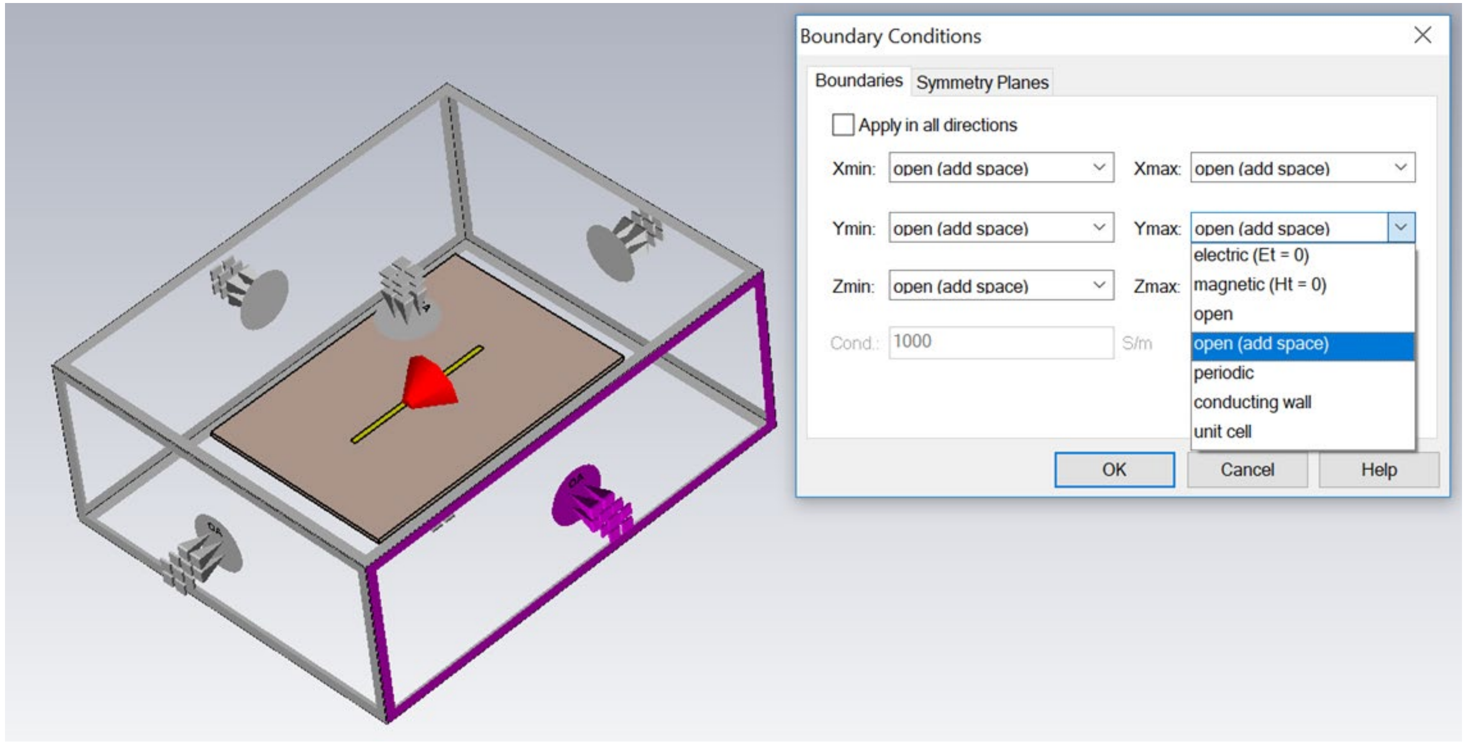
Periodic: Connects two opposite boundaries with a definable phase shift such that the calculation domain is simulated to be periodically expanded in the corresponding direction. Thus, changing one boundary to periodic always changes the opposite boundary to periodic as well.

If you select a periodic boundary condition you may enter a phase shift for this boundary in the Phase Shift/Scan Angle property page.

Simulating antenna models with periodic boundaries enables you to examine the near field influence among the individual antennas. However, note that the resulting structure represents an infinitely extended antenna pattern. Hence the farfield results are most accurate in conjunction with the definition of an (preferably large) antenna array in the farfield postprocessing. Please refer to the farfield overview for details.

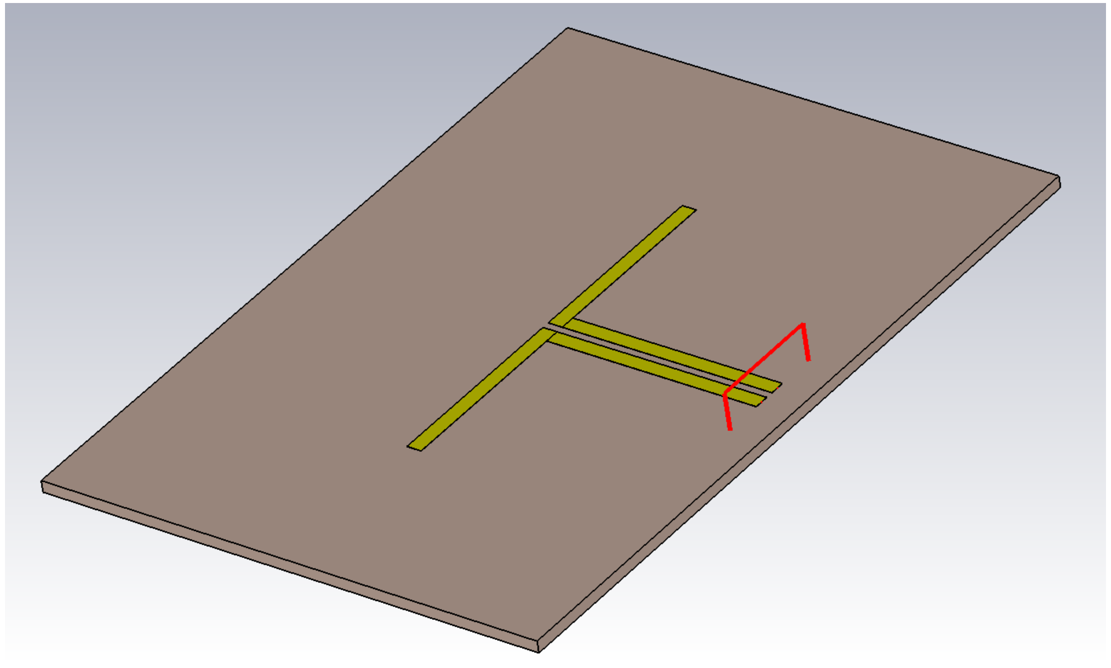
Conducting Wall: This boundary behaves like a wall of lossy metal material.

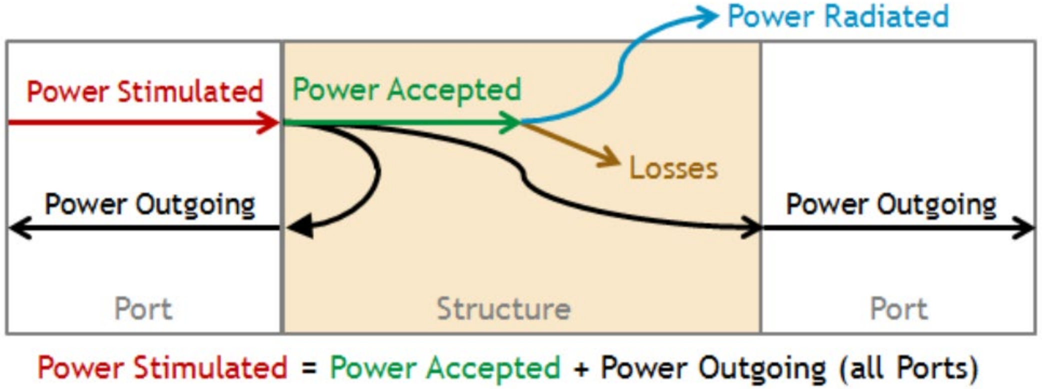
Unit Cell: A Unit Cell boundary condition is very similar to the Periodic boundary condition. In addition, a two-dimensional periodicity other than one in the direction of the coordinate axes can be defined (Boundary Conditions - Unit Cell). If there are open boundaries perpendicular to the Unit Cell boundaries, they are realized by Floquet modes, similar to modes of a wave guide port . The farfield considerations are the same as for the periodic boundaries.



Ports

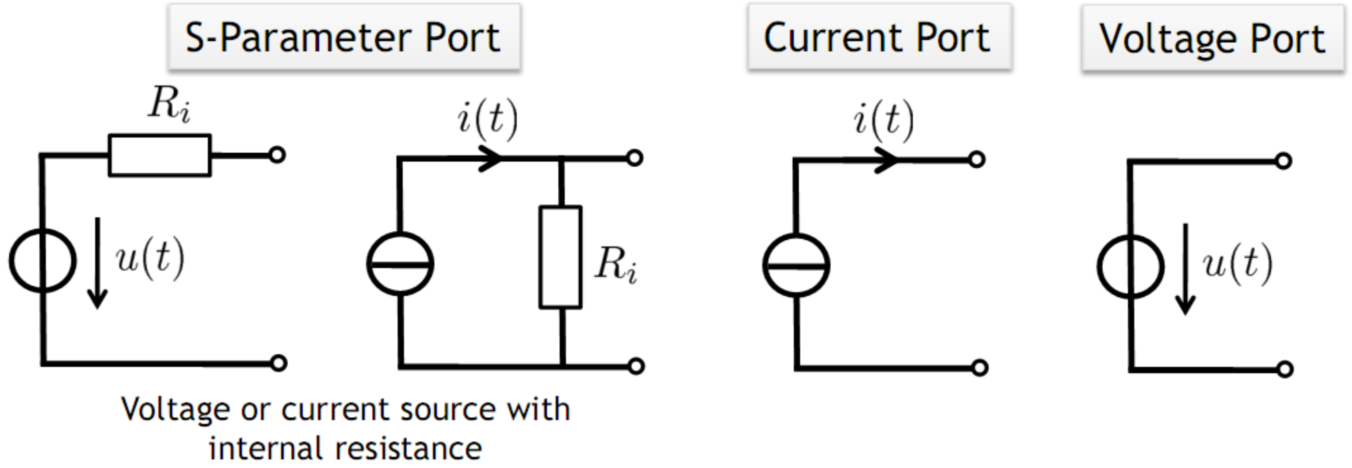
- Required to excite model
- Types:
  - Discrete Port
  - Waveguide Port
  - Plane Wave

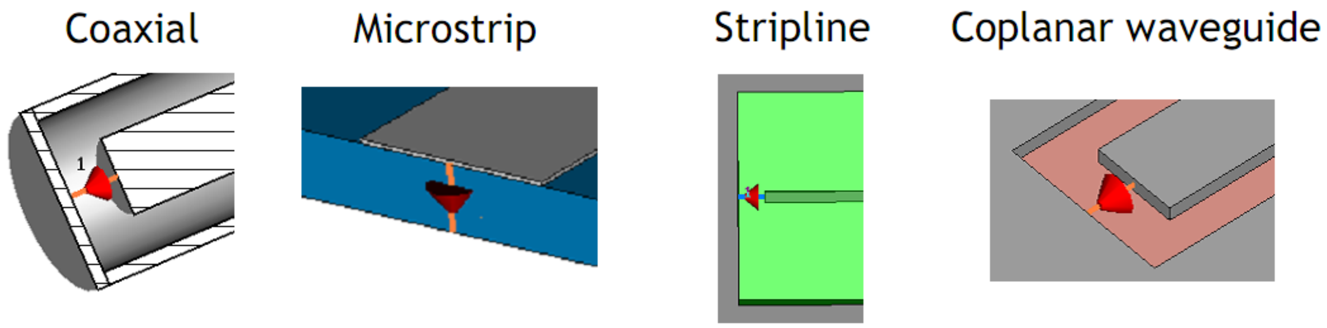




Discrete Port

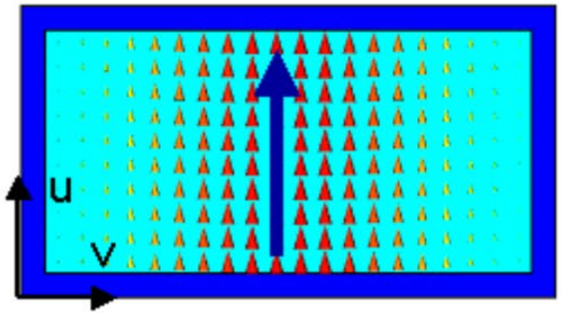
- Lumped element model
- Point/Edge/Face source



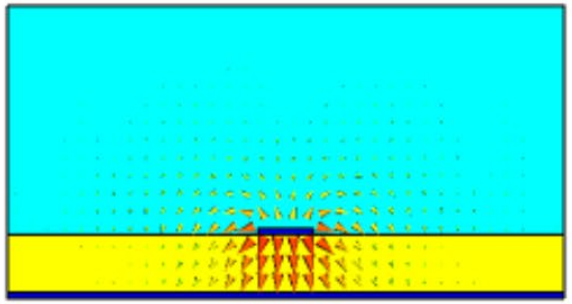


Waveguide Port

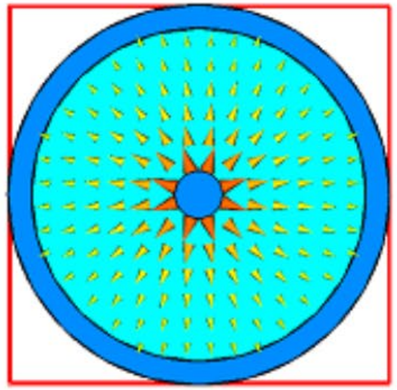
- Infinitely long waveguide
- Mode pattern waveguide should match mode pattern transmission line
- Fields should be enclosed by port



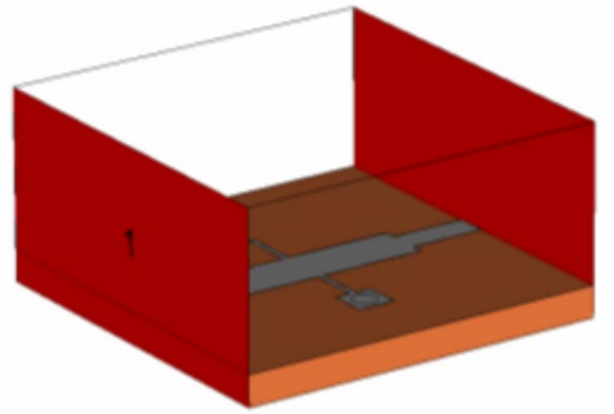
Rectangular hollow waveguide



Microstrip



Coaxial waveguide



Solvers

- Time Domain 

- Frequency Domain 

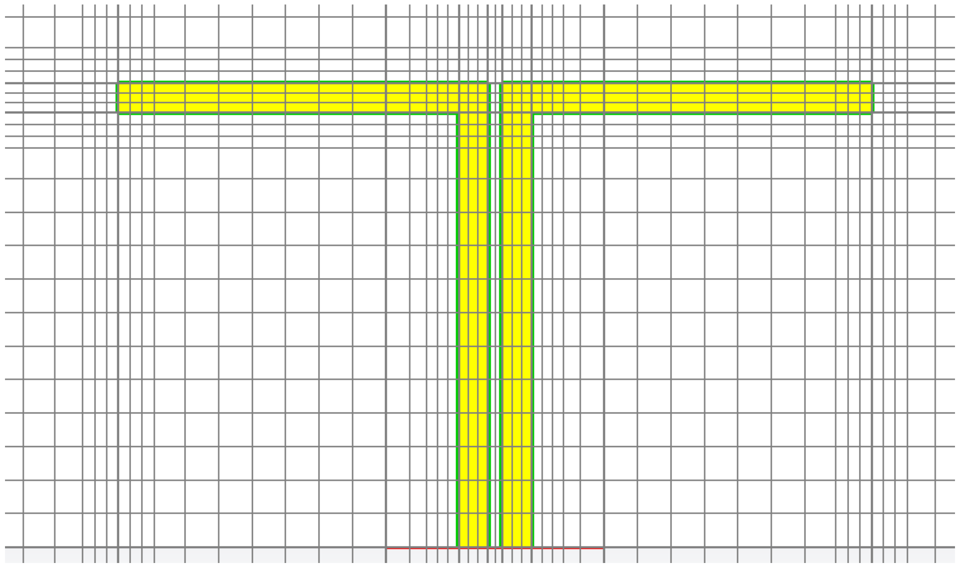
- Solvers solve Maxwell equations in a different way

Which solver is the “best”?

- Unique answer to this question is not easily possible as the performance and accuracy depend on many parameters:
  - Electrical size and geometry of the problem,
  - Material models and material parameters used,
  - Resonant behavior of the model,
  - Type of the mesh and the boundary conditions,
  - Architecture of the workstation used for the simulation,
  - etc.

Mesh

- Mesh type depends on solver
- More mesh cells at critical points
- Simulation time depends on number of mesh cells



Analysis of Results

1D results:

- S-parameters
- Input impedance
- Efficiency

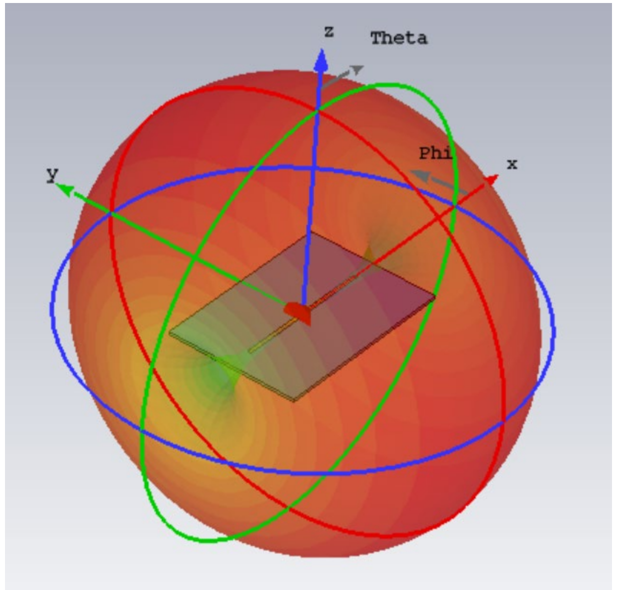
2D results:

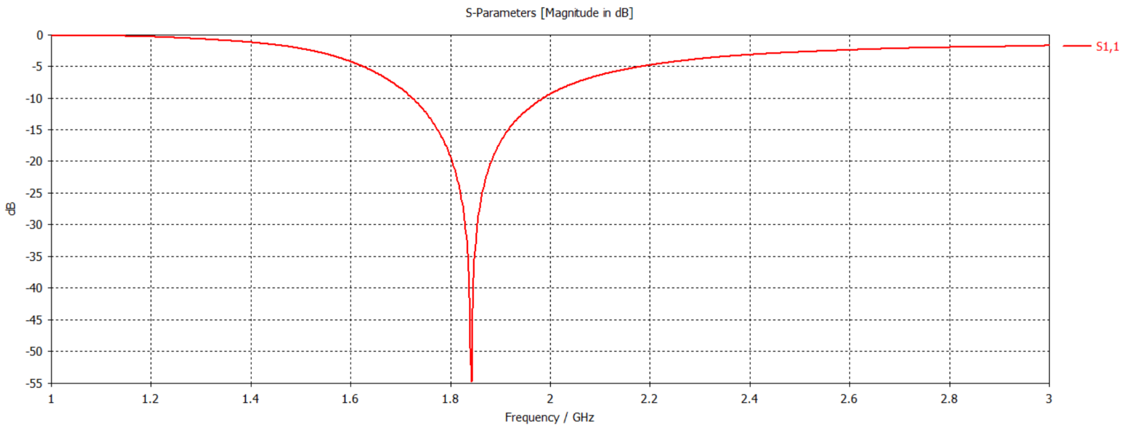
- Fields

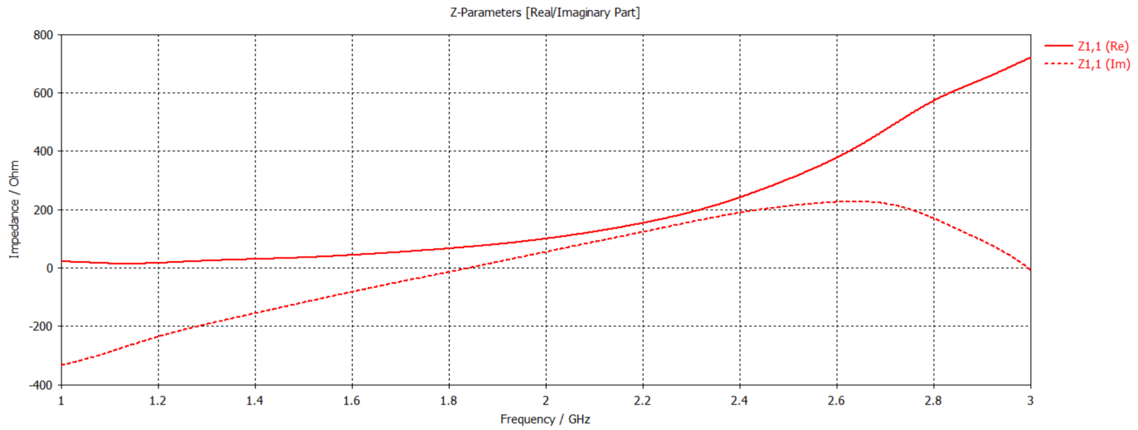
3D results:

- Radiation pattern

etc.

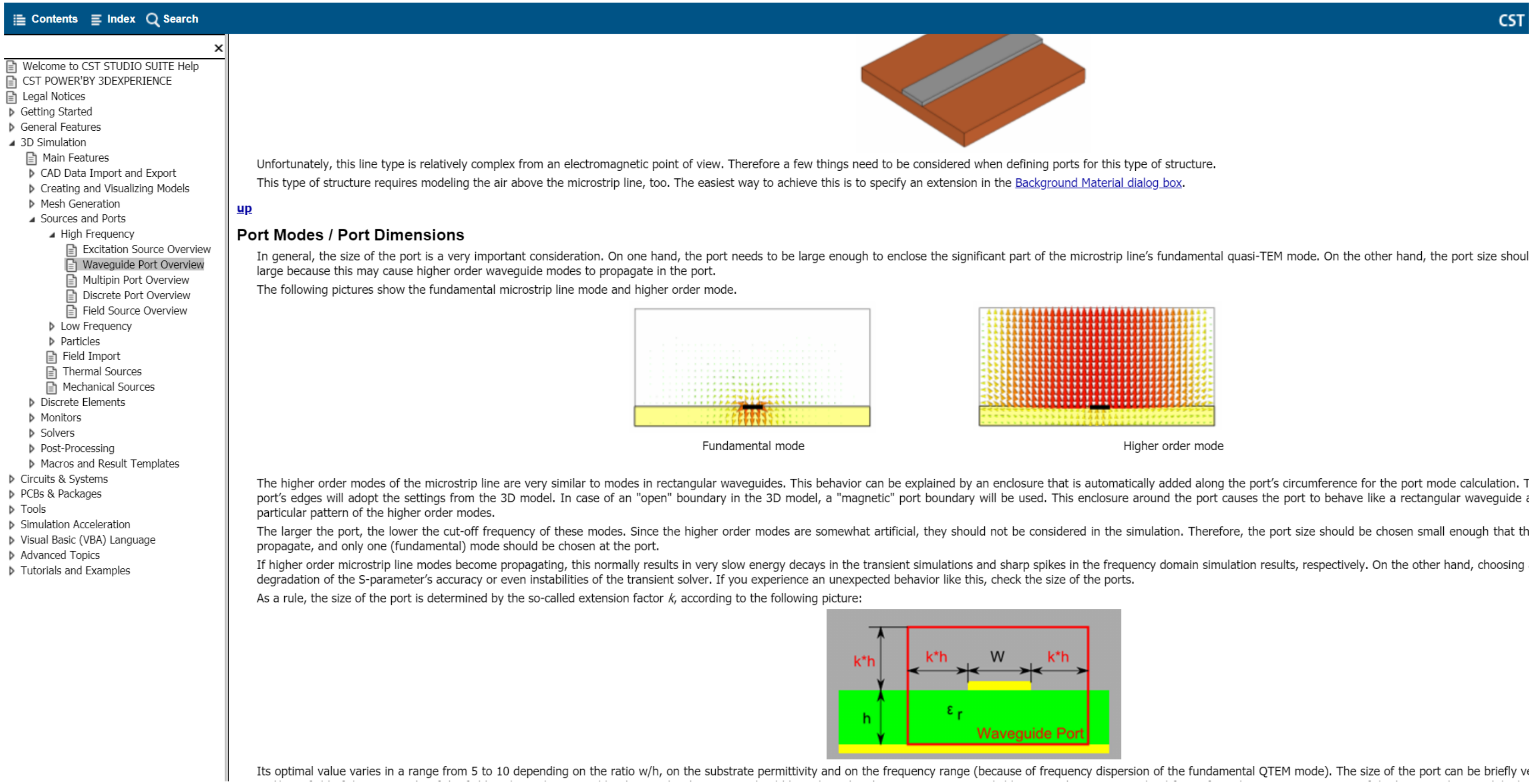






CST Help Function

- File → Help → Help Contents → 3D Simulation



Download CST (Student Version)

You can download the **student edition** of **CST Studio Suite** from https://www.3ds.com/products-services/simulia/products/cst-studio-suite/student-edition/

Microwave Engineering and Antennas

Radiated Fields: General Case

Bart Smolders, Professor

Department of Electrical Engineering

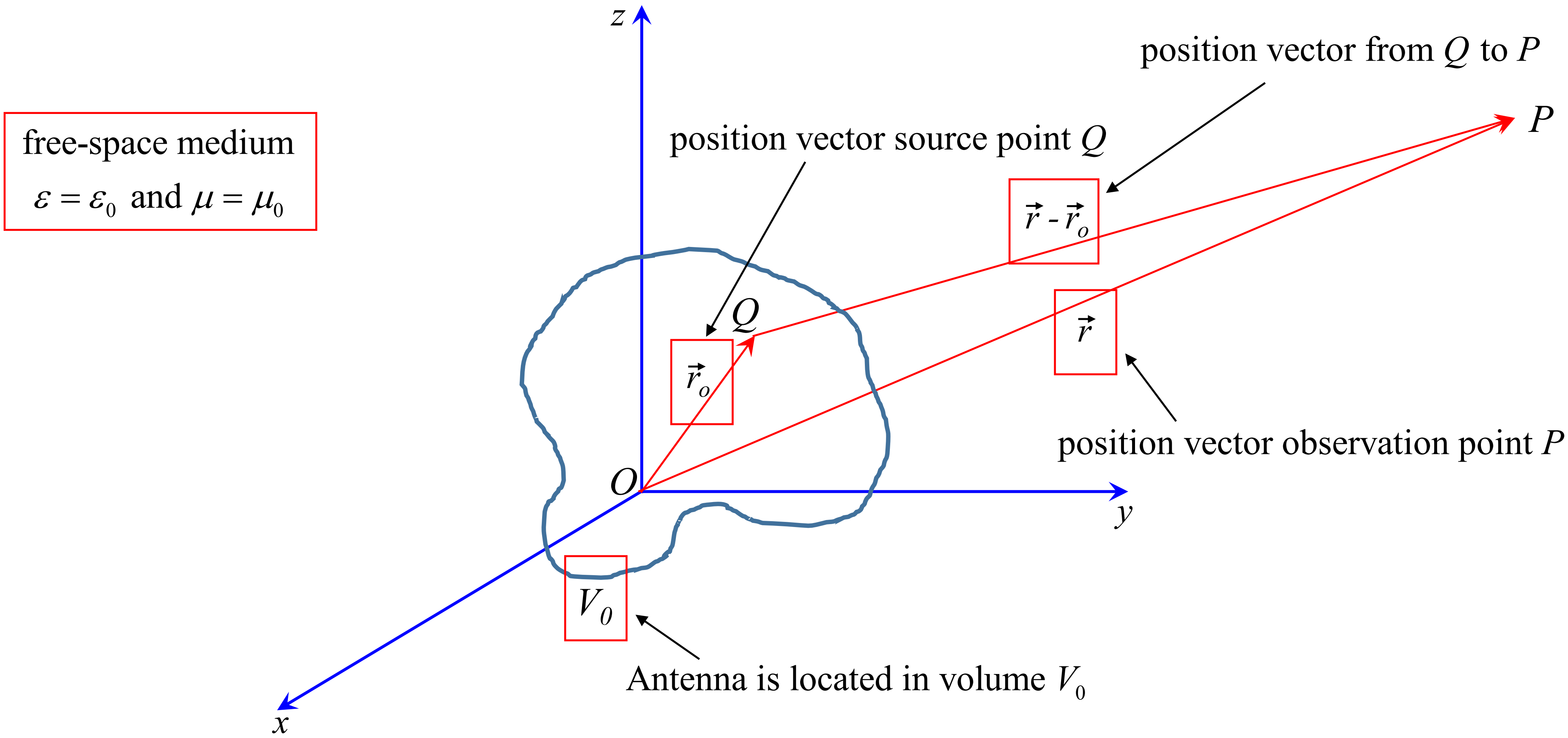
Center for Wireless Technology Eindhoven

Radiated Fields

Objective of this lecture

- Derive general formulation to calculate radiated fields from an antenna.
- Start with Maxwell's equations.
- Derive expression for E- and H-fields outside source region.

Configuration



Maxwell’s equations frequency domain

Electric field [V/m] Magnetic field [A/m]

Electric current distribution on antenna [A/m^2]

$$\nabla \times \boxed{\vec{E}(\vec{r})} = -j\omega\mu_0 \boxed{\vec{H}(\vec{r})}$$

$$\nabla \times \vec{H}(\vec{r}) = j\omega\varepsilon_0 \vec{E}(\vec{r}) + \boxed{\vec{J}_e(\vec{r})}$$

$$\boxed{\nabla \cdot \vec{H}(\vec{r}) = 0}$$

No magnetic sources

$$\nabla \cdot \vec{E}(\vec{r}) = \frac{\boxed{\rho_e(\vec{r})}}{\varepsilon_0}$$

Electric charge distribution on antenna [C/m^3]

Magnetic vector potential

convenient in notation lateron

magnetic vector potential

$$\left.\begin{aligned}\nabla\cdot\boxed{\vec{H}}&=0\\ \nabla\cdot\nabla\times\vec{A}_e&=0\end{aligned}\right\}\Longrightarrow\quad \vec{H}=\boxed{\frac{1}{\mu_0}}\nabla\times\boxed{\vec{A}_e}$$

Compact notation without $(\vec{r})$

$$\left.\begin{aligned}\nabla\times\vec{E}(\vec{r})&=-j\omega\mu_0\vec{H}(\vec{r})\\ \vec{H}&=\frac{1}{\mu_0}\nabla\times\vec{A}_e\end{aligned}\right\}\Longrightarrow\quad \vec{E}=-j\omega\vec{A}_e-\boxed{\nabla\phi_e}$$

Since $\nabla\times\nabla\phi_e=\vec{0}$

Approach

Introduce magnetic vector potential $\vec{A}_e$

$\Downarrow$

Helmholtz equation

$\Downarrow$

Lorenz gauge

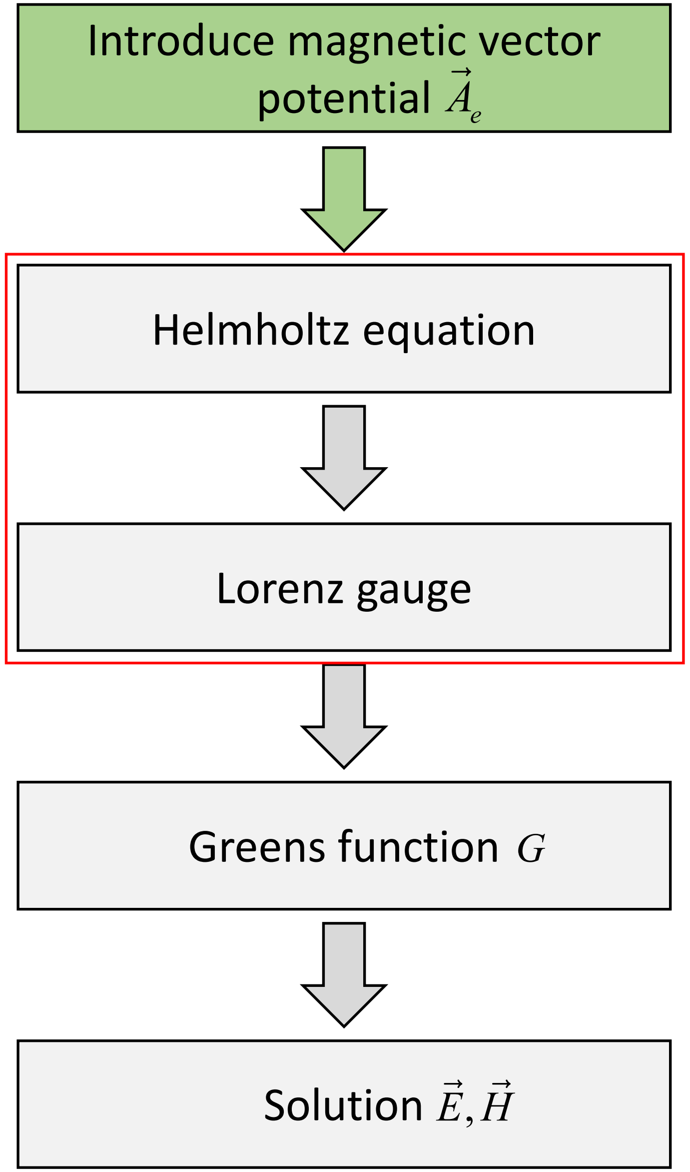
$\Downarrow$

Greens function G

$\Downarrow$

Solution $\vec{E}, \vec{H}$

Approach


Introduce magnetic vector potential $\vec{A}_e$
Helmholtz equation
Lorenz gauge
Greens function G
Solution $\vec{E}, \vec{H}$

Approach

Introduce magnetic vector potential $\vec{A}_e$

Helmholtz equation

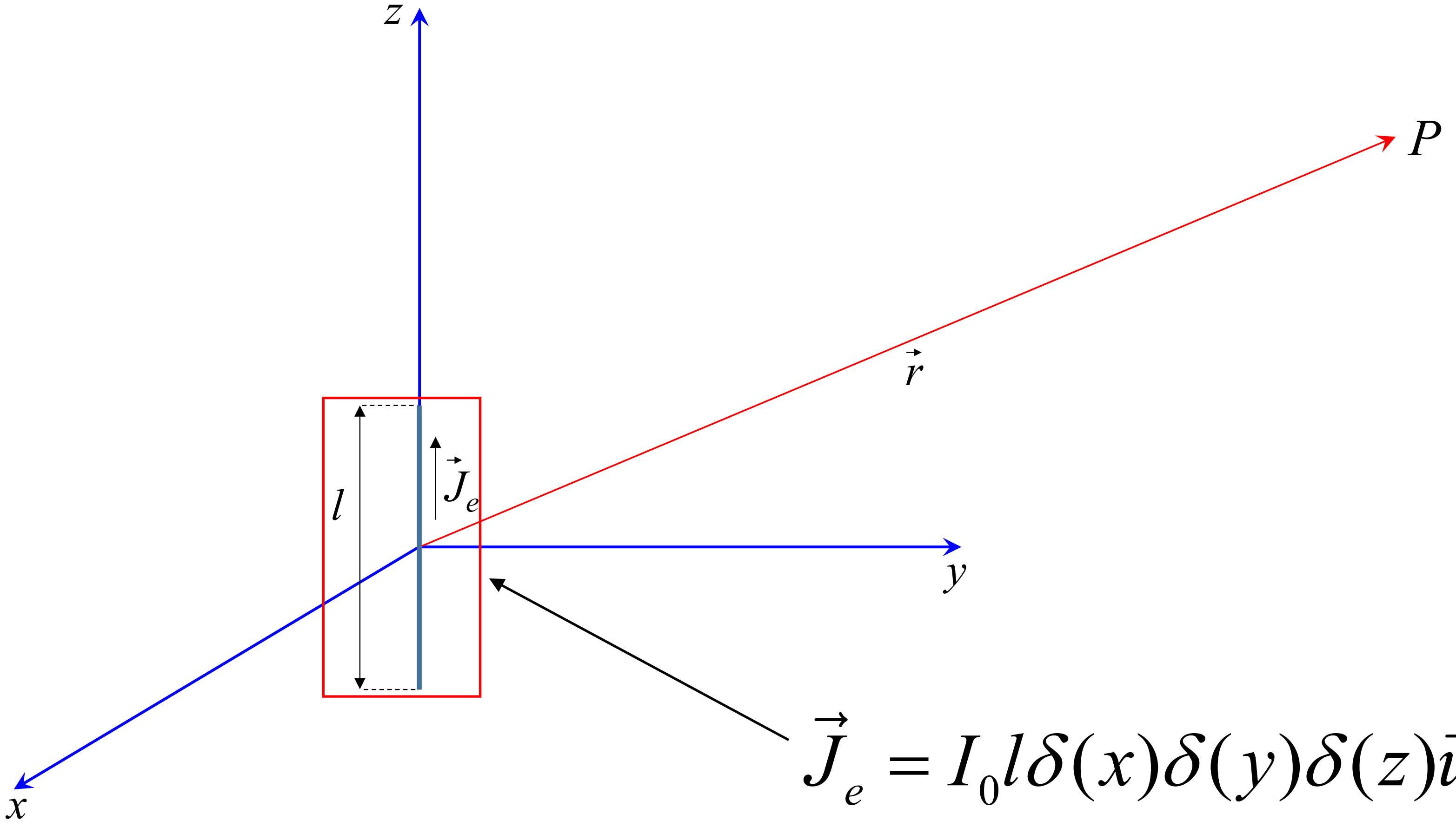
Lorenz gauge

Greens function G

Solution $\vec{E}, \vec{H}$

$$\nabla^2 \vec{A}_e + k_0^2 \vec{A}_e = -\mu_0 \vec{J}_e$$

Greens function: response of point source



Current z-directed only

$$\vec{J}_e = I_0 l \delta(x)\delta(y)\delta(z)\vec{u}_z = I_0 l \delta(\vec{r})\boxed{\vec{u}_z}$$

$$\Longrightarrow \quad \vec{A}_e = A_{ez}\vec{u}_z$$

Approach

Introduce magnetic vector potential $\vec{A}_e$

↓

Helmholtz equation

↓

Lorenz gauge

↓

Greens function G

↓

Solution $\vec{E}, \vec{H}$

$$\nabla^2 \vec{A}_e + k_0^2 \vec{A}_e = -\mu_0 \vec{J}_e$$

$$\vec{A}_e = \iiint_{V_0} \vec{J}_e(\vec{r}_0) \underbrace{\boxed{\frac{\mu_0 e^{-jk_0|\vec{r}-\vec{r}_0|}}{4\pi|\vec{r}-\vec{r}_0|}}}_{G(\vec{r},\vec{r}_0)} dV_0$$

General solution fields

$$\vec{H} = \frac{1}{\mu_0}\nabla \times \vec{A}_e$$

Lorenz gauge $\nabla \cdot \vec{A}_e = -j\omega\varepsilon_0\mu_0\phi_e$

$$\vec{E} = -j\omega\vec{A}_e - \nabla\boxed{\phi_e}$$

vector identity $\nabla^2\vec{A}_e = \nabla\nabla\cdot\vec{A}_e - \nabla\times\nabla\times\vec{A}_e$

$$= -j\omega\vec{A}_e - \frac{\boxed{\nabla\nabla\cdot\vec{A}_e}}{j\omega\varepsilon_0\mu_0}$$

Helmholtz equation $\nabla^2\vec{A}_e + k_0^2\vec{A}_e = -\mu_0\vec{J}_e$

$$= \frac{1}{j\omega\varepsilon_0\mu_0}\left[\nabla\times\nabla\times\vec{A}_e + \boxed{\nabla^2\vec{A}_e + k_0^2\vec{A}_e}\right]$$

Outside source region: $\vec{J}_e = 0$

$$= \frac{1}{j\omega\varepsilon_0\mu_0}\left[\nabla\times\nabla\times\vec{A}_e - \boxed{\mu_0\vec{J}_e}\right]$$

Summary of approach

$$\vec{A}_e(\vec{r}) = \iiint_{V_0} \vec{J}_e(\vec{r}_0)\frac{\mu_0 e^{-jk_0|\vec{r}-\vec{r}_0|}}{4\pi|\vec{r}-\vec{r}_0|}dV_0$$

$$\vec{H}(\vec{r}) = \frac{1}{\mu_0}\nabla\times\vec{A}_e(\vec{r})$$

$$\boxed{\vec{E}(\vec{r})} = \frac{1}{j\omega\varepsilon_0\mu_0}\left[\nabla\times\nabla\times\vec{A}_e(\vec{r})\right]$$

Outside source region

Introduce magnetic vector potential $\vec{A}_e$

Helmholtz equation

Lorenz gauge

Greens function G

Solution $\vec{E},\vec{H}$

Summary

- Starting point Maxwell’s equations.
- Defined a step-by-step approach to determine the fields outside source region.
- In the next weblecture we will simplify the expressions by using far-field approximations.

Microwave Engineering and Antennas

Radiated Fields: Far Field

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CWTe
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Radiated Fields: Far Field

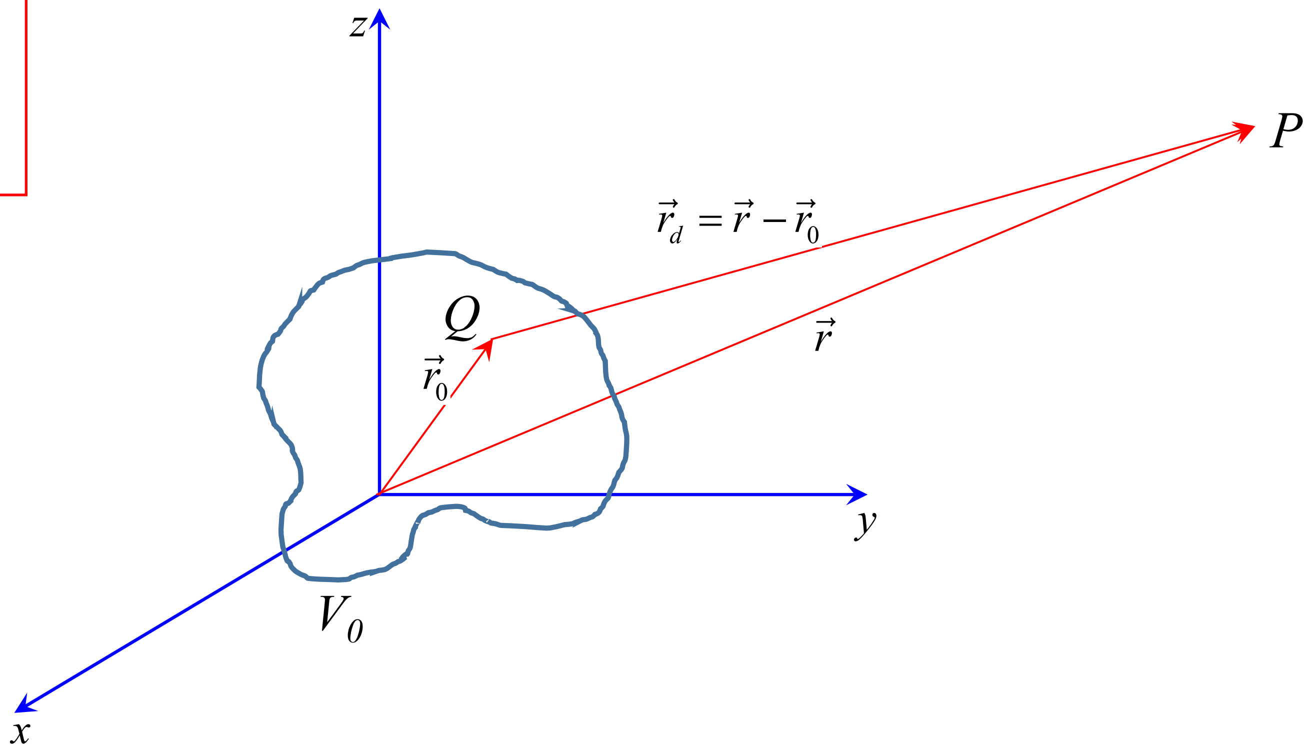
Objective of this lecture

- Introduce far-field approximations
- Derive the far-field condition
- Derive general expressions for the E- and H-fields in the far-field region of an antenna
- Determine radiation pattern

Starting point

$$\boxed{\begin{aligned}\vec{A}_e(\vec{r}) &= \iiint_{V_0} \vec{J}_e(\vec{r}_0)\frac{\mu_0 e^{-jk_0|\vec{r}-\vec{r}_0|}}{4\pi|\vec{r}-\vec{r}_0|}dV_0 \\ \vec{H}(\vec{r}) &= \frac{1}{\mu_0}\nabla\times\vec{A}_e(\vec{r})\end{aligned}}$$

$$\vec{E}(\vec{r}) = \frac{1}{j\omega\varepsilon_0\mu_0}\left[\nabla\times\nabla\times\vec{A}_e(\vec{r})\right]$$



Far-field approximation, step 1

Derivative w.r.t. $\vec{r}$ (observation point)

Integration over $\vec{r}_0$ (source points)

$$\left.\begin{aligned}\vec{A}_e(\vec{r}) &= \iiint_{V_0} \vec{J}_e(\vec{r}_0)\frac{\mu_0 e^{-jk_0|\vec{r}-\vec{r}_0|}}{4\pi|\vec{r}-\vec{r}_0|}dV_0 \\ \vec{H}(\vec{r}) &= \frac{1}{\mu_0}\nabla\times\vec{A}_e(\vec{r})\end{aligned}\right\}$$

$$\vec{H}(\vec{r}) = \iiint_{V_0} \boxed{\nabla_r}\times\vec{J}_e(\vec{r}_0)\boxed{\varphi(\vec{r},\vec{r}_0)}\boxed{dV_0}$$

$$\varphi(\vec{r},\vec{r}_0) = \frac{e^{-jk_0|\vec{r}-\vec{r}_0|}}{4\pi|\vec{r}-\vec{r}_0|}$$

$\boxed{\text{Step 1: rewrite } \nabla_r\times\vec{J}_e(\vec{r}_0)\varphi(\vec{r},\vec{r}_0)}$

Far-field approximation, step 2

$$\vec{H}(\vec{r}) = \iiint_{V_0} \boxed{\nabla_r \times \vec{J}_e(\vec{r}_0)\varphi(\vec{r},\vec{r}_0)} dV_0$$

$$\nabla_r \times \vec{J}_e(\vec{r}_0)\varphi(\vec{r},\vec{r}_0) = -\frac{1}{4\pi}\left(jk_0 + \frac{1}{\boxed{r_d}}\right)\frac{e^{-jk_0\boxed{r_d}}}{\boxed{r_d}}\vec{u}_{r_d} \times \vec{J}_e(\vec{r}_0)$$

$r_d = |\vec{r} - \vec{r}_0|$

$$\vec{H}(\vec{r}) = -\frac{1}{4\pi}\iiint_{V_0}\left(jk_0 + \frac{1}{r_d}\right)\frac{e^{-jk_0 r_d}}{r_d}\vec{u}_{r_d} \times \vec{J}_e(\vec{r}_0)\, dV_0$$

Far-field approximation, step 2

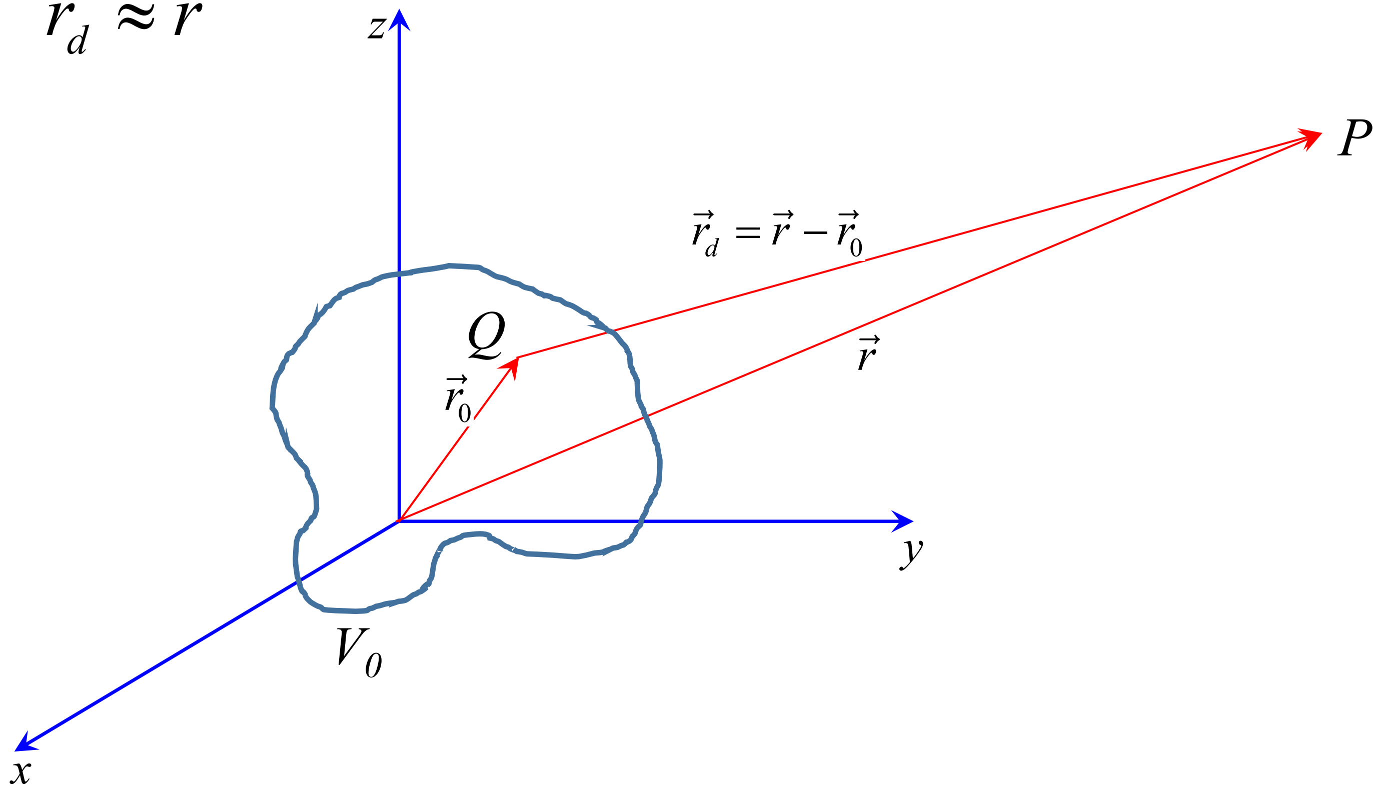
$$\vec{H}(\vec{r}) = -\frac{1}{4\pi}\iiint_{V_0}\left(\boxed{jk_0+\frac{1}{r_d}}\right)\frac{e^{-jk_0 r_d}}{\boxed{r_d}}\boxed{\vec{u}_{r_d}}\times\vec{J}_e(\vec{r}_0)\,dV_0$$

Assume $|\vec{r}| \gg |\vec{r}_0|$

$$\vec{u}_{r_d} \approx \vec{u}_r$$

$$r_d \approx r$$

$$jk_0+\frac{1}{r_d} \approx jk_0$$



Far-field approximation, step 2

$$\vec{H}(\vec{r}) = \frac{-jk_0}{4\pi r}\vec{u}_r \times \iiint_{V_0} \vec{J}_e(\vec{r}_0)\boxed{e^{-jk_0 r_d}}\,dV_0$$

Assume $|\vec{r}| \gg |\vec{r}_0|$

periodic function!

Far-field approximation, step 2

$$\boxed{\vec{H}(\vec{r})} = \frac{-jk_0 \boxed{e^{-jk_0 r}}}{\boxed{4\pi r}} \vec{u}_r \times \iiint_{V_0} \vec{J}_e(\vec{r}_0) e^{\boxed{jk_0 \vec{u}_r \cdot \vec{r}_0}} dV_0 \qquad r > \frac{2D^2}{\lambda_0}$$

$\vec{H}\cdot\vec{u}_r = 0$ so no radial component

spherical expansion

Phasor $e^{j\omega t}$, time harmonic behavior

phasor, propagation in $+r$ direction

phase difference between source point $\vec{r}_0$ and origin coordinate system

$$\vec{H}(\vec{r}) = \frac{e^{-jk_0 r}}{r}\left(\boxed{\vec{u}_\theta H_\theta(\theta,\phi)} + \boxed{\vec{u}_\phi H_\phi(\theta,\phi)}\right)$$

Far-field approximation, step 3

$$\vec{E}(\vec{r}) = \frac{1}{j\omega\varepsilon_0\mu_0}\left[\nabla\times\nabla\times\vec{A}_e(\vec{r})\right]$$

$$\Downarrow \longleftarrow \boxed{\text{far-field approximation } r > \frac{2D^2}{\lambda_0}}$$

$$\boxed{\vec{E}(\vec{r})} = \frac{-k_0^2}{j\omega\varepsilon_0}\frac{e^{-jk_0 r}}{4\pi r}\vec{u}_r\times\vec{u}_r\times\iiint_{V_0}\vec{J}_e(\vec{r}_0)e^{jk_0\vec{u}_r\cdot\vec{r}_0}dV_0$$

$$\Downarrow$$

$\vec{E}\cdot\vec{u}_r = 0$ so no radial component

$$\vec{E}(\vec{r}) = \frac{e^{-jk_0 r}}{r}\left(\vec{u}_\theta E_\theta(\theta,\phi) + \vec{u}_\phi E_\phi(\theta,\phi)\right)$$

Far-field approximation

Summary

extra $\vec{u}_r \times$

$$\vec{E}(\vec{r}) = \frac{-k_0^2}{j\omega\varepsilon_0}\frac{e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \boxed{\vec{u}_r \times} \iiint_{V_0} \vec{J}_e(\vec{r}_0) e^{jk_0 \vec{u}_r \cdot \vec{r}_0} dV_0$$

$$\vec{H}(\vec{r}) = \frac{-jk_0 e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \iiint_{V_0} \vec{J}_e(\vec{r}_0) e^{jk_0 \vec{u}_r \cdot \vec{r}_0} dV_0$$

TEM wave

$$\boxed{\vec{H}(\vec{r}) = \frac{1}{\boxed{Z_0}}\vec{u}_r \times \vec{E}(\vec{r})}$$

$Z_0 = 120\pi = 377\ \Omega$

TU/e

Far field radiation pattern

Time-average Poynting vector

$$\left.\begin{aligned}\vec{S}_p(\vec{r}) &= \frac{1}{2}\mathrm{Re}\left[\vec{E}(\vec{r})\times\vec{H}^*(\vec{r})\right] \\ \vec{H}(\vec{r}) &= \frac{1}{Z_0}\vec{u}_r\times\vec{E}(\vec{r})\end{aligned}\right\} \qquad \vec{S}_p(\vec{r}) = \frac{1}{2Z_0}\left|\vec{E}(\vec{r})\right|^2 \boxed{\vec{u}_r} \quad \left[\mathrm{W/m^2}\right]$$

Power flow in the r-direction

Radiation pattern

Radiated power per element of solid angle

$$F(\theta,\phi) = \frac{P(\theta,\phi)}{P_{\max}} = \frac{\left|r^2\vec{S}_p(\vec{r})\right|}{P_{\max}}$$

Summary

- Far-field expression for the E- and H-fields have been derived.
- This holds for any antenna.
- From this we can determine the radiation pattern and other characteristics.
- Next web lectures we will show how are general expressions can be used for all kind of antennas.

Microwave Engineering and Antennas

Electric Dipole Antenna

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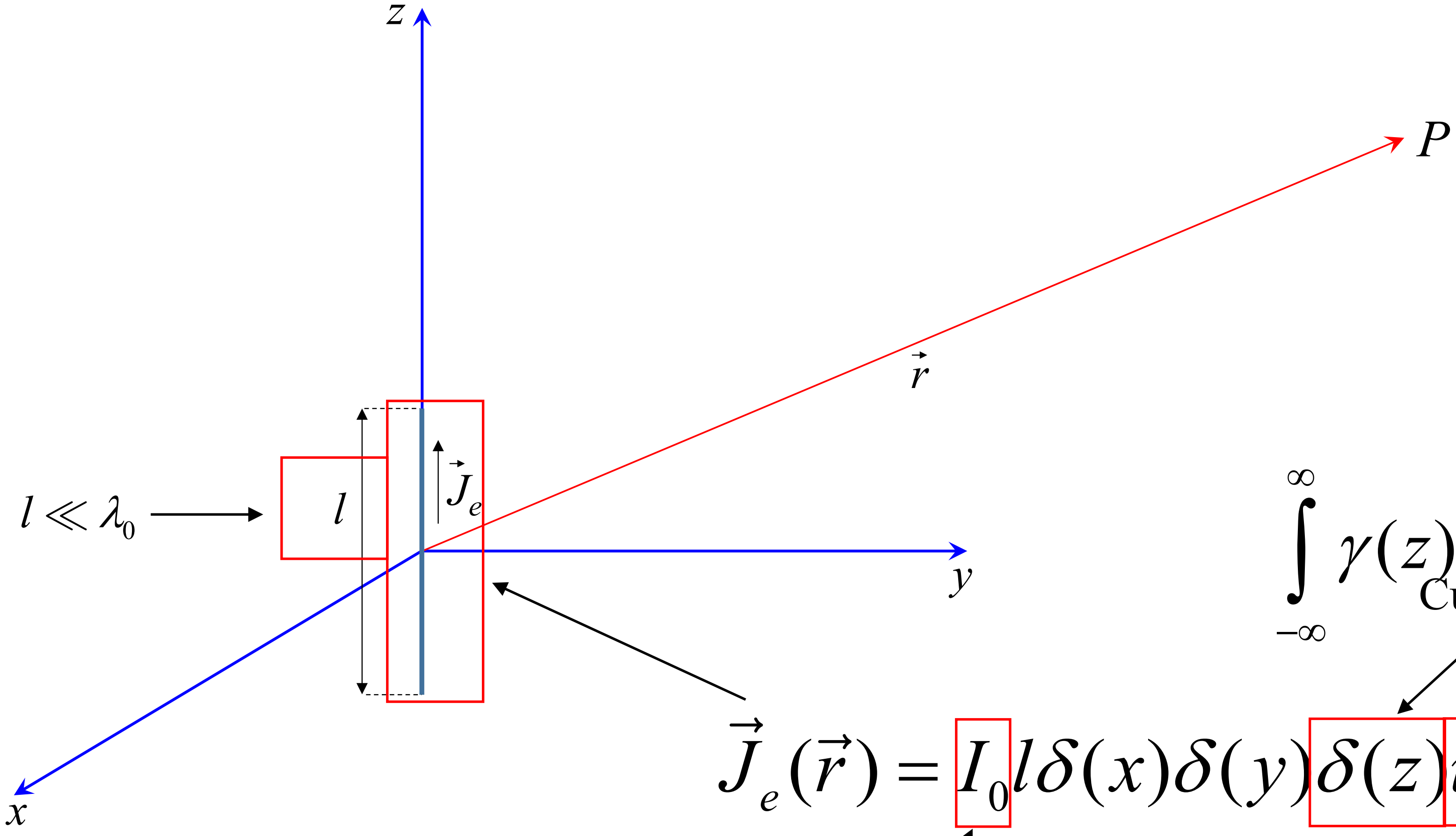
Electric dipole antenna

Objective of this lecture

- Apply the general formulation to determine the far-field of an electric dipole
- Plot the radiation pattern
- Calculate the total radiated power and the radiation resistance

Current distribution on antenna



$$\int_{-\infty}^{\infty} \gamma(z)\delta(z - z_0)dz = \gamma(z_0)$$

Current z-directed only

$$\vec{J}_e(\vec{r}) = \boxed{I_0} l\delta(x)\delta(y)\boxed{\delta(z)}\boxed{\vec{u}_z} = I_0 l\delta(\vec{r})\vec{u}_z$$

Total current along dipole

Apply our recipe

Far-field expression

$$\vec{H}(\vec{r}) = \frac{-jk_0 e^{-jk_0 r}}{4\pi r} \boxed{\vec{u}_r \times \iiint_{V_0}} \boxed{\vec{J}_e(\vec{r}_0)} e^{jk_0 \vec{u}_r \cdot \vec{r}_0} dV_0$$

$$\vec{E}(\vec{r}) = Z_0 \vec{H}(\vec{r}) \times \vec{u}_r$$

$$\vec{J}_e(\vec{r}_0) = I_0 l \delta(\vec{r}_0) \vec{u}_z$$

$$\vec{u}_r \times \vec{u}_z = -\vec{u}_\phi \sin\theta$$

$$\vec{H}(\vec{r}) = \frac{jk_0 I_0 l e^{-jk_0 r}}{4\pi r} \sin\theta \boxed{\vec{u}_\phi}$$

$$\boxed{\vec{E}(\vec{r})} = \frac{jk_0 Z_0 I_0 l e^{-jk_0 r}}{4\pi r} \sin\theta \boxed{\vec{u}_\theta}$$

Pointing vector

$$\vec{S}_p(\vec{r}) = \frac{1}{2Z_0} \left| \vec{E}(\vec{r}) \right|^2 \vec{u}_r = \frac{1}{2} \frac{k_0^2 Z_0 (I_0 l)^2}{(4\pi r)^2} \sin^2\theta \vec{u}_r$$

Radiation properties

Radiation pattern

$$F(\theta) = \frac{P(\theta)}{P_{\max}} = \frac{\left| r^2 \vec{S}_p(\vec{r}) \right|}{\boxed{P_{\max}}} = \sin^2\theta$$

Maximum at $\theta = \pi / 2$

Total radiated power

$$P_t = \iint_{sphere} \boxed{\vec{S}_p(\vec{r}) \cdot \vec{u}_r r^2} d\Omega$$

$$\vec{S}_p(\vec{r}) \cdot \vec{u}_r r^2 = \frac{1}{2} \frac{k_0^2 Z_0 (I_0 l)^2}{(4\pi)^2} \sin^2\theta$$

$$d\Omega = \sin\theta d\theta d\phi$$

standard integral $\int_0^\pi \sin^3\theta d\theta = \frac{4}{3}$

$$P_t = \frac{1}{2} \frac{k_0^2 Z_0 (I_0 l)^2}{(4\pi)^2} \int_0^{2\pi} \boxed{\int_0^\pi \sin^3\theta d\theta} d\phi$$

$$= \frac{k_0^2 Z_0 (I_0 l)^2}{12\pi}$$

Radiation properties

Directivity function and Directivity/Gain

$$D(\theta) = \frac{P(\theta)}{P_t / 4\pi} = \frac{3}{2}\sin^2\theta \quad \Rightarrow \quad D = \frac{3}{2} \quad \Rightarrow \quad \boxed{G = \frac{3}{2}}$$

1.76 dBi

Input impedance

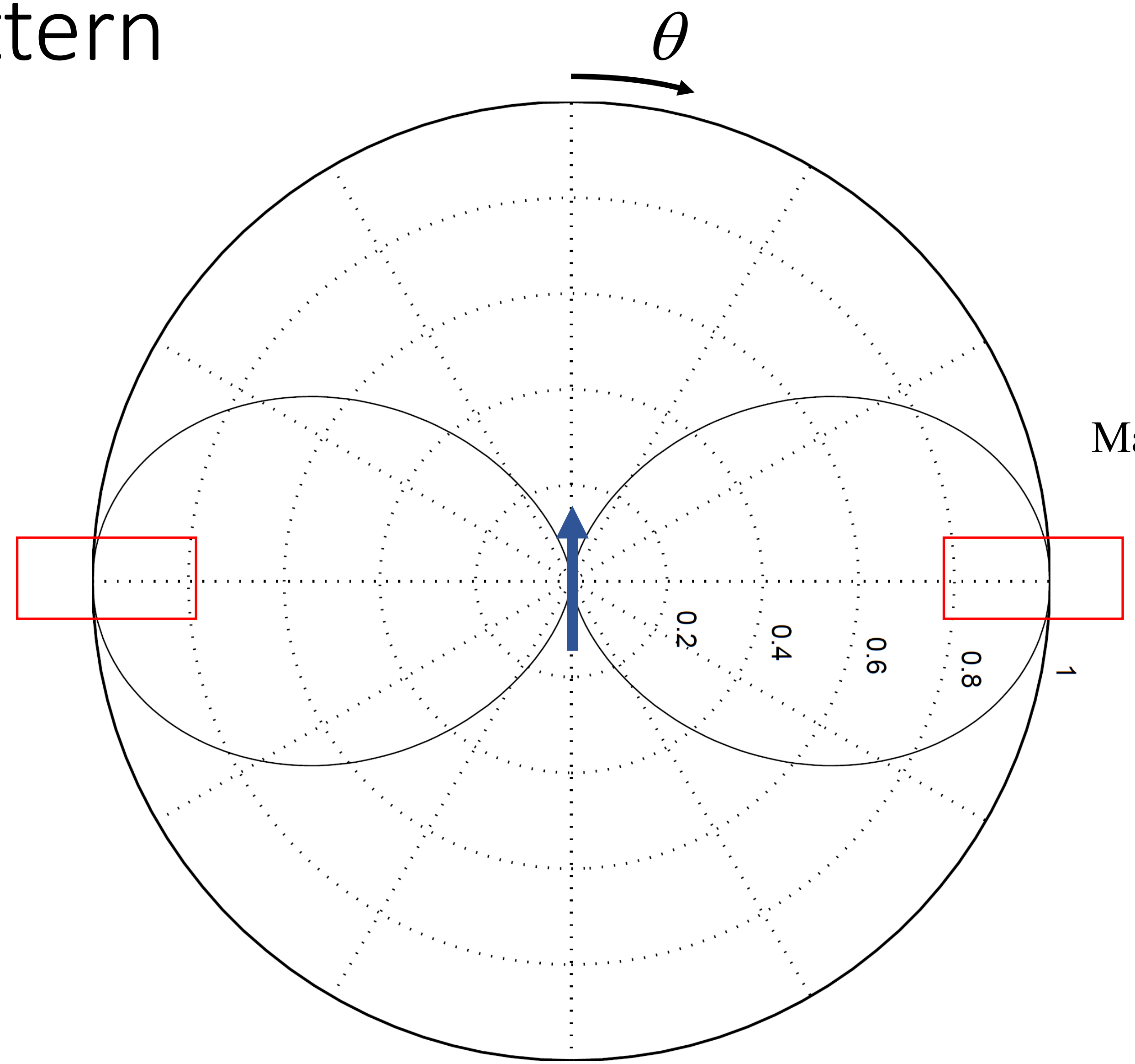
$$Z_a = R_a = \boxed{\frac{P_t}{\frac{1}{2}|I_0|^2}} = \frac{k_0^2 Z_0 l^2}{6\pi} = 80\pi^2 \boxed{\left(\frac{l}{\lambda_0}\right)^2}$$

since $P_t = \frac{1}{2} R_a |I_0|^2$

$l \ll \lambda_0$, so R_a very small!

Radiation pattern

Polar diagram



Maximum radiation at $\theta = \dfrac{\pi}{2}$

Summary

- The radiation properties of an ideal electric dipole have been calculated.
- The dipole has an antenna Gain of 1.76 dBi.
- We have shown the radiation pattern using a polar diagram.
- Main draw-back of an electric dipole is its very low R_a.
- In the next web lecture we will look at resonant wire antennas, which have a much higher R_a.

Microwave Engineering and Antennas

Wire Antennas

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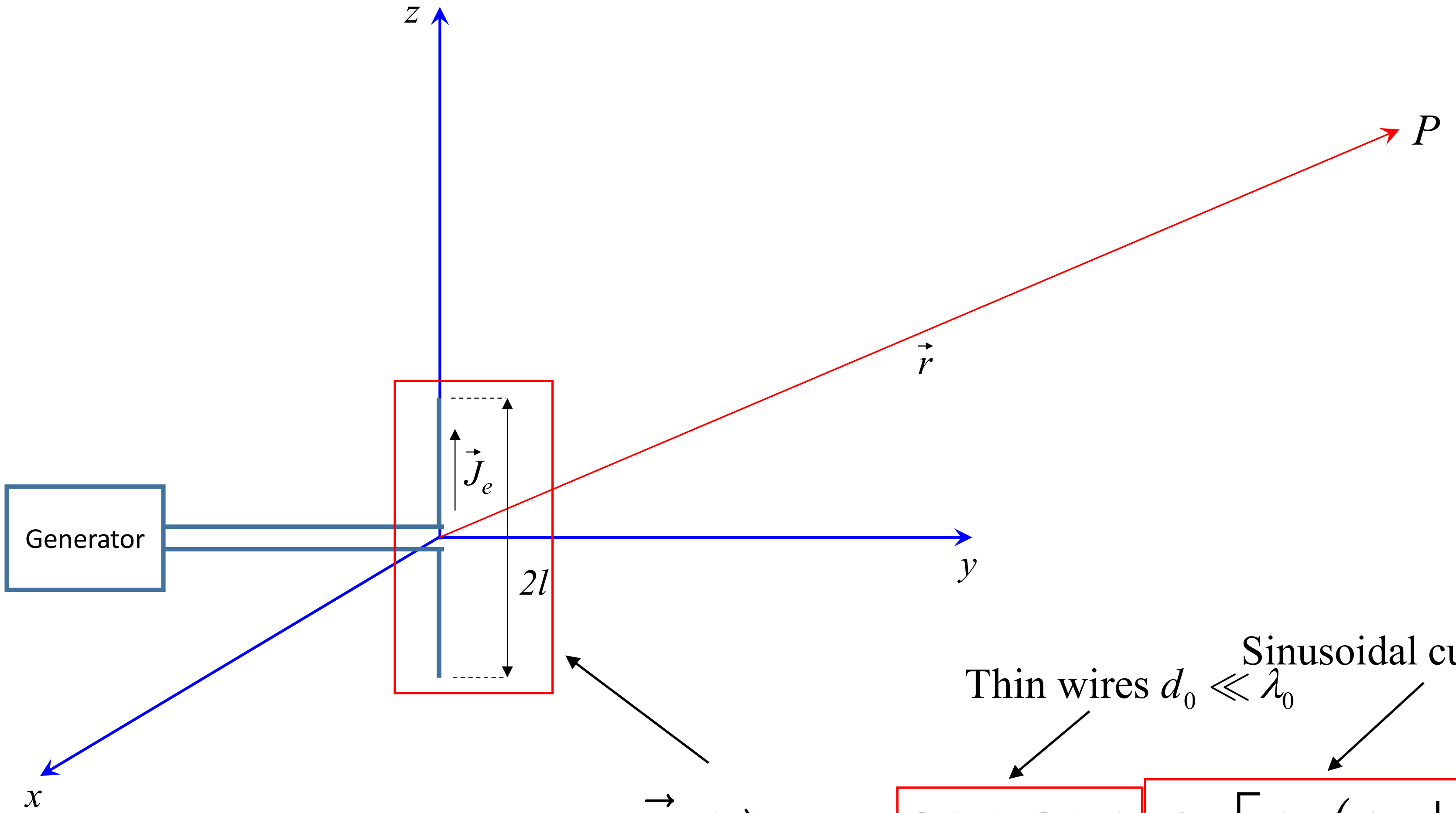
Wire antennas

Objective of this lecture

- Apply the general formulation to analyze wire antennas.
- We will consider dipoles, the monopole and the folded dipole.
- Show that wire antennas have a much higher R_a as compared to the electric dipole.

Current distribution on wire antenna



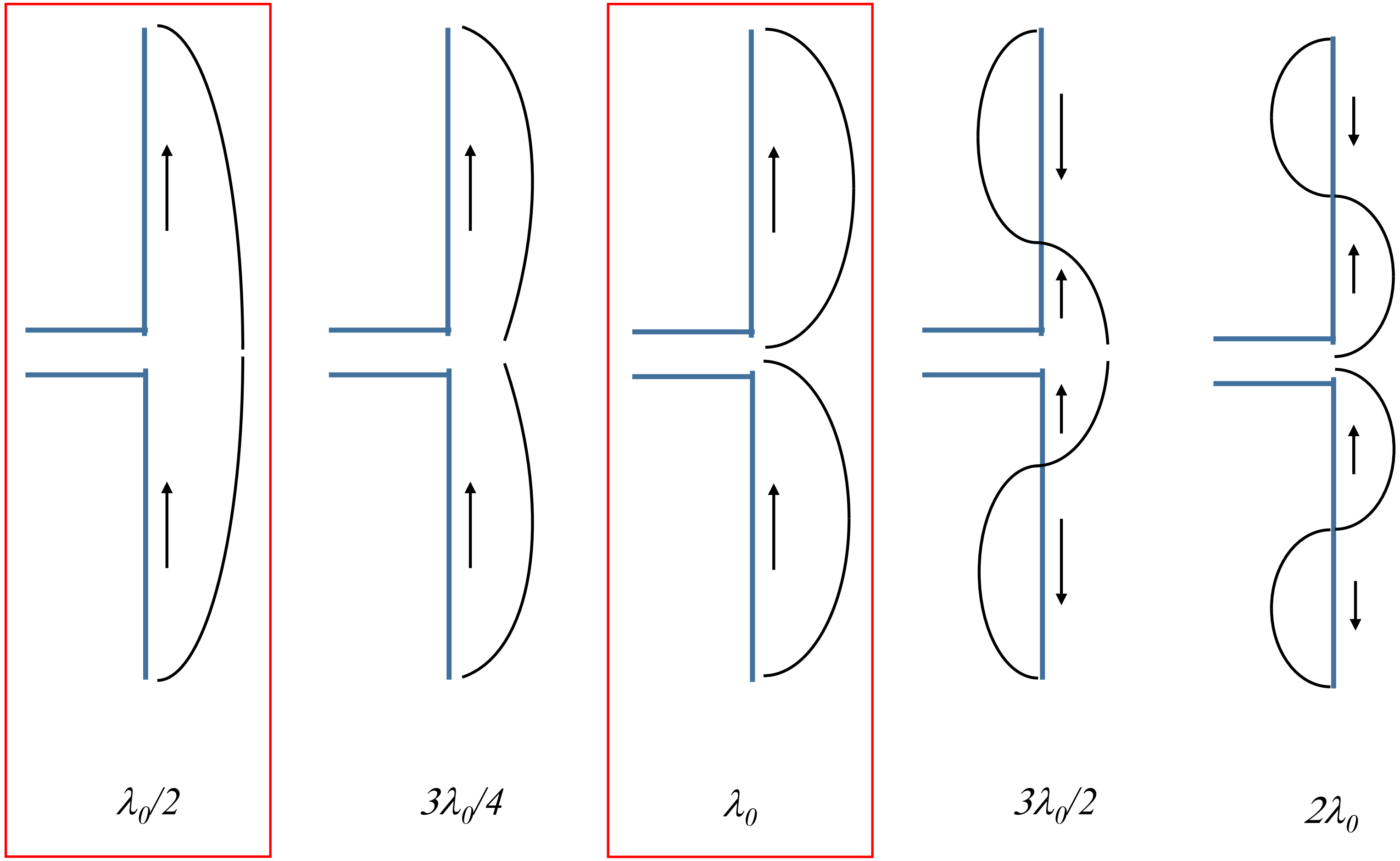
Thin wires $d_0 \ll \lambda_0$

Sinusoidal current distribution at resonance

Current z-directed only

$$\vec{J}_e(\vec{r}) = I_0 \boxed{\delta(x)\delta(y)} \boxed{\sin\left[k_0\left(l-|z|\right)\right]} \boxed{\vec{u}_z} \text{ for } -l \le z \le l$$

Current distribution various lengths



Apply our recipe

Far-field expression

$$\left.\begin{aligned}
&\vec{E}(\vec{r}) = \frac{-k_0^2}{j\omega\varepsilon_0}\frac{e^{-jk_0 r}}{4\pi r}\boxed{\vec{u}_r \times \vec{u}_r \times \iiint_{V_0}\boxed{\vec{J}_e(\vec{r}_0)}}e^{jk_0\vec{u}_r\cdot\vec{r}_0}dV_0\\
&\vec{J}_e(\vec{r}_0) = I_0\delta(x_0)\delta(y_0)\sin\left[k_0\left(l-|z_0|\right)\right]\vec{u}_z\\
&\vec{u}_r \times \vec{u}_r \times \vec{u}_z = \vec{u}_\theta \sin\theta
\end{aligned}\right\} \quad \vec{E}(\vec{r}) = \boxed{E_\theta}\vec{u}_\theta$$

$$E_\theta = \frac{-k_0^2 I_0}{j\omega\varepsilon_0}\frac{e^{-jk_0 r}}{4\pi r}\sin\theta\underbrace{\boxed{\int_{-l}^{l}\sin\left[k_0\left(l-|z_0|\right)\right]e^{jk_0 z_0\cos\theta}dz_0}}_{I_{z_0}}$$

Solving the integral

$$I_{z_0} = \int_{-l}^{l} \sin\left[k_0\left(l - \boxed{|z_0|}\right)\right] e^{jk_0 z_0 \cos\theta} dz_0$$

Substitute $\tilde{z}_0 = -z_0$

$$= \int_{0}^{l} \sin\left[k_0\left(l - z_0\right)\right] e^{jk_0 z_0 \cos\theta} dz_0 + \boxed{\int_{-l}^{0}} \sin\left[k_0\left(l + z_0\right)\right] e^{jk_0 z_0 \cos\theta} \boxed{dz_0}$$

$$= \int_{0}^{l} \sin\left[k_0\left(l - z_0\right)\right] \boxed{e^{jk_0 z_0 \cos\theta}} dz_0 + \int_{0}^{l} \sin\left[k_0\left(l - z_0\right)\right] \boxed{e^{-jk_0 z_0 \cos\theta}} dz_0$$

$$= \boxed{2} \int_{0}^{l} \boxed{\sin\left[k_0\left(l - z_0\right)\right]} \boxed{\cos\left(k_0 z_0 \cos\theta\right)} dz_0 = \frac{2\left[\cos\left(k_0 l \cos\theta\right) - \cos\left(k_0 l\right)\right]}{k_0 \sin^2\theta}$$

$$2\sin\alpha\cos\beta = \sin(\alpha-\beta) + \sin(\alpha+\beta)$$

Fields and radiation pattern

Far-field expression

$$E_\theta = \frac{-k_0^2 I_0}{j\omega\varepsilon_0}\frac{e^{-jk_0 r}}{4\pi r}\sin\theta\left(I_{z_0}\right) = jZ_0 I_0 \boxed{\frac{e^{-jk_0 r}}{2\pi r}}\boxed{\left[\frac{\cos\left(k_0 l\cos\theta\right)-\cos\left(k_0 l\right)}{\sin\theta}\right]}$$

Spherical wave expansion $\qquad$ θ-dependence

Pointing vector

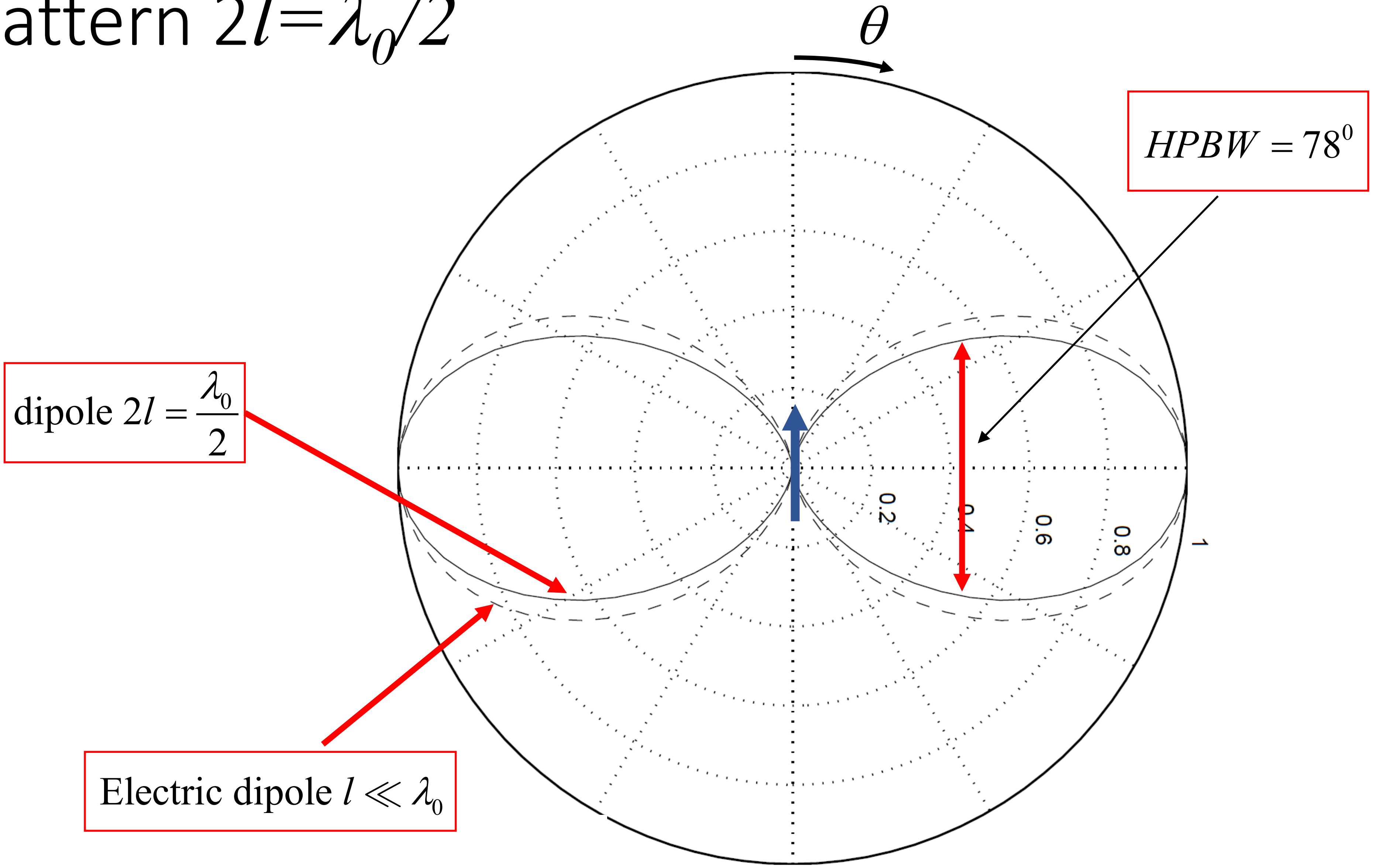
$$\vec{S}_p = \frac{1}{2Z_0}\left|\vec{E}\right|^2 \vec{u}_r = \frac{Z_0\left|I_0\right|^2}{8\pi^2 r^2}\left[\frac{\cos\left(k_0 l\cos\theta\right)-\cos\left(k_0 l\right)}{\sin\theta}\right]^2 \vec{u}_r$$

Radiation pattern

$$F(\theta) = \frac{P(\theta)}{P_{\max}} = \frac{\left|r^2\vec{S}_p(\theta)\right|}{P_{\max}}$$

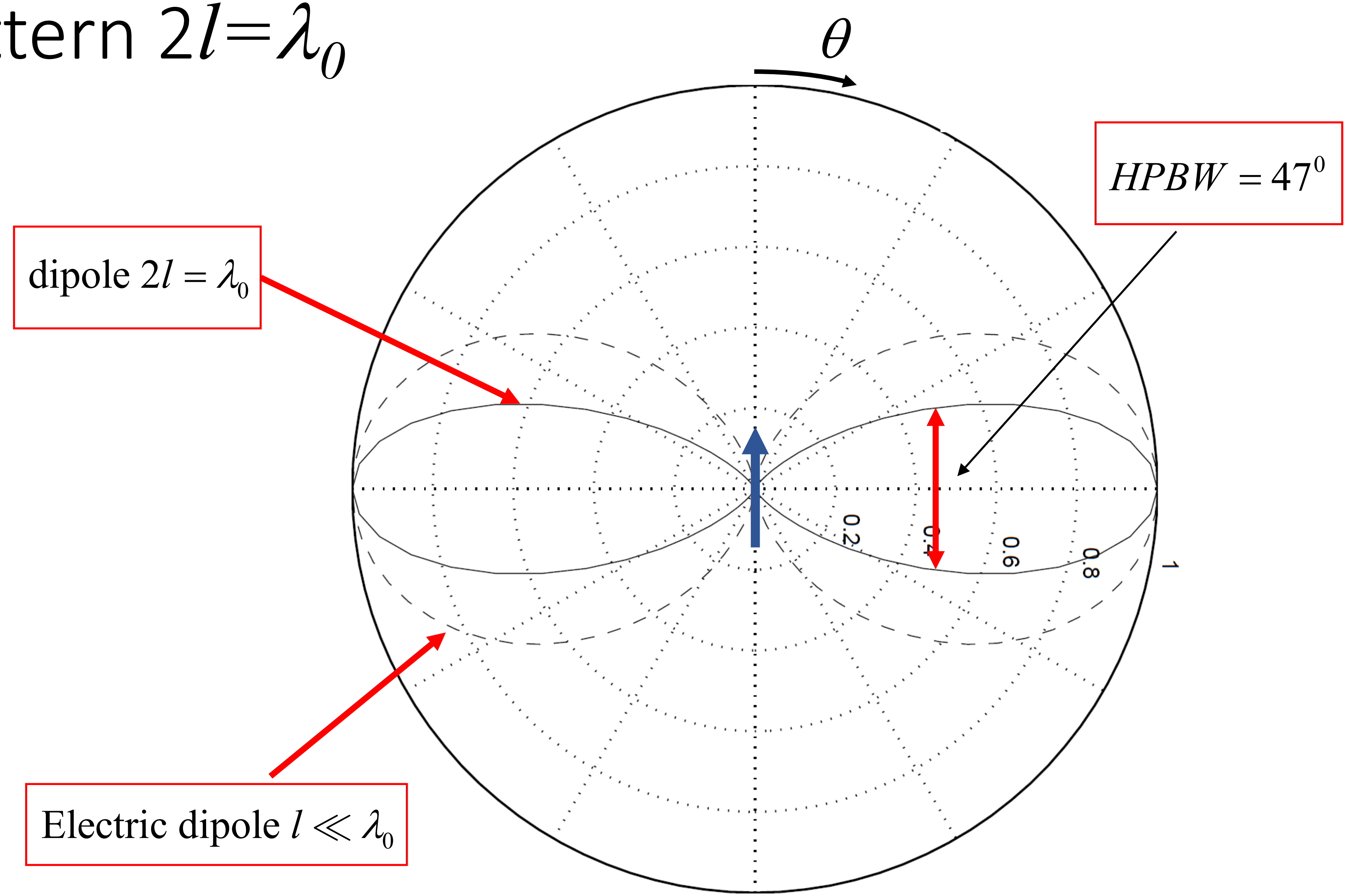
Radiation pattern $2l=\lambda_0/2$

Polar diagram



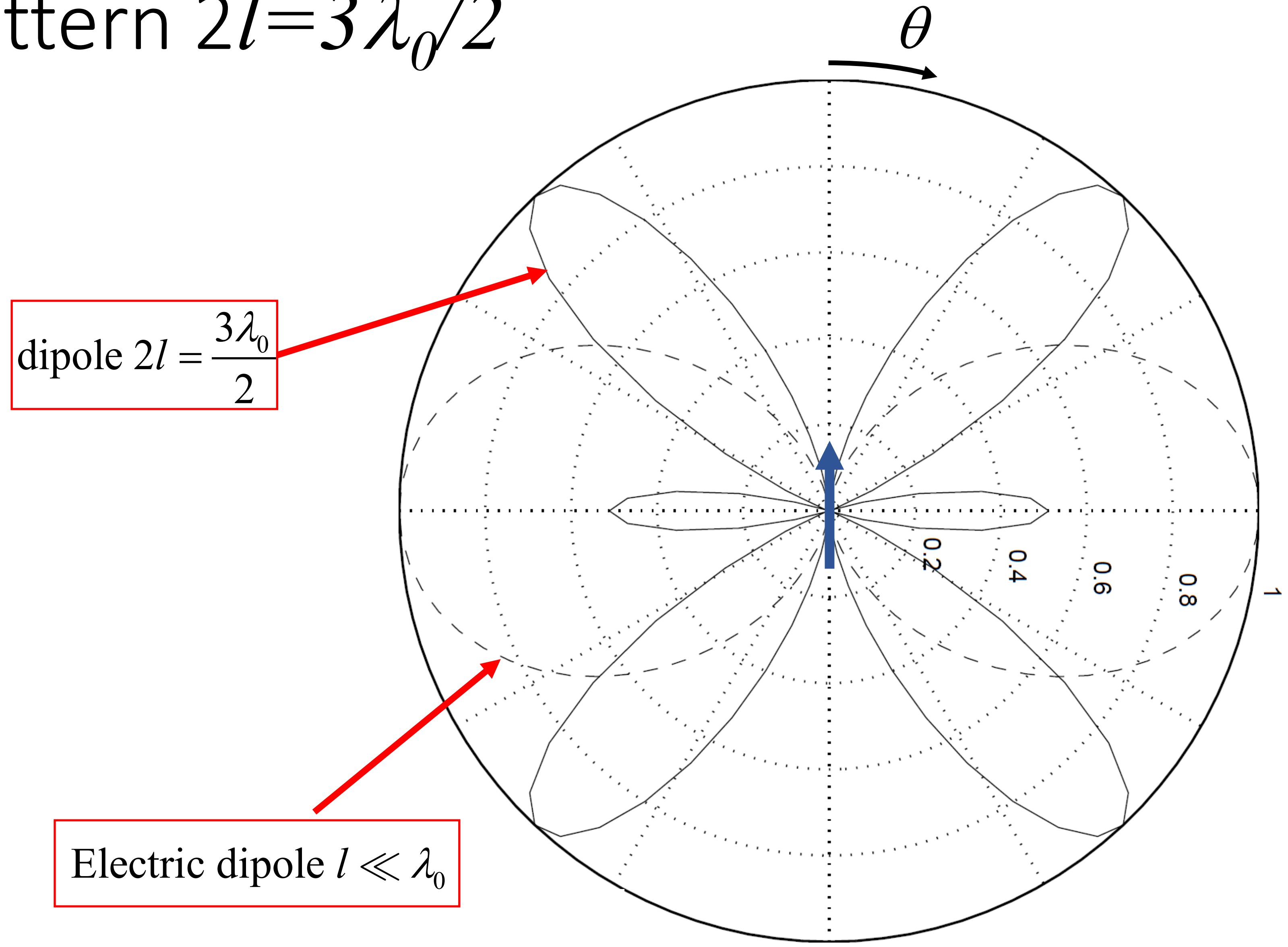
Radiation pattern $2l=\lambda_0$

Polar diagram



Radiation pattern $2l=3\lambda_0/2$

Polar diagram



Radiation properties

Total radiated power

$$P_t = \iint\limits_{sphere} \vec{S}_p(\vec{r}) \cdot \vec{u}_r r^2 d\Omega$$

$$= \frac{Z_0 |I_0|^2}{8\pi^2} \boxed{\int_0^{2\pi}} \int_0^{\pi} \left[\frac{\cos(k_0 l \cos\theta) - \cos(k_0 l)}{\sin\theta} \right]^2 \sin\theta d\theta \boxed{d\phi}$$

$$= \frac{Z_0 |I_0|^2}{8\pi^2} \boxed{2\pi} \boxed{\int_0^{\pi} \left[\frac{\cos(k_0 l \cos\theta) - \cos(k_0 l)}{\sin\theta} \right]^2 \sin\theta d\theta}$$

$$= \frac{1}{2} 73.1 |I_0|^2 \text{ for } \frac{\lambda_0}{2} \text{ dipole}$$

$I_p = 1.218$ when $2l = \frac{\lambda_0}{2}$

Radiation properties $\boxed{\lambda_0/2 \text{ dipole}}$

Directivity and antenna gain

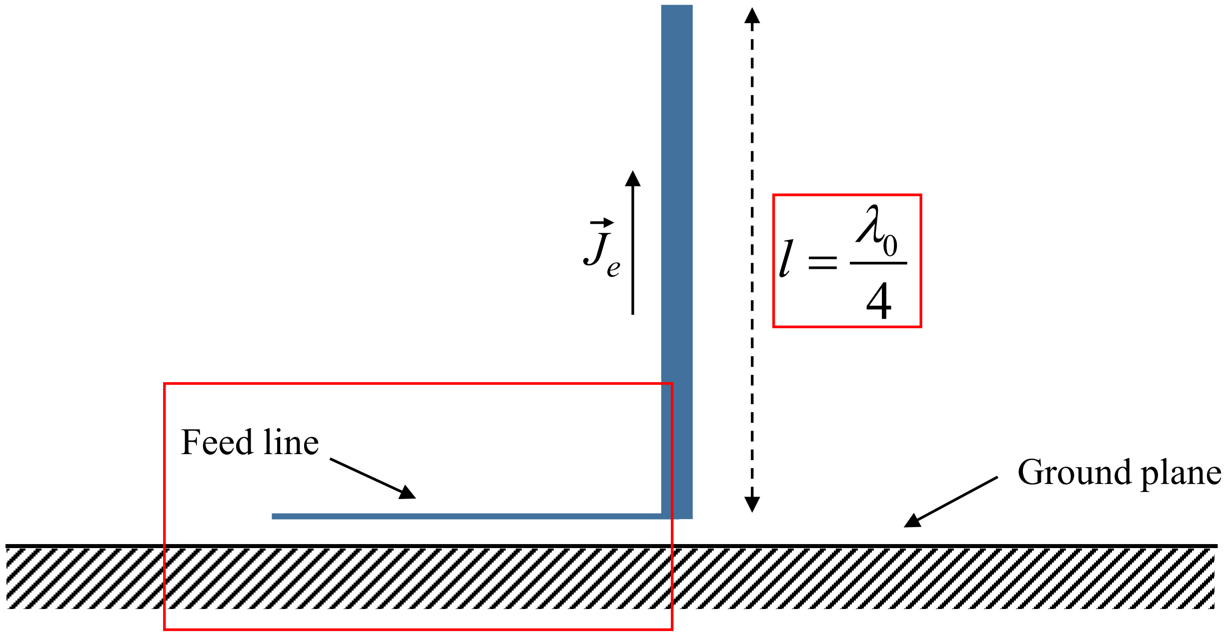
$$D = \frac{\max(P(\theta))}{P_t / 4\pi} = \frac{\dfrac{Z_0 |I_0|^2}{8\pi^2}}{\dfrac{73.1|I_0|^2/2}{4\pi}} = 1.64 \Rightarrow \boxed{G = 1.64} \text{ for perfect conductors}$$

2.15 dBi

Input impedance

$$Z_a = R_a = \frac{P_t}{\frac{1}{2}|I_0|^2} = \boxed{73.1\ \Omega}$$

Monopole antenna

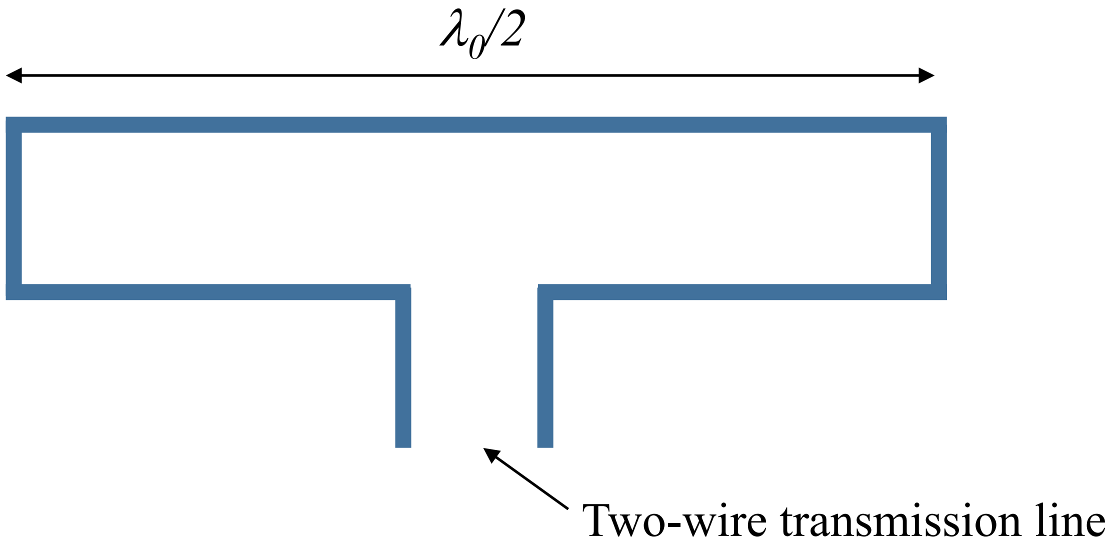


$$P_t = \frac{1}{2} P_{t,dipole} = \frac{1}{4} 73.1 |I_0|^2$$

$$D = 3.28 \text{ and } D_{dB} = 5.15 \text{ dBi}$$

$$Z_a = R_a = 36.5\ \Omega$$

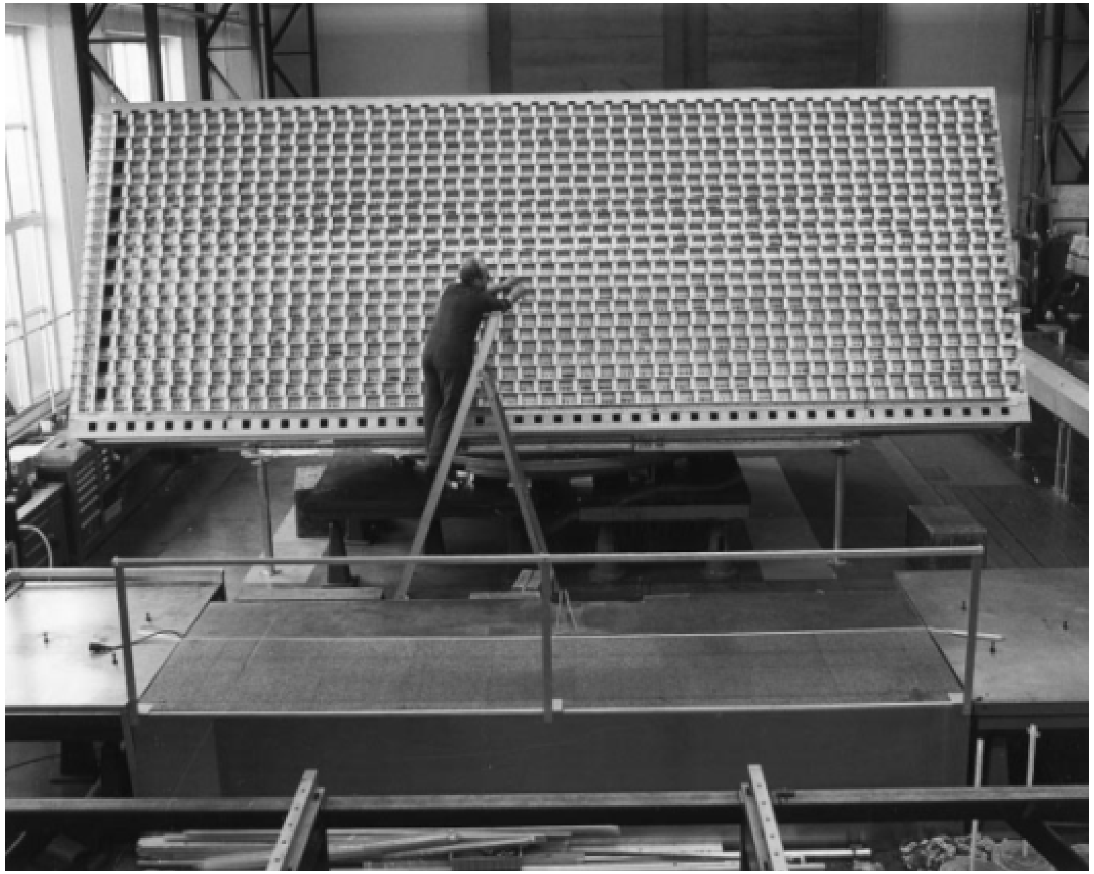
Folded Dipole



$$Z_a = R_a = 4R_{a,dipole} = 292\ \Omega$$

Radiation pattern similar to $\dfrac{\lambda_0}{2}$ dipole

L-Band phased-array radar with 1024 folded dipoles



M.C. Van Beurden, A.B. Smolders, IEEE Trans. AP, 2002, pp.1266-1273

Summary

- The radiation properties of wire antennas have been calculated.
- The half-wavelength dipole has a R_a=73 Ω, much more useful than the electric dipole.
- The monopole antenna and folded dipole have been introduced.

Microwave Engineering and Antennas

Loop Antennas and Magnetic Dipoles

Bart Smolders, Professor

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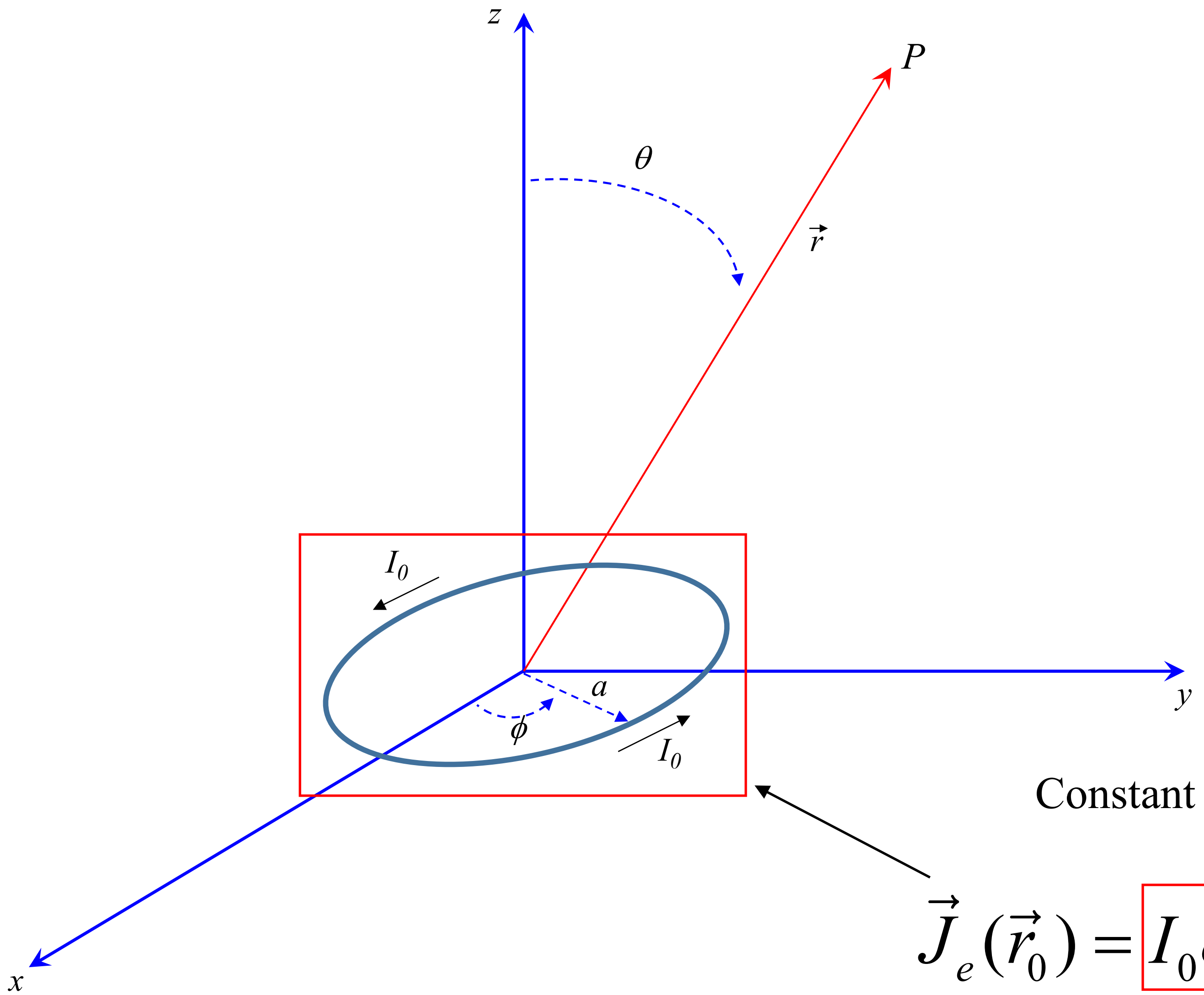
Loop antennas

Objective of this lecture

- Apply the general formulation to analyze loop antennas.
- We will show the analogy between a small loop antenna and an electric dipole.
- Introduce a perfect circularly-polarized antenna.

Current distribution on loop antenna



Current ϕ_0-directed

$\vec{u}_{\phi_0} = -\vec{u}_x \sin\phi_0 + \vec{u}_y \sin\phi_0$

Thin wires $d_0 \ll \lambda_0$

Constant current along loop, $a \ll \lambda_0$

$$\vec{J}_e(\vec{r}_0) = \boxed{I_0}\,\boxed{\delta\left(\sqrt{x_0^2 + y_0^2} - a\right)}\,\delta(z_0)\,\boxed{\vec{u}_{\phi_0}}$$

Apply our recipe

Far-field expression

$$\vec{H}(\vec{r}) = \frac{-jk_0 e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \iiint_{V_0} \boxed{\vec{J}_e(\vec{r}_0)} e^{jk_0 \boxed{\vec{u}_r \cdot \vec{r}_0}} dV_0$$

$$\vec{E}(\vec{r}) = Z_0 \vec{H}(\vec{r}) \times \vec{u}_r$$

$$\vec{J}_e(\vec{r}_0) = I_0 \delta\left(\sqrt{x_0^2 + y_0^2} - a\right)\delta(z_0)\vec{u}_{\phi_0}$$

$$\vec{u}_r \cdot \vec{r}_0 = a\cos(\boxed{\phi} - \boxed{\phi_0})\sin\theta$$

observation point P

Source point on loop

$$\vec{H} = \frac{-jk_0 a I_0 e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \boxed{\vec{I}_{\phi_0}}$$

$$\vec{I}_{\phi_0} = \int_0^{2\pi} \vec{u}_{\phi_0} e^{jk_0 a \boxed{\cos(\phi - \phi_0)} \sin\theta} d\phi_0$$

$$= \int_0^{2\pi} \vec{u}_{\phi_0} e^{jk_0 a \boxed{\cos\phi_0} \sin\theta} d\phi_0$$

$$= 2\pi j \boxed{J_1(k_0 a \sin\theta)}\vec{u}_\phi$$

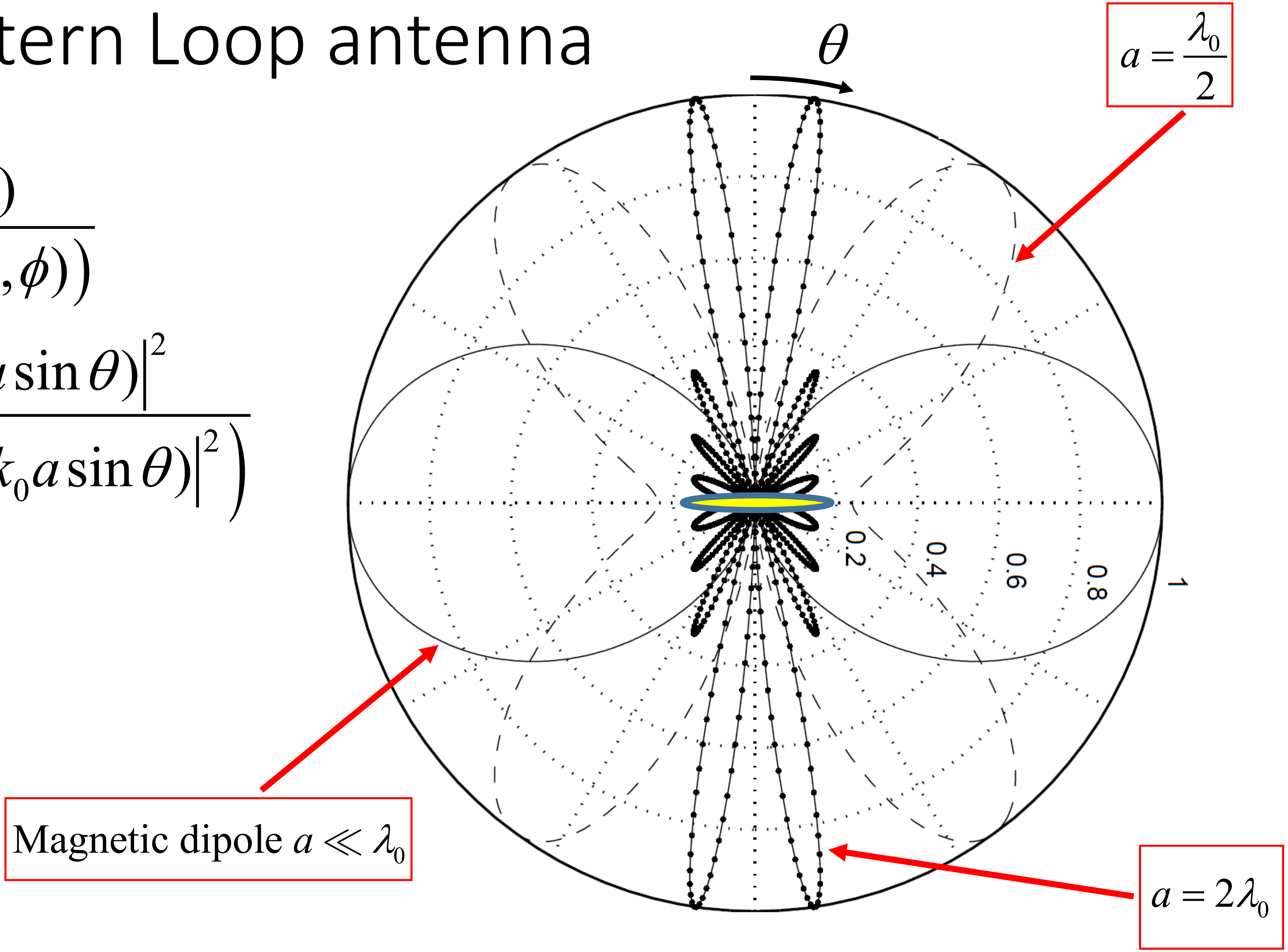
Due to symmetry, field not dependent on ϕ

We can use ϕ=0

Radiation pattern Loop antenna

$$F(\theta) = \frac{P(\theta,\phi)}{\max\left(P(\theta,\phi)\right)}$$

$$= \frac{\left|J_1(k_0 a \sin\theta)\right|^2}{\max\left(\left|J_1(k_0 a \sin\theta)\right|^2\right)}$$



Magnetic dipole

Small loop

$$J_1\left(k_0 a\sin\theta\right)\approx\frac{1}{2}k_0 a\sin\theta \qquad a \ll \lambda_0$$

$$\boxed{H_\theta = -\frac{k_0^2\left(\pi a^2 I_0\right)e^{-jk_0 r}}{4\pi r}\sin\theta \qquad E_\phi = \frac{k_0^2 Z_0\left(\pi a^2 I_0\right)e^{-jk_0 r}}{4\pi r}\sin\theta}$$

$$\boxed{H_\phi = \frac{jk_0 I_0 l e^{-jk_0 r}}{4\pi r}\sin\theta \qquad E_\theta = \frac{jk_0 Z_0 I_0 l e^{-jk_0 r}}{4\pi r}\sin\theta}$$

Magnetic dipole

Electric dipole

Magnetic+Electric dipole combined

Combining
magnetic+electric dipole

Choose $I_0 l = -k_0 \pi a^2 I_0$ $\Rightarrow$ $\boxed{E_\phi = jE_\theta}$

$$H_\theta = -\frac{k_0^2\left(\pi a^2 I_0\right)e^{-jk_0 r}}{4\pi r}\sin\theta$$

$$E_\phi = \frac{\boxed{k_0^2 Z_0\left(\pi a^2 I_0\right)}e^{-jk_0 r}}{4\pi r}\sin\theta$$

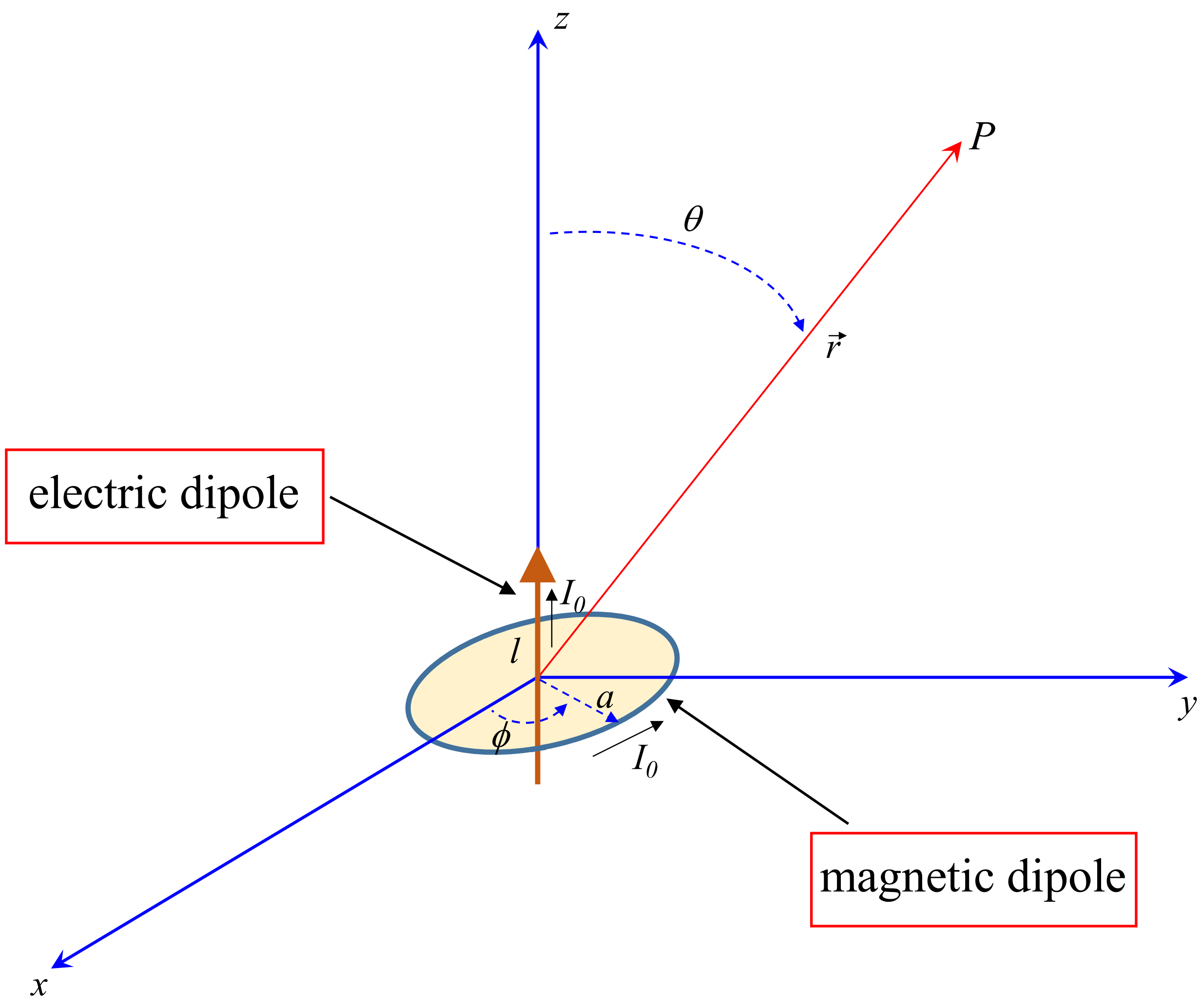
Magnetic dipole

$$H_\phi = \frac{jk_0 I_0 l e^{-jk_0 r}}{4\pi r}\sin\theta$$

$$E_\theta = \frac{\boxed{jk_0 Z_0 I_0 l}e^{-jk_0 r}}{4\pi r}\sin\theta$$

Electric dipole

Magnetic+Electric dipole combined



dipole moment

$\boxed{p = I_0 l} = -k_0 \pi a^2 I_0$ | $p = I_0 l = k_0 \pi a^2 I_0$

$$E_\phi = jE_\theta \qquad\qquad E_\phi = -jE_\theta$$

LHCP — Left-Hand Circular Polarization

RHCP — Right-Hand Circular Polarization

Summary

- Loop antennas with uniform current distribution have been analyzed.
- The magnetic dipole is a small loop antenna.
- Combining the electric and magnetic dipole provides an antenna with perfect circular-polarization properties.

Microwave Engineering and Antennas

Magnetic Sources and the Equivalence Principle

Bart Smolders, Professor

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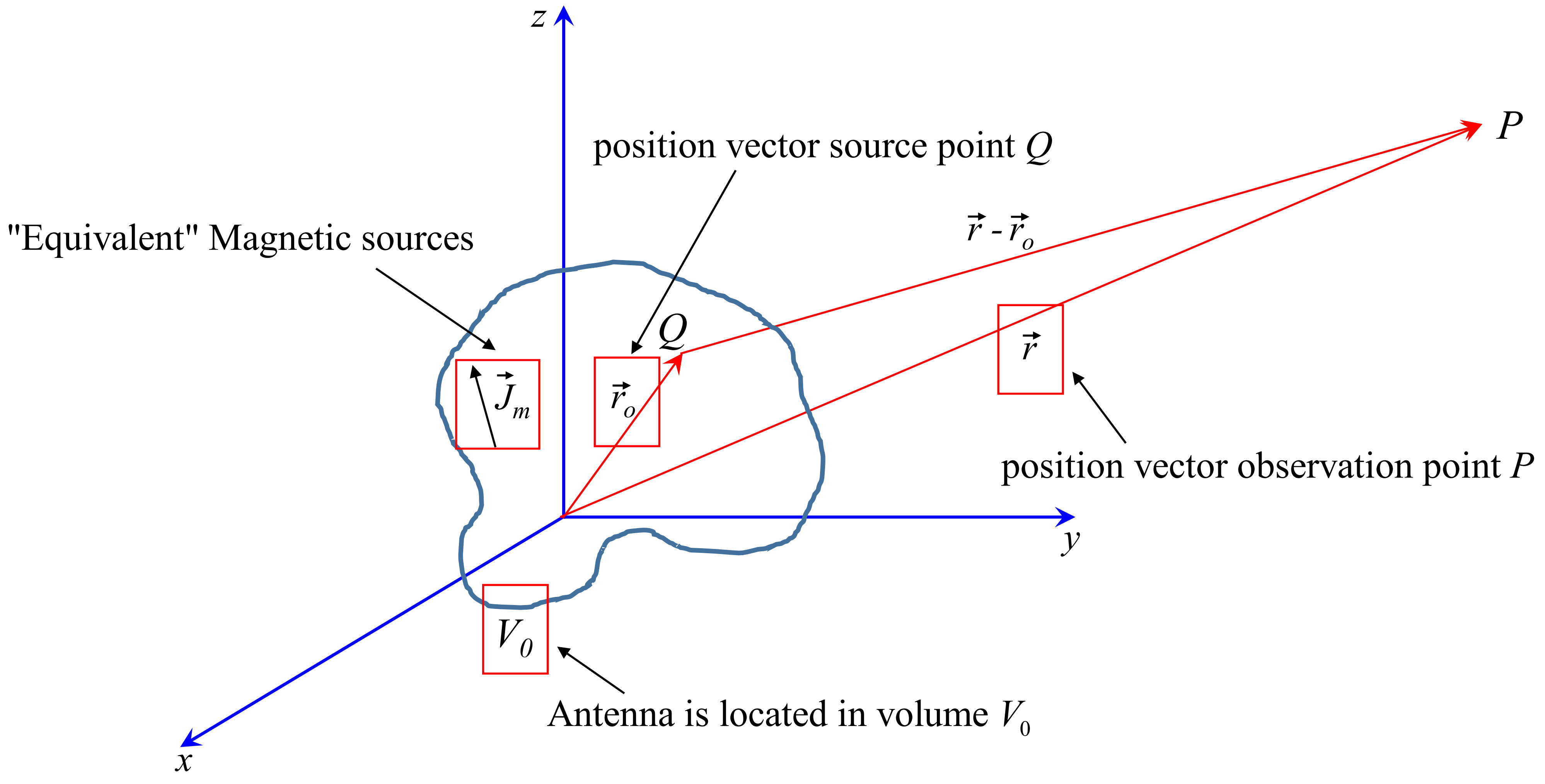
Magnetic sources and Equivalence principle

Objective of this lecture

- Extension of Maxwell's equations with magnetic sources.
- Introduce the equivalence principle.
- Formulate a new recipe to determine the radiation properties of aperture antennas

Magnetic sources



Maxwell’s equations with magnetic sources

Equivalent magnetic current distribution in V_0

$$\nabla \times \vec{E}(\vec{r}) = -j\omega\mu_0 \vec{H}(\vec{r}) - \boxed{\vec{J}_m(\vec{r})}$$

$$\nabla \times \vec{H}(\vec{r}) = j\omega\varepsilon_0 \vec{E}(\vec{r})$$

$$\nabla \cdot \vec{H}(\vec{r}) = \frac{\boxed{\rho_m(\vec{r})}}{\mu_0}$$

Magnetic charge distribution in V_0

$$\boxed{\nabla \cdot \vec{E}(\vec{r}) = 0}$$

No electric sources, define electric vector potential $\vec{A}_m$

Approach

Introduce electric vector potential $\vec{A}_m$

$\downarrow$

Helmholtz equation

$\downarrow$

Lorenz gauge

$\downarrow$

Greens function G

$\downarrow$

Solution $\vec{E}, \vec{H}$

Approach

Introduce electric vector potential $\vec{A}_m$

↓

Helmholtz equation

↓

Lorenz gauge

↓

Greens function G

↓

Solution $\vec{E}, \vec{H}$

Approach

Introduce electric vector potential $\vec{A}_m$

Helmholtz equation

Lorenz gauge

Greens function G

Solution $\vec{E}, \vec{H}$

Approach

Introduce electric vector potential $\vec{A}_m$

Helmholtz equation

Lorenz gauge

Greens function G

Solution $\vec{E}, \vec{H}$

Approach

Introduce electric vector potential $\vec{A}_m$

$\Downarrow$

Helmholtz equation

$\Downarrow$

Lorenz gauge

$\Downarrow$

Greens function G

$\Downarrow$

Solution $\vec{E}, \vec{H}$ $\Rightarrow$ Far-field approximations

Far-field from sources

Magnetic sources

magnetic current distribution

$$\vec{E}(\vec{r}) = \frac{jk_0 e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \iiint_{V_0} \boxed{\vec{J}_m(\vec{r}_0)} e^{jk_0 \vec{u}_r \cdot \vec{r}_0} dV_0$$

$$\vec{H}(\vec{r}) = \frac{1}{\boxed{Z_0}}\vec{u}_r \times \vec{E}(\vec{r})$$

377 Ω

Electric sources

electric current distribution

$$\vec{E}(\vec{r}) = \frac{-k_0^2}{j\omega\varepsilon_0}\frac{e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \vec{u}_r \times \iiint_{V_0} \boxed{\vec{J}_e(\vec{r}_0)} e^{jk_0 \vec{u}_r \cdot \vec{r}_0} dV_0$$

$$\vec{H}(\vec{r}) = \frac{1}{Z_0}\vec{u}_r \times \vec{E}(\vec{r})$$

Far-field approximation

Electric and Magnetic sources

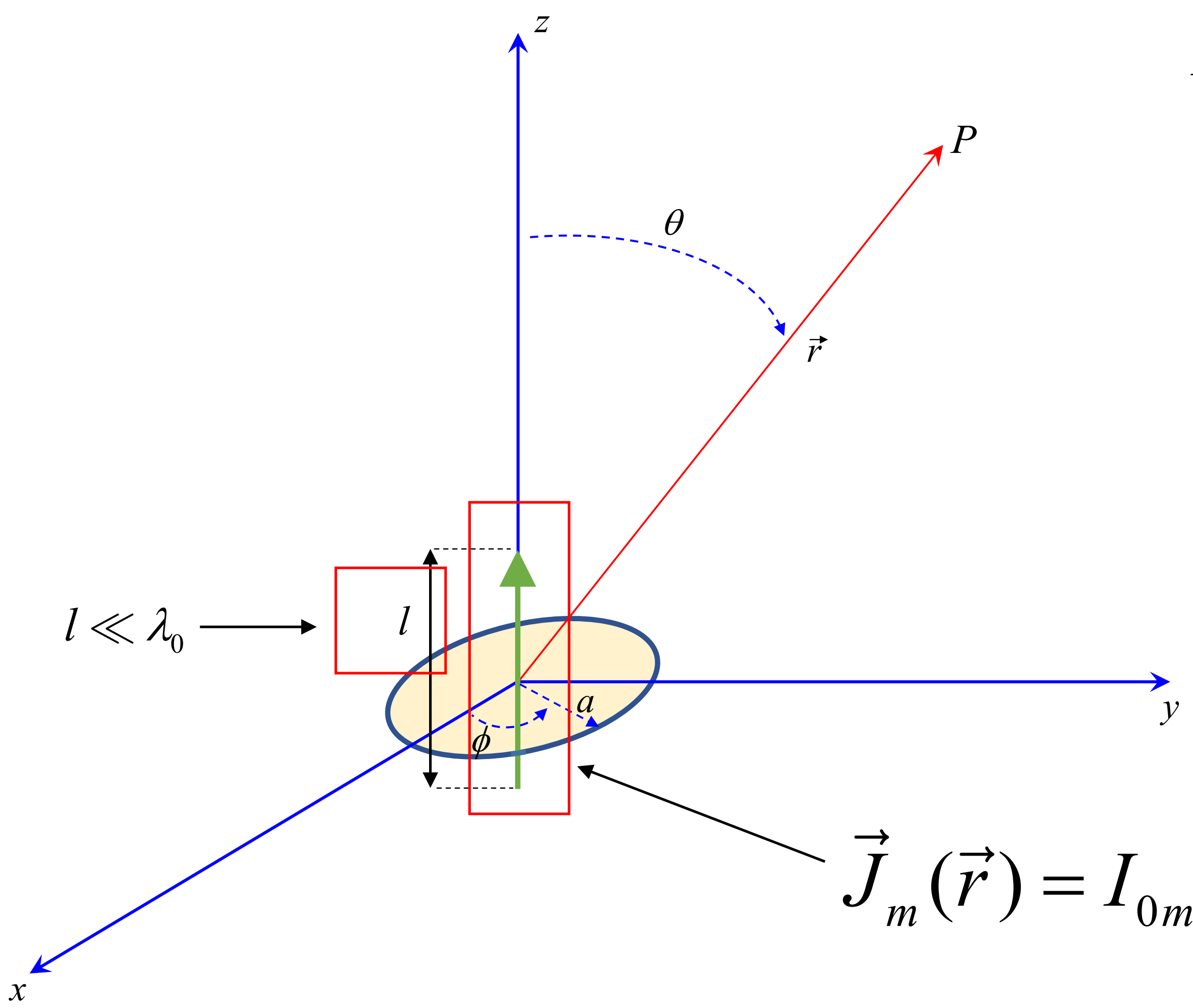
magnetic current distribution

$$\vec{E}(\vec{r}) = \frac{jk_0 e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \iiint_{V_0} \left(\boxed{\vec{J}_m(\vec{r}_0)} + Z_0 \vec{u}_r \times \boxed{\vec{J}_e(\vec{r}_0)} \right) e^{jk_0 \vec{u}_r \cdot \vec{r}_0} dV_0$$

electric current distribution

$$\vec{H}(\vec{r}) = \frac{1}{Z_0}\vec{u}_r \times \vec{E}(\vec{r})$$

Magnetic dipole



$$\vec{E}(\vec{r}) = \frac{jk_0 e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \iiint_{V_0} \vec{J}_m(\vec{r}_0) e^{jk_0 \vec{u}_r \cdot \vec{r}_0} dV_0$$

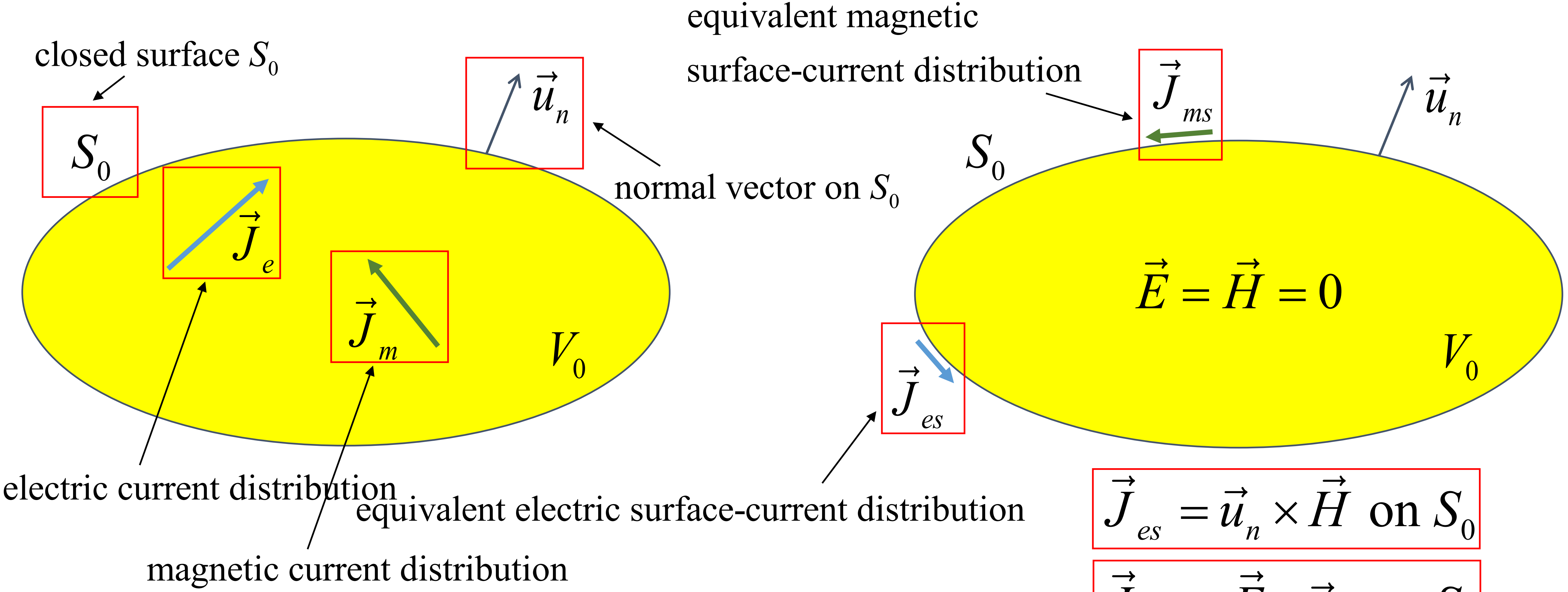
$$\boxed{= \frac{-jk_0 I_{0m} l e^{-jk_0 r}}{4\pi r}\sin\theta \vec{u}_\phi}$$

Similar to small loop of electric current!

$$\vec{J}_m(\vec{r}) = I_{0m} l \delta(x)\delta(y)\delta(z)\vec{u}_z$$

Equivalence principle

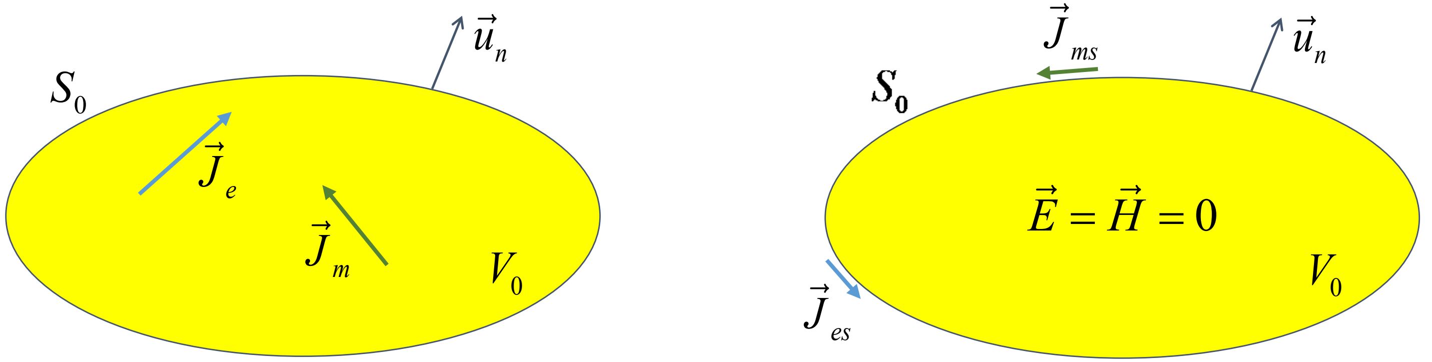
Original problem | **Equivalent problem**



$$\boxed{\vec{J}_{es} = \vec{u}_n \times \vec{H} \text{ on } S_0}$$

$$\boxed{\vec{J}_{ms} = \vec{E} \times \vec{u}_n \text{ on } S_0}$$

Radiated fields using equivalence principle



$$\vec{E}(\vec{r}) = \frac{jk_0 e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \iint_{S_0} \left(\vec{J}_{ms}(\vec{r}_0) + Z_0 \vec{u}_r \times \vec{J}_{es}(\vec{r}_0)\right) e^{jk_0 \vec{u}_r \cdot \vec{r}_0} \boxed{dS_0}$$

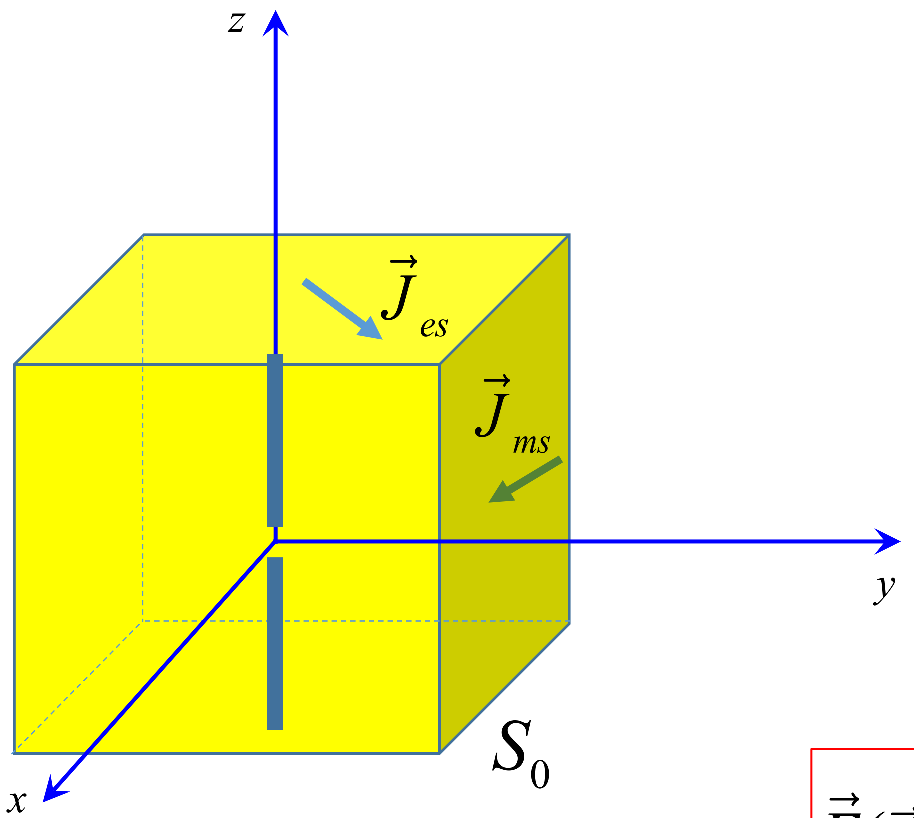
surface integral over S_0

$$= \frac{jk_0 e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \iint_{S_0} \left(\boxed{\vec{E} \times \vec{u}_n} + Z_0 \vec{u}_r \times \boxed{\vec{u}_n \times \vec{H}}\right) e^{jk_0 \vec{u}_r \cdot \vec{r}_0} dS_0$$

tangential fields on S_0

$$\vec{H}(\vec{r}) = \frac{1}{Z_0}\vec{u}_r \times \vec{E}(\vec{r})$$

Application of equivalence principle (1)



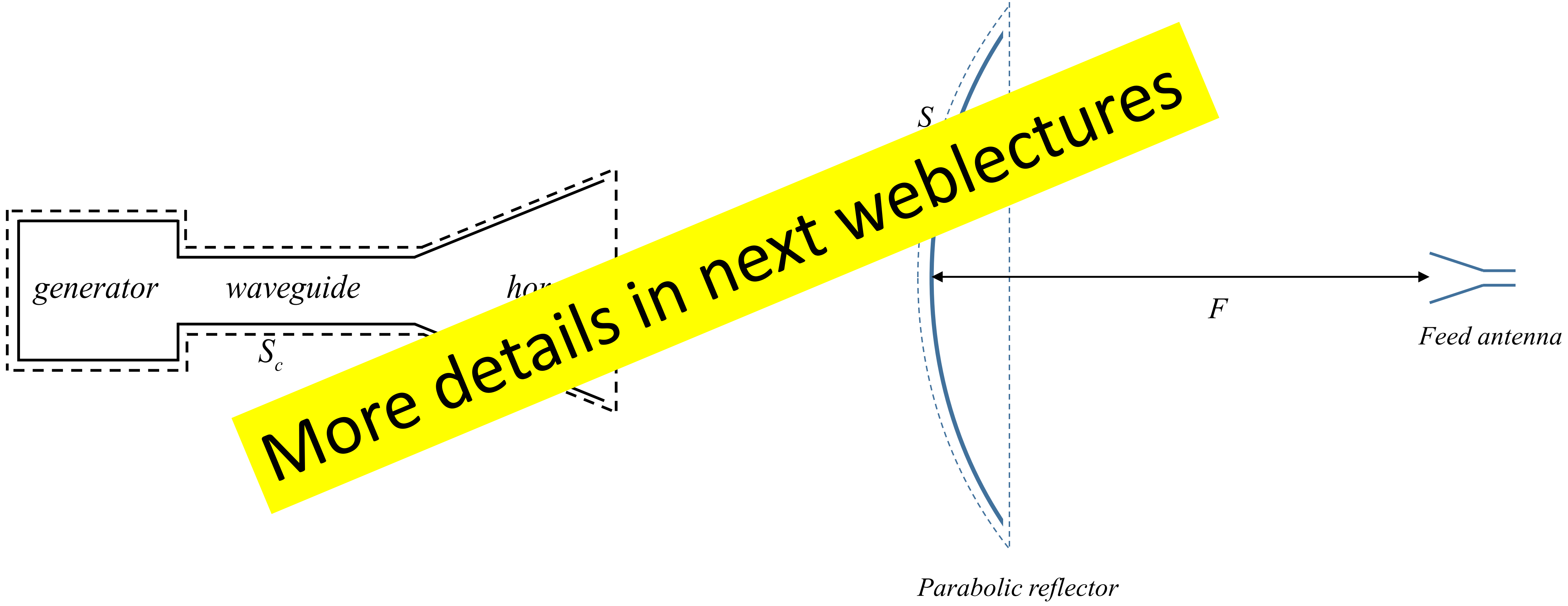
Procedure used in EM solvers

- Define antenna problem
- Define a box with closed surface S_0
- Solve EM fields in Box, e.g. with FEM
- Determine tangential fields on S_0
- Calculate far-fields

$$\boxed{\vec{E}(\vec{r}) = \frac{jk_0 e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \iint_{S_0} \left(\vec{J}_{ms}(\vec{r}_0) + Z_0 \vec{u}_r \times \vec{J}_{es}(\vec{r}_0)\right) e^{jk_0 \vec{u}_r \cdot \vec{r}_0} dS_0}$$

Application of equivalence principle (2)

Analysis of aperture antennas



Summary

- Magnetic sources have been introduced.
  - Duality principle.
  - Magnetic dipole.
- Equivalence concept was introduced.
  - Useful concept to analyze aperture antennas.

Summary

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  - Duality concept.
  - Magnetic dipole.
- Equivalence concept was introduced.
  - Useful concept to analyze aperture antennas.

Microwave Engineering and Antennas

Horn Antennas

Bart Smolders, Professor

Department of Electrical Engineering

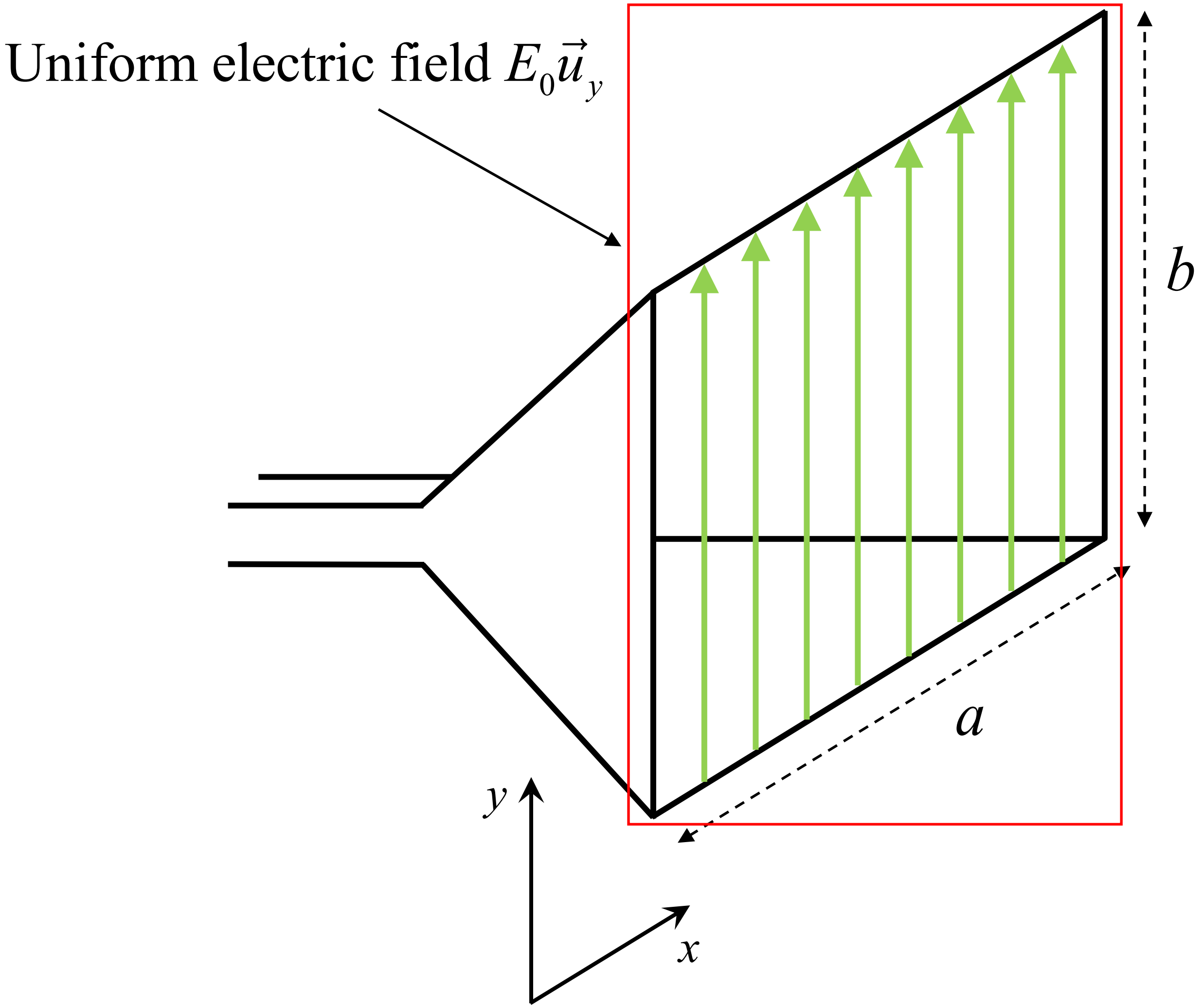
Center for Wireless Technology Eindhoven

Horn antennas

Objective of this lecture

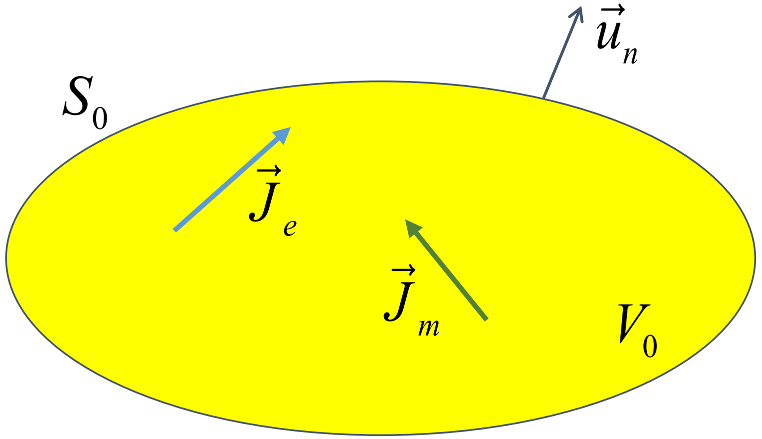
- Use the equivalence concept and apply our far-field recipe to horn antennas.
- Calculation of the radiation properties of a uniform rectangular aperture.

Horn antenna model

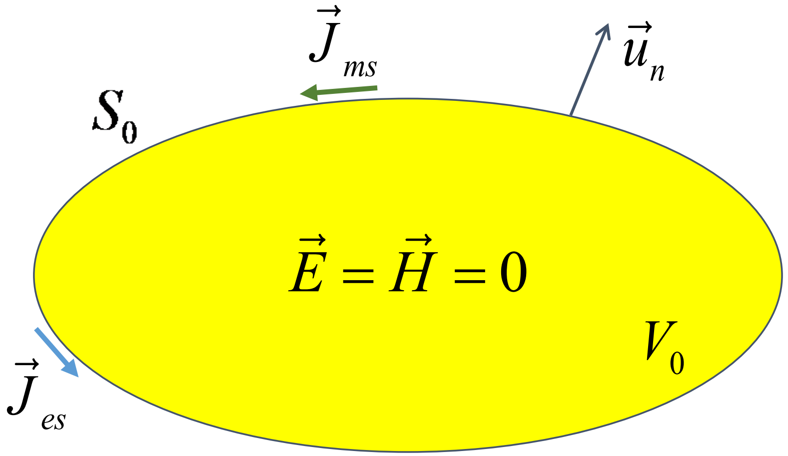


Apply equivalence concept and our recipe (1)

Original problem



Equivalent problem



Far-field

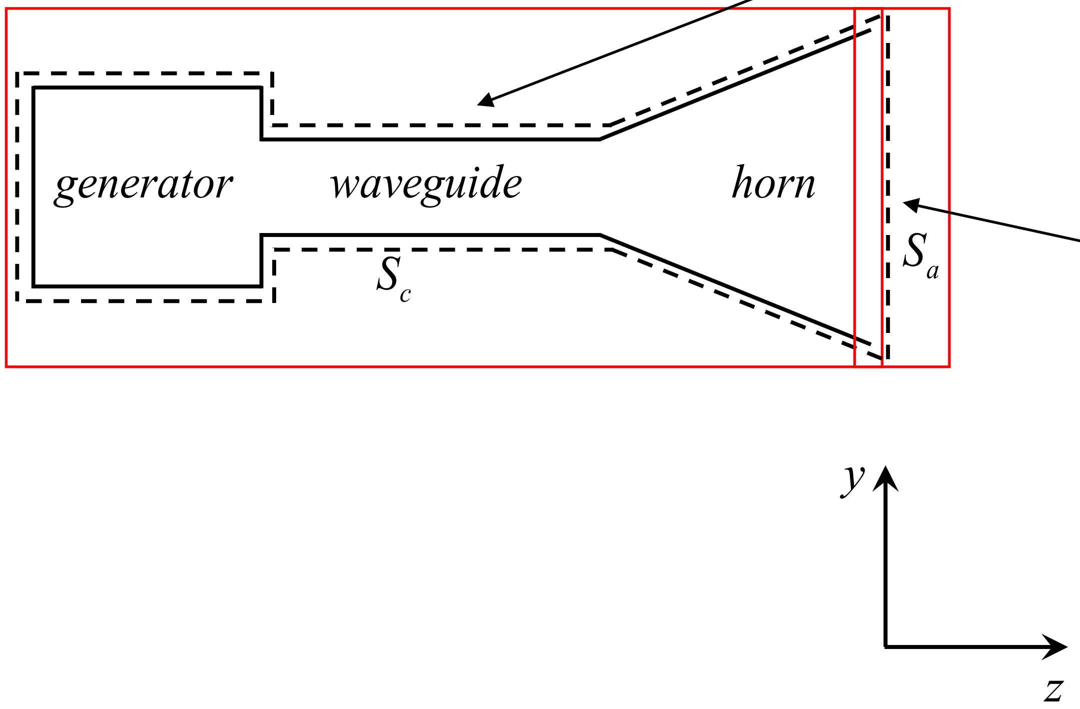
$$\vec{E}(\vec{r}) = \frac{jk_0 e^{-jk_0 r}}{4\pi r} \vec{u}_r \times \iint_{S_0} \left(\boxed{\vec{E} \times \vec{u}_n} + Z_0 \vec{u}_r \times \boxed{\vec{u}_n \times \vec{H}} \right) e^{jk_0 \vec{u}_r \cdot \vec{r}_0} \boxed{dS_0}$$

Determine tangential fields on S_0

define closed surface S_0

Apply equivalence concept and our recipe (2)

Select closed surface $S_a + S_c$



On S_c

$$\vec{E} \times \vec{u}_n = \vec{0}$$

$$\vec{H} \times \vec{u}_n = \vec{0}$$

On S_a

$$\vec{E}_a = E_0 \vec{u}_y$$

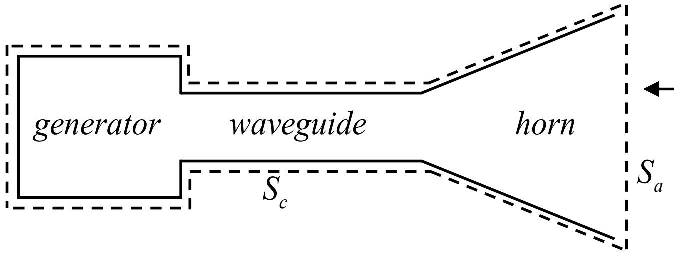
$$\vec{H}_a = -\frac{E_0}{Z_0} \vec{u}_x$$

$\Rightarrow$

$$\vec{E}_a \times \vec{u}_n = E_0 \vec{u}_x$$

$$\vec{u}_n \times \vec{H}_a = -\frac{E_0}{Z_0} \vec{u}_y$$

Apply equivalence concept and our recipe (3)



$$\boxed{\vec{E}_a \times \vec{u}_n = E_0 \vec{u}_x}$$

$$\boxed{\vec{u}_n \times \vec{H}_a = -\frac{E_0}{Z_0}\vec{u}_y}$$

$$\vec{E}(\vec{r}) = \frac{jk_0 e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \iint_{S_0}\left(\boxed{\vec{E}\times\vec{u}_n} + Z_0\vec{u}_r \times \boxed{\vec{u}_n \times \vec{H}}\right) e^{jk_0 \vec{u}_r\cdot\vec{r}_0} dS_0$$

$$= \frac{jk_0 e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \boxed{\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}}\left(E_0\vec{u}_x - \vec{u}_r \times E_0 \vec{u}_y\right) e^{jk_0 \vec{u}_r\cdot\vec{r}_0} dx_0 dy_0$$

$$= \frac{jk_0 e^{-jk_0 r}}{4\pi r}\left(\vec{L}_m + \vec{L}_e\right)$$

integration over aperture S_a

Far-field expression uniform aperture

$$\vec{E} = \frac{jk_0 e^{-jk_0 r}}{4\pi r}\left(\vec{L}_m + \vec{L}_e\right)$$

$$= \frac{jk_0 abE_0 e^{-jk_0 r}}{4\pi r}\left(\boxed{\vec{u}_\theta \sin\phi} + \boxed{\vec{u}_\phi \cos\phi}\right)(\cos\theta + 1)\left[\operatorname{sinc}\left(\boxed{\frac{k_0 au}{2}}\right)\right]\left[\operatorname{sinc}\left(\boxed{\frac{k_0 bv}{2}}\right)\right]$$

$$= \boxed{\vec{E}_\theta \vec{u}_\theta} + \boxed{\vec{E}_\phi \vec{u}_\phi}$$

$E_\phi = 0$ in $\phi = \dfrac{\pi}{2}$ plane (E-plane)

$u = \sin\theta\cos\phi$

$v = \sin\theta\sin\phi$

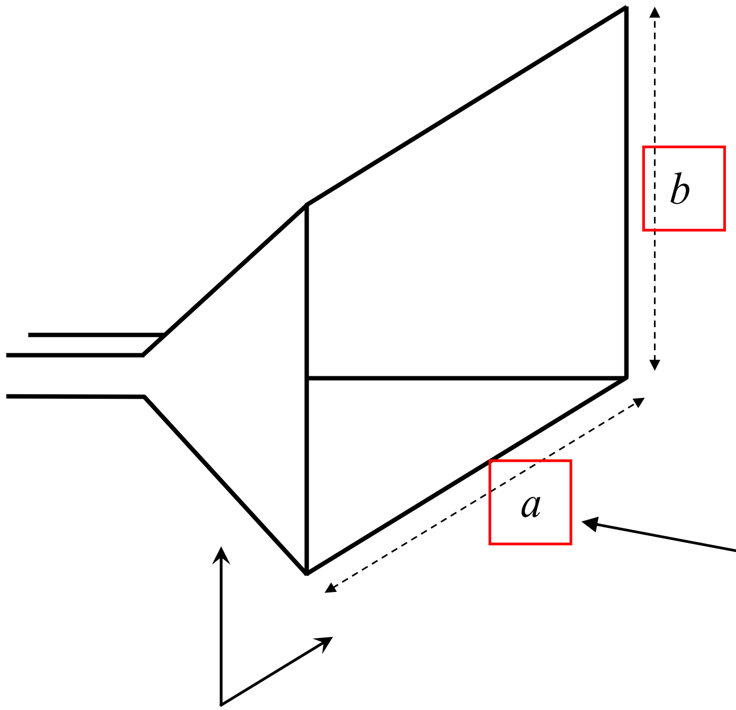
Radiation pattern

$E_\theta = 0$ in $\phi = 0$ plane (H-plane)

$$F(\theta,\phi) = \frac{|E_\theta(\theta,\phi)|^2 + |E_\phi(\theta,\phi)|^2}{\boxed{|E_\theta(0,0)|^2 + |E_\phi(0,0)|^2}}$$

maximum value at $(\theta,\phi) = (0,0)$

Properties horn antenna with uniform aperture



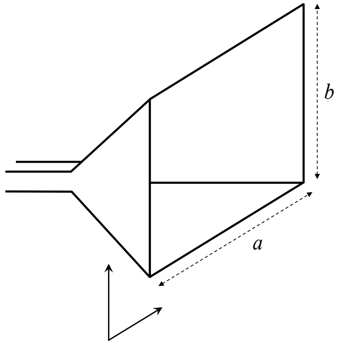
First sidelobe at -13.2 dB

Increased a gives smaller beam in xz plane

Increased b gives smaller beam in yz plane

$$F(\theta,\phi)=\frac{1}{4}(\cos\theta+1)^2\boxed{\left[\operatorname{sinc}\left(\boxed{\frac{k_0 a u}{2}}\right)\right]^2\left[\operatorname{sinc}\left(\boxed{\frac{k_0 b v}{2}}\right)\right]^2}$$

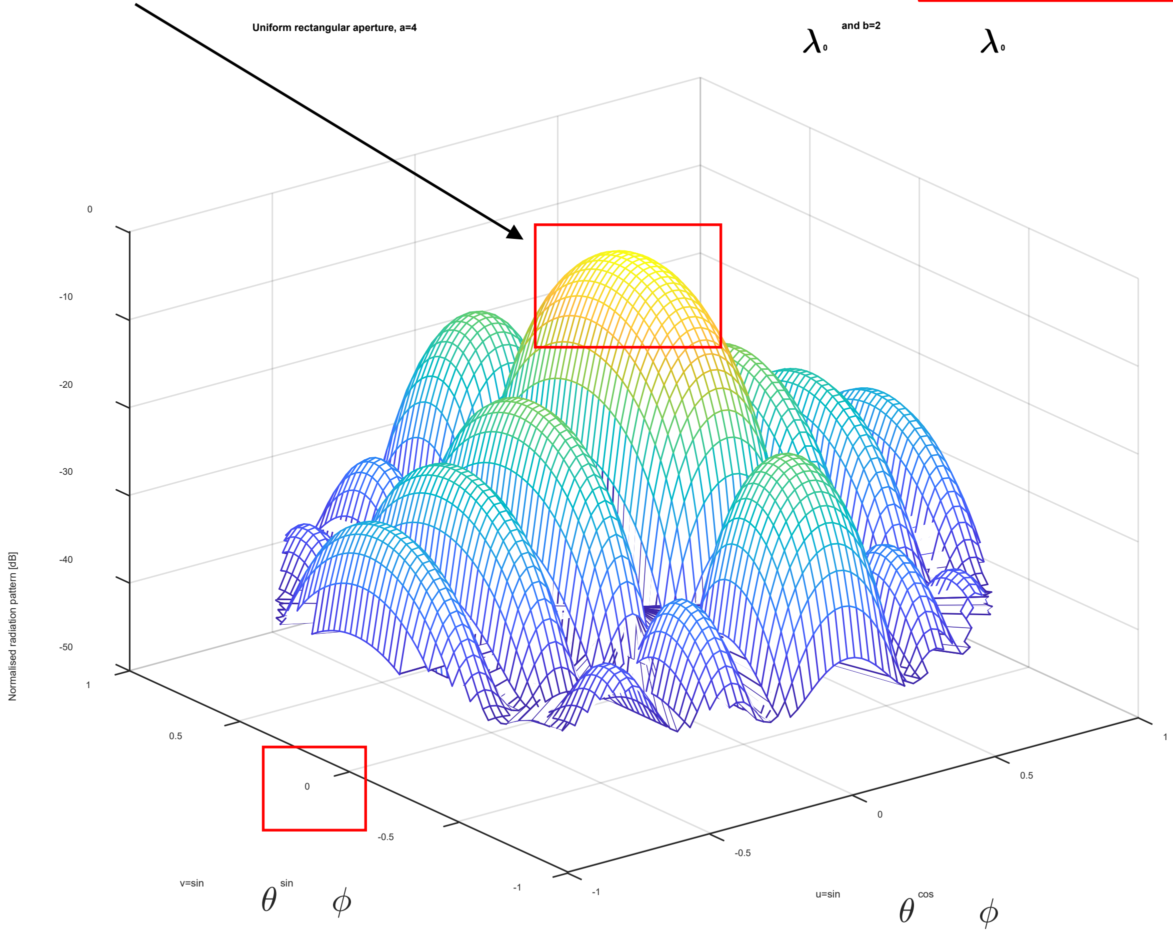
Example horn antenna

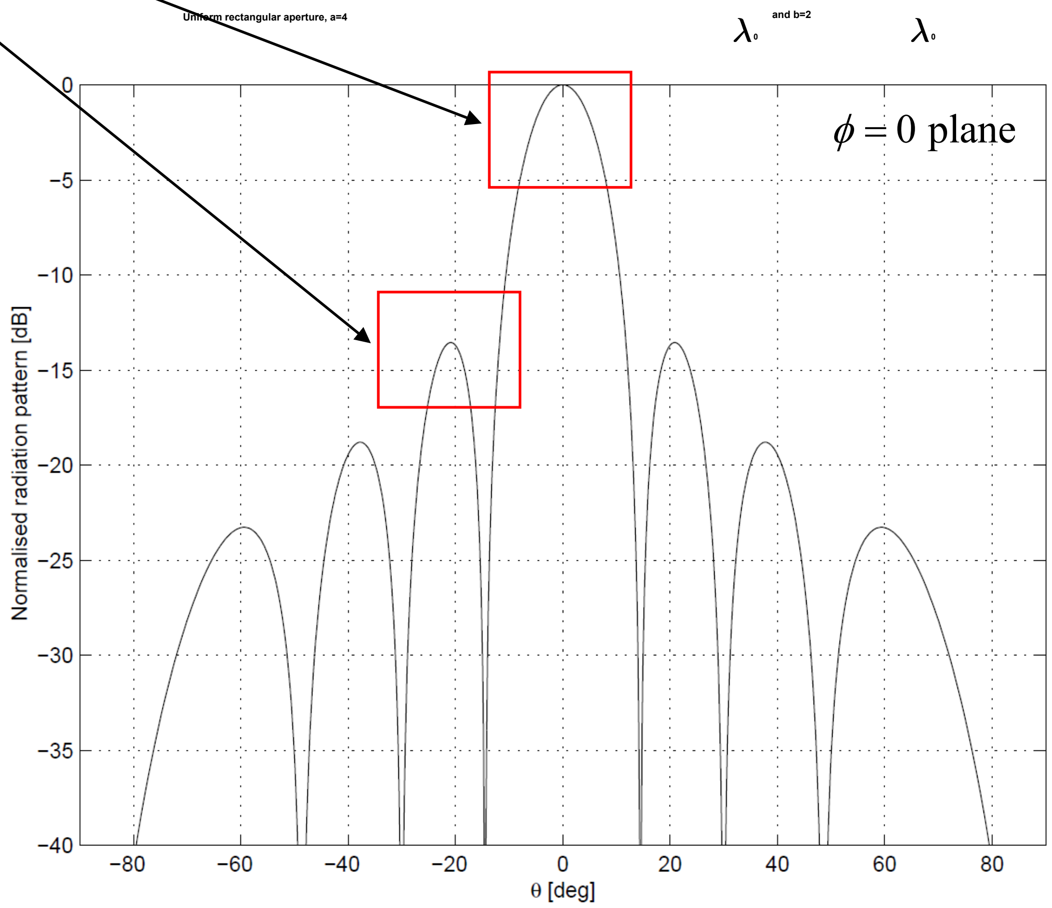


$$\text{Directivity } D = 4\pi \frac{ab}{\lambda_0^2} \to 20 \text{ dBi}$$

$$HPBW = 2\arcsin\left(\frac{0.433\lambda_0}{a}\right) = 12.4^0$$

First sidelobe at -13.26 dB





Summary

- Horn antennas can be modelled as a uniform aperture.
- Using the tangential fields in the aperture, the radiation properties are found.
- First sidelobe at -13.2 dB.
- More realistic aperture fields can be determined with an EM solver.

Microwave Engineering and Antennas

Reflector Antennas

Bart Smolders, Professor

Department of Electrical Engineering

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Reflector antennas

Objective of this lecture

- Use the equivalence concept and apply our far-field recipe on reflector antennas.
- Determine radiation properties of circular apertures.
- Introduce tapering.

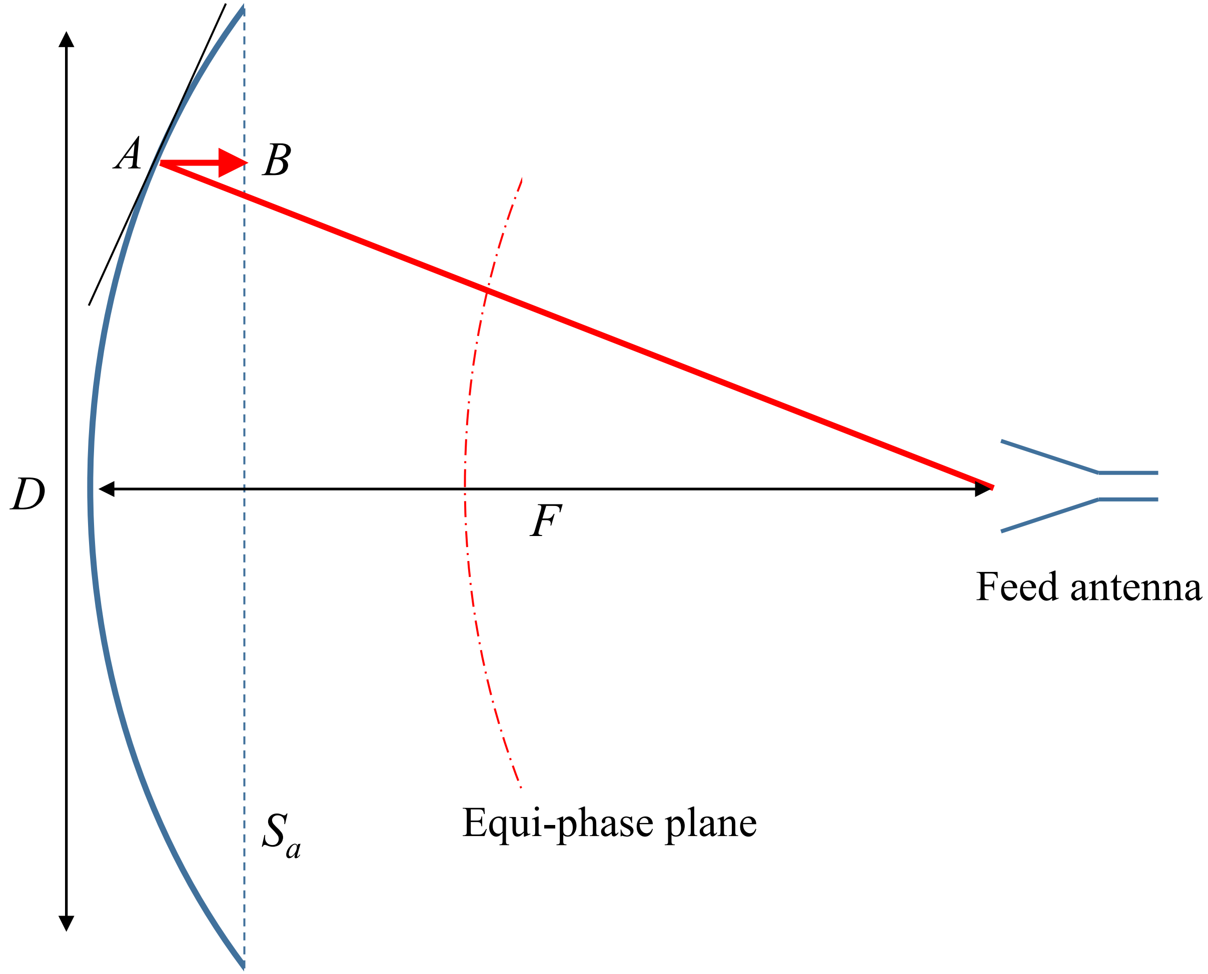
Reflector antennas

Applications

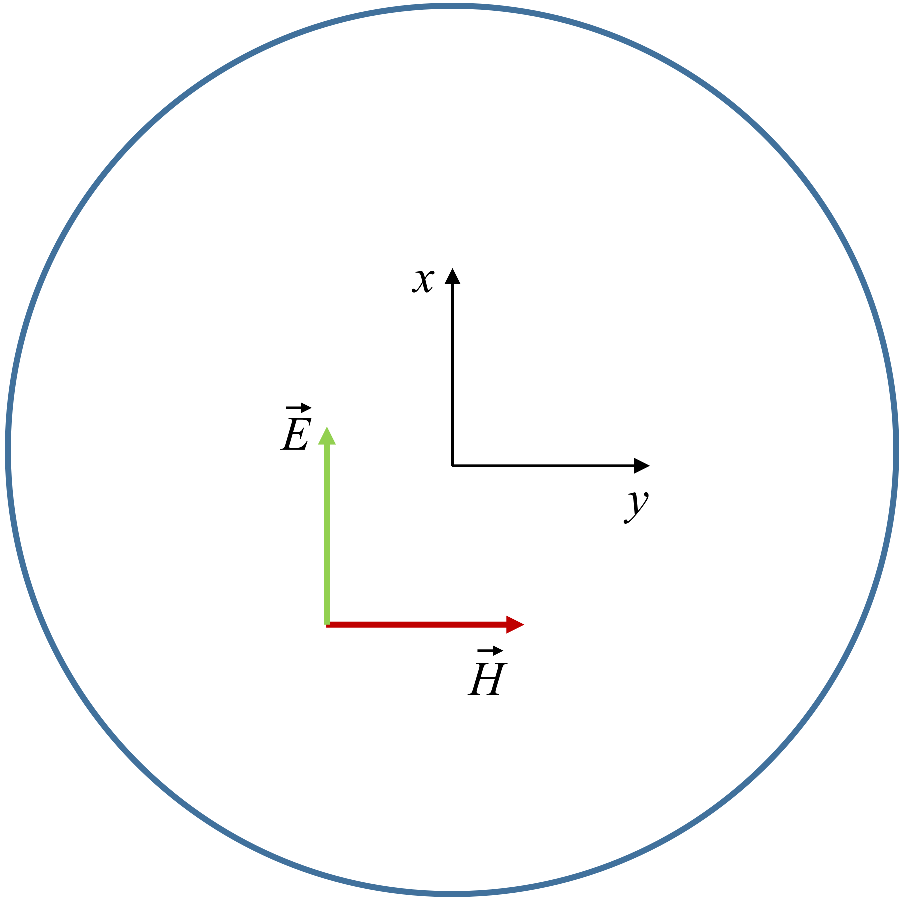
- Radio telescopes
- Radar
- Satellite-TV reception

Reflector antenna model

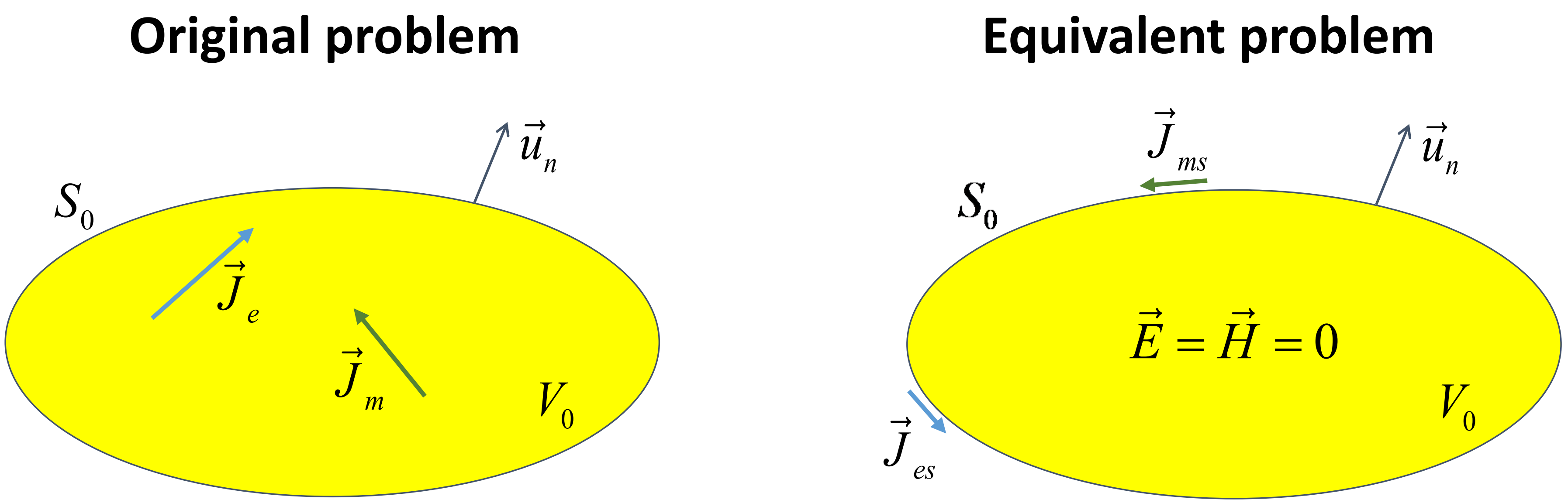
Parabolic reflector



Uniform circular aperture



Apply equivalence concept and our recipe

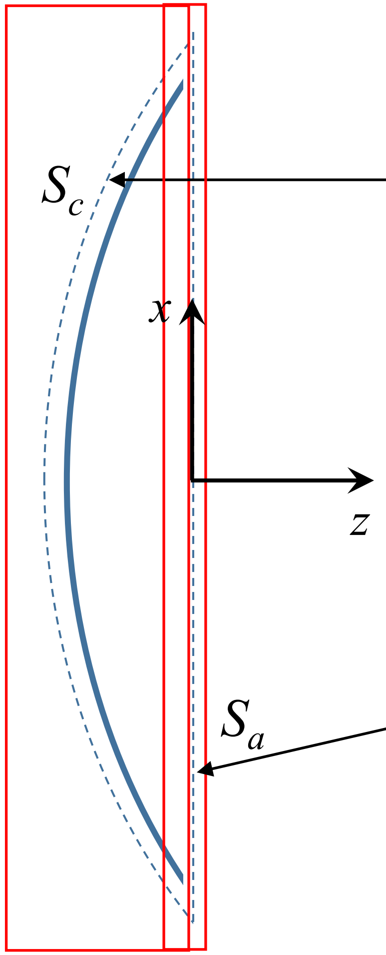


Far-field

$$\vec{E}(\vec{r}) = \frac{jk_0 e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \iint_{S_0}\left(\boxed{\vec{E}\times\vec{u}_n} + Z_0\vec{u}_r \times \boxed{\vec{u}_n \times \vec{H}}\right) e^{jk_0\vec{u}_r\cdot\vec{r}_0}\,\boxed{dS_0}$$

Determine tangential fields on S_0 define closed surface S_0

Select closed surface



On S_c

$$\vec{E} \times \vec{u}_n = \vec{0}$$

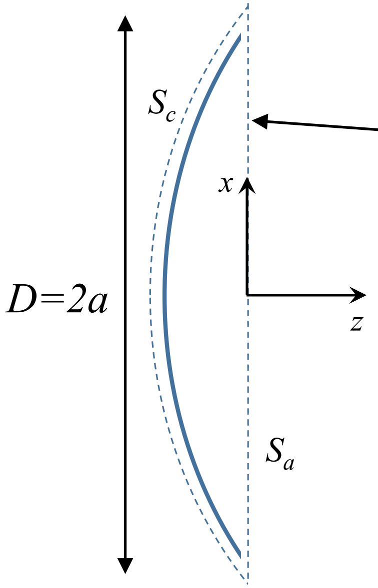
$$\vec{H} \times \vec{u}_n = \vec{0}$$

On S_a

$$\vec{E}_a = E_0 \vec{u}_x \qquad \Rightarrow \qquad \vec{E}_a \times \vec{u}_n = -E_0 \vec{u}_y$$

$$\vec{H}_a = \frac{E_0}{Z_0} \vec{u}_y \qquad \qquad \vec{u}_n \times \vec{H}_a = -\frac{E_0}{Z_0} \vec{u}_x$$

Determine integral over aperture S_a



$$\boxed{\vec{E}_a \times \vec{u}_n = -E_0 \vec{u}_y}$$

$$\boxed{\vec{u}_n \times \vec{H}_a = -\frac{E_0}{Z_0}\vec{u}_x}$$

$$\vec{E}(\vec{r}) = \frac{jk_0 e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \iint_{S_0}\left(\boxed{\vec{E}\times\vec{u}_n} + Z_0 \vec{u}_r \times \boxed{\vec{u}_n \times \vec{H}}\right) e^{jk_0 \vec{u}_r \cdot \vec{r}_0}\, dS_0$$

$$= \frac{-jk_0 e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \boxed{\int_0^{2\pi}\int_0^{a}}\left(E_0 \vec{u}_y + \vec{u}_r \times E_0 \vec{u}_x\right) e^{jk_0 \vec{u}_r \cdot \vec{r}_0}\, \boxed{r_0\, dr_0\, d\phi_0}$$

integration over circular aperture S_a

Far-field uniform circular aperture

$$\vec{E} = \frac{-jk_0 e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \int_0^{2\pi}\int_0^{a}\left(E_0\vec{u}_y + \vec{u}_r \times E_0\vec{u}_x\right)e^{jk_0\vec{u}_r\cdot\vec{r}_0}\, r_0\, dr_0\, d\phi_0$$

$$= \frac{ja^2 k_0 E_0 e^{-jk_0 r}}{2r}(1+\cos\theta)\left(\boxed{\vec{u}_\theta\cos\phi} - \boxed{\vec{u}_\phi\sin\phi}\right)\frac{J_1(k_0 a\sin\theta)}{k_0 a\sin\theta}$$

$$= \boxed{E_\theta\vec{u}_\theta} + \boxed{E_\phi\vec{u}_\phi}$$

$E_\phi = 0$ in $\phi = 0$ plane (E-plane)

$E_\theta = 0$ in $\phi = \frac{\pi}{2}$ plane (H-plane)

Radiation pattern

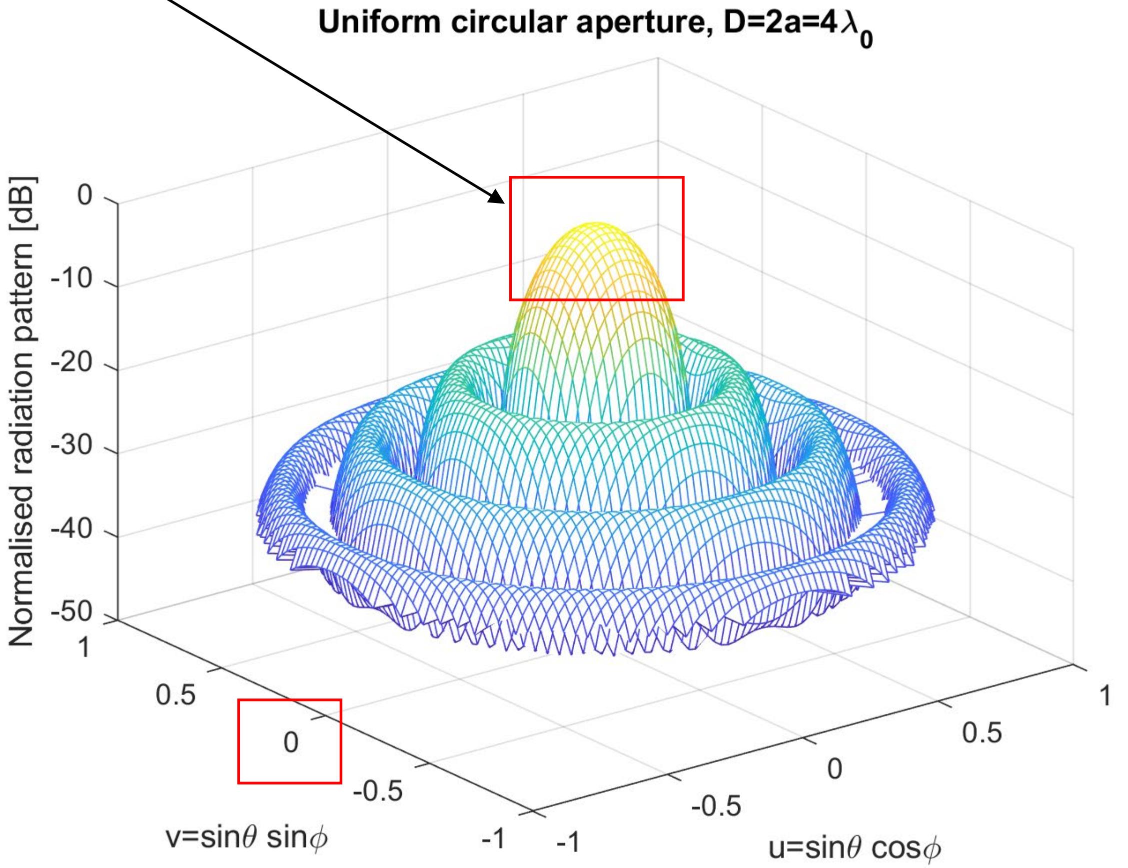
$$F(\theta,\phi) = \frac{\left|E_\theta(\theta,\phi)\right|^2 + \left|E_\phi(\theta,\phi)\right|^2}{\left|E_\theta(0,0)\right|^2 + \left|E_\phi(0,0)\right|^2} = (1+\cos\theta)^2\left[\frac{J_1(k_0 a\sin\theta)}{k_0 a\sin\theta}\right]^2$$

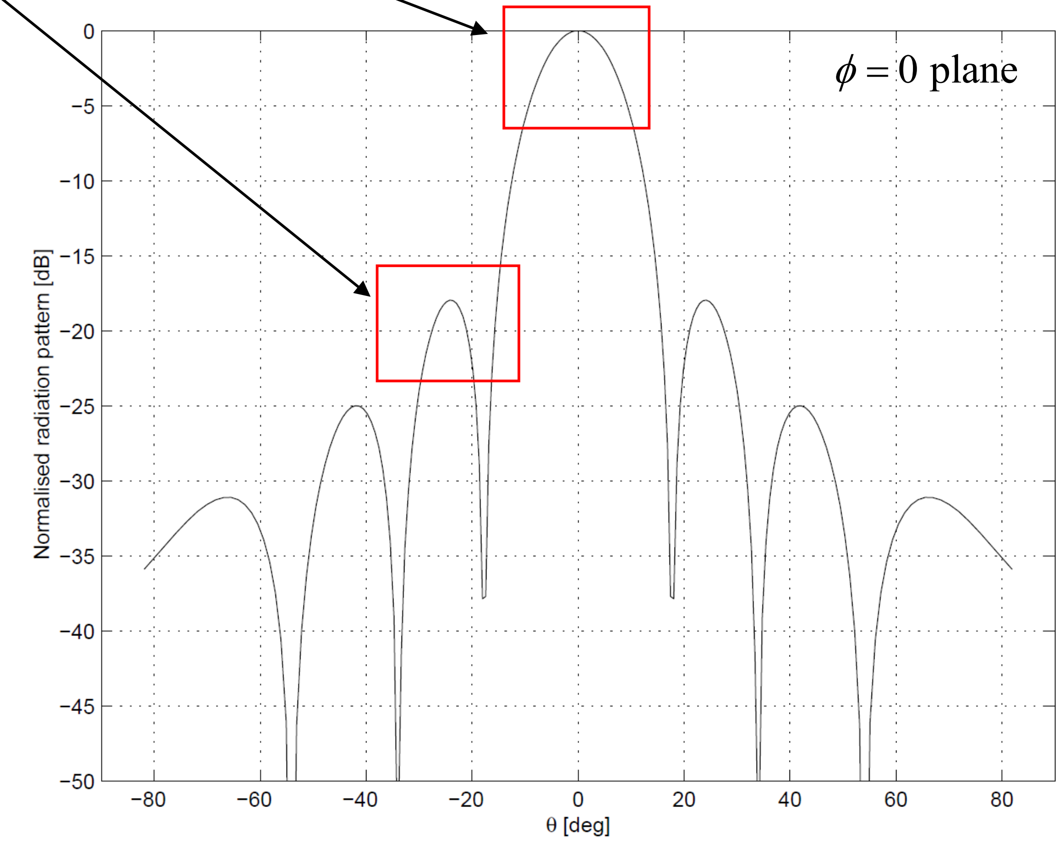
Example reflector antenna ($D=4\lambda_0$)

$$\text{Directivity } D = \left(\frac{2\pi a}{\lambda_0}\right)^2 \rightarrow 22 \text{ dBi}$$

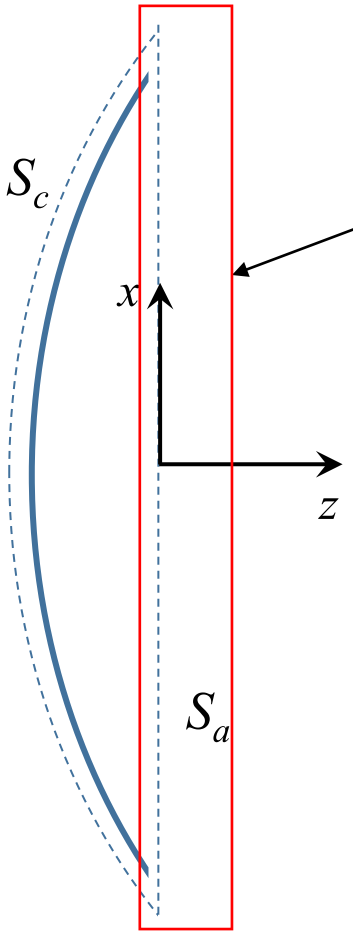
$$HPBW = 1.02\frac{\lambda_0}{2a} = 14.6^0$$

First sidelobe at -17.6 dB





Tapering



$$\vec{E}_a = W(r_0)\vec{u}_x$$

$$\vec{H}_a = \frac{W(r_0)}{Z_0}\vec{u}_y$$

$$\boxed{W(r_0) = E_0\left[1-\left(\frac{r_0}{a}\right)^2\right]^p}$$

use special feed (horn or FPA)

$$= 2\pi a^2 E_0 2^p p!\frac{J_{p+1}(k_0 a\sin\theta)}{(k_0 a\sin\theta)^{p+1}}$$

$$\vec{E} = \frac{-jk_0 e^{-jk_0 r}}{4\pi r}(1+\cos\theta)(\vec{u}_\theta\cos\phi - \vec{u}_\phi\sin\phi)\boxed{\int_0^{2\pi}\int_0^a E_0\left[1-\left(\frac{r_0}{a}\right)^2\right]^p e^{jk_0\vec{u}_r\cdot\vec{r}_0} r_0 dr_0 d\phi_0}$$

TU/e

Tapering, effect on antenna parameters

$$W(r_0) = E_0\left[1-\left(\frac{r_0}{a}\right)^2\right]^p$$

Antenna parameter	$\boxed{p=0}$	$\boxed{p=1}$	$\boxed{p=2}$
3-dB beam width in degrees	$29.2\frac{\lambda_0}{a}$	$36.4\frac{\lambda_0}{a}$	$42.1\frac{\lambda_0}{a}$
Beam width between zeros in degrees	$69.9\frac{\lambda_0}{a}$	$93.4\frac{\lambda_0}{a}$	$116.3\frac{\lambda_0}{a}$
First sidelobe level [dB]	$\boxed{-17.6}$	$\boxed{-24.6}$	$\boxed{-30.6}$
Directivity	$\boxed{\left(\frac{2\pi a}{\lambda_0}\right)^2}$	$\boxed{0.75\left(\frac{2\pi a}{\lambda_0}\right)^2}$	$\boxed{0.56\left(\frac{2\pi a}{\lambda_0}\right)^2}$

Summary

- The radiation properties of reflector antennas with and without tapering have been derived.
- Tapering reduces the first sidelobe level, but the aperture efficiency will be lower.
- This can be compensated by increasing the size of the reflector.

Microwave Engineering and Antennas

Microstrip Antennas

Bart Smolders, Professor

Department of Electrical Engineering

Center for Wireless Technology Eindhoven

Microstrip antennas

Objective of this lecture

- Introduce microstrip antennas.
- Use cavity model to find the resonating modes and resonance frequencies.
- Determine radiation properties using the equivalence principle.
- Discuss properties of microstrip antennas, including bandwidth.

Properties and applications

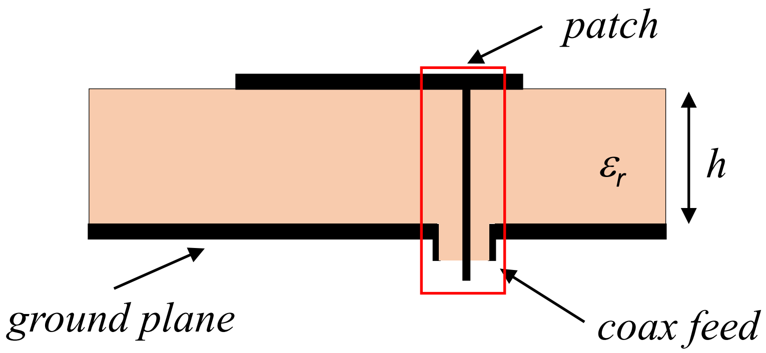
Properties

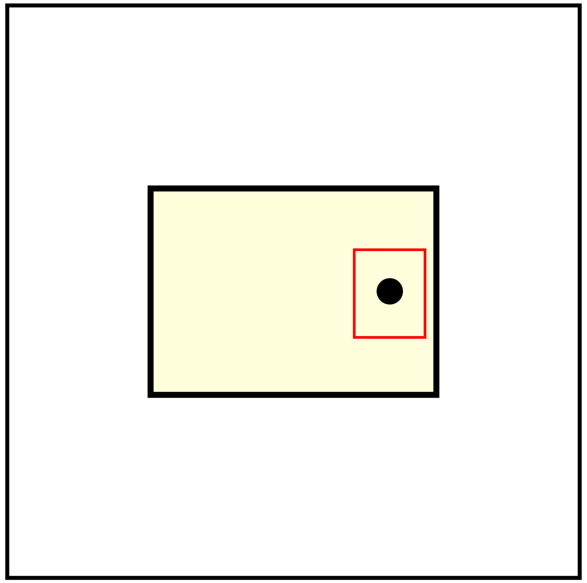
- Low profile
- Integration on PCB/laminate
- Low-cost

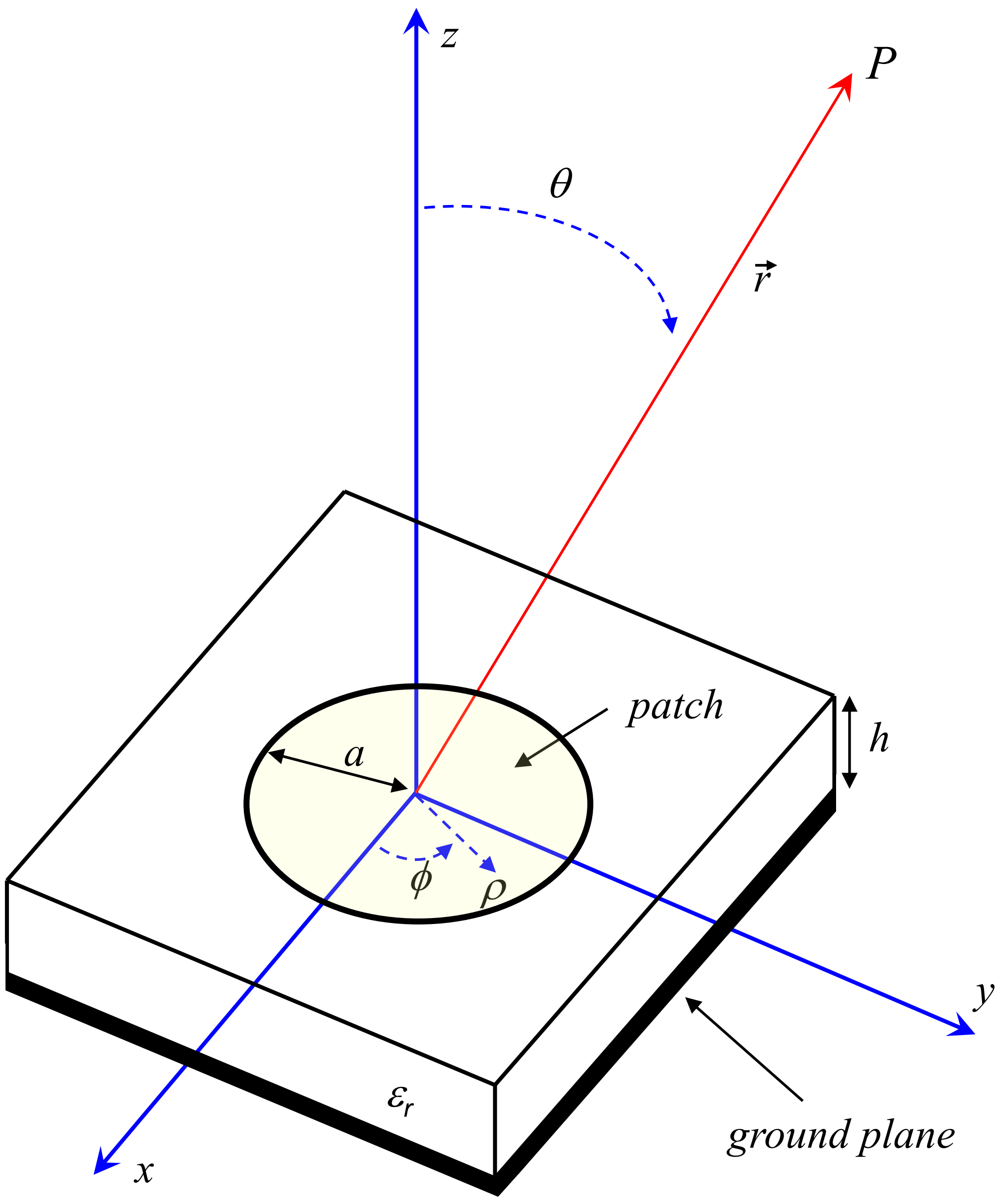
Applications

- Phased arrays, e.g. 5G mm-wave
- GPS receivers
- Satellite communication

Microstrip antenna

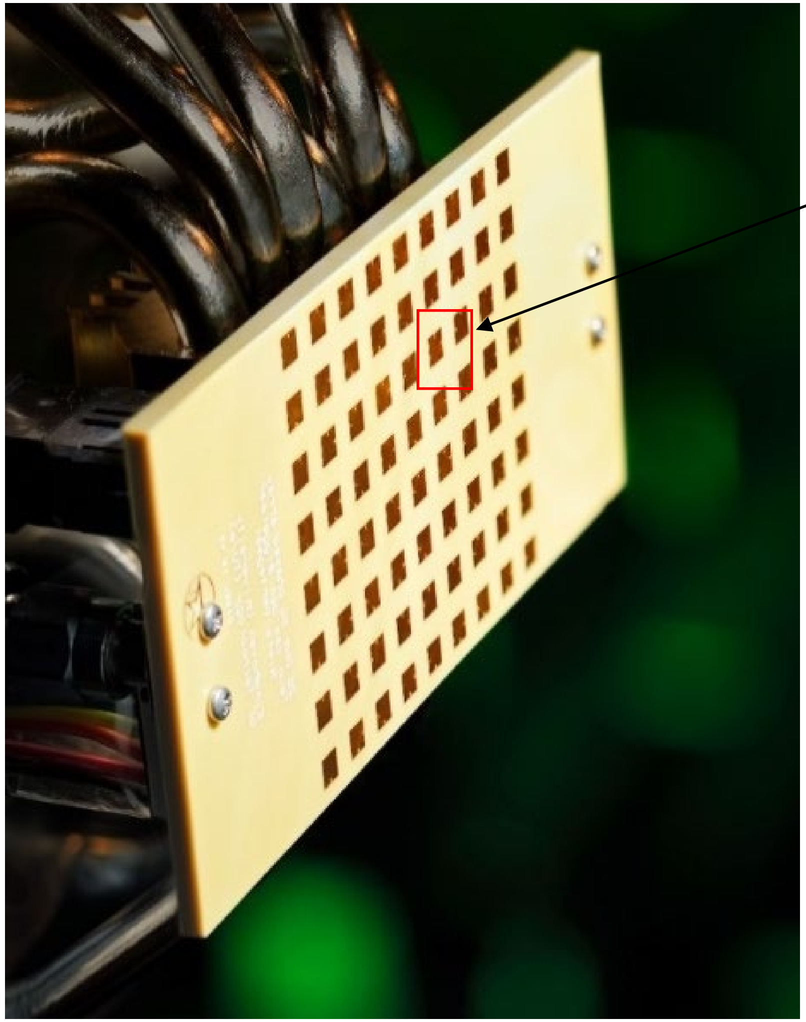


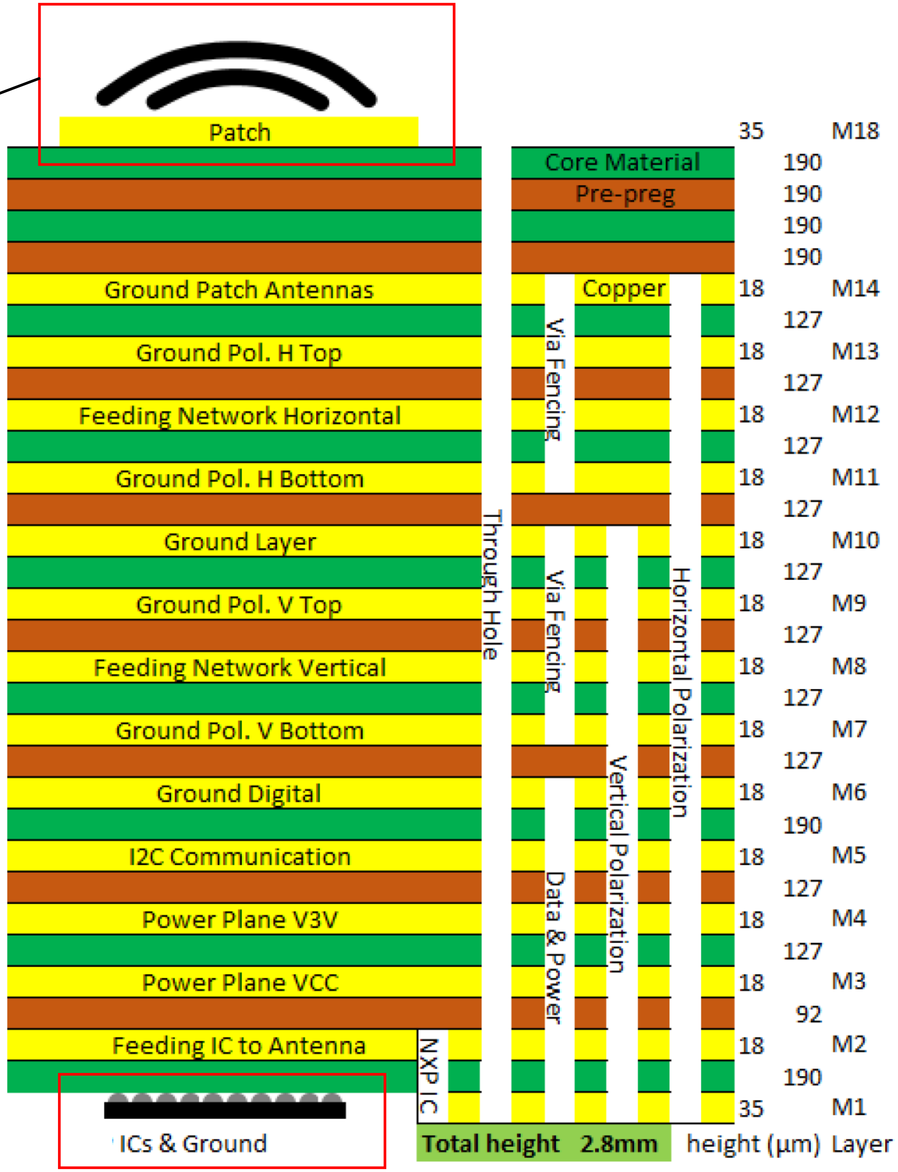




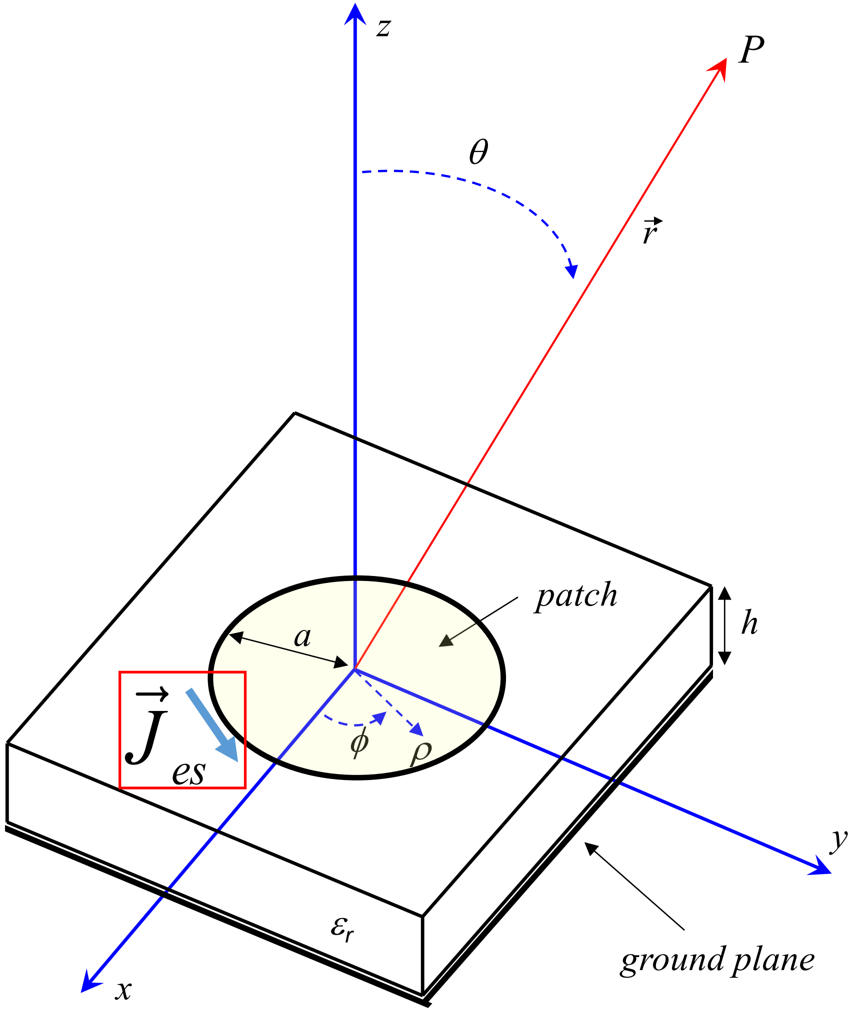
TU/e

Microstrip antenna example





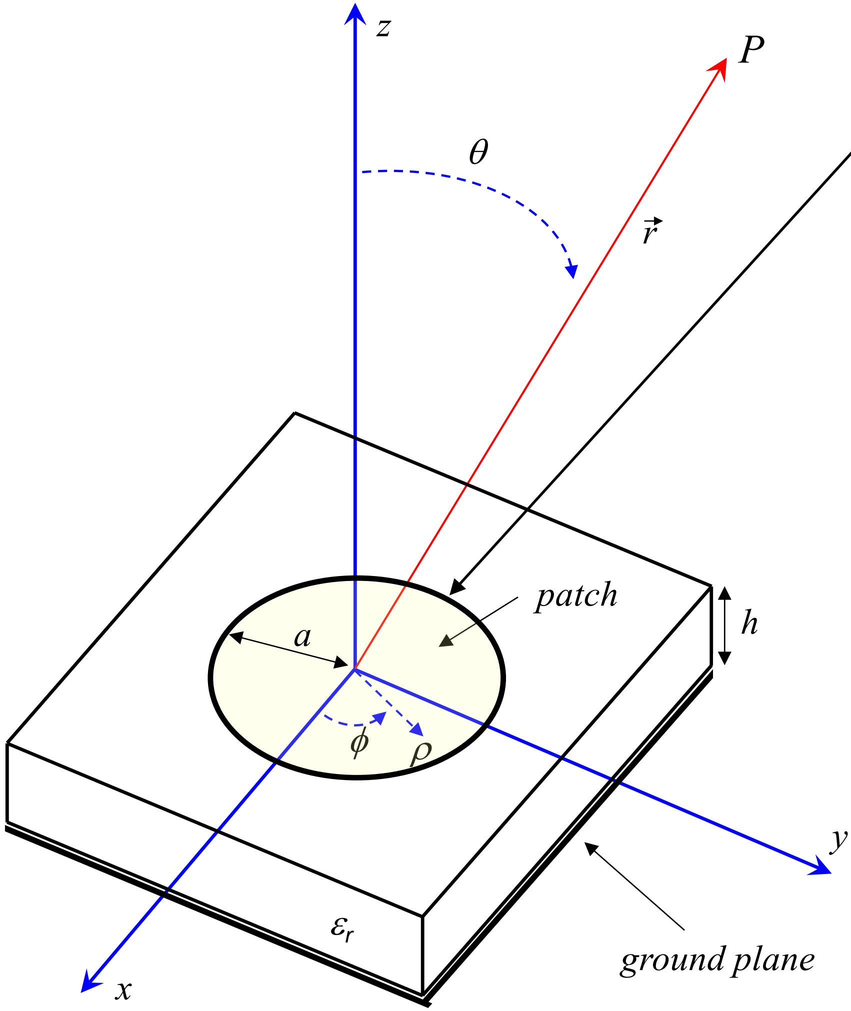
Cavity model



Assumptions

- E-field is concentrated between circular patch and ground plane (cavity).
- $h \ll \lambda_0$: E-field not z-dependent.
- In cavity: $\vec{E} = E_z \vec{u}_z$
- Surface current along edge $\rho = a$ parallel to edge, so $\vec{H} \times \vec{u}_n = \vec{0}$ on sidewall of cavity.

Cavity model



On sidewall cylindrical cavity

$$\left.\begin{aligned}\vec{H}\times\vec{u}_n &= \vec{0}\\ \vec{H} &= \frac{-1}{j\omega\mu_0}\nabla\times\vec{E}\end{aligned}\right\}\quad \boxed{\frac{\partial E_z}{\partial\rho} = 0 \text{ for } \rho = a}$$

Solve Helmholtz equation in cavity

$$\nabla^2\vec{E} + k^2\vec{E} = \vec{0}$$

wavenumber in cavity $k_0\sqrt{\varepsilon_r}$

order Bessel function

$$\Rightarrow\quad E_z = \boxed{E_0} J_{\boxed{n}}(\boxed{k}\rho)\cos(\boxed{n}\phi)$$

$$H_\rho = \frac{n}{j\omega\mu_0\rho}E_0 J_n(k\rho)\sin(n\phi)$$

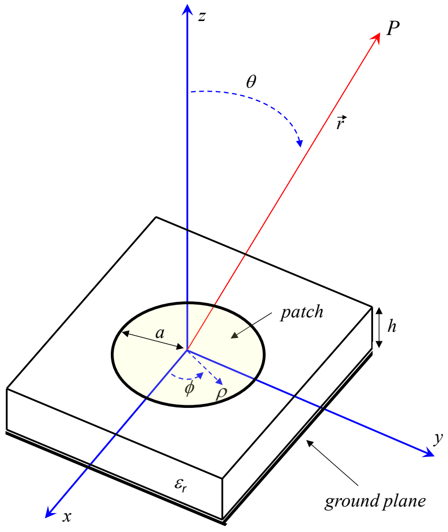
Due to voltage feed $V_0 = hE_0$

$$H_\phi = \frac{k}{j\omega\mu_0}E_0\frac{\partial J_n(k\rho)}{\partial\rho}\cos(n\phi)$$

Resonance condition

On sidewall ($\rho = a$) $\vec{H} \times \vec{u}_n = \vec{0}$

$$H_\phi = \frac{k}{j\omega\mu_0} E_0 \frac{\partial J_n(ka)}{\partial \rho} \cos(n\phi) = 0$$



Resonance condition

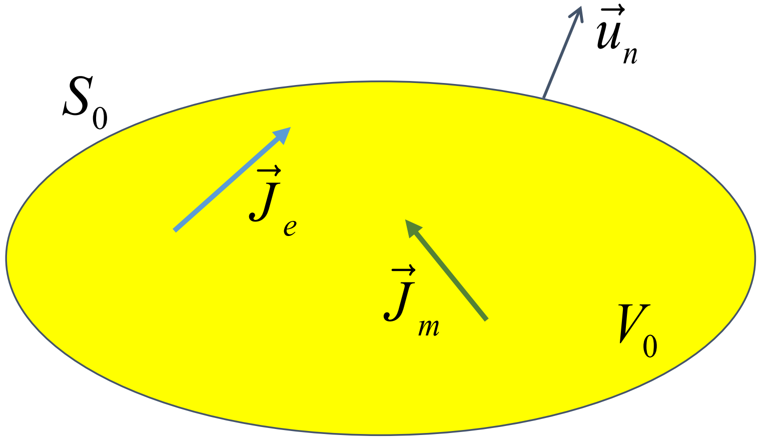
$$\frac{\partial J_n(ka)}{\partial \rho} = 0$$

first mode resonance: $f_{01} = \dfrac{1.841c}{2\pi a\sqrt{\varepsilon_r}}$

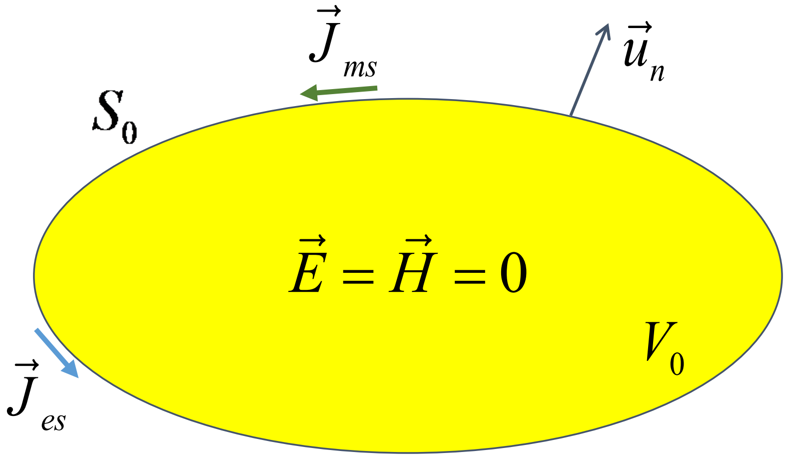
Mode *n*	***ka***
1	1.841
2	3.054
0	3.832
3	4.201

Apply equivalence concept and our recipe

Original problem



Equivalent problem



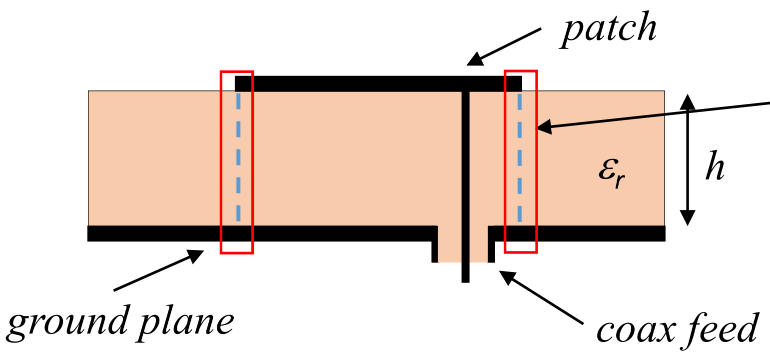
Far-field

$$\vec{E}(\vec{r}) = \frac{jk_0 e^{-jk_0 r}}{4\pi r}\vec{u}_r \times \iint_{S_0} \left(\boxed{\vec{E} \times \vec{u}_n} + Z_0 \vec{u}_r \times \boxed{\vec{u}_n \times \vec{H}} \right) e^{jk_0 \vec{u}_r \cdot \vec{r}_0} \boxed{dS_0}$$

Determine tangential fields on S_0

define closed surface S_0

Far field



On sidewall cavity

$$\vec{J}_{ms} = \boxed{2}\vec{E} \times \vec{u}_n = 2E_z \vec{u}_\phi$$

accounts for the image due to groundplane

Far-field (first mode *n=1*)

$$\vec{E}(\vec{r}) = \frac{jk_0 e^{-jk_0 r}}{4\pi r} \vec{u}_r \times \iint_{S_0} 2E_z \vec{u}_\phi e^{jk_0 \vec{u}_r \cdot \vec{r}_0} dS_0 = E_\theta \vec{u}_\theta + E_\phi \vec{u}_\phi$$

$$E_\theta = \boxed{F(r)} \cos\phi \left[J_2(k_0 a \sin\theta) - J_0(k_0 a \sin\theta) \right]$$

$$\boxed{F(r) = \frac{jhak_0 E_0 J_1(ka) e^{-jk_0 r}}{2r}}$$

$$E_\phi = F(r) \cos\theta \sin\phi \left[J_2(k_0 a \sin\theta) + J_0(k_0 a \sin\theta) \right]$$

Example radiation pattern

Circular patch

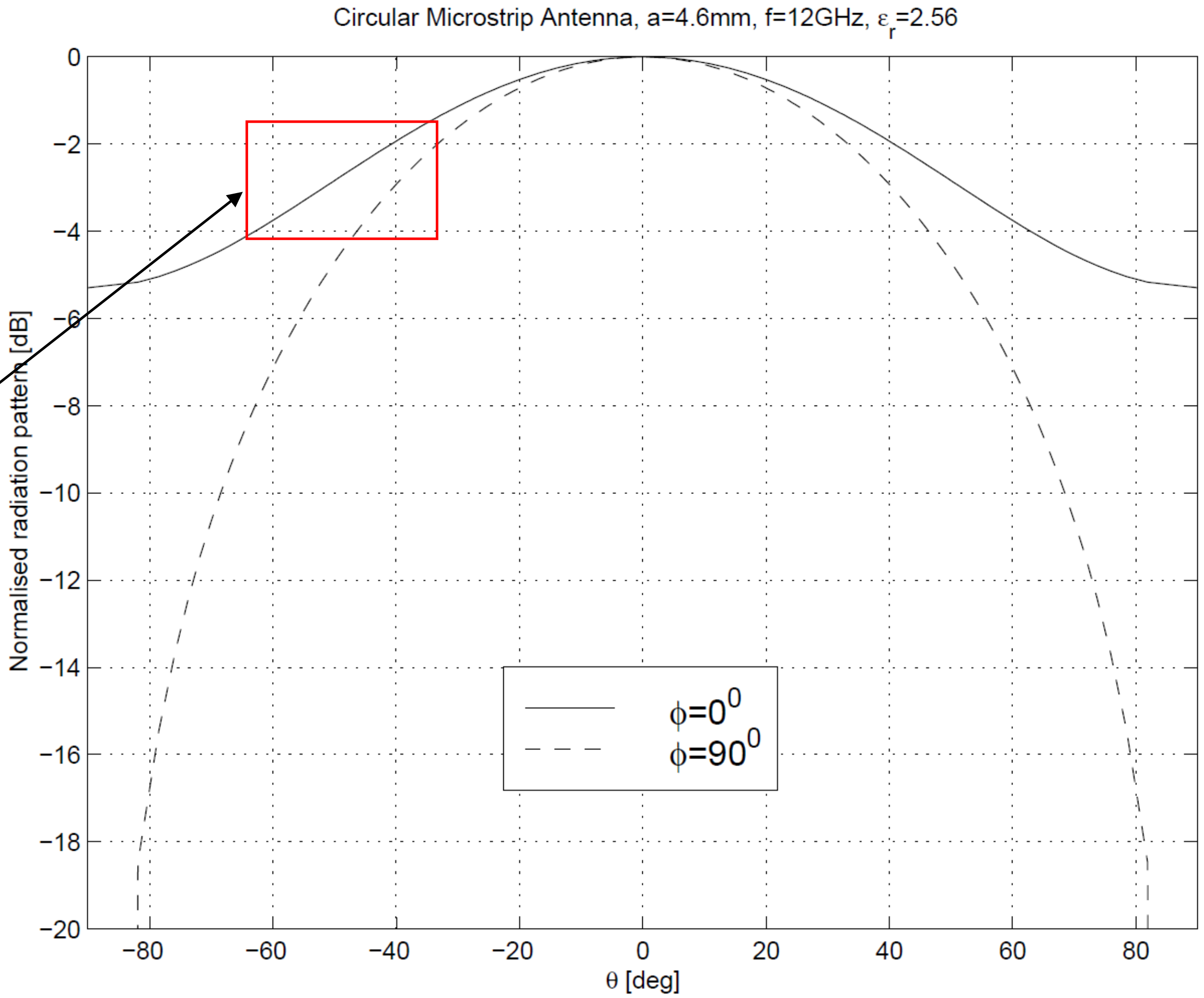
$\varepsilon_r = 2.56$

$a = 4.6$ mm

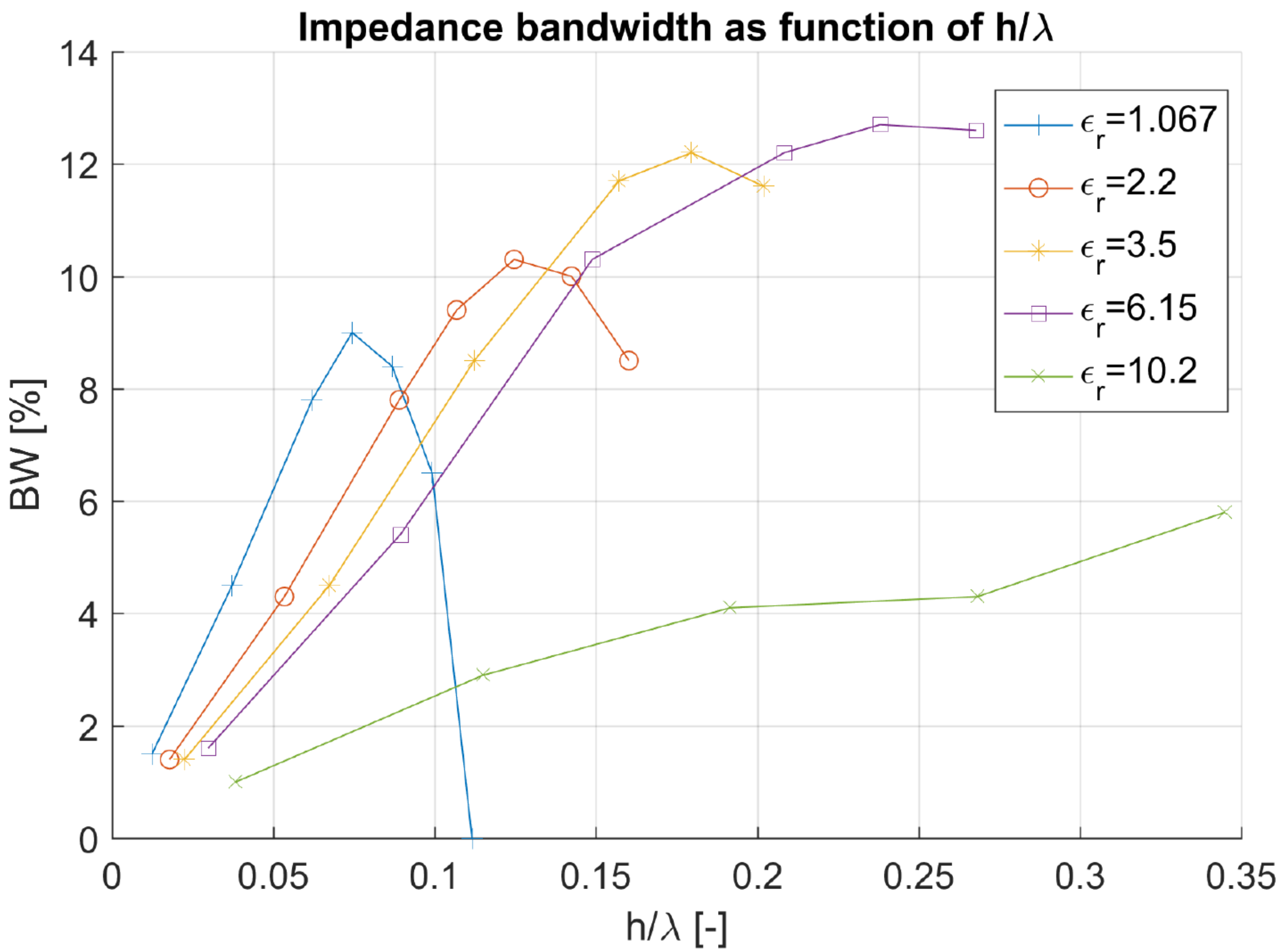
$f_{res} \approx 12$ GHz

Feed in $\phi = 0^0$ plane

HPBW $= 104^0$ in $\phi = 0^0$ plane



Impedance bandwidth (using numerical model)



Summary

- Microstrip antennas are low-profile and easy to integrate in PCB/laminate technologies.
- We derived analytic expression for the far-field pattern and resonance frequency.
- Microstrip antennas provide a wide HPBW, so ideal to use in phased-arrays/MIMO solutions.
- Bandwidth depends on h and ε_r.

Microwave Engineering and Antennas

Linear Arrays with real (non-ideal) Antennas

Bart Smolders, Professor

Department of Electrical Engineering

Center for Wireless Technology Eindhoven

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Linear array of real antennas

Objective of this lecture

- Extension of linear array theory to include non-ideal antenna elements.
- Introduce the concept of embedded element pattern.
- Show examples of microstrip and waveguide arrays .

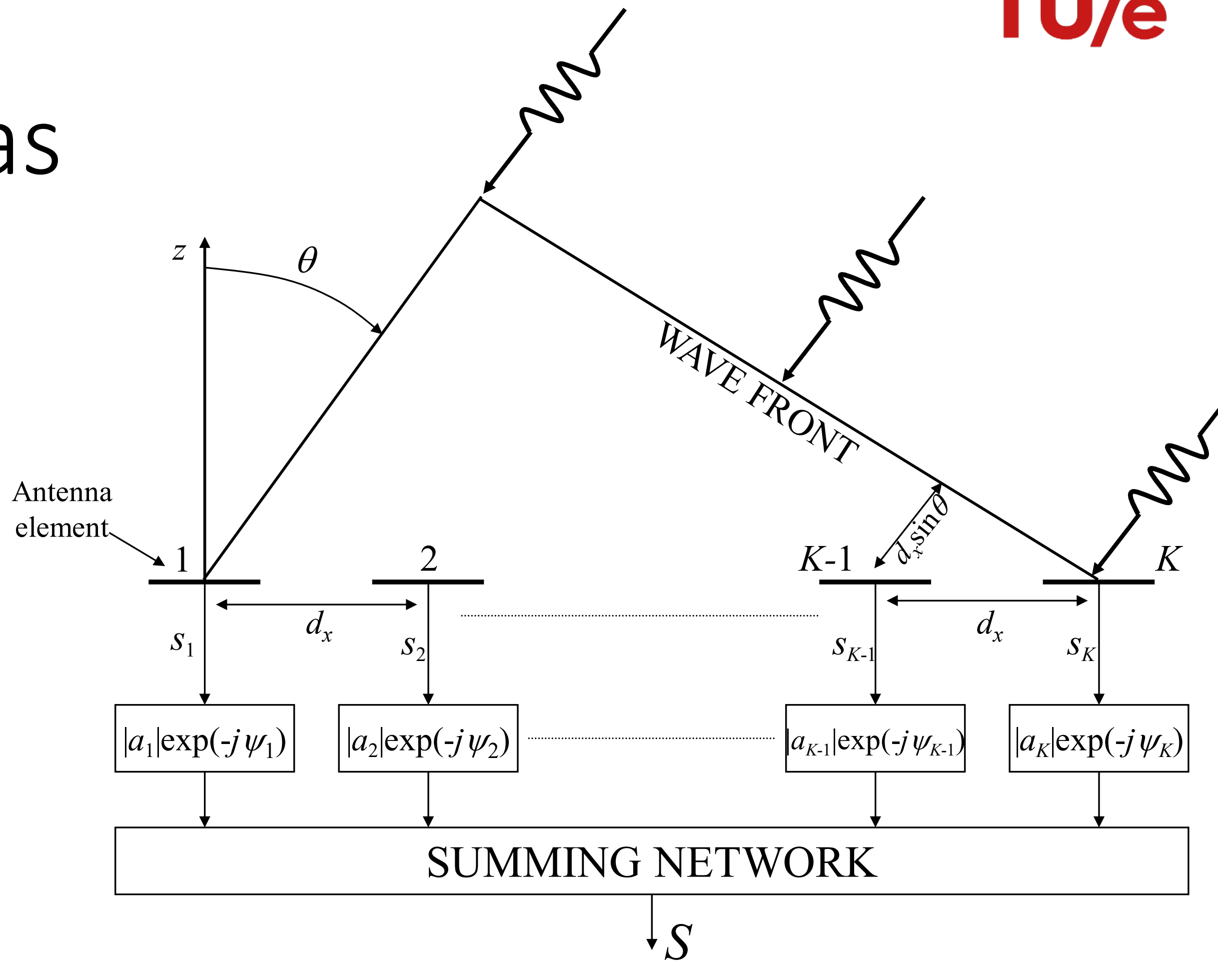
Linear array of real antennas

Isotropic elements

$$\boxed{S(\theta)} = \sum_{k=1}^{K} |a_k| e^{jk_0(k-1)d_x(\sin\theta - \sin\boxed{\theta_0})}$$

array factor (both TX and RX case)

scan angle



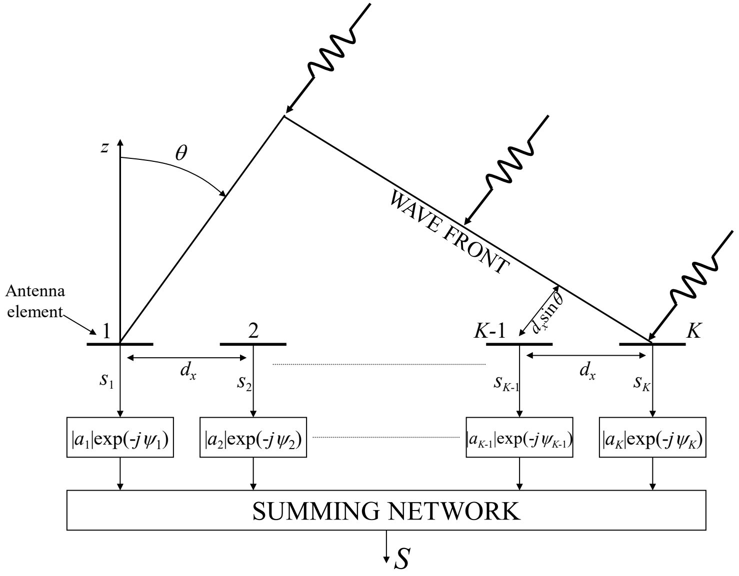
Real antenna elements

$$\boxed{\vec{S}(\theta,\phi)} = \sum_{k=1}^{K} |a_k| \boxed{\vec{f}_k(\theta,\phi)} e^{jk_0(k-1)d_x(\sin\theta - \sin\theta_0)}$$

radiated field total array

radiated field element k (element factor or embedded element pattern)

Linear array of real antennas



Without mutual coupling and identical antennas

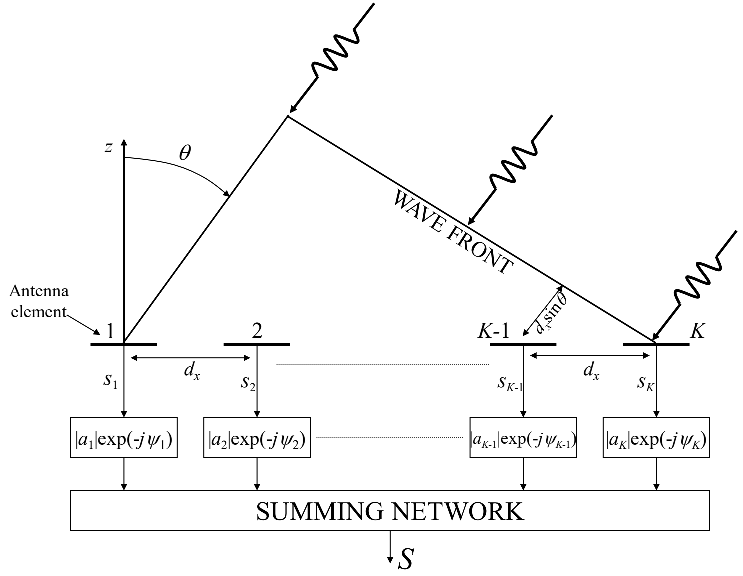
$$\vec{S}(\theta,\phi) = \boxed{\vec{f}(\theta,\phi)} \sum_{k=1}^{K} |a_k| e^{jk_0 (k-1) d_x (\sin\theta - \sin\theta_0)}$$

independent of k

Normalized radiation pattern

$$F(\theta,\phi) = \frac{\left|\vec{S}(\theta,\phi)\right|^2}{\left|\vec{S}(\theta_0,\phi_0)\right|^2}$$

Example: Linear array of microstrip antennas



Element factor along $\phi = 0^0$ plane

$$\vec{E}(\theta) = \boxed{A_0}\left[J_2(k_0 a \sin\theta) - J_0(k_0 a \sin\theta)\right]\boxed{\vec{u}_\theta}$$

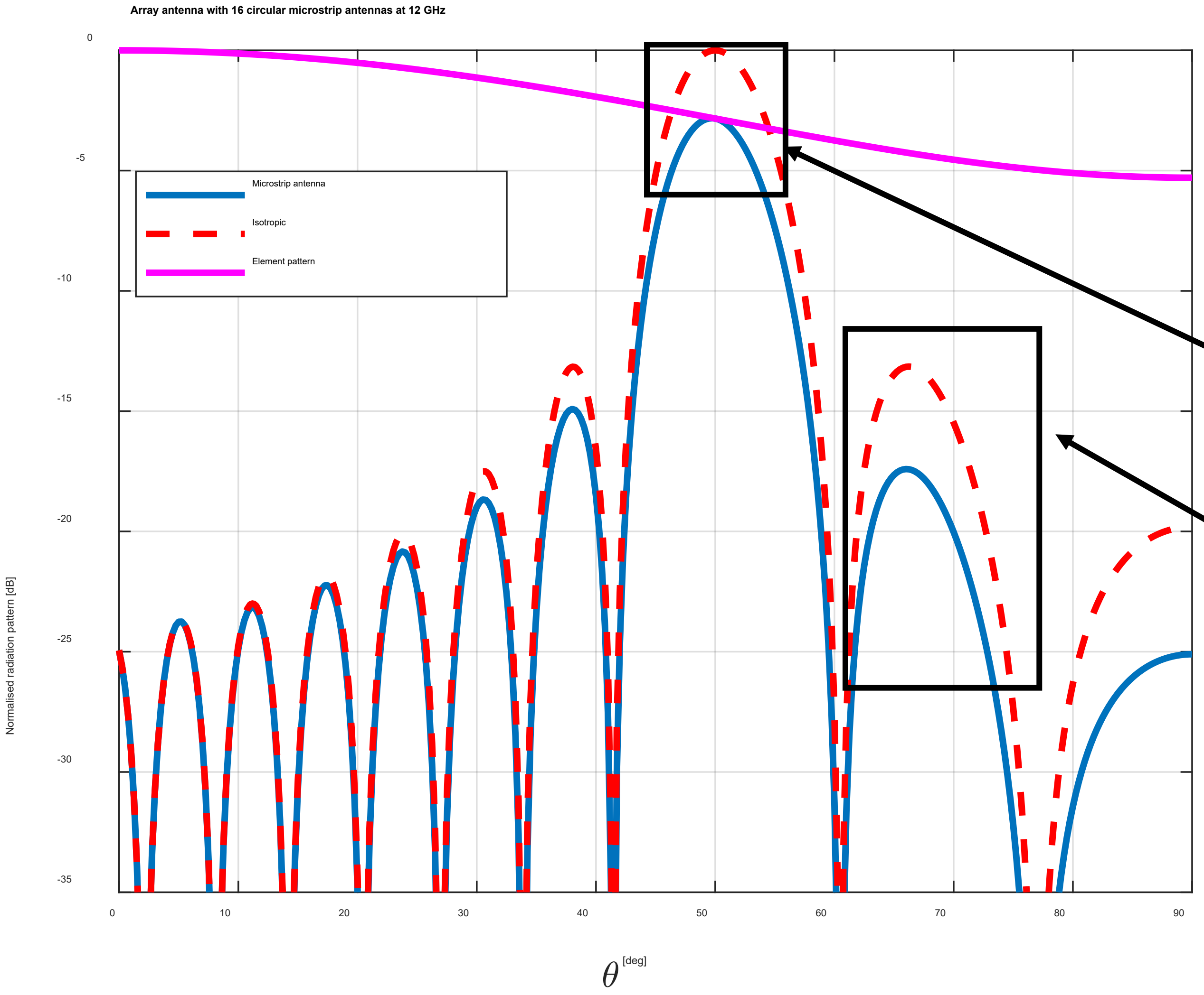
only θ component

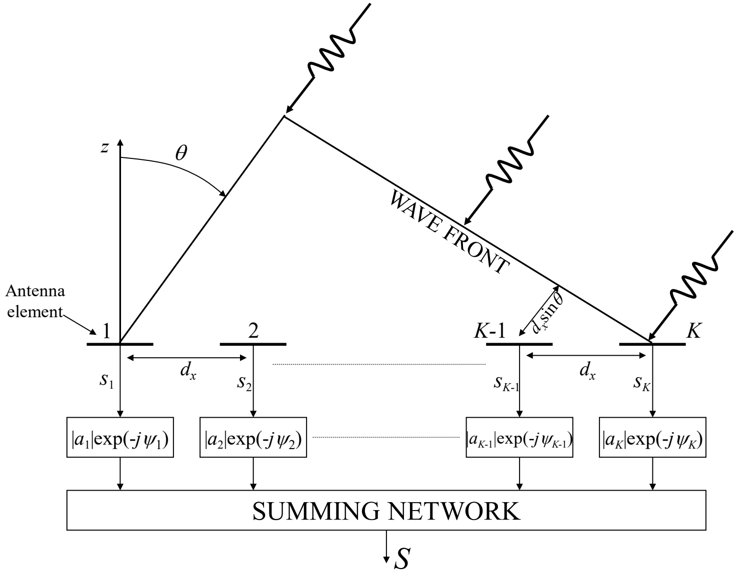
Total radiated field by array along $\phi = 0^0$ plane

$$\vec{S}(\theta) = \left[J_2(k_0 a \sin\theta) - J_0(k_0 a \sin\theta)\right]\vec{u}_\theta \sum_{k=1}^{K} \boxed{|a_k|} e^{jk_0 (k-1) d_x (\sin\theta - \sin\theta_0)}$$

Includes A_0

Linear array of circular microstrip antennas, 50^0 scan.

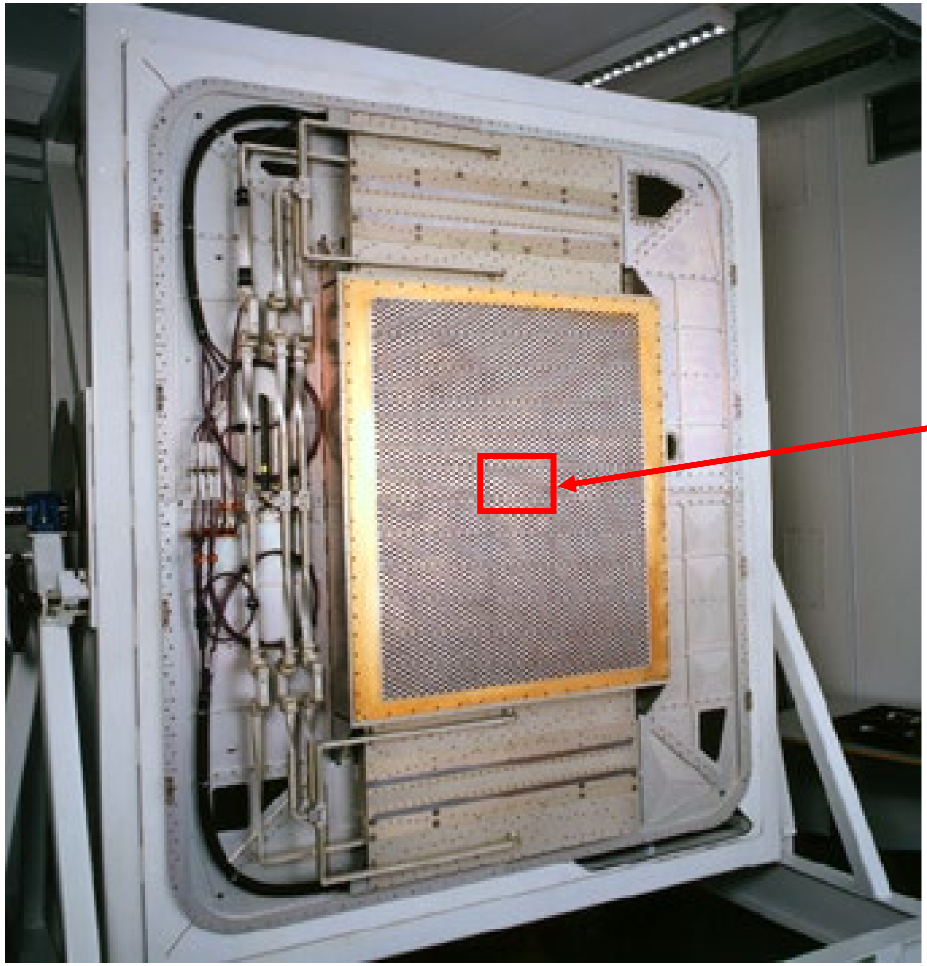


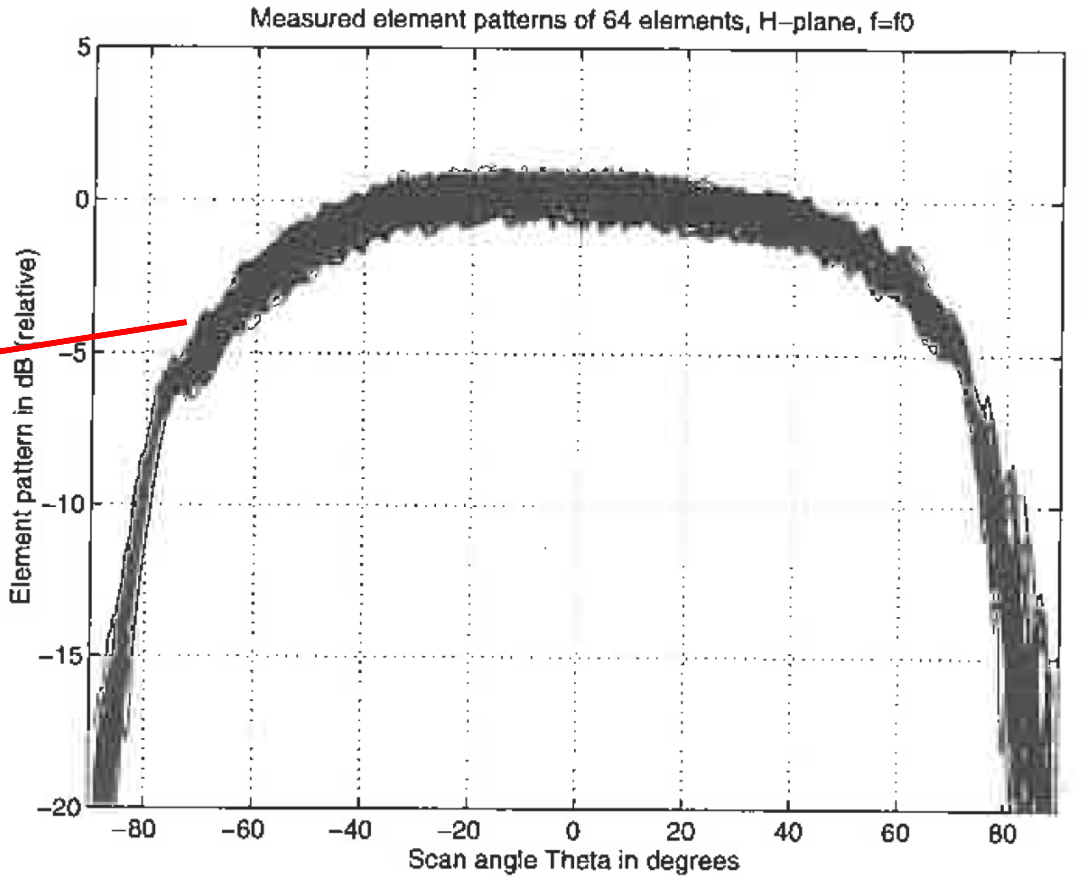


Reduction of Gain due to element pattern

Lower sidelobes due to element pattern

Example: Large X-band waveguide array





From: A.B. Smolders, “Design and construction of a broadband wide-scan angle phased-array antenna with 4096 radiating elements” Proceedings of International Symposium on Phased Array Systems and Technology 1996, pp.87-92.

Summary

- Extension towards linear arrays of real antennas.
- Element factor is a vector (e.g. E-field).
- Example of microstrip array and waveguide array was shown.

Microwave Engineering and Antennas

Method of Moments (MoM)

Bart Smolders, Professor

Department of Electrical Engineering

Center for Wireless Technology Eindhoven

Method of Moments (MoM)

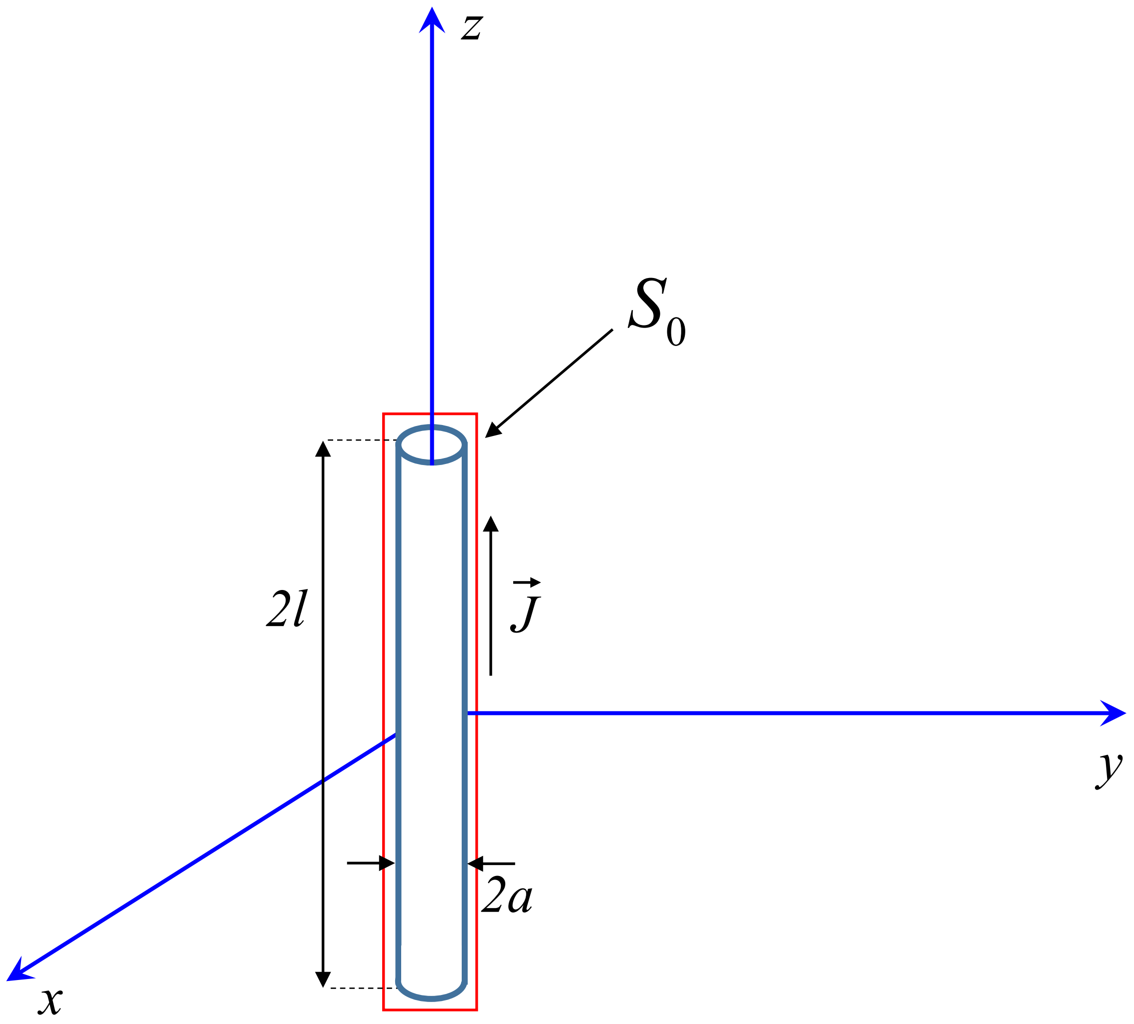
Objective of this lecture

- Determine current distribution on antennas using MoM.
- Apply MoM on a linear wire antenna.
- Show an example obtained with a commercial tool.

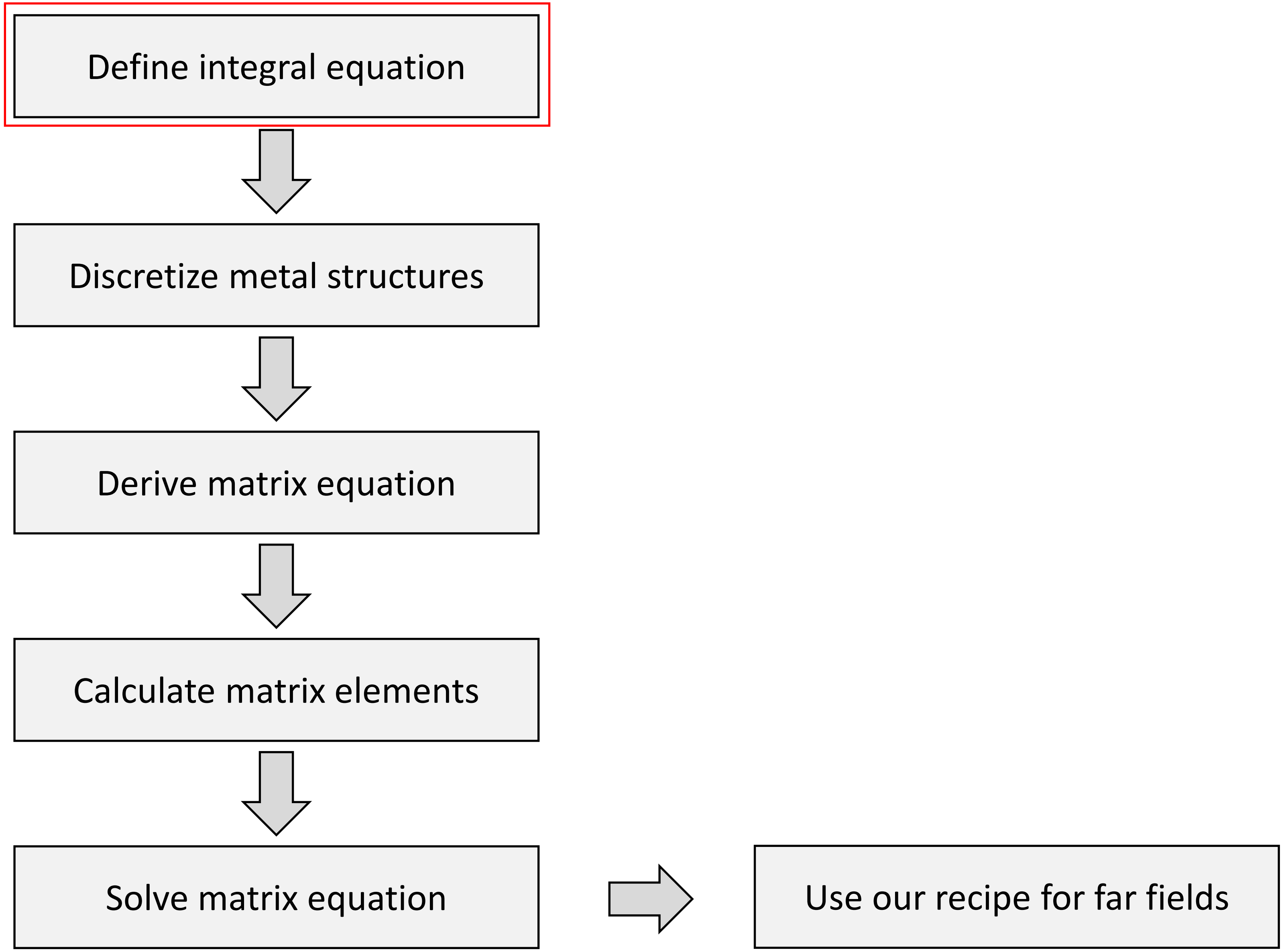
Test case: wire antenna

Properties

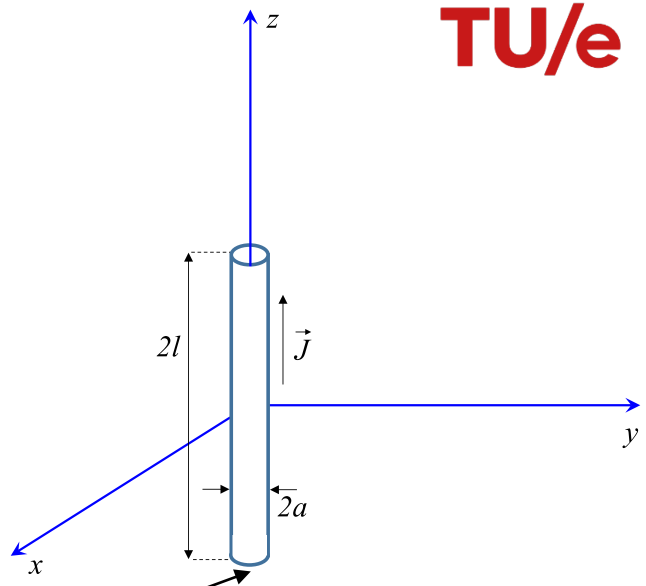
- Center-fed dipole of length $2l$
- Metal cylinder diameter $2a$
- Electrically thin
- S_0 is perfect electric conductor (PEC)



Approach


Define integral equation
Discretize metal structures
Derive matrix equation
Calculate matrix elements
Solve matrix equation
Use our recipe for far fields

Integral equation



Boundary condition: total tangential *E*-field on PEC is zero.

$$\boxed{\vec{u}_n} \times \vec{E}^{tot} = \boxed{\vec{u}_n \times (\boxed{\vec{E}^{ex}} + \boxed{\vec{E}^{s}}) = 0,} \qquad \text{on } S_o$$

Normal vector on cylinder Excitation field Scattered field

Integration over cylinder

$$\vec{E}^{s} = -j\omega\mu_0 \iint_{S_0} \boxed{G(\vec{r},\vec{r}_0)} \vec{J}(\vec{r}_0) dS_0 - \nabla\left(\frac{j}{\omega\varepsilon_0} \nabla \cdot \boxed{\iint_{S_0}} G(\vec{r},\vec{r}_0) \boxed{\vec{J}(\vec{r}_0)} dS_0 \right)$$

Greens function

(Unknown) current distribution on antenna

$$\Rightarrow \quad \boxed{\vec{u}_n \times \vec{E}^{ex} = \vec{u}_n \times j\omega\mu_0 \iint_{S_0} G(\vec{r},\vec{r}_0) \vec{J}(\vec{r}_0) dS_0 + \vec{u}_n \times \nabla\left(\frac{j}{\omega\varepsilon_0} \nabla \cdot \iint_{S_0} G(\vec{r},\vec{r}_0) \vec{J}(\vec{r}_0) dS_0 \right)}$$

Approach

Define integral equation

↓

Discretize metal structures

↓

Derive matrix equation

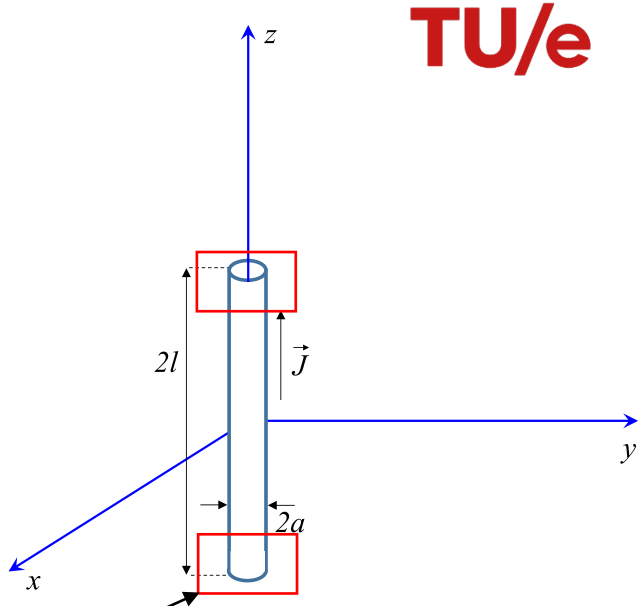
↓

Calculate matrix elements

↓

Solve matrix equation → Use our recipe for far fields

Discretize metal structures



Expansion of unknown current on S_0:

$$\vec{J}(x,y,z) = \sum_{n=1}^{\boxed{N_{\max}}} \boxed{I_n}\,\boxed{\vec{J}_n(x,y,z)}$$

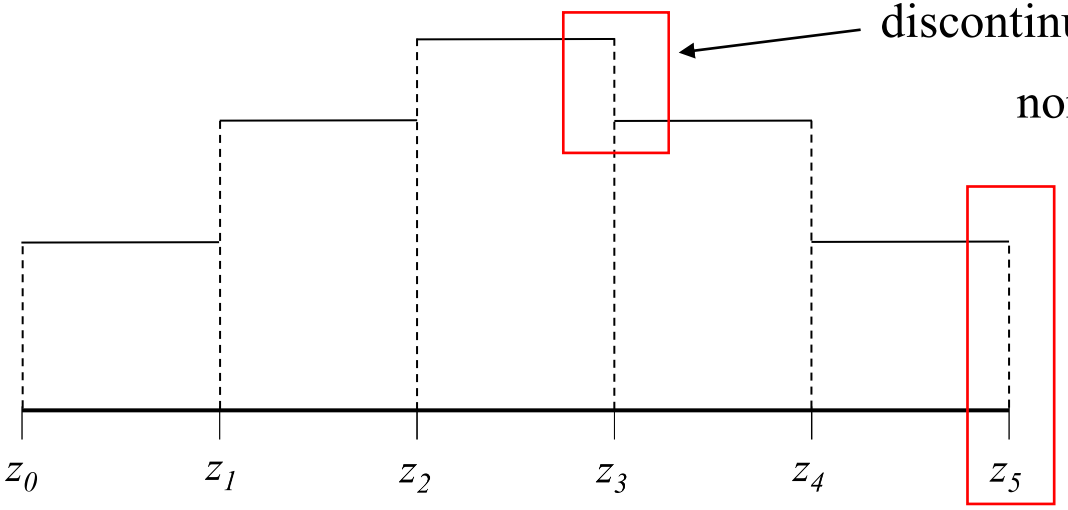
total number of modes

unknown mode coefficients basis function

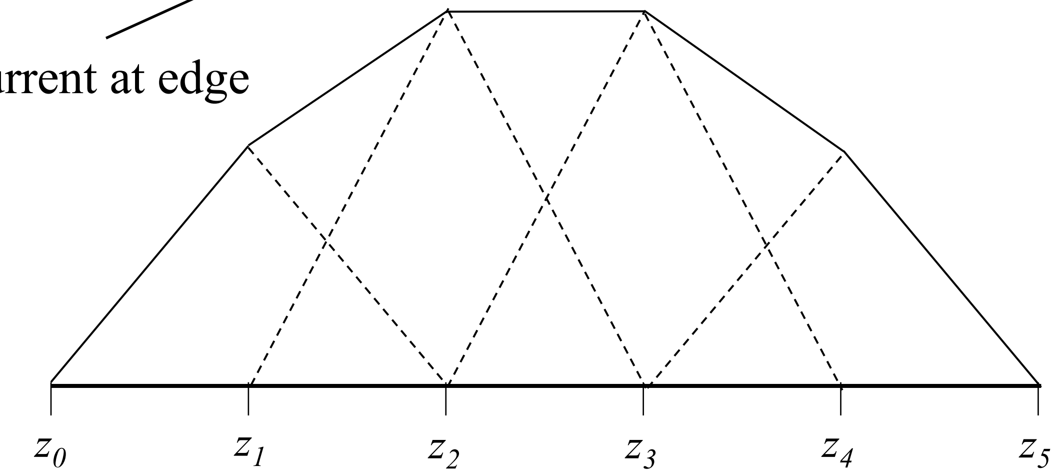
Piece-wise constant (PWC) basis functions

discontinuity

non-zero current at edge



Piece-wise linear (PWL) basis functions



Approach

Define integral equation

↓

Discretize metal structures

↓

Derive matrix equation

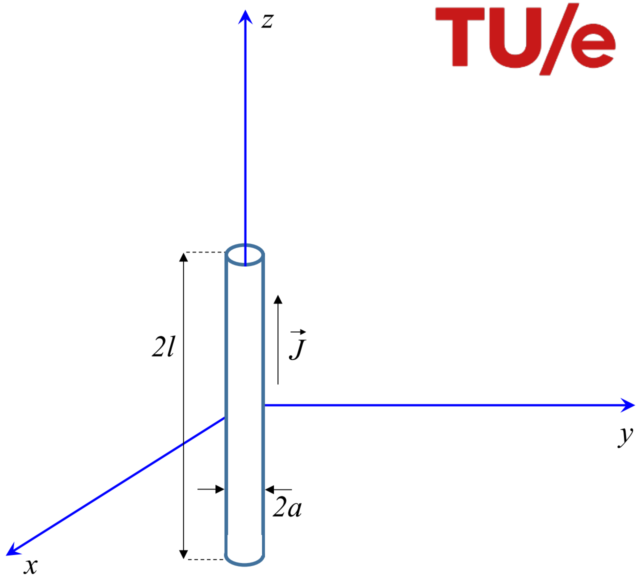
↓

Calculate matrix elements

↓

Solve matrix equation → Use our recipe for far fields

Matrix equation



Use superposition concept:

$$\vec{E}^s = \sum_{n=1}^{N_{\max}} I_n \boxed{\vec{E}_n^s}$$

Field due to basis function $\vec{J}_n$

Apply boundary condition:

$$\vec{u}_n \times \Big(\sum_{n=1}^{N_{\max}} I_n \vec{E}_n^s + \vec{E}^{ex}\Big) = 0, \qquad \text{on } S_o$$

Introduce a residue:

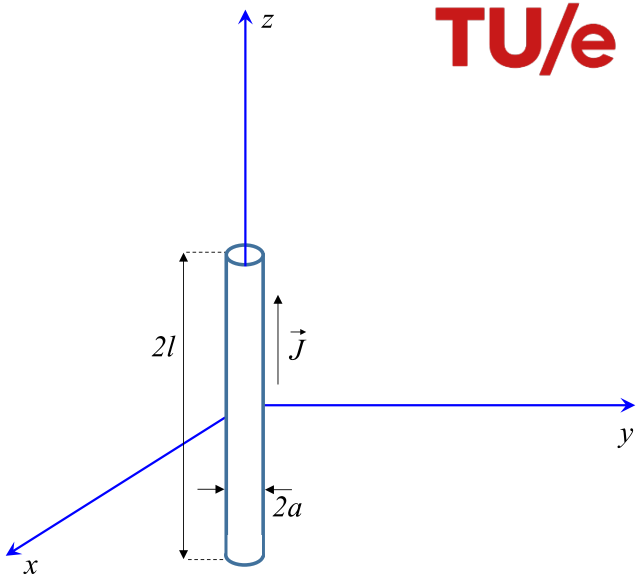
MoM: weigh residue to zero

Weighting/Test function

$$\vec{R} = \vec{u}_n \times \Big(\sum_{n}^{N_{\max}} I_n \vec{E}_n^s + \vec{E}^{ex}\Big) \quad \Rightarrow \quad \iint_{S_m} \vec{R} \cdot \boxed{\vec{J}_m} dS = 0, \quad \text{for } m = 1 \ldots N_{\max}$$

$$\Rightarrow \boxed{\sum_{n=1}^{N_{\max}} I_n Z_{mn} + V_m^{ex} = 0, \quad \text{for } m = 1 \ldots N_{\max}}$$

Matrix equation



$$\sum_{n=1}^{N_{\max}} I_n Z_{mn} + V_m^{ex} = 0, \quad \text{for } m = 1 \ldots N_{\max}$$

$$\boxed{[Z]}[I] + \boxed{[V^{ex}]} = [0]$$

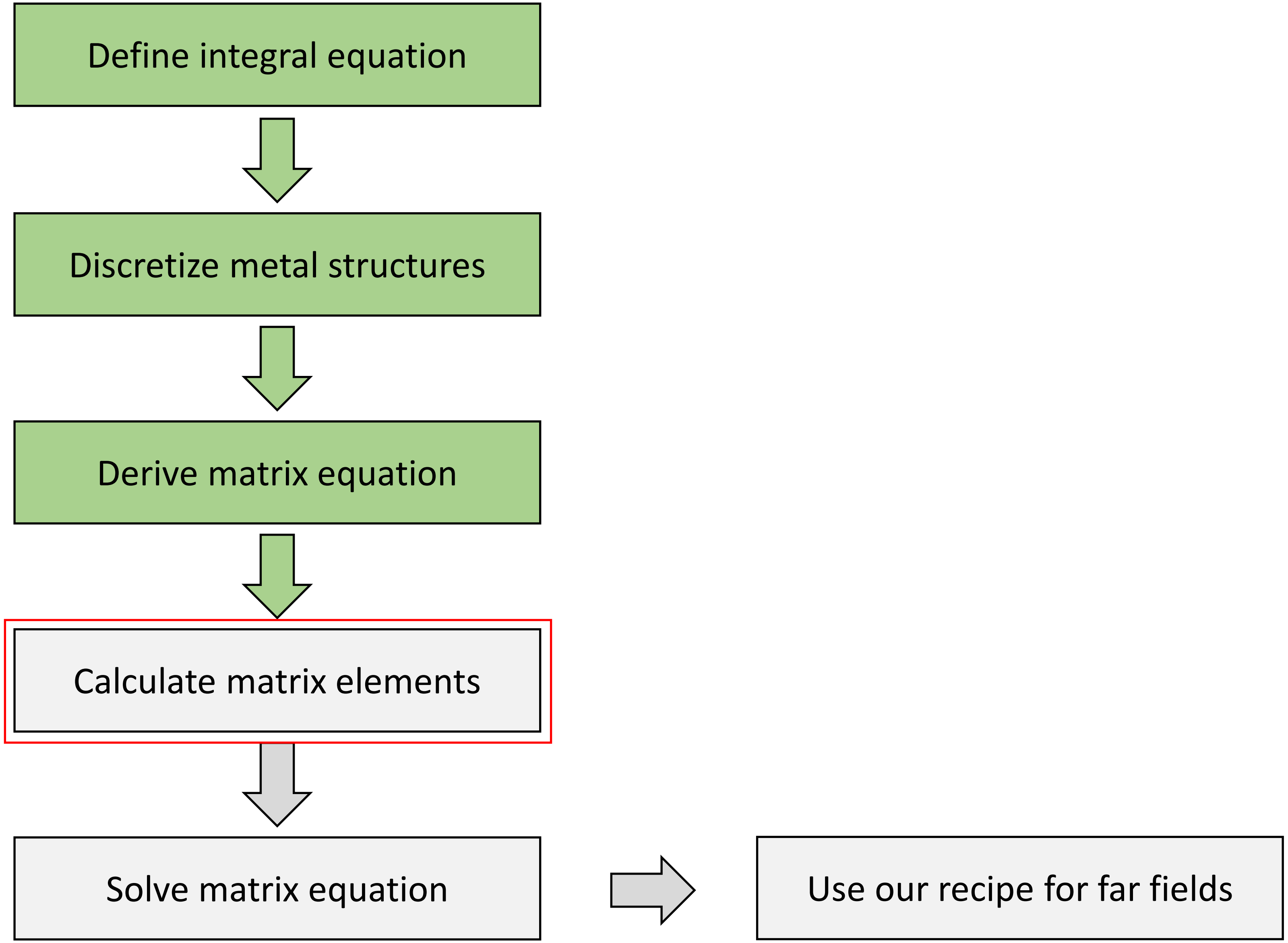
$N_{\max} \times N_{\max}$ matrix $\quad N_{\max}$ vector

Elements of the matrix and excitation vector:

$$Z_{mn} = \iint_{S_m} \vec{E}_n^s \cdot \vec{J}_m dS$$

$$V_m^{ex} = \iint_{S_m} \vec{E}^{ex} \cdot \vec{J}_m dS$$

Approach


Define integral equation
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Test case: wire antenna

Expansion functions on cylinder $\boxed{r = a}$

$$\vec{J}_n = \boxed{\vec{u}_z} \frac{1}{2\pi a} \boxed{J_{zn}(z)} \delta(r-a)$$

z-directed and z-dependent only

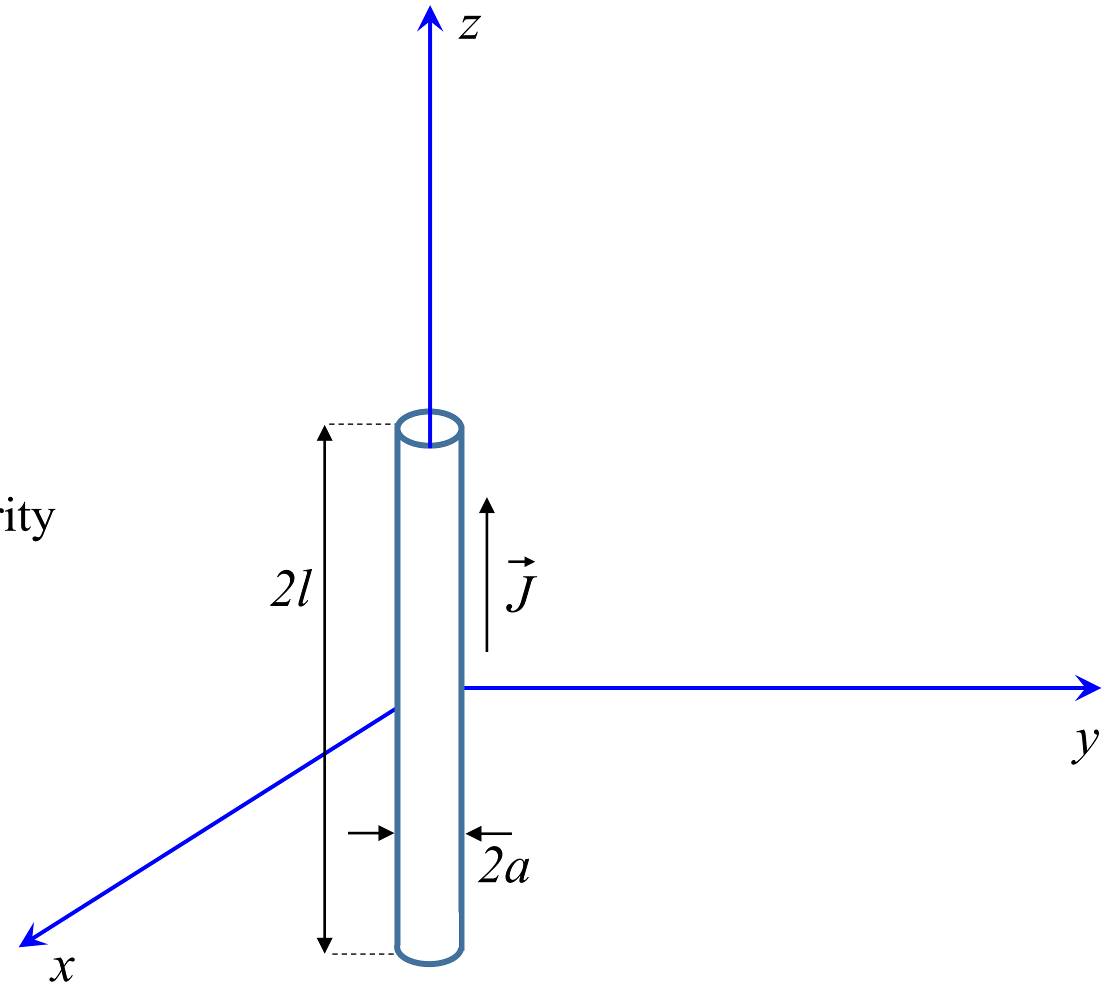
avoids singularity

Test functions on axis $\boxed{r = 0}$

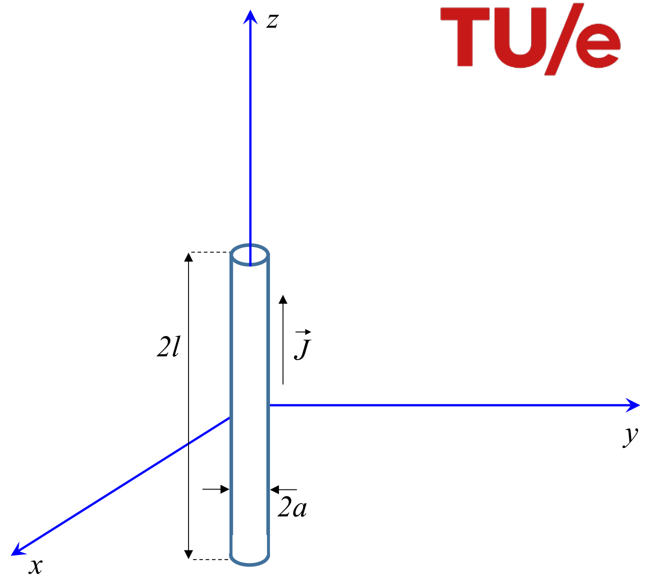
$$\vec{J}_m = \vec{u}_z J_{zm}(z) \delta(r)$$

PWC basis functions:

$$J_{zn}(z) = \begin{cases} 1 & z_{n-1} < z < z_{n+1} \\ 0 & \text{elsewhere} \end{cases}$$



Test case: wire antenna



Matrix elements

$$Z_{mn} = -j\omega\mu_0 \int_{z_{m-1}}^{z_{m+1}} \int_{z_{n-1}}^{z_{n+1}} \left(1 + \frac{1}{k_0^2}\frac{d^2}{dz^2}\right) \frac{e^{-jk_0R_a}}{4\pi R_a} dz_0 dz$$

Excitation vector

$$V_m^{ex} = \iint_{S_m} \vec{E}^{ex} \cdot \vec{J}_m dS$$

$$\vec{E}^{ex} = \boxed{V_g}\,\delta(z - \boxed{z_g})\vec{u}_z$$

generator voltage location of voltage generator

$$J_m(z) = \begin{cases} 1 & z_{m-1} < z < z_{m+1} \\ 0 & \text{elsewhere} \end{cases}$$

$$[V^{ex}] = \begin{pmatrix} 0 \\ .. \\ V_g \\ .. \\ 0 \end{pmatrix}$$

Approach

Define integral equation

↓

Discretize metal structures

↓

Derive matrix equation

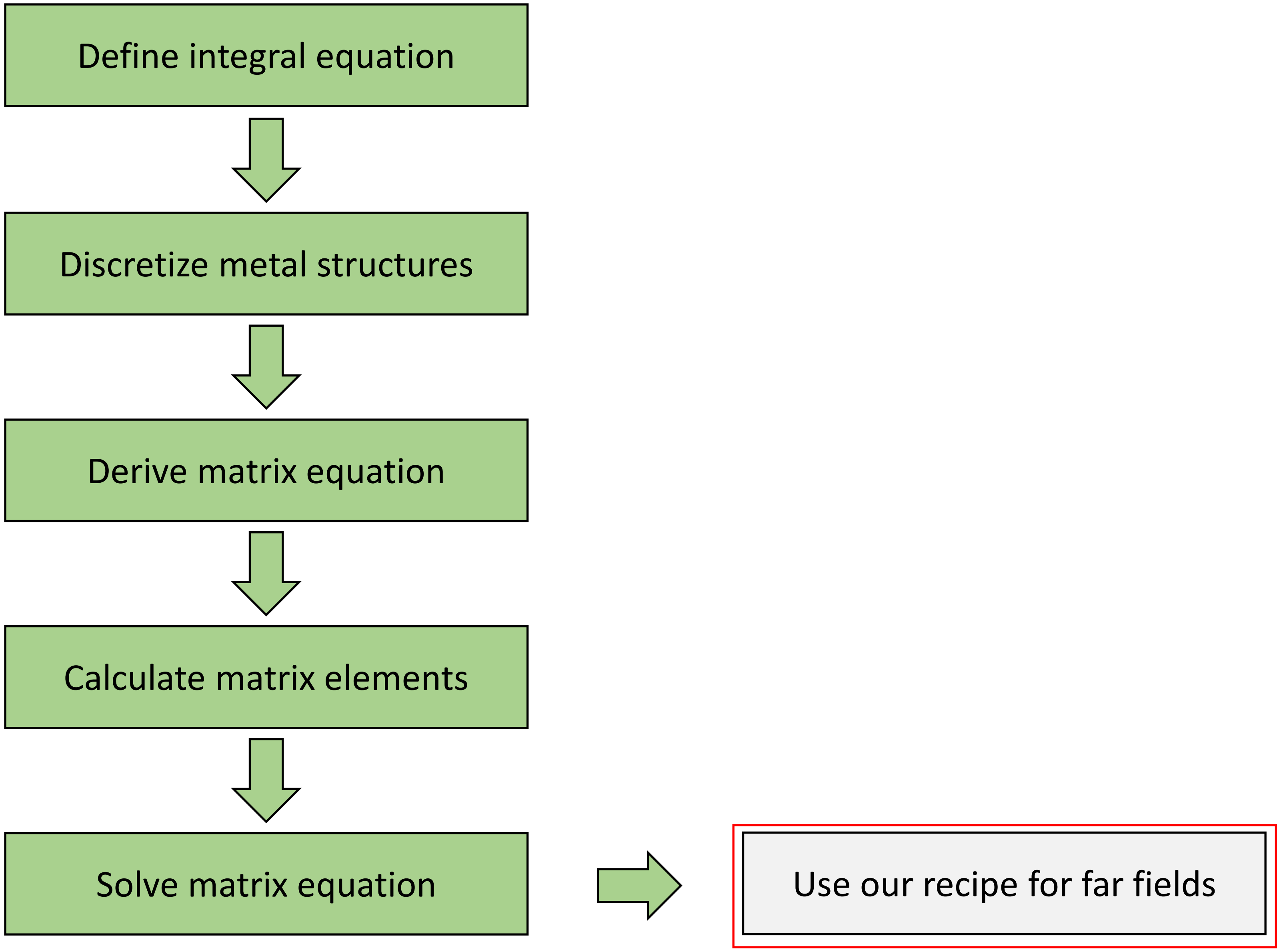
↓

Calculate matrix elements

↓

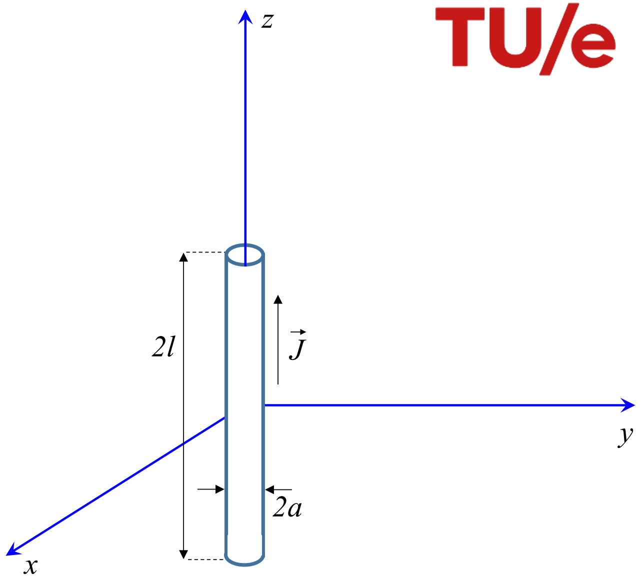
Solve matrix equation → Use our recipe for far fields

Approach



Define integral equation
Discretize metal structures
Derive matrix equation
Calculate matrix elements
Solve matrix equation
Use our recipe for far fields

Test case: wire antenna



Current distribution on wire

$$\vec{J}(z) = \sum_{n=1}^{N_{\max}} \boxed{I_n} \vec{J}_n(z)$$

Determined by solving matrix equation

Far field

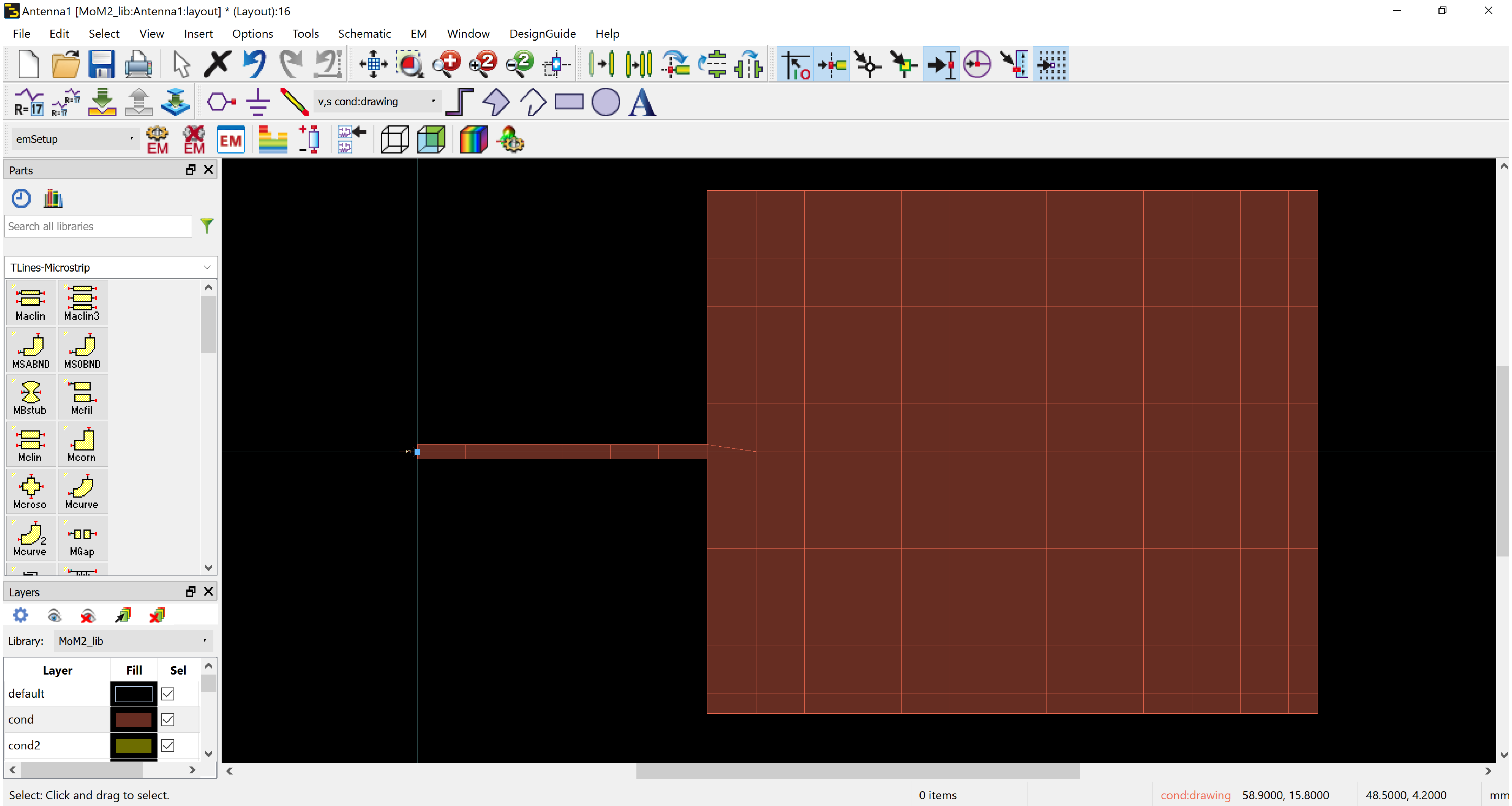
$$\vec{E}(\vec{r}) = \frac{-k_0^2}{j\omega\varepsilon_0} \frac{e^{-jk_0 r}}{4\pi r} \vec{u}_r \times \vec{u}_r \times \iiint_{V_0} \vec{J}(\vec{r}_0) e^{jk_0 \vec{u}_r \cdot \vec{r}_0} dV_0$$

Antenna impedance

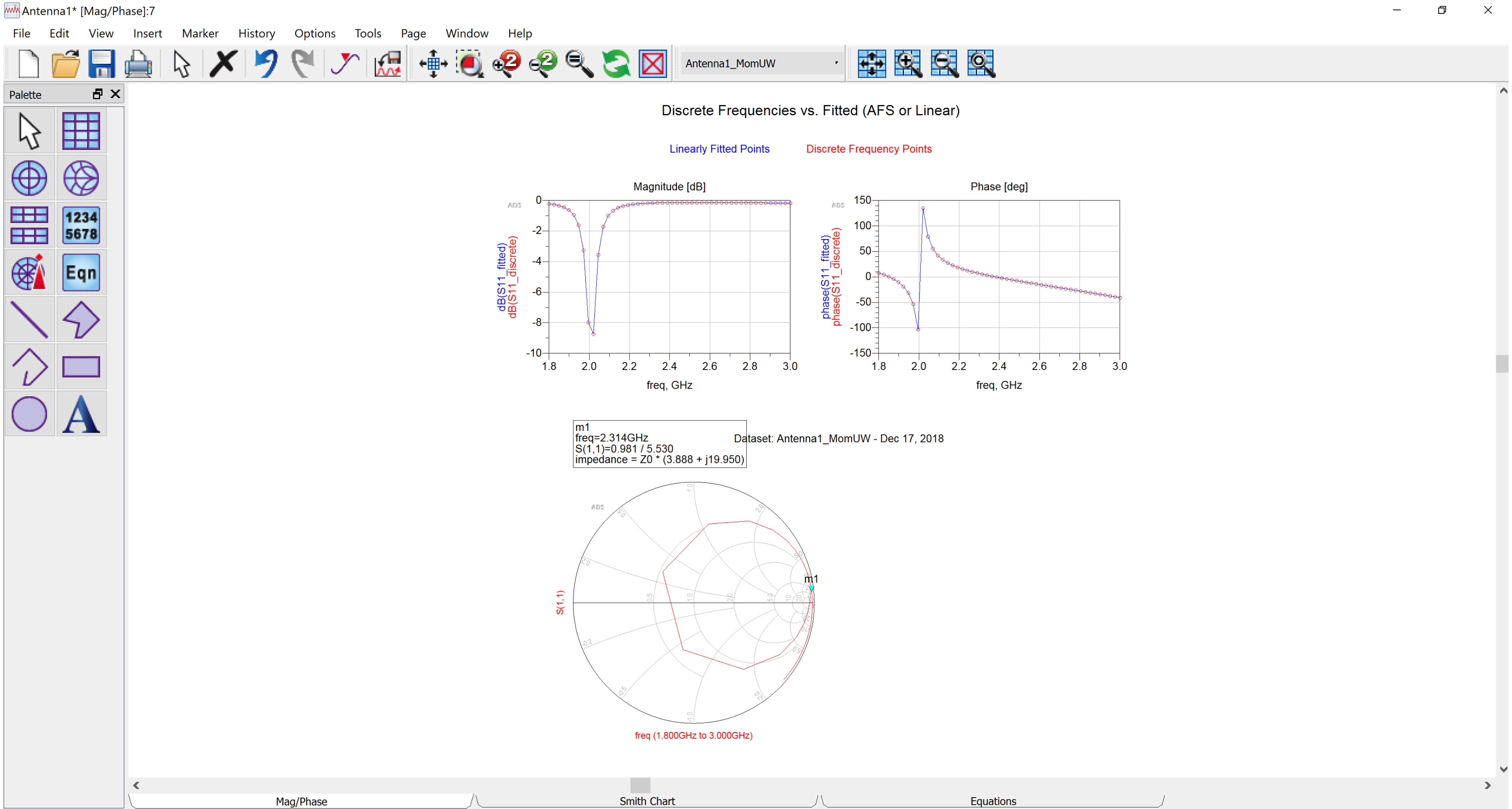
$$Z_a = \frac{V_g}{\boxed{I_i}}$$

with I_i the modecoeficient of segment i at $z = z_g$

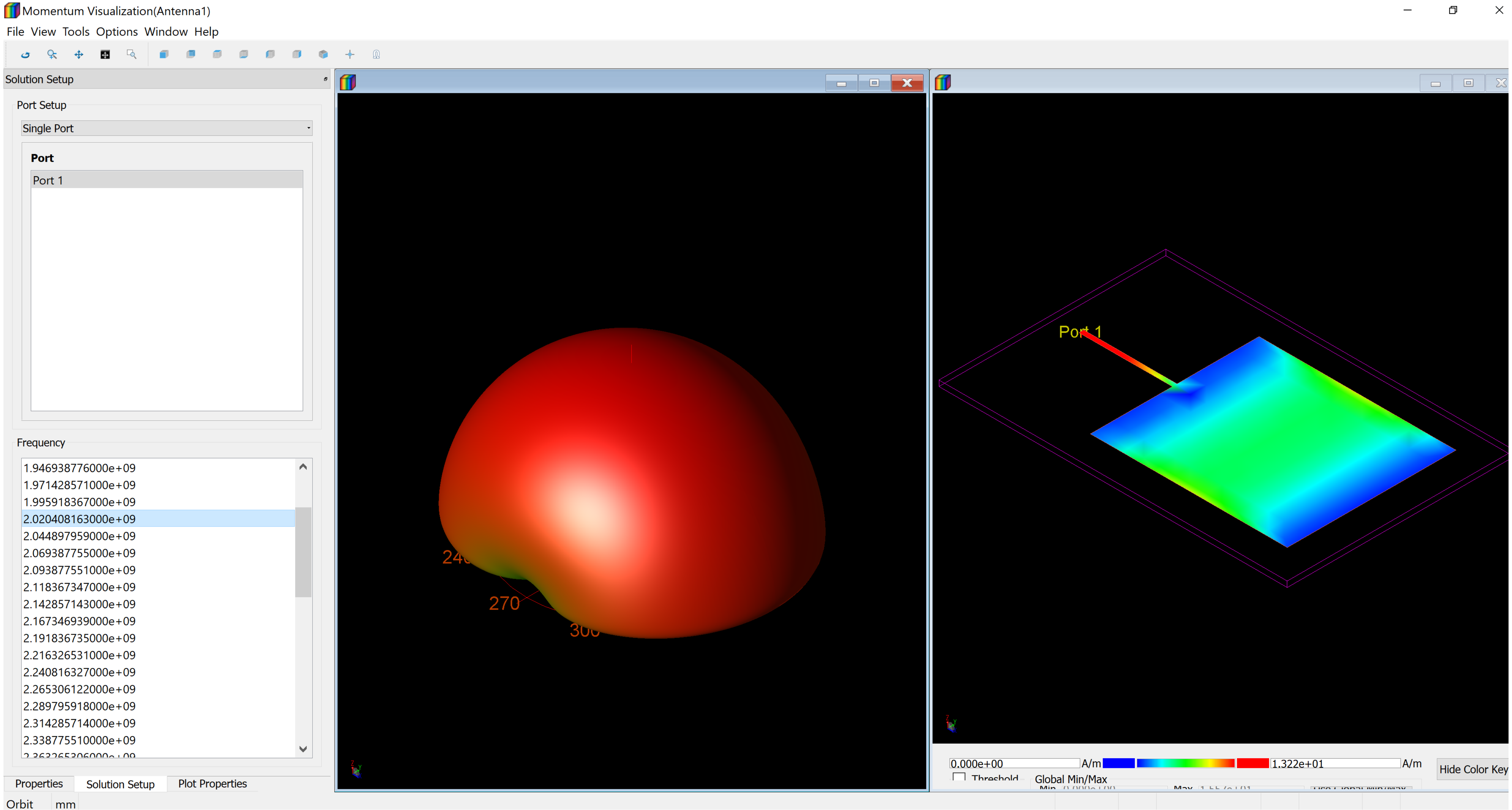
MoM in professional tool: ADS



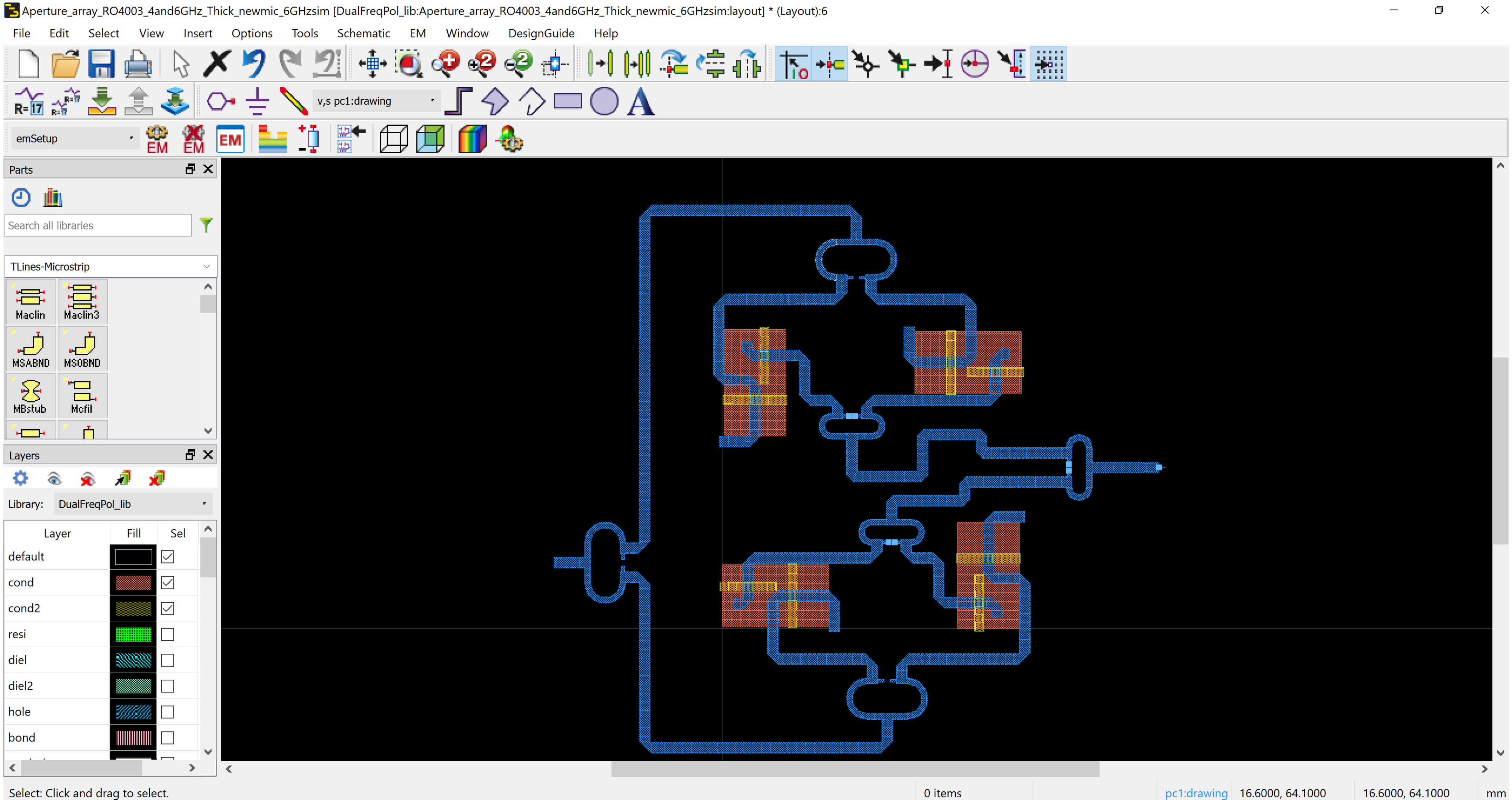
Input impedance



Far-field and current distribution



Antenna and microwave circuits co-simulation



Summary

- MoM can be used to calculate the current distribution on antennas.
- Once the current distribution is known, we can apply our far-field recipe.
- Example using professional tool ADS.

Microwave Engineering and Antennas

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Power Gain – Part I

The Concept of Power Transfer

Note: In these slide we have used the peak power in the definition of $P_{\mathrm{av,s}}$ and related power definitions.
In the book and quizzes we will use the time-average (rms) value with an additional factor ½.

Domine Leenaerts, Professor

Department of Electrical Engineering

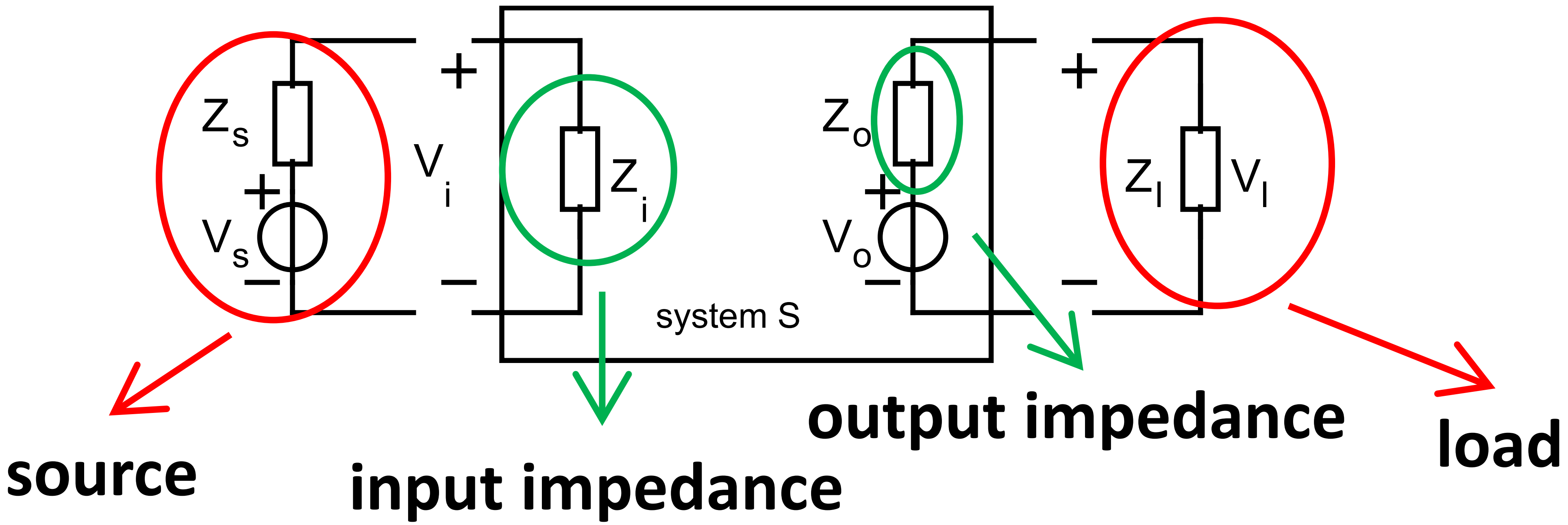
Center for Wireless Technologies Eindhoven

Power Gain:
The Concept of Power Transfer

Objective of this lecture

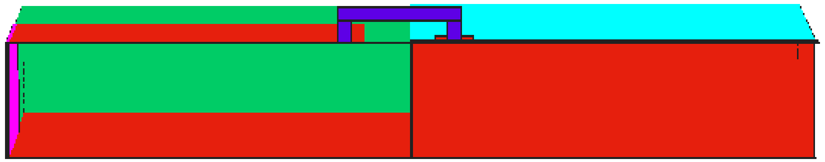
- Introduce power transfer in a system
- Difference between available power, delivered power and dissipated power
- Definition of Power Gain

A generic electronic system, e.g. an RF amplifier

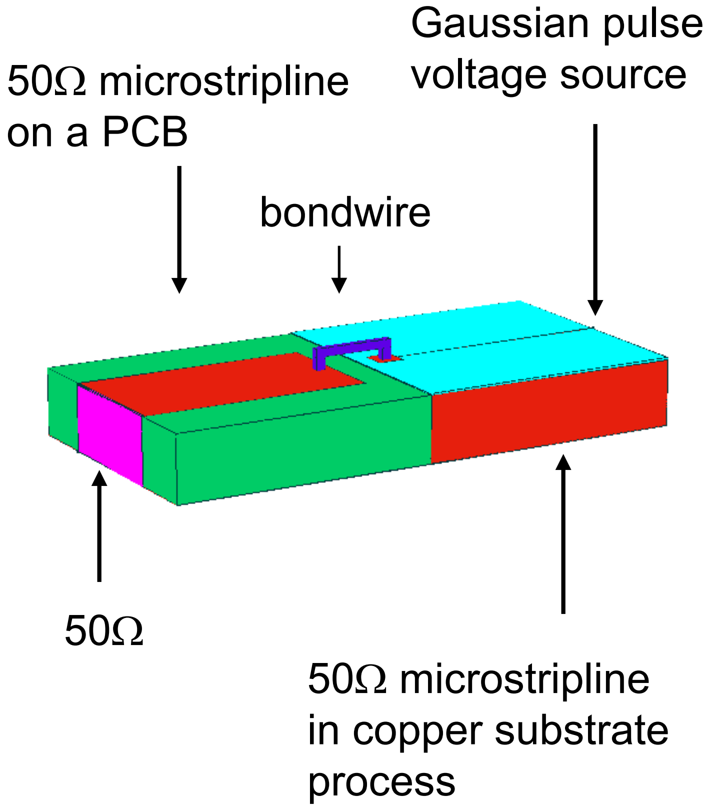


- Note: we do not discuss the inside of the system, i.e. black-box approach

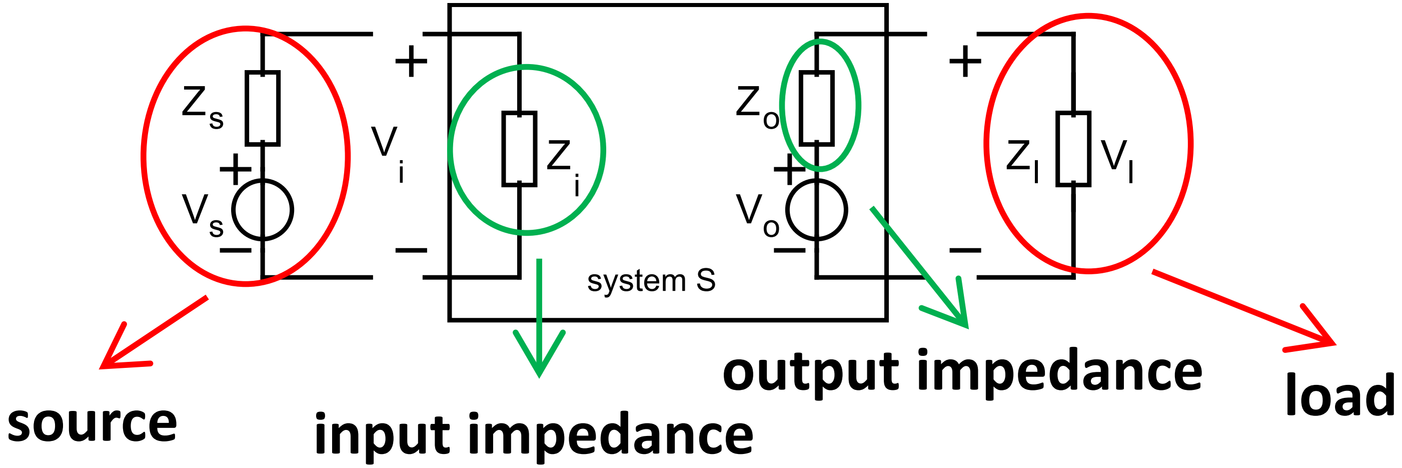
Power Transfer – the importance of matching



E-field strength in vertical plane (cross-section), Value of |E| in dB[V/m]



Available Power from a source: P_{av}

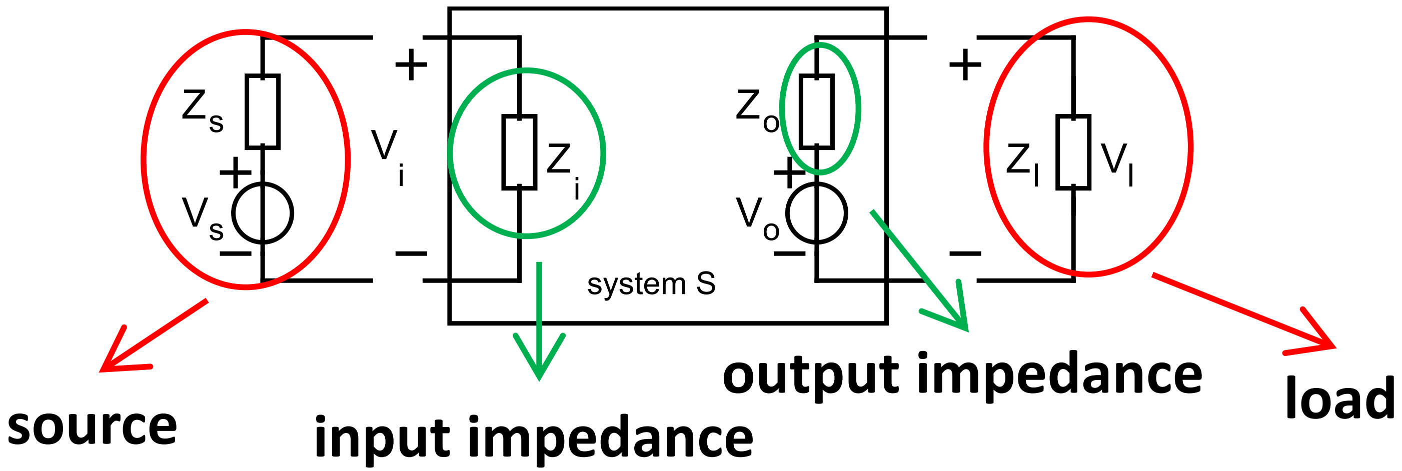


$$P_{av,s} = \frac{R_i}{(R_i + R_s)^2} {V_s}^2 \bigg|_{R_i = R_s} = \frac{{V_s}^2}{4R_s}$$

Available (input) power:

signal power that would be extracted from source V_s by a load conjugately matched to the output of the source

Power Delivered to a load: P_{del}

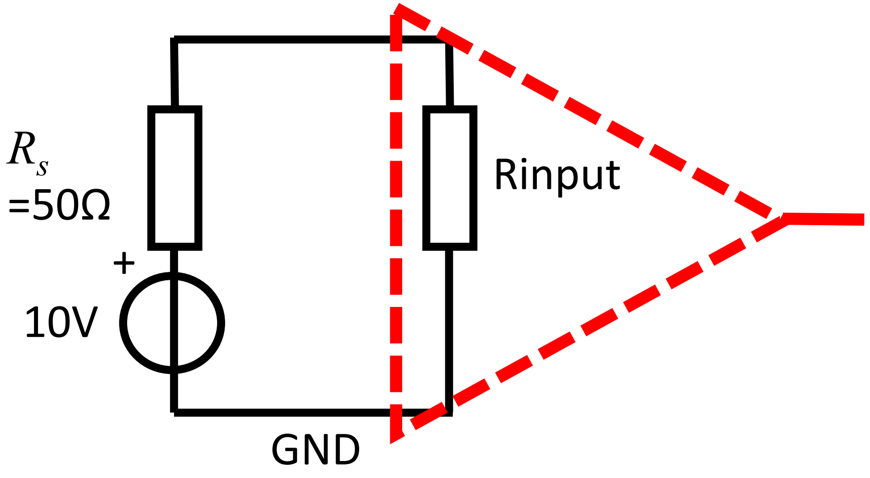


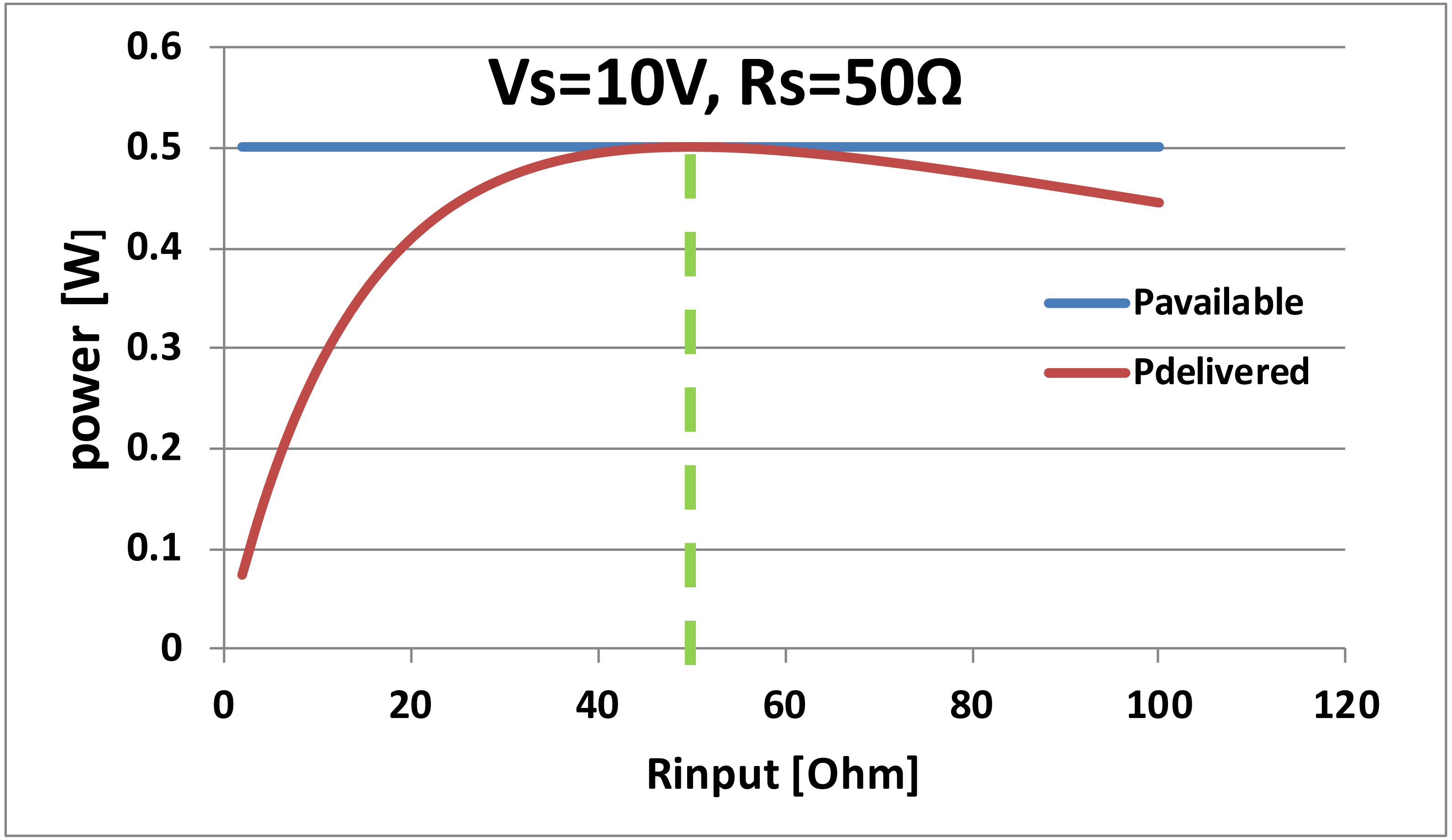
$$V_i = \frac{R_i}{R_i + R_s} V_s$$

$$P_{del,i} = \frac{R_i}{(R_i + R_s)^2} {V_s}^2$$

$$\boxed{P_{del,i} \leq P_{av,s}}$$

Example



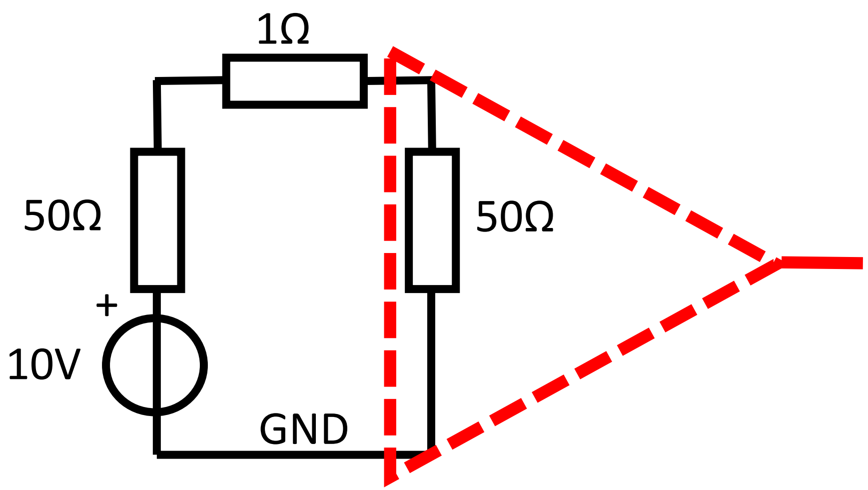


$$P_{av,s} = \frac{V_s{}^2}{4R_s} = \frac{100}{200} = 0.5$$

$$P_{del,i} = \frac{R_i}{(R_i + R_s)^2} V_s{}^2 = \frac{100R_i}{(R_i + 50)^2}$$

$R_{input} = R_s \rightarrow$ power match: $P_{av} = P_{del}$

Difference between power delivered and power dissipated



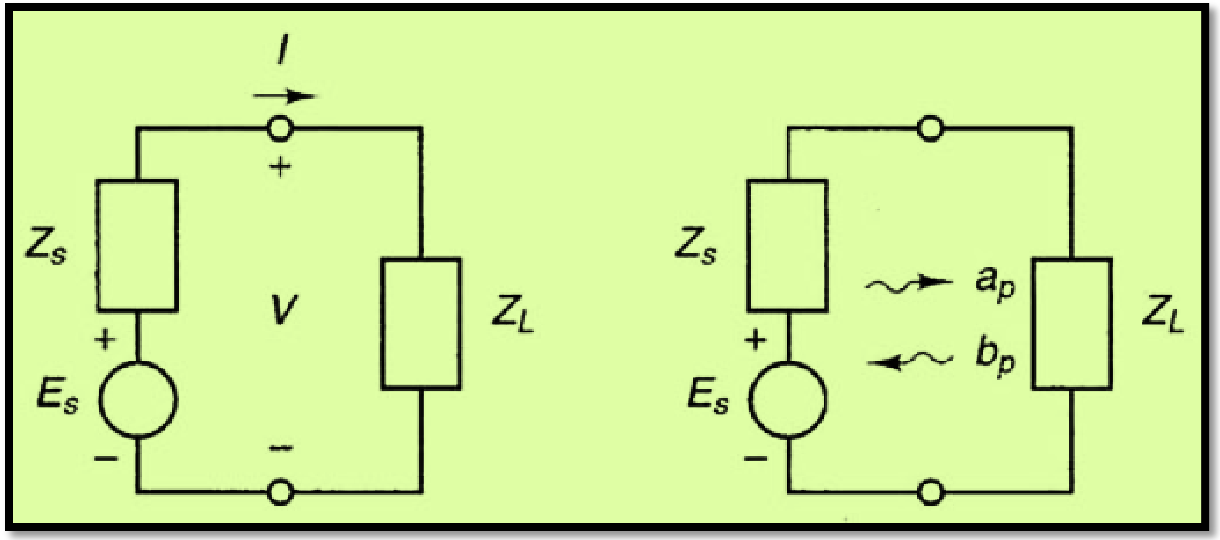
$$P_{av} = \frac{V^2}{4R_s} = \frac{100}{4 \cdot 50} = 0.5W$$

$$P_{del} = \frac{R_l \ V^2}{(R_s + R_l + R_{series})^2} = \frac{50 \cdot 100}{(50 + 1 + 50)^2} = 0.49W$$

$$P_{diss} = i^2 R_{series} = \left(\frac{10}{101}\right)^2 \cdot 1 = 0.01W$$

$$P_{av} = P_{del} + P_{diss}$$

Power Transfer using waves



$$v = \frac{V}{\sqrt{R_s}}$$

$$i = \sqrt{R_s} I$$

$$R_s = Re[Z_s]$$

$$a_p = \frac{1}{2\sqrt{R_s}}(V + Z_s I)$$

$$b_p = \frac{1}{2\sqrt{R_s}}(V - Z_s^{\,*}\ I)$$

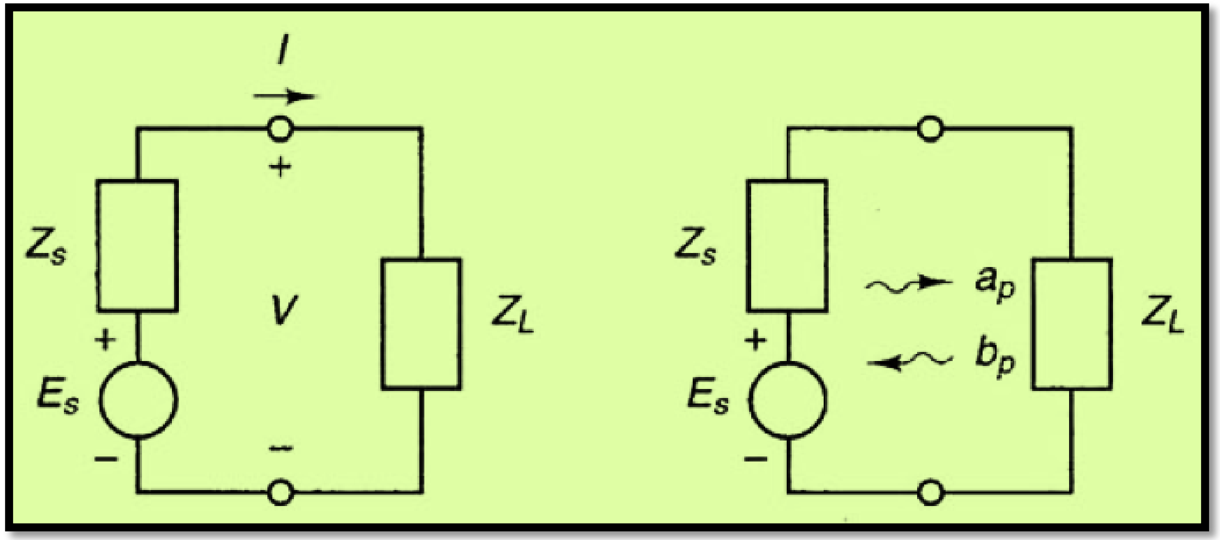
Incident wave

$$a_p = \frac{1}{2}(v + i)$$

Reflected wave

$$b_p = \frac{1}{2}(v - i)$$

Available Power using waves



$$a_p = \frac{1}{2\sqrt{R_S}}(V + Z_S I) = \frac{E_S}{2\sqrt{R_S}}$$

$$|a_p|^2 = \frac{|E_S|^2}{4R_s}$$

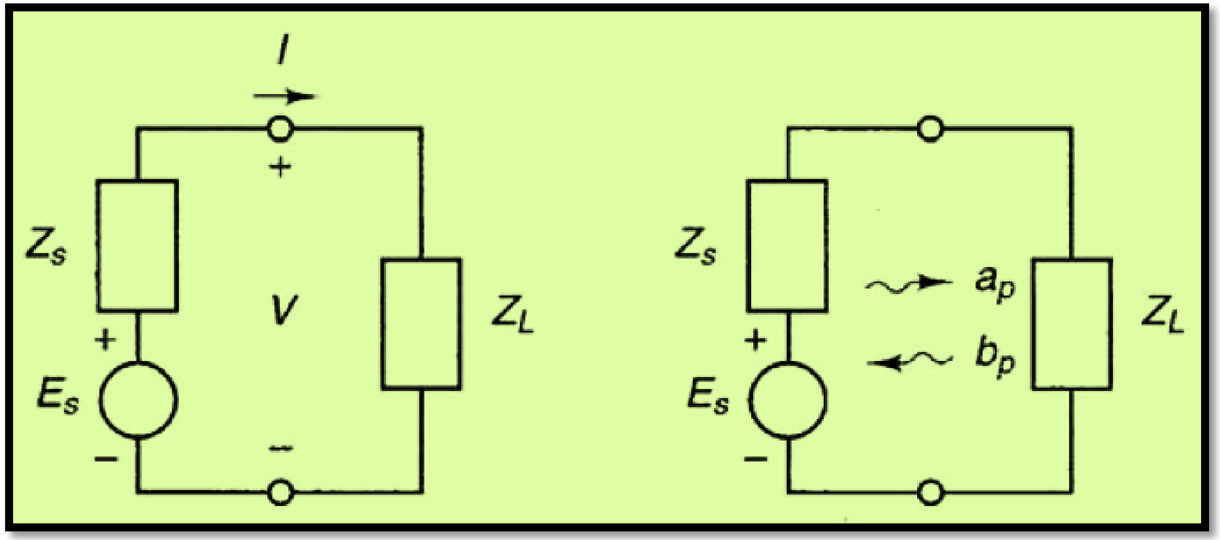
$$V = E_S - Z_S I$$

Available Power: $Z_L = Z_S$

(optimal power transfer)

$$P_{av,s} = |a_p|^2 = \frac{|E_s|^2}{4R_s}$$

Delivered Power using waves parameters



$$V = E_S - Z_s I$$

$$a_p = \frac{1}{2\sqrt{Z_s}}(V + Z_s I)$$

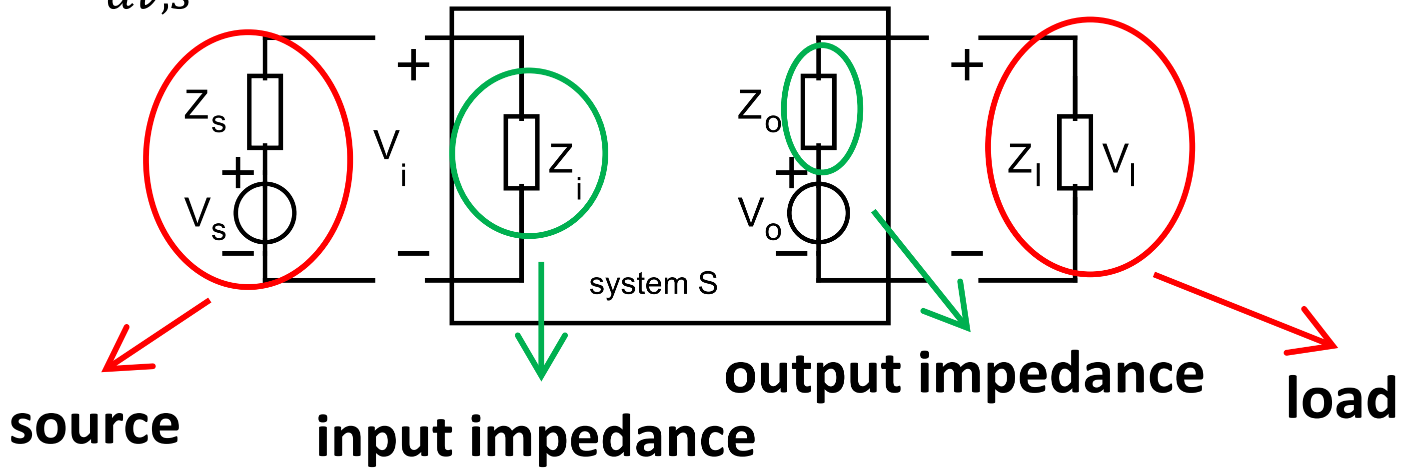
$$b_p = \frac{1}{2\sqrt{Z_s}}(V - Z_s I)$$

$$P_{del} = \left(\left|a_p\right|^2 - \left|b_p\right|^2\right)$$

$$P_{del} = \underbrace{\left|a_p\right|^2}_{P_{av,s}}\left(1 - \left|\Gamma_p\right|^2\right)$$

Four definitions of Power Gain

- Available (power) Gain $G_{av} = \frac{P_{av,o}}{P_{av,s}}$ — mismatch conditions at input
- Delivered (power) Gain $G_{del} = \frac{P_{del,l}}{P_{del,i}}$ — mismatch conditions at output
- Maximum (power) Gain $G_{max} = \frac{P_{av,o}}{P_{del,i}}$ — optimal power transfer
- Transducer (power) Gain $G_T = \frac{P_{del,l}}{P_{av,s}}$ — mismatch at input and output



Summary

- Transfer of power is influenced by impedance matching conditions
- Available power, delivered power and dissipated power
- Definition of power gain is based on transfer of power from source to load

Microwave Engineering and Antennas

Power Gain – Part II

Note: In these slide we have used the peak power in the definition of $P_{\mathrm{av,s}}$ and related power definitions.
In the book and quizzes we will use the time-average (rms) value with an additional factor ½.

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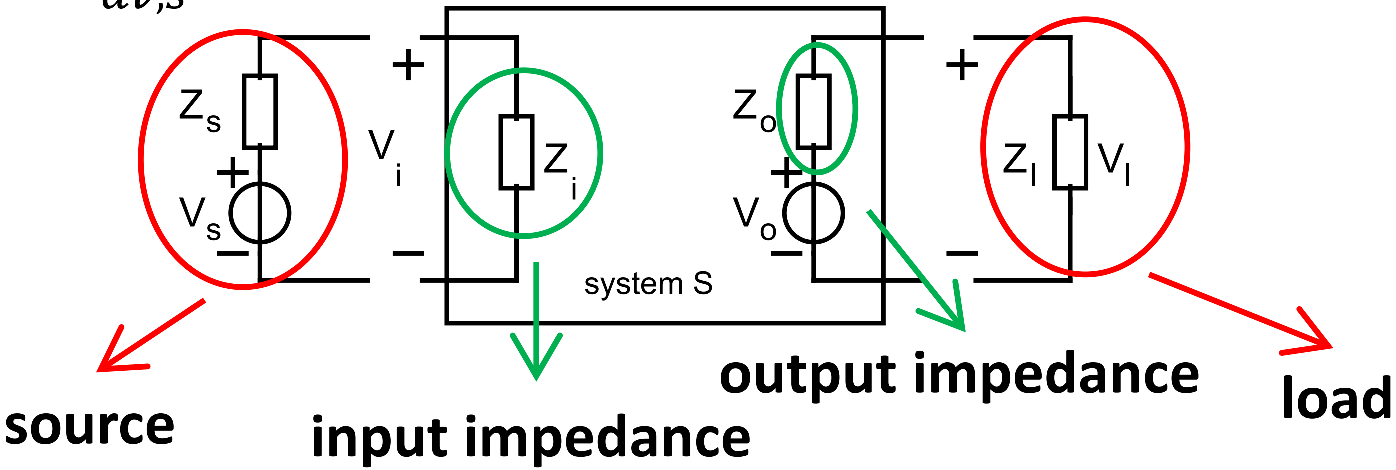
Power Gain – Part II

Objective of this lecture

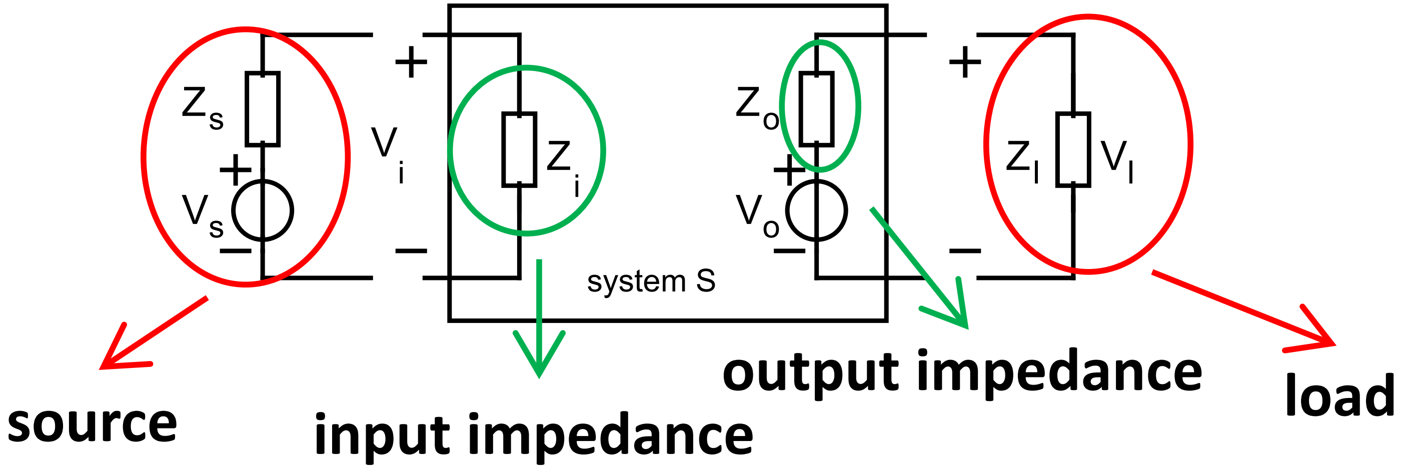
- Introduce the 4 definitions of power gain
- Provide an example to show the differences

Four definitions of Power Gain

- Available (power) Gain $G_{av} = \frac{P_{av,o}}{P_{av,s}}$ mismatch conditions at input
- Delivered (power) Gain $G_{del} = \frac{P_{del,l}}{P_{del,i}}$ mismatch conditions at output
- Maximum (power) Gain $G_{max} = \frac{P_{av,o}}{P_{del,i}}$ optimal power transfer
- Transducer (power) Gain $G_{T} = \frac{P_{del,l}}{P_{av,s}}$ mismatch at input and output



Available and Delivered (Power) Gain



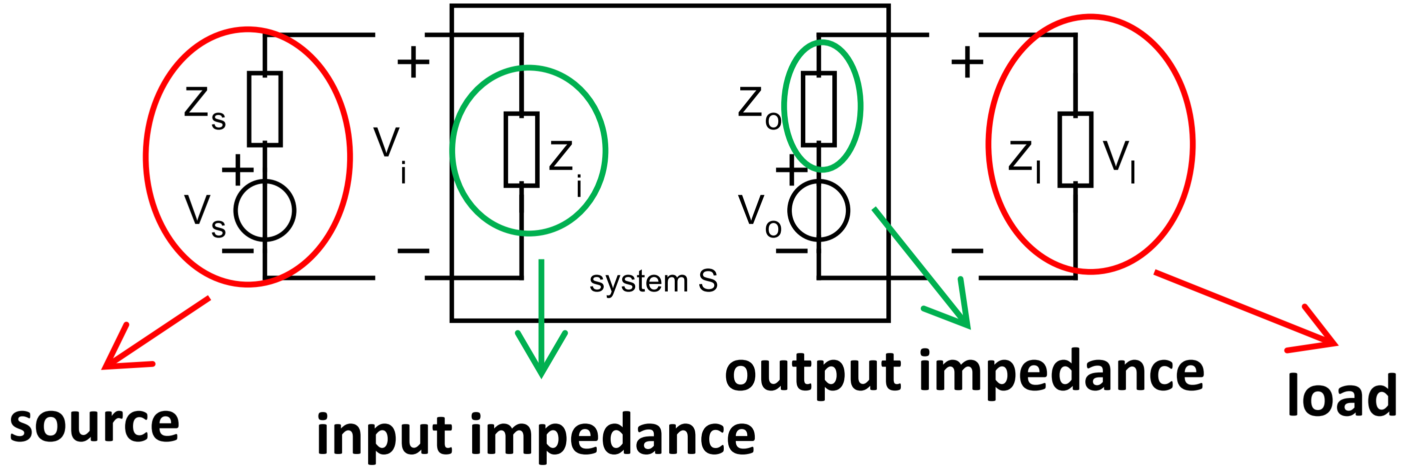
Z_o=0 Ω; Z_i = ∞ Ω:

$$A_v = \frac{V_o}{V_i} = \frac{V_l}{V_s}$$

$$G_{av} = \frac{P_{av,o}}{P_{av,s}} = \left(\frac{R_i}{(R_i + R_s)}\right)^2 \frac{{A_v}^2 R_s}{R_o}$$

$$G_{del} = \frac{P_{del,l}}{P_{del,i}} = {A_v}^2 \frac{R_i R_l}{(R_l + R_o)^2}$$

Maximum and Transducer (Power) Gain



$$A_v = \frac{V_o}{V_i} = \frac{V_l}{V_s}$$

$$G_{max} = \frac{P_{av,o}}{P_{del,i}} = A_v{}^2 \frac{R_i}{4R_o}$$

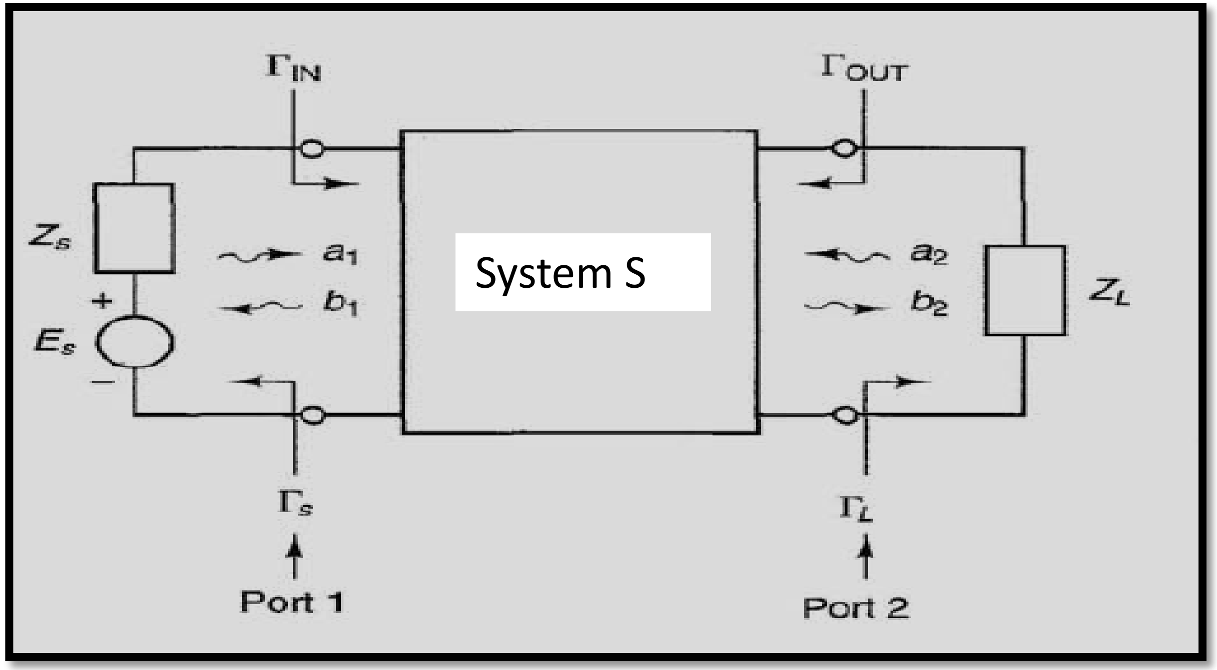
$$G_T = \frac{P_{del,l}}{P_{av,s}} = \left(\frac{R_i}{(R_i + R_s)}\right)^2 A_v{}^2 \frac{4R_l R_s}{(R_l + R_o)^2}$$

$$G_T \xrightarrow[R_i=R_s]{} G_{del}$$

$$G_T \xrightarrow[R_l=R_o]{} G_{av}$$

$$G_T \xrightarrow[R_l=R_o,\, R_s=R_i]{} G_{max}$$

Power Gain and scatter parameters: reflection

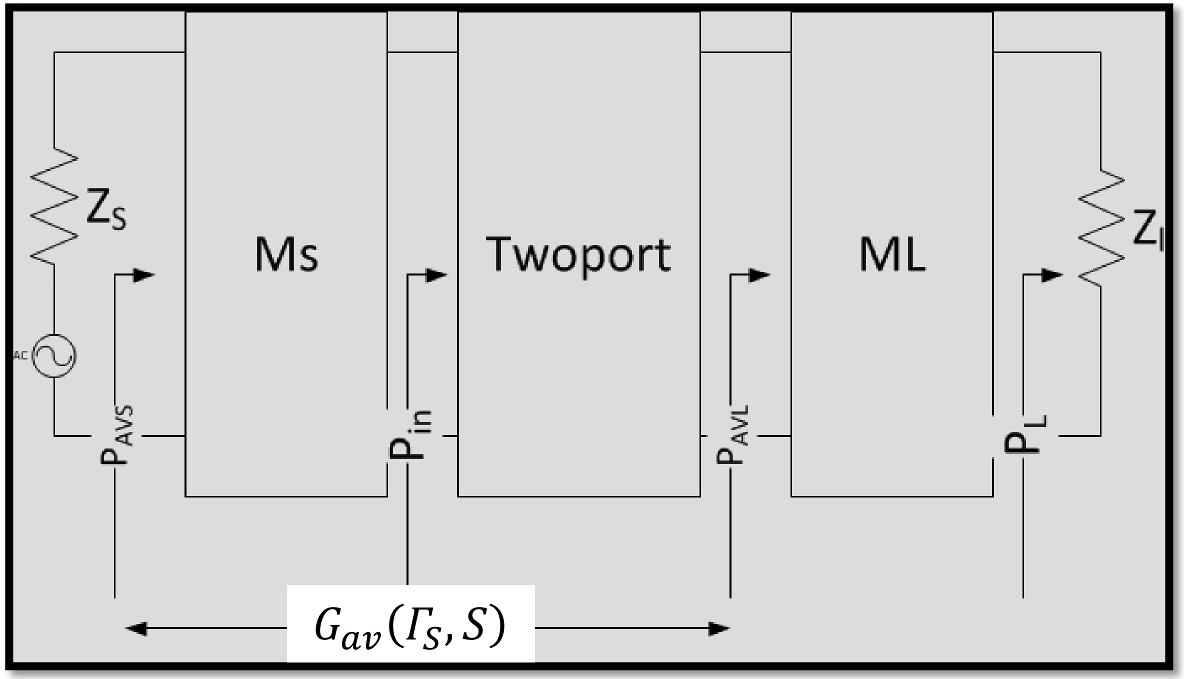


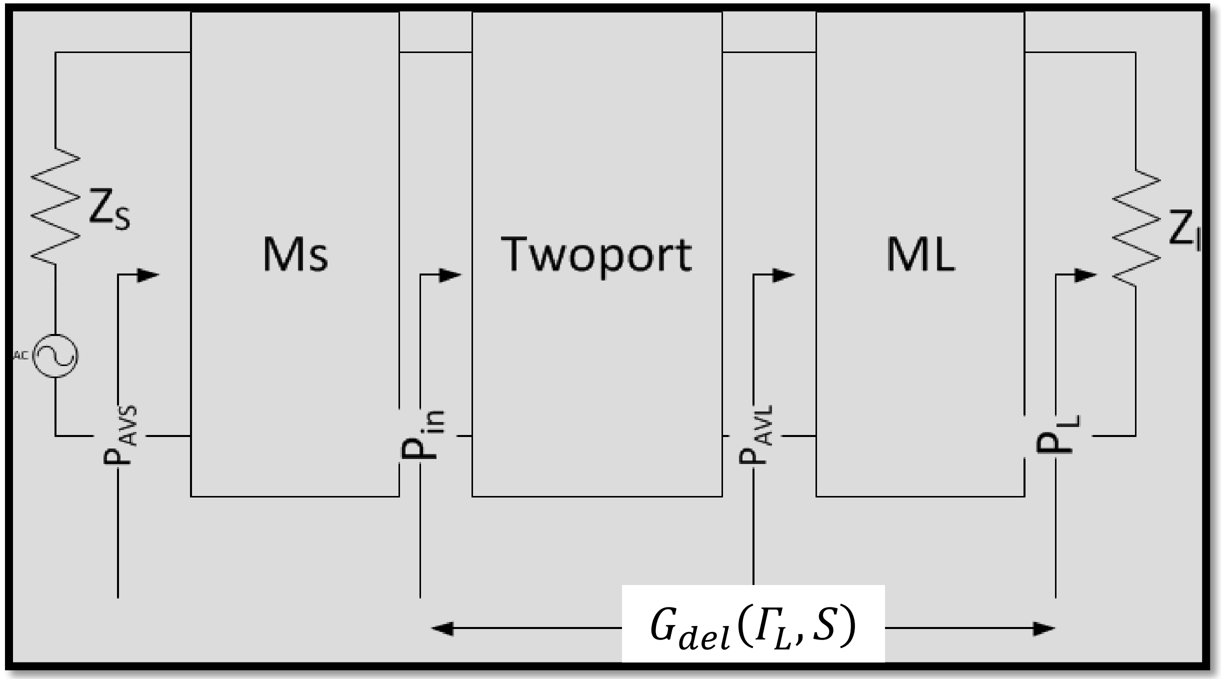
$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}}{\frac{1}{\Gamma_L} - S_{22}}$$

$$\Gamma_{out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12}S_{21}}{\frac{1}{\Gamma_S} - S_{11}}$$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Available, and delivered Power Gain

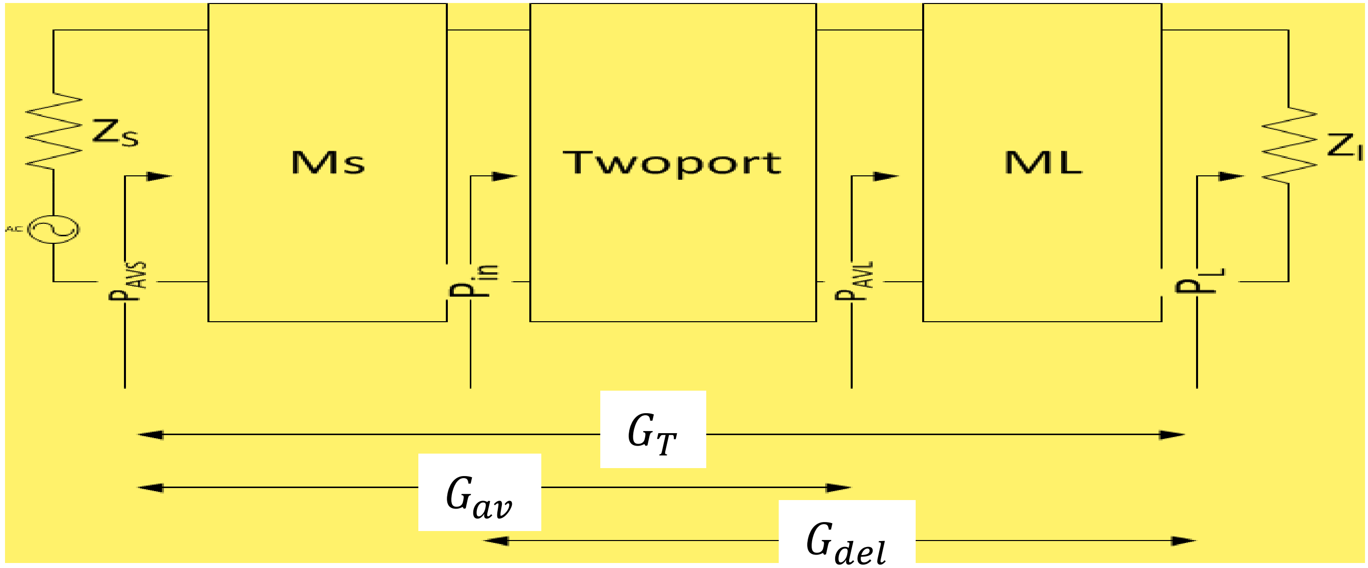




$$G_{av}(\Gamma_S, S) = \frac{P_{av,l}}{P_{av,s}} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2 \frac{1}{1-|\Gamma_{out}|^2}$$

$$G_{del}(\Gamma_L, S) = \frac{P_{del,l}}{P_{del,s}} = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}|S_{21}|^2 \frac{1}{1-|\Gamma_{in}|^2}$$

Relation between all power gains



$$G_T(\Gamma_S, \Gamma_L, S) = \frac{P_{del,l}}{P_{av,s}} = \frac{P_{del,s}}{P_{av,s}} \frac{P_{del,l}}{P_{del,s}}$$
$$= M_S G_{del}(\Gamma_L, S)$$

$$G_T(\Gamma_S, \Gamma_L, S) \leq G_{del}(\Gamma_L, S)$$

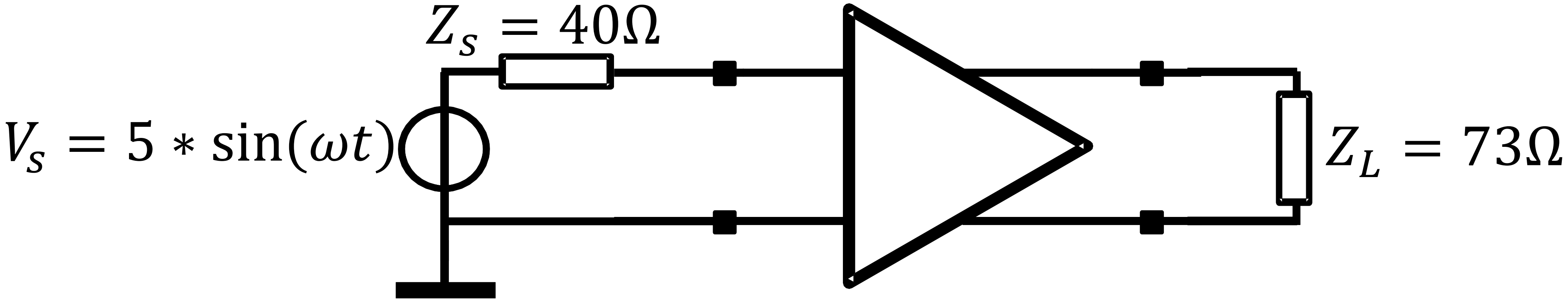
$$M_S = \frac{P_{del,in}}{P_{av,s}} = \frac{(1 - |\Gamma_{in}|^2)(1 - |\Gamma_S|^2)}{|1 - \Gamma_{in}\Gamma_S|^2}$$

$$G_T(\Gamma_S, \Gamma_L, S) = \frac{P_{del,l}}{P_{av,s}} = \frac{P_{av,l}}{P_{av,s}} \frac{P_{del,l}}{P_{av,l}}$$
$$= G_{av}(\Gamma_S, S) M_L$$

$$M_L = \frac{P_{del,l}}{P_{av,l}} = \frac{(1 - |\Gamma_{Out}|^2)(1 - |\Gamma_L|^2)}{|1 - \Gamma_{out}\Gamma_L|^2}$$

$$G_T(\Gamma_S, \Gamma_L, S) \leq G_{av}(\Gamma_S, S)$$

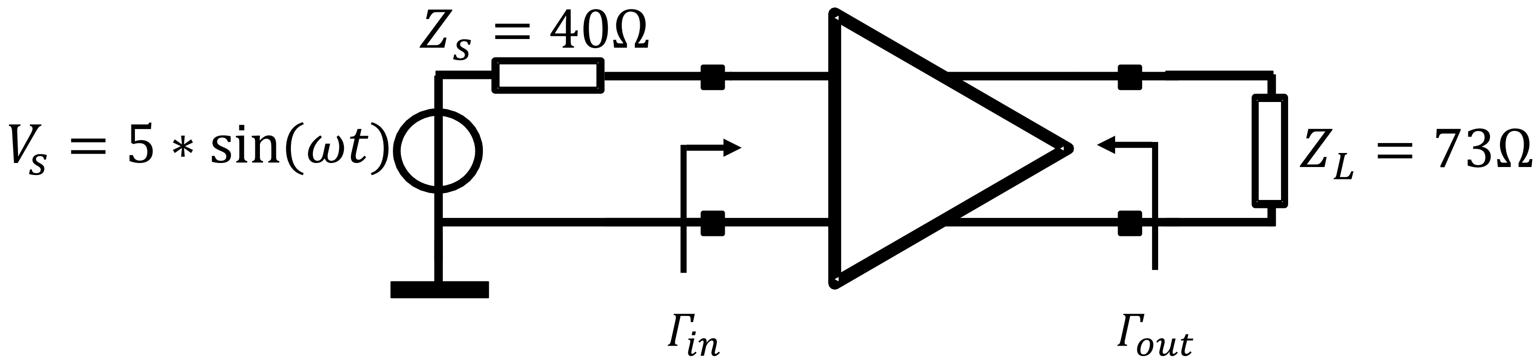
Example: an RF amplifier



$$s_{11} = 0.61, 165^o$$
$$s_{12} = 0.05, 42^o$$
$$s_{21} = 3.72, 59^o$$
$$s_{22} = 0.45, -48^o$$

$$Z_0 = 50\Omega$$

Example: an RF amplifier



$$\Gamma = \frac{Z - Z_0}{Z + Z_0}, then\ \Gamma_s = -0.111, \Gamma_L = 0.187$$

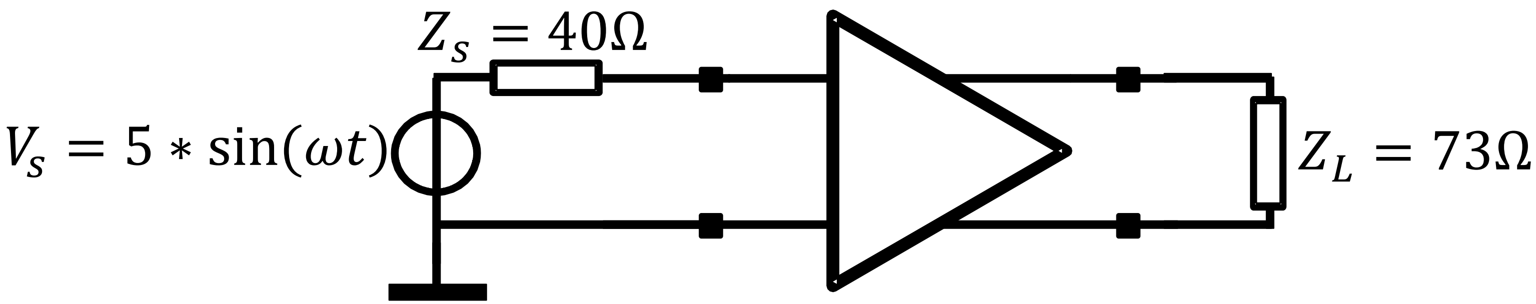
$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}}{\frac{1}{\Gamma_L} - S_{22}} = -0.594 + j0.194$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}}{\frac{1}{\Gamma_s} - S_{11}} = 0.305 - j0.356$$

$$Z_{in} = 11 + j7.54\ [\Omega]$$

$$Z_{out} = 63 - j58.4[\Omega]$$

Example: an RF amplifier



$$G_{max}(S) = 41.5 \; or \; 16.18dB$$

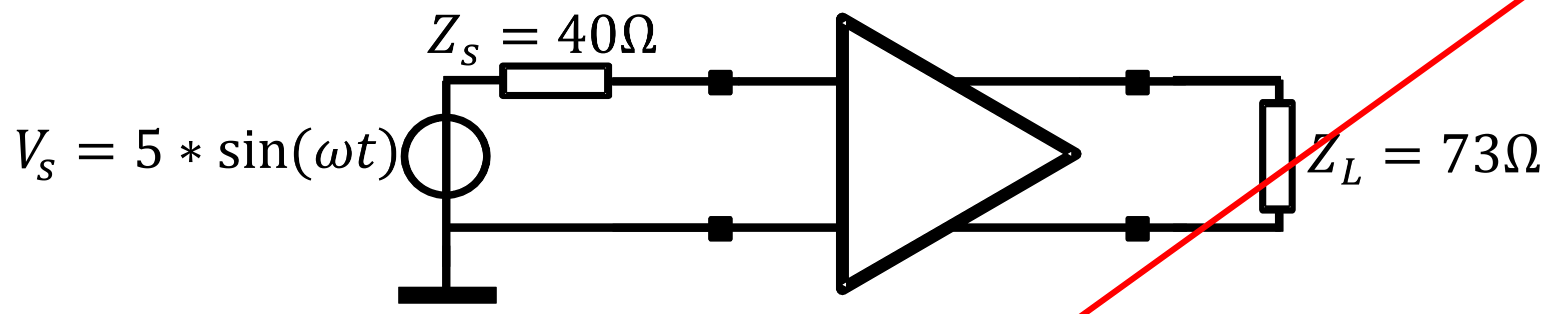
$$G_{av}(\Gamma_S, S) = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1}{1 - |\Gamma_{out}|^2} = 20.05 \; or \; 13.02dB$$

$$G_{del}(\Gamma_L, S) = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} |S_{21}|^2 \frac{1}{1 - |\Gamma_{in}|^2} = 24.49 \; or \; 13.89dB$$

$$G_T(\Gamma_S, \Gamma_L, S) = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} |S_{21}|^2 \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_{in}\Gamma_s|^2} = 16.89 \; or \; 12.27dB$$

Example: an RF amplifier

Note: The time average P_{av} = 78 mW or 18.93 dBm. Time average P_{del} = 31.2 dBm



- The available power $P_{av} = \frac{|V_S|^2}{4R_S}$ = 156 mW or P_{av} = 21.93 dBm.
- The power delivered to the load is the available power multiplied by the transducer gain. This results in $P_{del} = P_{av}G_T$ = 2.63W or expressed in dBm,

$$P_{del}(dBm) = G_T(\Gamma_S, \Gamma_L, S)P_{av}(dBm) = 21.93 + 12.27 = 34.2dBm$$

Summary

- 4 definitions of power gain, depending on the impedance matching conditions.
- Matching networks at between source and input (M_s) and output and load (M_L) influences the power transfer from source to load
- Transducer (power) gain is equal to or smaller than the available (power) gain or delivered (power) gain

Microwave Engineering and Antennas

Noise – Part I
The Concept of noise figure

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Noise – Part I
The Concept of noise figure

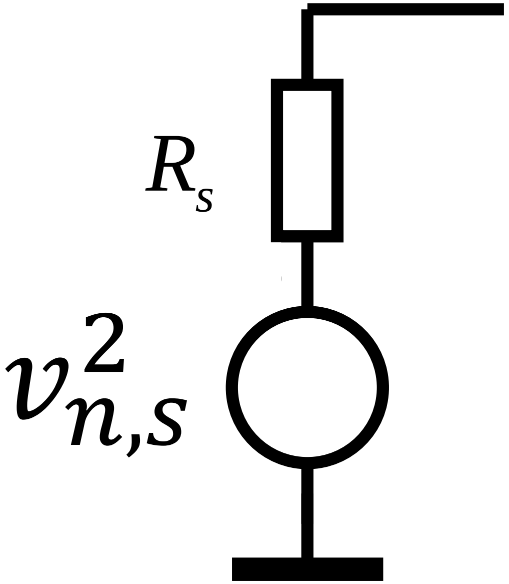
Objective of this lecture

- Discuss available noise power
- Explain the concept of noise figure
- Discuss the formula of Friis

Available noise power

$$\overline{v_{noise,R_s}}^2 = 4 \cdot k \cdot T_o \cdot R_s \cdot B \quad [V^2]$$

$$T_o = 290^o K \quad k = 1.38 \cdot 10^{-23} J/K$$

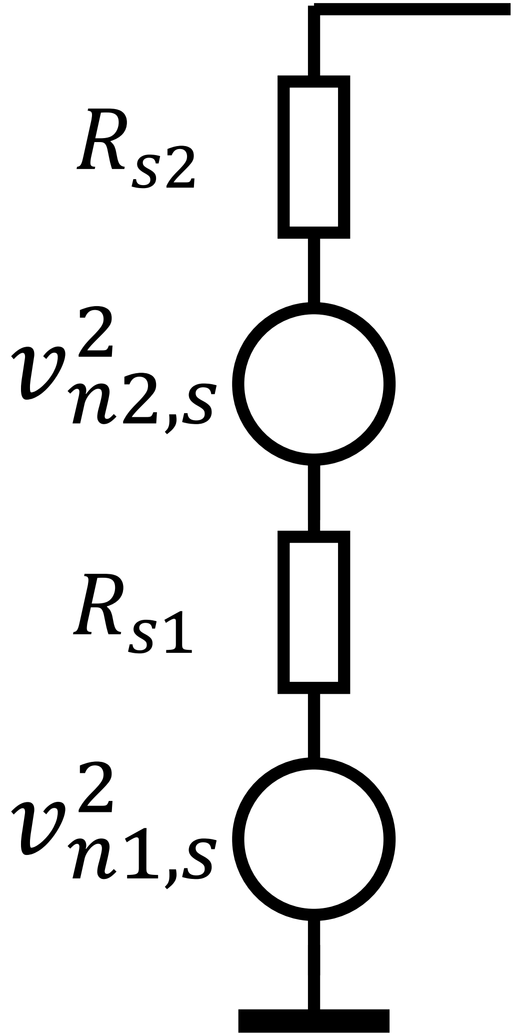


$$N_{av} = \frac{\overline{v_{noise,R_s}}^2}{4 \cdot R_s} = \frac{4 \cdot k \cdot T_o \cdot R_s \cdot B}{4 \cdot R_s} = k \cdot T_o \cdot B \quad [W]$$

At room temperature: $N_{av} = -174\ [dBm/Hz]$

… and independent of the impedance!

Network of noise sources



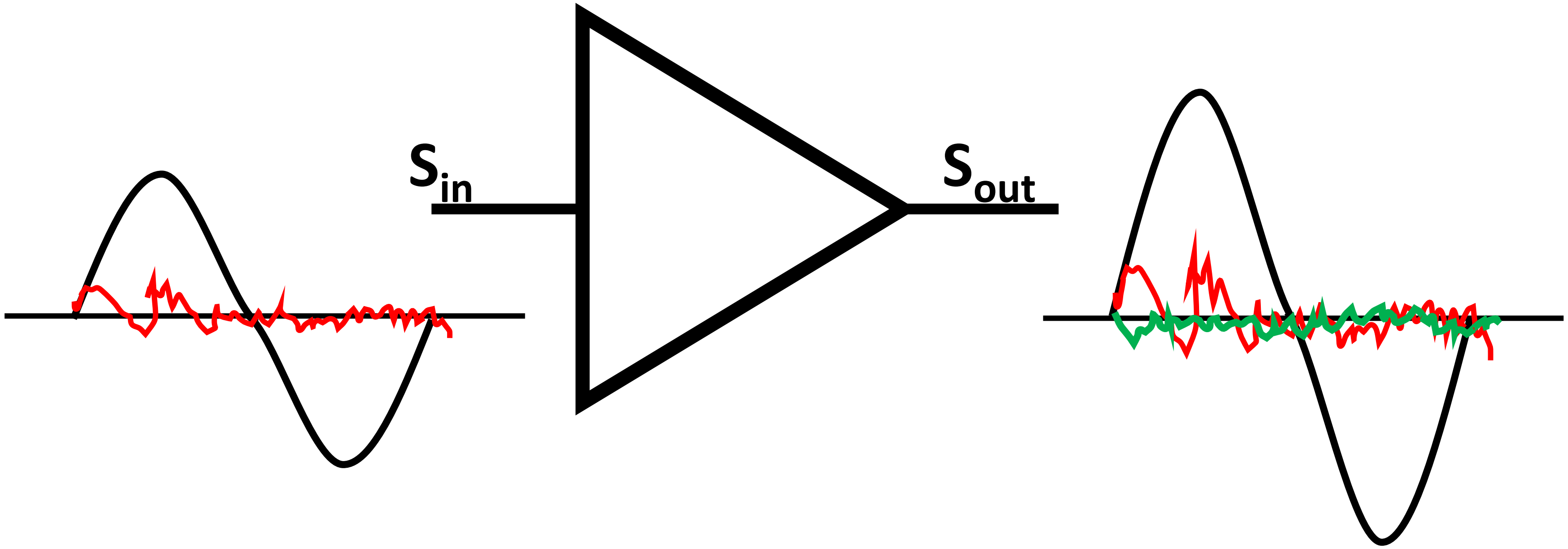
$$\overline{v_{n,s}}^2 = \overline{v_{n1,s}}^2 + \overline{v_{n2,s}}^2$$

$$\overline{v_{n,s}}^2 = 4kT_oB(R_1 + R_2)$$

$$N_{av} = \frac{4kTB(R_1 + R_2)}{4 \cdot (R_1 + R_2)} = kT_oB$$

Similar result if sources are parallel to each other

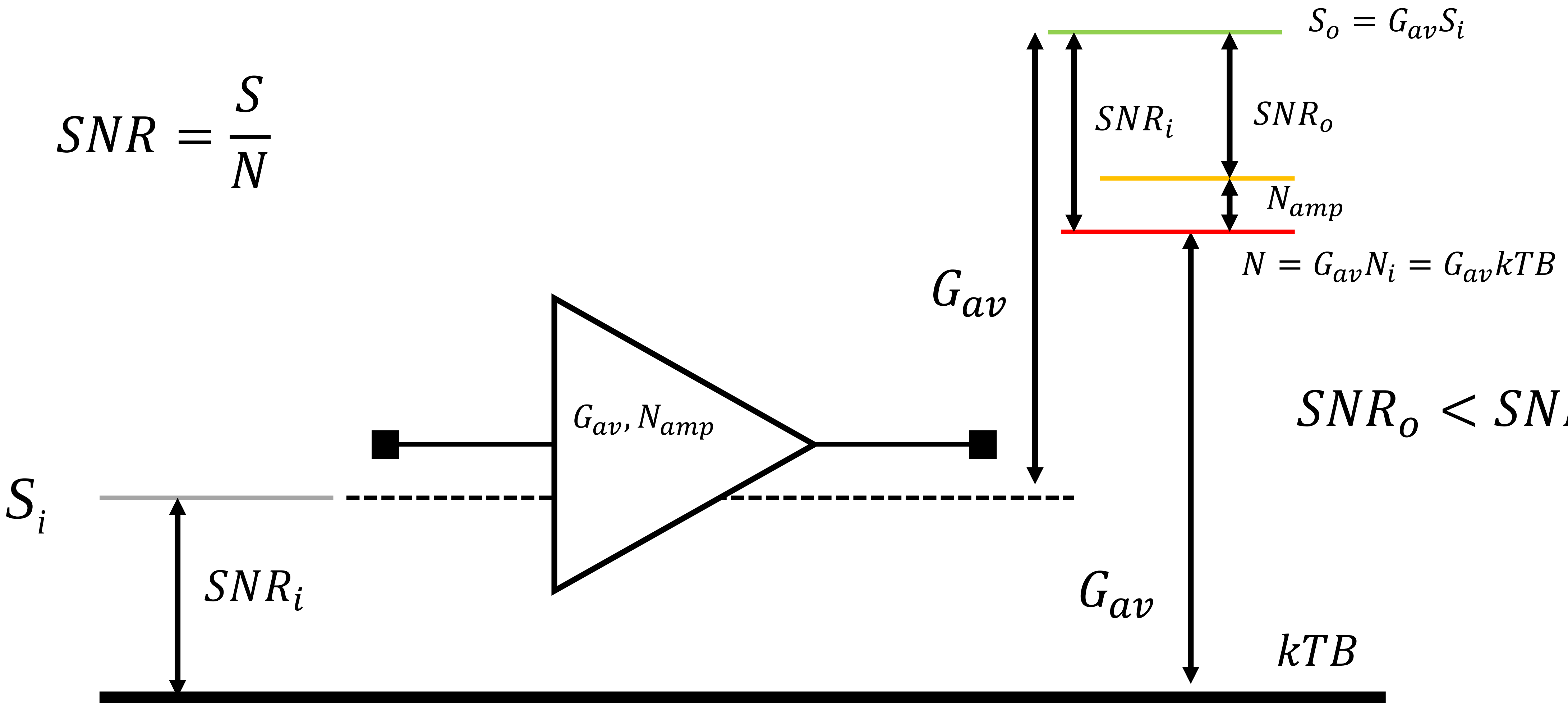
An amplifier amplifies all signals present at input



$$S_{out} = G_{av} \cdot (S_{in} + N_{in}) + N_{amp}$$

Noise of amplifier degrades signal-to-noise ratio

$$SNR = \frac{S}{N}$$



$$SNR_o < SNR_i$$

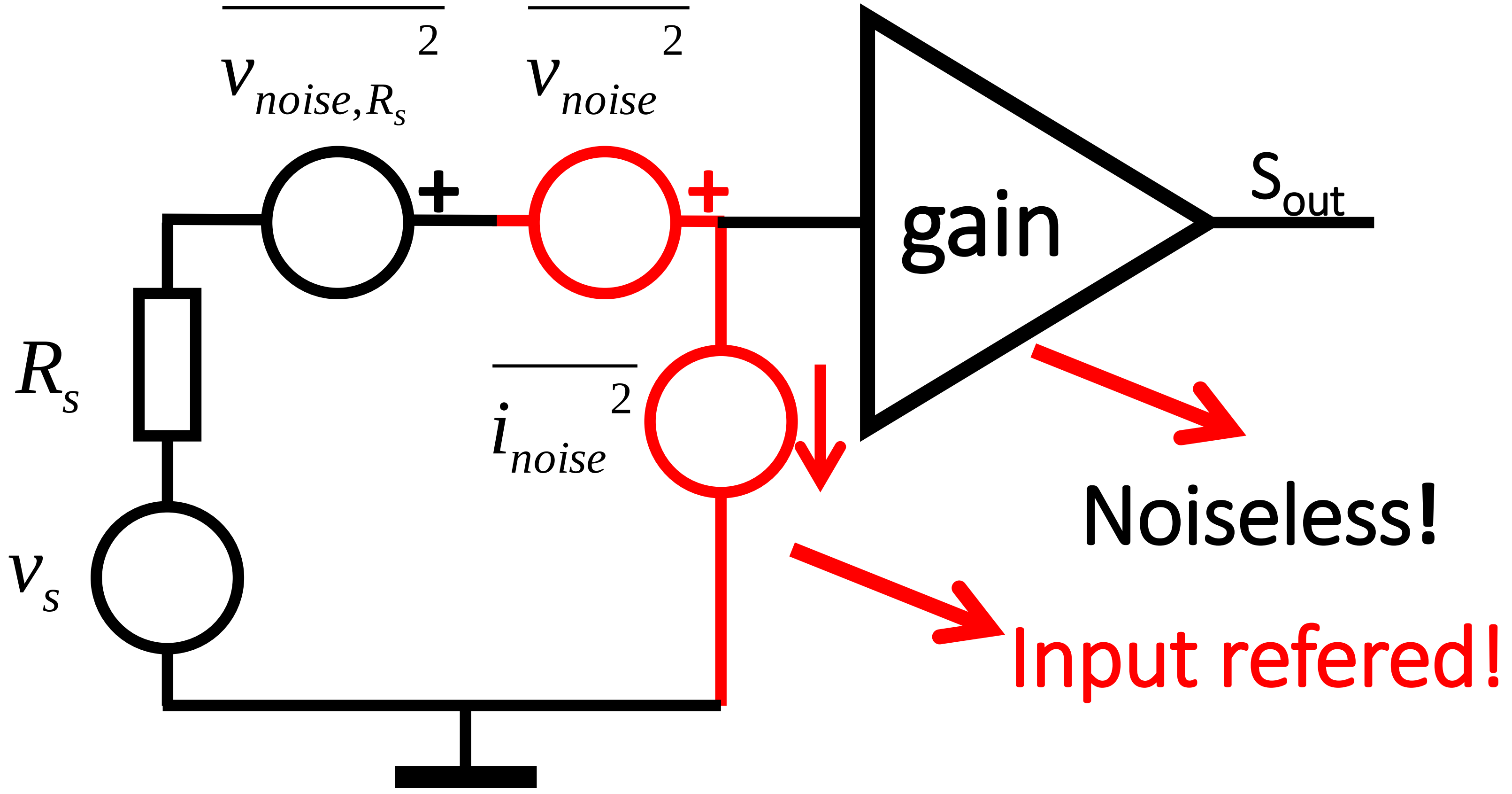
Noise Factor, Noise Figure

- SNR degradation by noise is expressed by the noise factor F

$$F = \frac{\left(\frac{S}{N}\right)_{in}}{\left(\frac{S}{N}\right)_{out}} = \frac{\frac{S_{in}}{N_{in}}}{\frac{G_{av}S_{in}}{N_{out}}} = \frac{N_{out}}{G_{av} \cdot N_{in}} = 1 + \frac{N_{amp}}{G_{av} \cdot kTB}$$

- The noise figure is simple the $10\log_{10}$ of the noise factor
- One question remains: why the available power gain G_{av}?

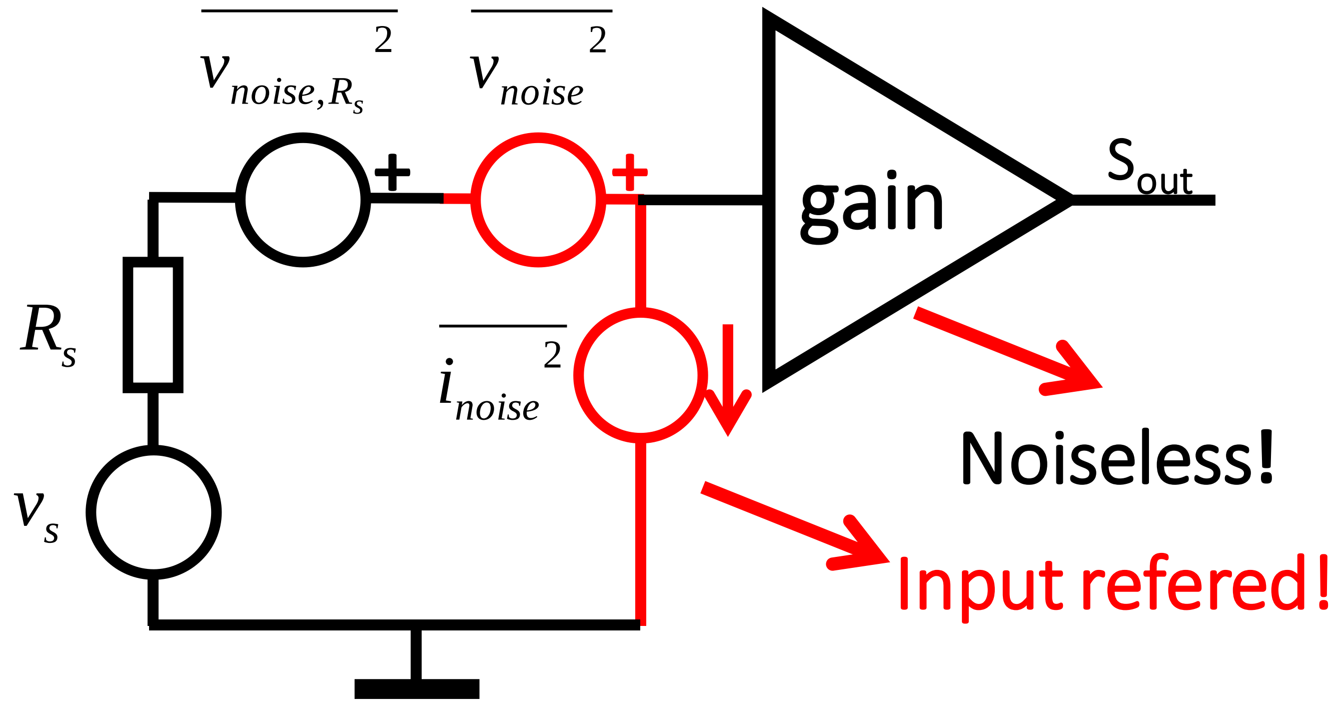
Concept of input referred noise



$$\overline{v_{noise,R_s}}^2 = 4 \cdot k \cdot T_o \cdot R_s \cdot B \quad [V^2]$$

$$T_o = 290^o K \quad k = 1.38 \cdot 10^{-23} J/K$$

Signal-to-noise ratio at input and output



$$SNR_{input} = \frac{v_s^2}{\overline{v_{noise,R_s}}^2}$$

$$SNR_{output} = \frac{G_{av} \cdot (v_s^2/4R_s)}{N_{out}} = \frac{G_{av} \cdot (v_s^2/4R_s)}{G_{av} \cdot \left(\overline{v_{noise,R_s}}^2 + \overline{[v_{noise} + i_{noise} \cdot R_s]^2}\right)/4R_s}$$

Hence the noise factor becomes:

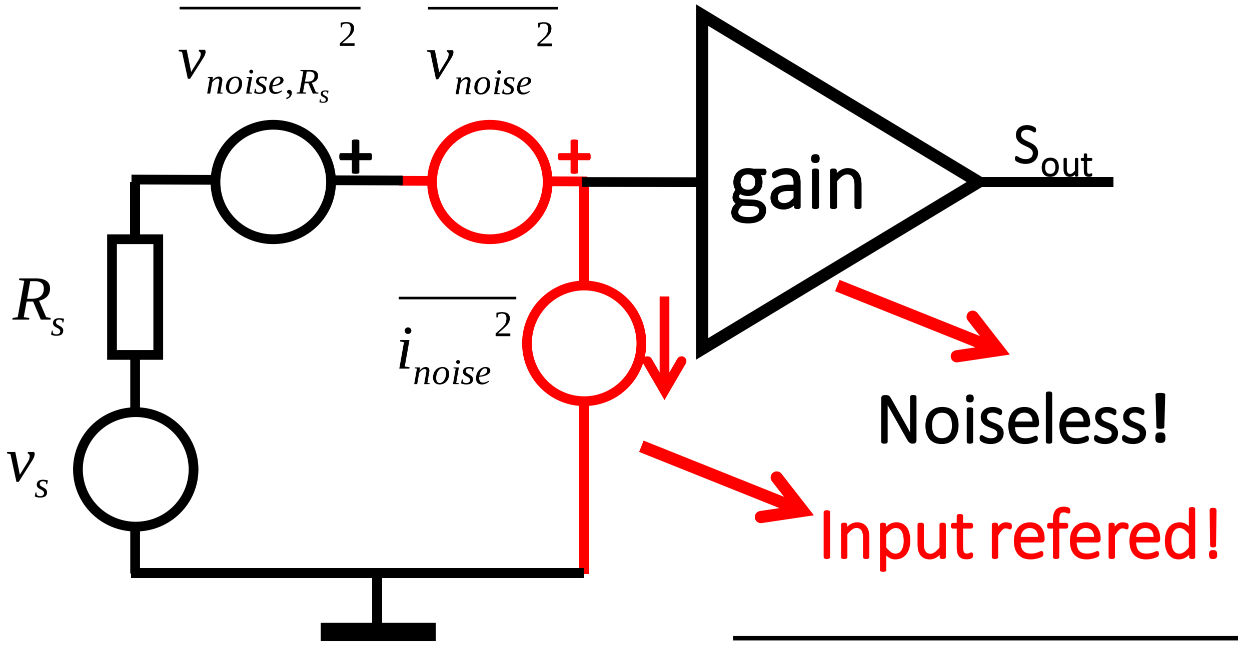
$$F = \frac{SNR_{input}}{SNR_{output}} = 1 + \frac{\overline{[v_{noise} + i_{noise} \cdot R_s]^2}}{\overline{v_{noise,R_s}}^2} = 1 + \frac{N_{amp}}{G_{av}}\frac{1}{N_{in}} = \frac{N_{out}}{G_{av}N_{in}}$$

It is important to note that noise factor is based on the <u>available power gain,</u> not just gain

$$N_{amp} = G_{av}(F-1)kTB$$

Is the noise at the output of the amplifier due to its own noise, referred to kTB

Minimum noise factor

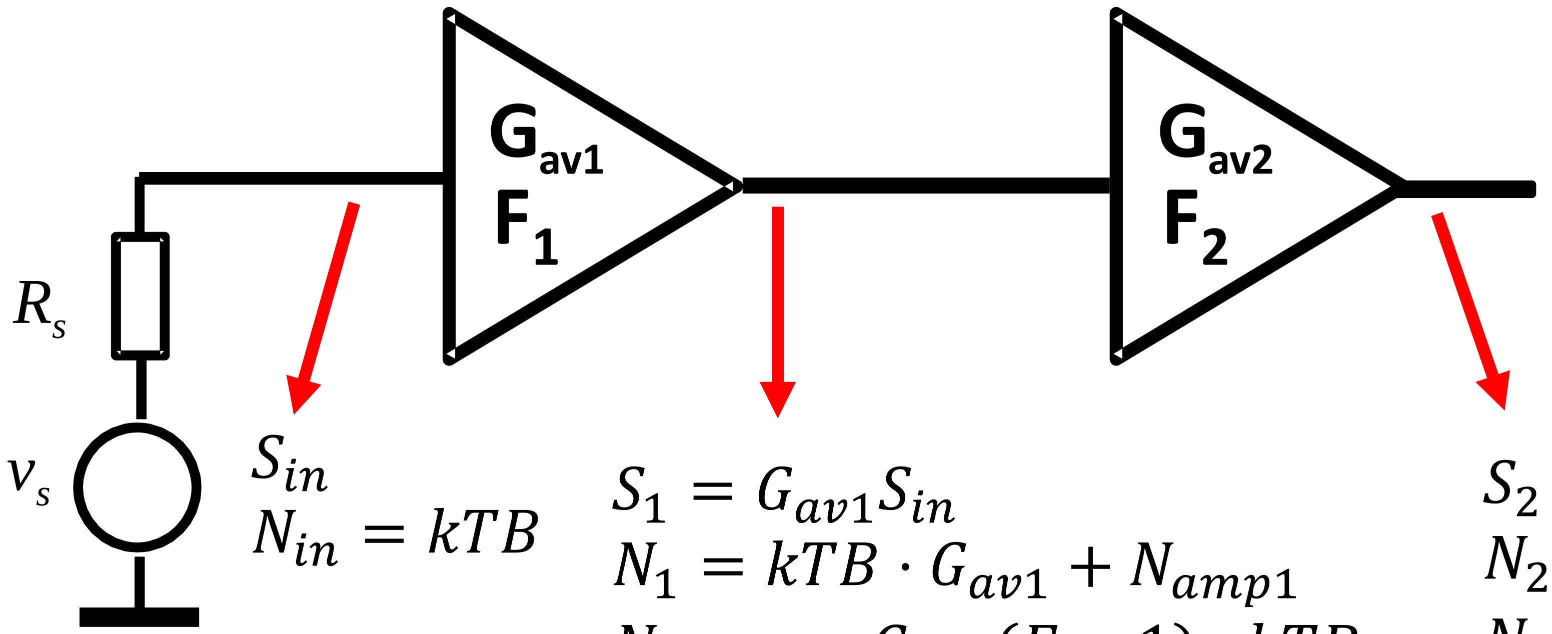


$$\overline{v_{noise,R_s}}^2 = 4 \cdot k \cdot T_o \cdot R_s \cdot B$$
$$\overline{v_{noise}}^2 = 4 \cdot k \cdot T_o \cdot R_{amp,v} \cdot B$$
$$\overline{i_{noise}}^2 = 4 \cdot k \cdot T_o \cdot G_{amp,i} \cdot B$$

$$F = 1 + \frac{\overline{[v_{noise} + i_{noise} \cdot R_s]^2}}{\overline{v_{noise,R_s}}^2} = 1 + \frac{R_{amp,v}}{R_s} + G_{amp,i} \cdot R_s$$

$$F_{\min} = 1 + 2\sqrt{R_{amp,v} \cdot G_{amp,i}} \qquad R_{s,opt} = \sqrt{\frac{R_{amp,v}}{G_{amp,i}}}$$

Noise in cascaded amplifiers (Formula of Friis)



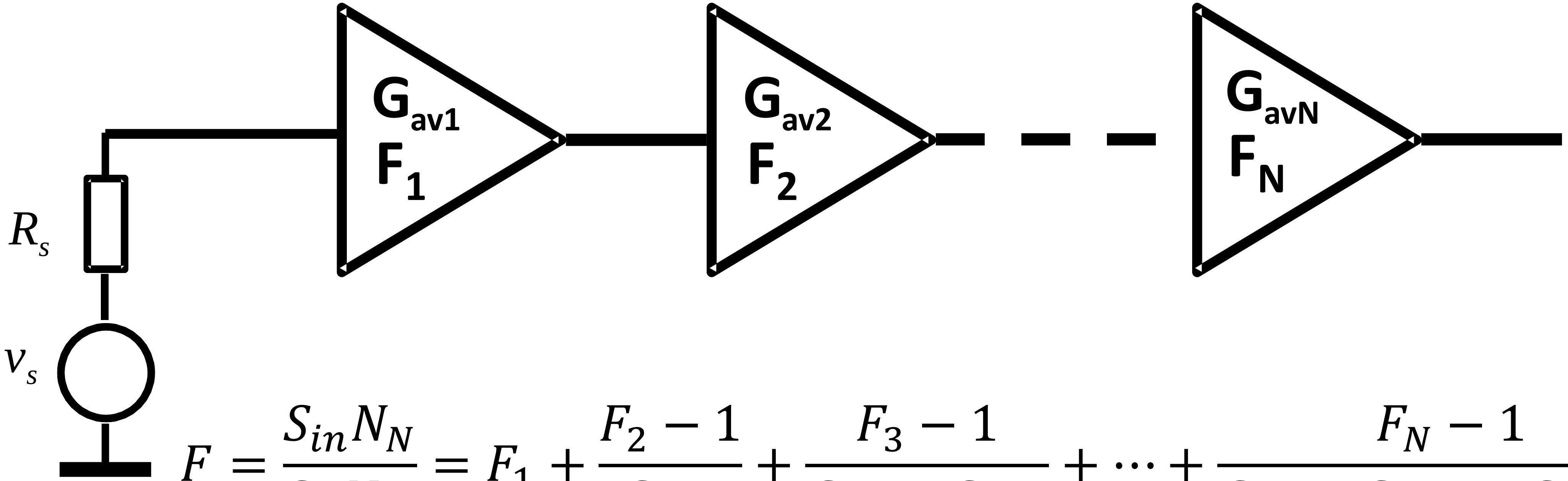
S_{in}
$N_{in} = kTB$

$S_1 = G_{av1} S_{in}$
$N_1 = kTB \cdot G_{av1} + N_{amp1}$
$N_{amp1} = G_{av1}(F-1) \cdot kTB$

$S_2 = G_{av1} G_{av2} S_{in}$
$N_2 = G_{av2} \cdot N_1 + N_{amp2}$
$N_{amp2} = G_{av2} \cdot kTB \cdot (F_2 - 1)$

$$F = \frac{\left(\frac{S_{in}}{N_{in}}\right)}{\left(\frac{S_2}{N_2}\right)} = \frac{S_{in} N_2}{S_2 N_{in}} = \frac{1}{G_{av1} G_{av2}} \cdot \frac{(G_{av1} G_{av2} F_1 kTB + (F_2 - 1) G_{av2} kTB)}{kTB} = F_1 + \frac{F_2 - 1}{G_{av1}}$$

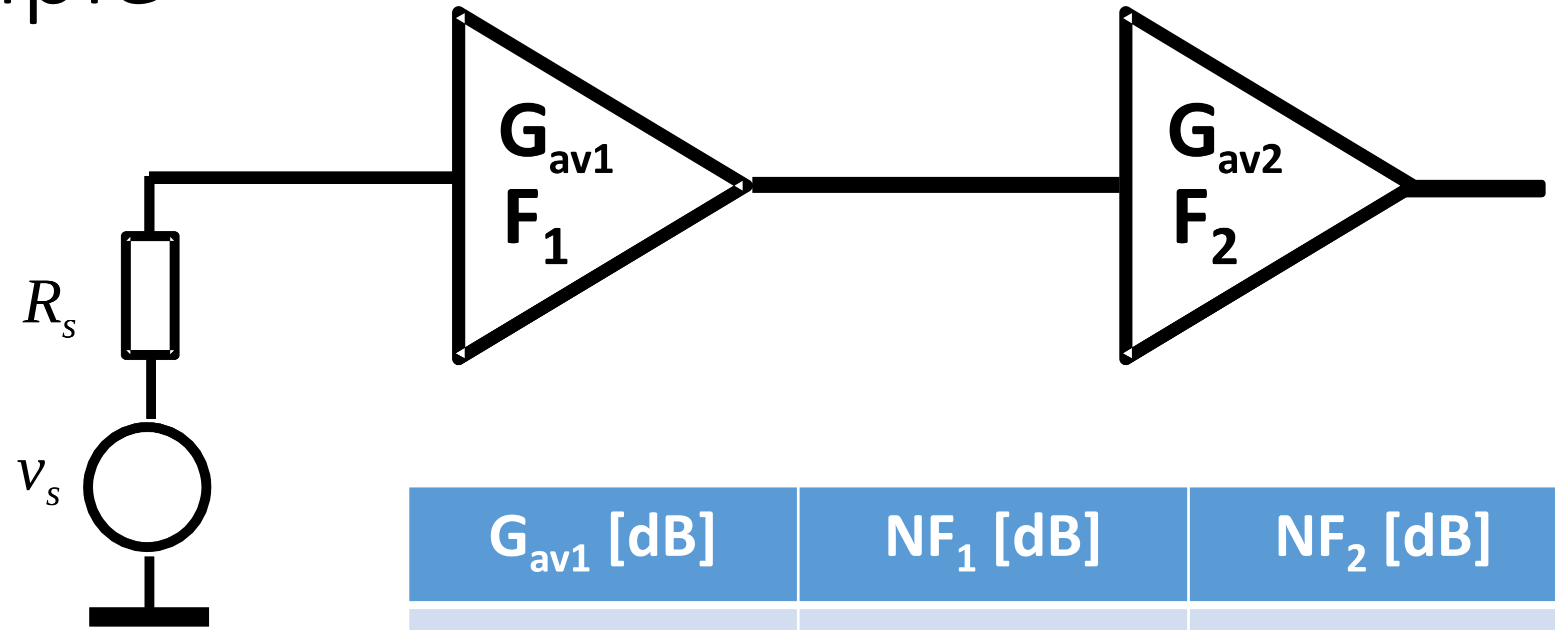
Noise in cascades amplifiers



$$F = \frac{S_{in}N_N}{S_N N_{in}} = F_1 + \frac{F_2 - 1}{G_{av1}} + \frac{F_3 - 1}{G_{av1} \cdot G_{av2}} + \cdots + \frac{F_N - 1}{G_{av1} \cdot G_{av2} \cdots G_{avN-1}}$$

Note 1: the gain of first stage is very important
Note 2: the noise performance of the first stage is very important

Example



G_{av1} [dB]	NF_1 [dB]	NF_2 [dB]	NF_{tot} [dB]
10	3	3	3.21
10	1	3	1.33
20	1	3	1.03
20	1	10	1.30

Summary

- Available noise power is kTB
- Noise factor is the ratio of SNR between input and output
- Minimum noise factor and optimal noise impedance
- Formula of Friis

Microwave Engineering and Antennas

Noise – Part II

lossy network, noise matching

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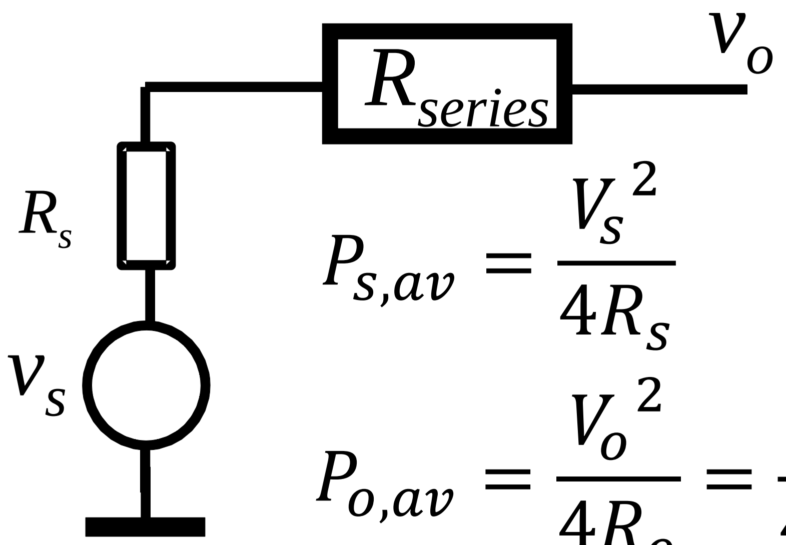
Noise – Part II
examples

Objective of this lecture

- Noise factor of a lossy network
- The impact of resistive matching to overall NF
- Example of a matching network for NF optimization

Available gain of resistive networks



$$P_{s,av} = \frac{{V_s}^2}{4R_s}$$

$$P_{o,av} = \frac{{V_o}^2}{4R_o} = \frac{{V_s}^2}{4(R_s + R_{series})}$$

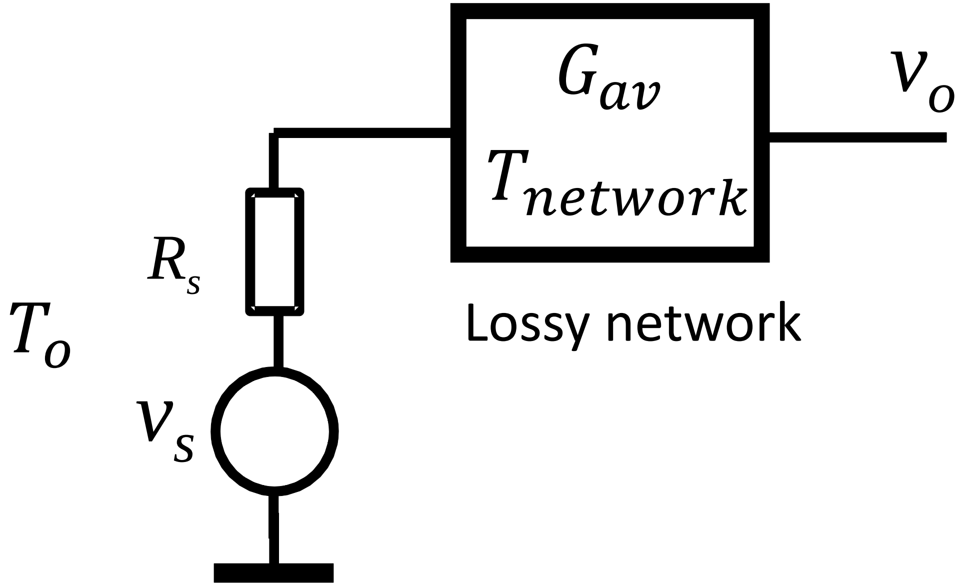
$$G_{av} = \frac{P_{o,av}}{P_{s,av}} = \frac{R_s}{R_s + R_{series}}$$

G_{av} is maximized when $R_s \gg R_{series}$

For a shunt network, one can derive that $G_{av} = \frac{P_{o,av}}{P_{s,av}} = \frac{R_{shunt}}{R_s + R_{shunt}}$

G_{av} is then maximized when $R_{shunt} \gg R_s$

Noise factor of a lossy network

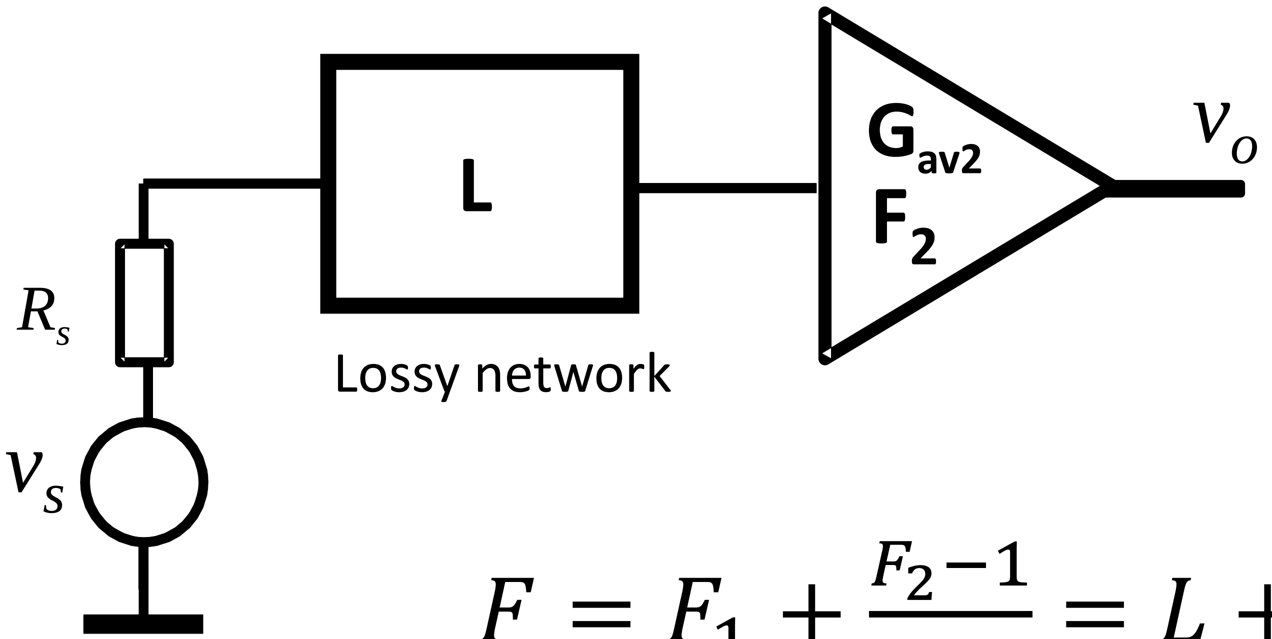


Define L as the loss, then $L = \dfrac{1}{G_{av}}$

$$F = \frac{SNR_{input}}{SNR_{output}} = \frac{N_{out}}{G_{av}N_{in}} = \frac{kT_{network}B + G_{av}\cdot k(T_o - T_{network})B}{G_{av}\cdot k(T_o)B} = 1 + \frac{T_{network}}{T_o}\left(\frac{1}{G_{av}} - 1\right)$$

If both temperatures are equal, then $F = 1 + \dfrac{T_{network}}{T_o}\left(\dfrac{1}{G_{av}} - 1\right) = \dfrac{1}{G_{av}} = L$

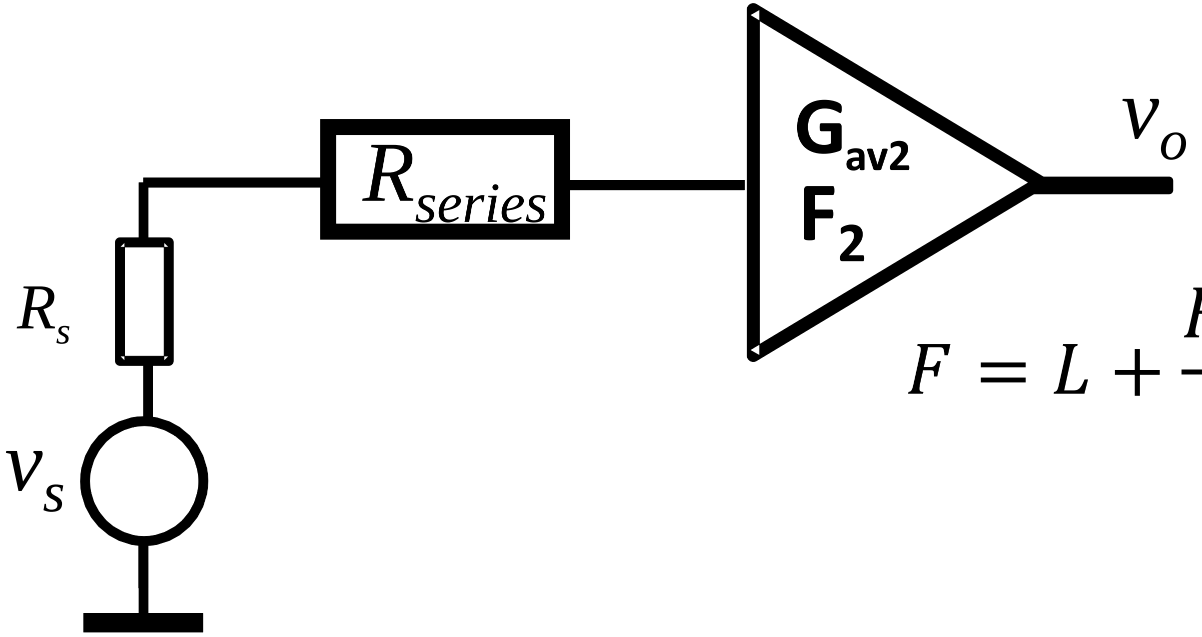
Impact of lossy network in cascaded network



$$F = F_1 + \frac{F_2 - 1}{G_{av1}} = L + \frac{F_2 - 1}{G_{av1}} = L + \frac{F_2 - 1}{1/L} = L \cdot F_2$$

$$NF = L[dB] + NF_2$$

Resistors in input matching networks adversely impact NF



$$F = L + \frac{F_2 - 1}{G_{av1}} = \frac{R_s + R_{series}}{R_s} + \frac{F_2 - 1}{\left(\frac{R_s}{R_s + R_{series}}\right)}$$

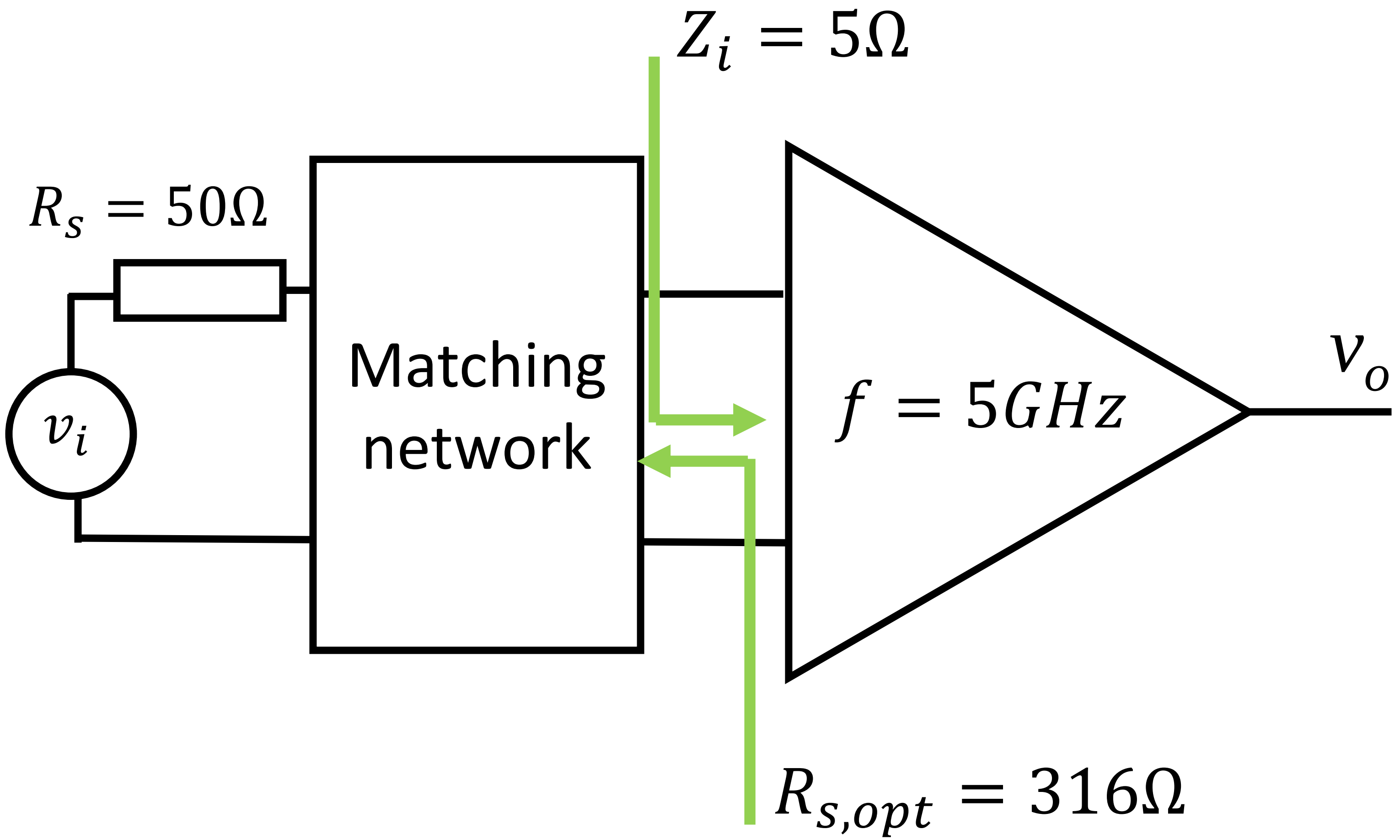
$R_s = 50\Omega, R_{series} = 100\ \Omega$

$F_2 = 2; NF_2 = 3dB$

$$F = \frac{50 + 100}{50} + \frac{2 - 1}{(0.33)} = 6 \qquad NF = 7.8\text{dB}$$

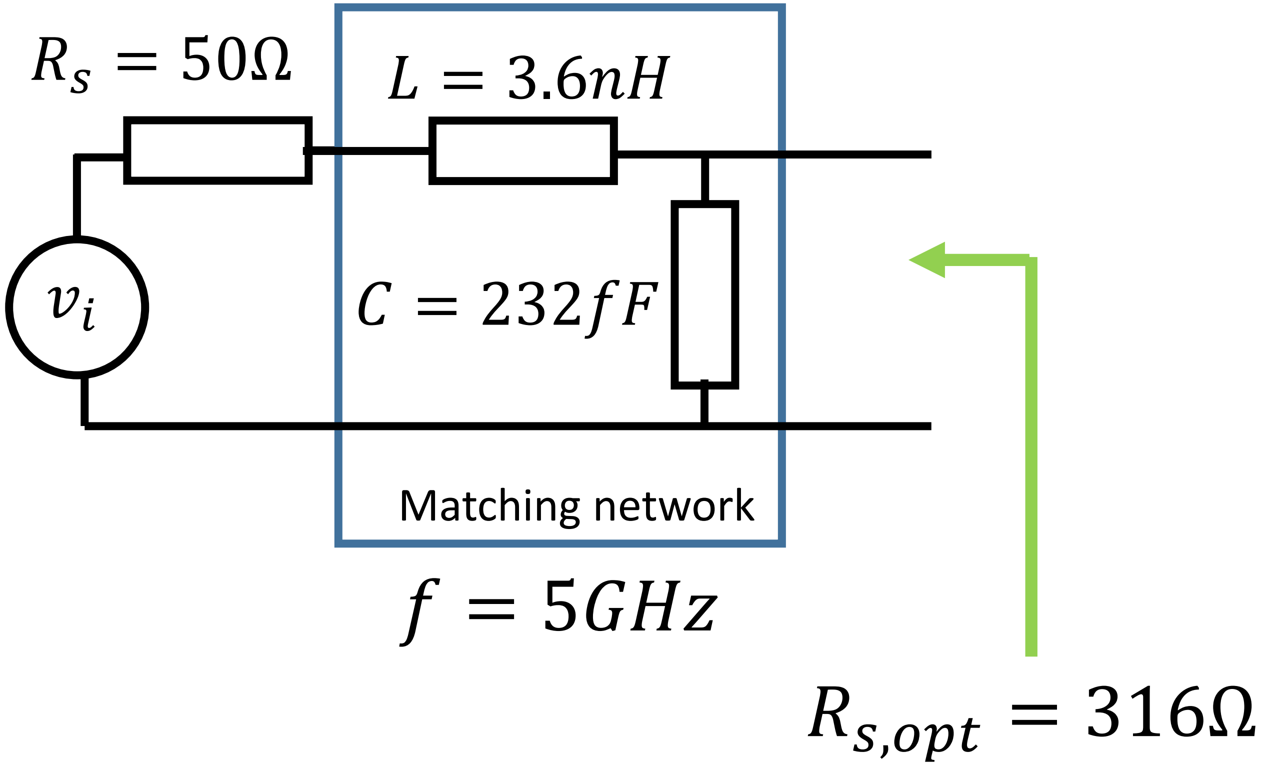
Uplift of almost 5dB, just by a resistor!!!

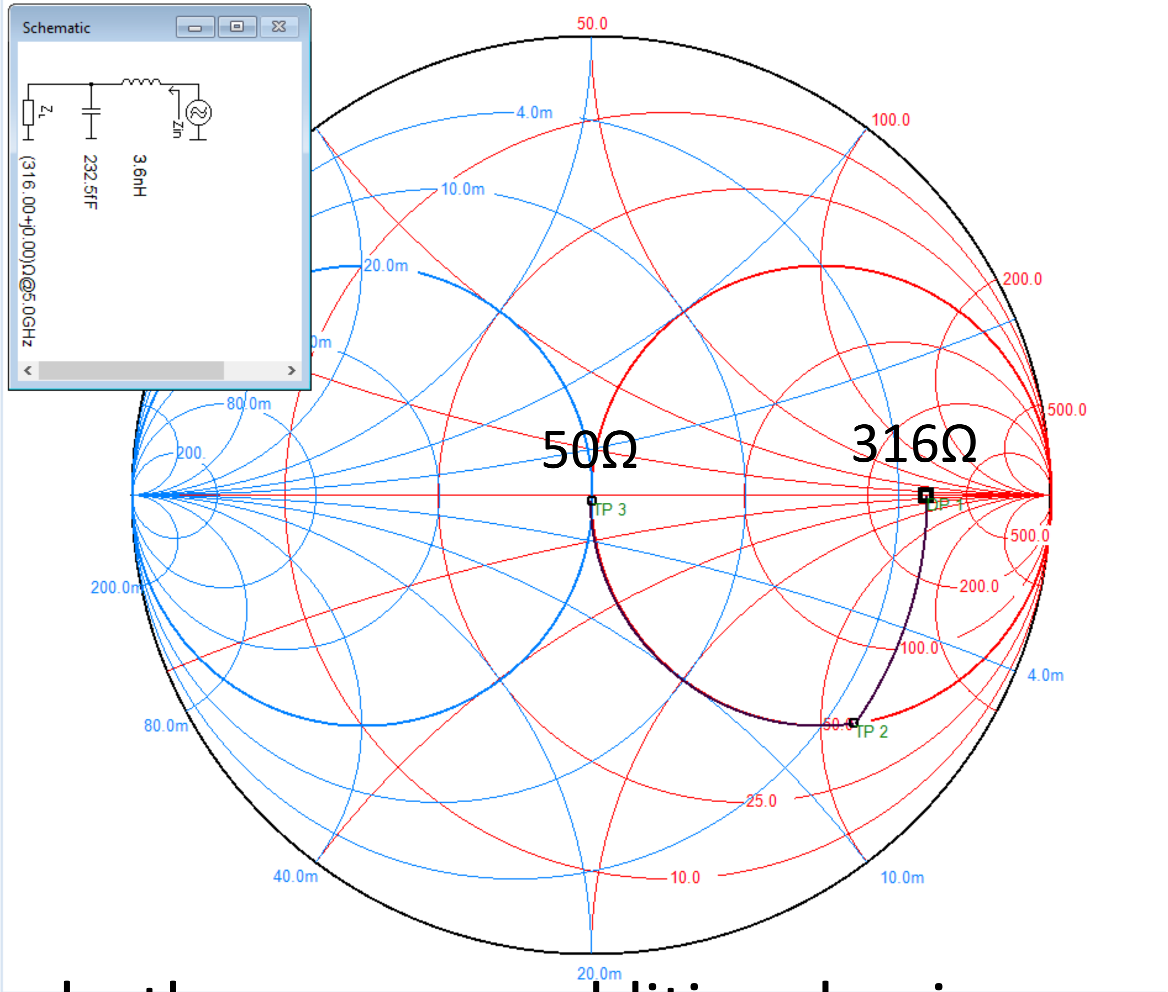
What would be a good matching network for noise?



Using a series resistor of 266Ω would be a very bad idea, knowing Friss (see earlier slide).

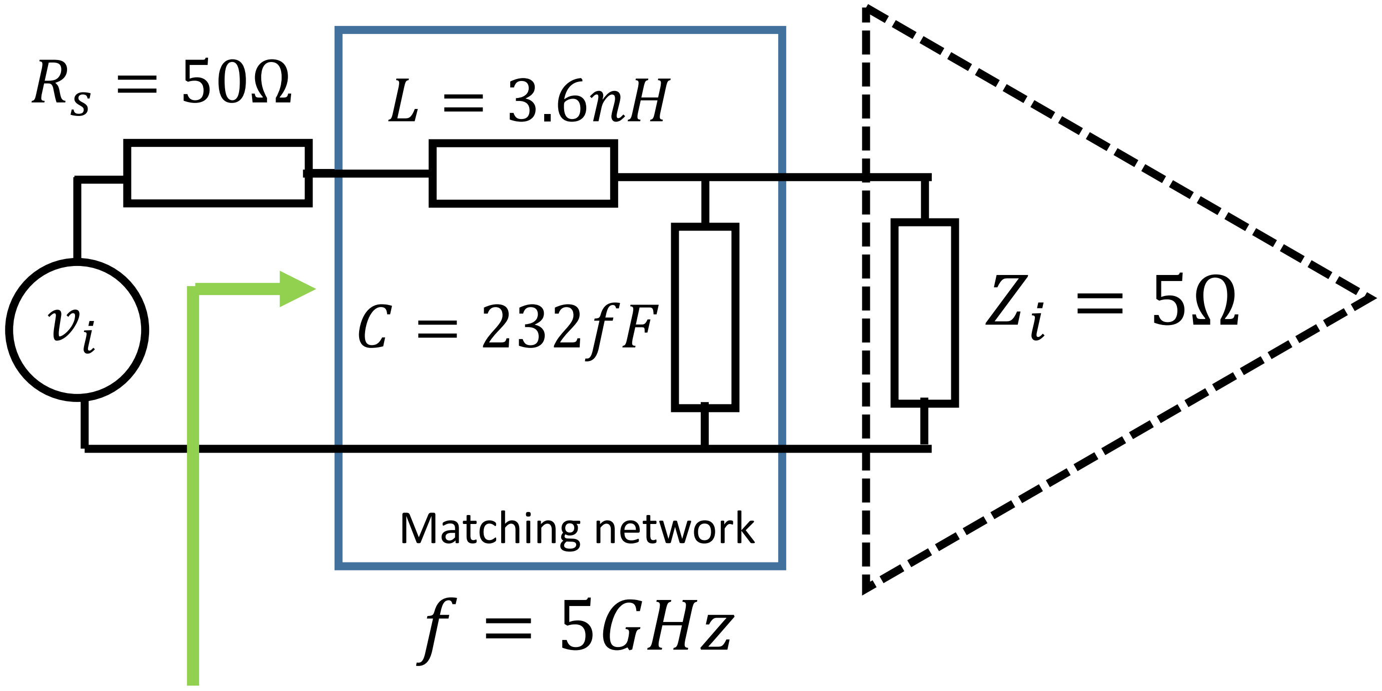
What would be a good matching network for noise performance?





As we use (ideal) reactive components only, there are no additional noise contributions

What is the impact to power match?



$$f = 5GHz$$

$$Z_{input} = 5 + j115\Omega$$

Impedance mismatch: no optimal power transfer

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Summary

- Noise factor of a lossy network
- Impact of resistive noise matching
- Noise matching is not the same as matching for optimal power transfer

Power match: $Z_s = Z_{in}^*$

Noise match: $Z_s = Z_{opt}$

Microwave Engineering and Antennas

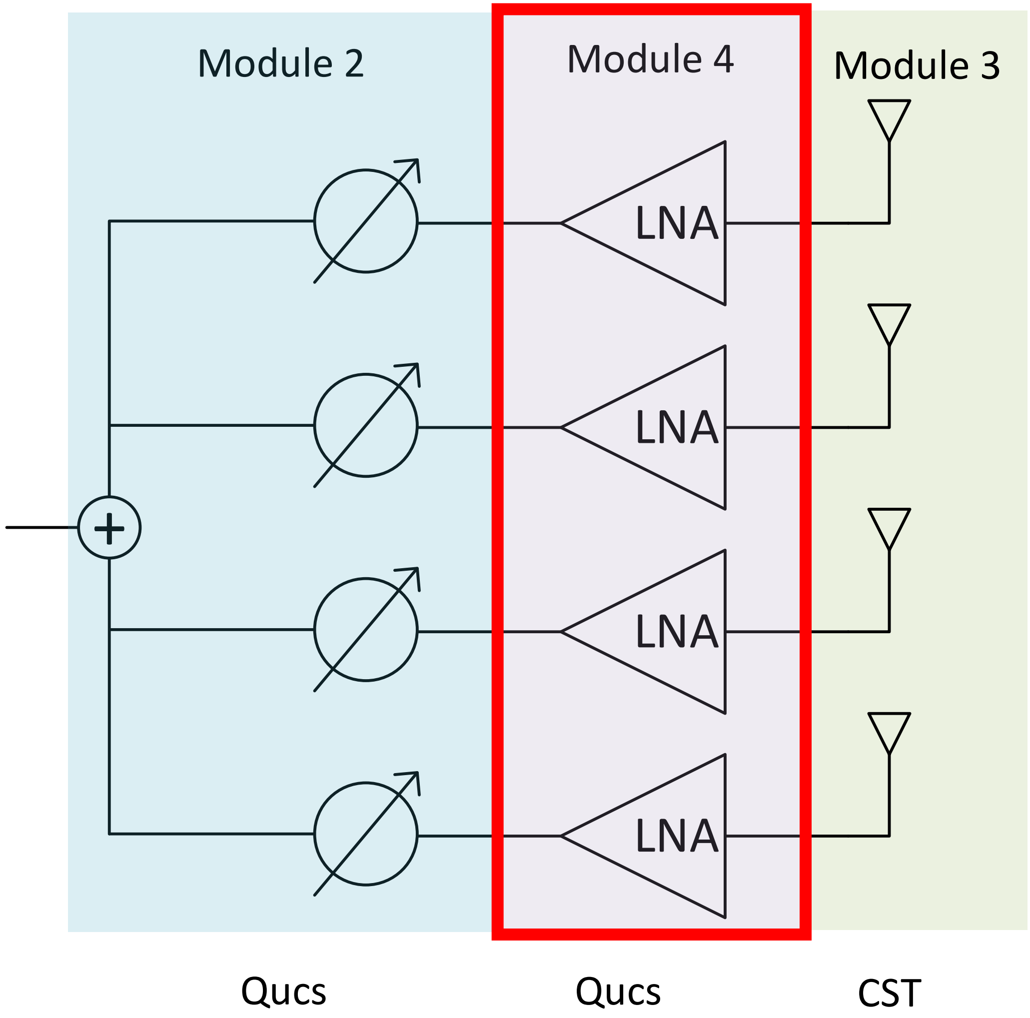
Hands-On Design: Low-Noise Amplifier

Ulf Johannsen, Assistant Professor

Department of Electrical Engineering

Center for Wireless Technology Eindhoven

Hands-On Design Assignment



4-Element Analogue Beamforming Receiver-Array

- 5.8 GHz ISM band
- ±45˚ beamsteering range
- Main-lobe antenna gain of 9dBi for entire beamsteering range
- System noise figure < 2.7dB

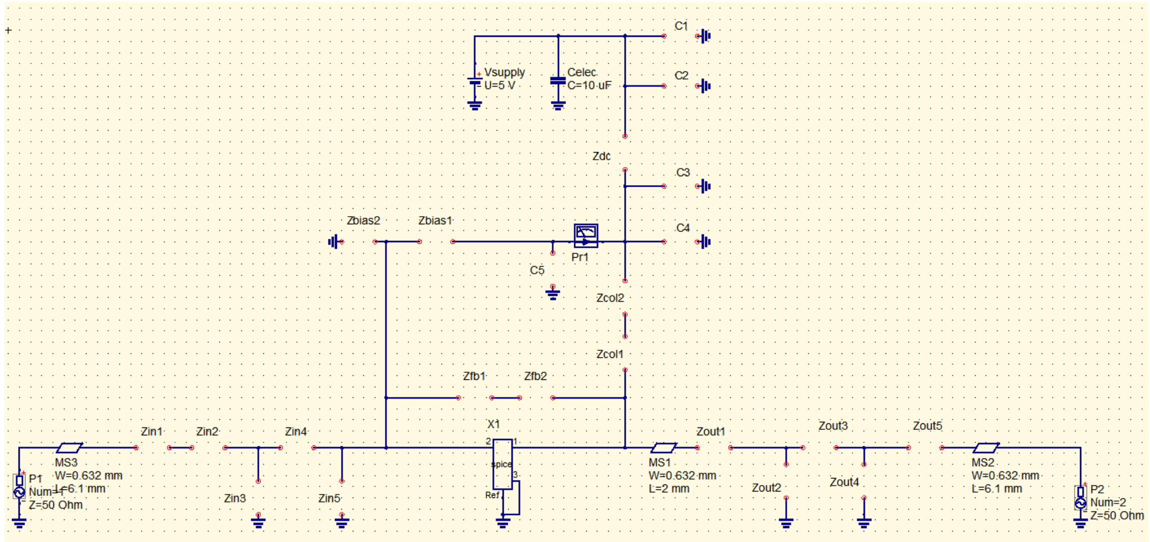
Qucs

- Quite Universal Circuit Simulator (Qucs) is a free-software electronics circuit simulator under GPL.
- You can **download** a copy from http://qucs.sourceforge.net/
- A **short description of mathematical functions** that can be used in Qucs equation (including some standard microwave engineering parameters) can be found here: https://web.mit.edu/qucs_v0.0.19/docs/en/mathfunc.html

Task and Design Requirements

- Design a low-noise amplifier
- The overall design, shown on the bottom right, must fulfil the following requirements:

ID	Parameter	Requirement
1	Operating bandwidth	5.725 – 5.875 GHz
2	Port match	$S_{11} < -10dB$ @ $Z_0 = 50\Omega$ $S_{22} < -10dB$ @ $Z_0 = 50\Omega$
3	Transducer Power Gain	$S_{21} > 15dB$ @ $Z_0 = 50\Omega$
4	Noise figure	$F < 2dB$
5	Stability	Unconditional



Rules and Tips

- **You may use lumped elements or distributed elements** (transmission lines) to achieve your design goals.
- When using lumped elements, you **may use the resistor, capacitor and inductor models from the Qucs library**.
- Show your **genuine attempt** to design what was asked of you. For a genuine attempt you
  - have **all design choices** as long as you design the components yourself in Qucs or CST (student version). Commercial **off-the-shelf solutions are not permitted**.
  - must upload **<u>all</u> necessary design files** and a **design report** that describes and explains the design.
  - **must use the BFU730F transistor** provided on Coursera, i.e. use one of the Touchstone files for different bias conditions. Do not forget to add **circuitry to achieve the chosen bias condition** (the transistor/touchstone-file is not affected by this, but it will have an affect on the matching networks).
  - **must use a 0.508 mm thick RO4350B substrate** from Rogers Corporation as PCB for your design: https://www.rogerscorp.com/Advanced%20Connectivity%20Solutions/RO4000%20Series%20Laminates/RO4350B%20Laminates

Rubric for peer review

Excellent (6 pts)	Very good (5 pts)	Good (4 pts)	Fair (3 pts)	Sufficient (2)	Insufficient (0 pts)
• The report describes **all** design aspects in detail, such that the reader/reviewer can implement the design in Qucs her-/himself. • **All** design aspects are explained clearly and correctly. • The design achieves the specified requirements **without exception**.	• The report describes almost all design aspects in detail. Only **minor details are missing** such that the reviewer **can implement the design** in Qucs her-/himself. • Most design aspects are explained clearly and correctly. **Only minor inaccuracies** are present. The understanding is not significantly affected by this. • The design achieves the specified requirements with **one exception**.	• The report describes most design aspects in detail. **A few details are missing** such that the reviewer may not be able to implement **some minor details** of the design in Qucs her-/himself. • There are **only a few inaccuracies/inconsistencies** in the explanation of the design aspects. The understanding of a few aspects of the design is slightly affected. • The design achieves the specified requirements with **two exceptions**.	• The report describes most design aspects in detail. **Some details are missing** such that the reviewer may **not** be able to implement **every aspect of the design correctly** in Qucs her-/himself. • There are **some inaccuracies/inconsistencies** in the explanation of the design aspects. Not every aspect of the design is entirely clear. • The design achieves the specified requirements with **three exceptions**.	• The report describes most design aspects in detail. **Several details are missing** such that the reviewer **struggles** to implement **the design correctly** in Qucs her-/himself. • There are **several inaccuracies/inconsistencies or some mistakes** in the explanation of the design aspects. About half of the design aspects are not entirely clear. • The design achieves the specified requirements with **four exceptions**.	• The report provides insufficient detail such that the reader/**reviewer cannot implement the design** in Qucs her-/himself. • There are **major design aspects missing** or are **mostly incorrectly** explained. • The **design does not achieve any of the requirements.**

Microwave Engineering and Antennas

Stability – part I
two-port conditions

Domine Leenaerts, Professor

Department of Electrical Engineering

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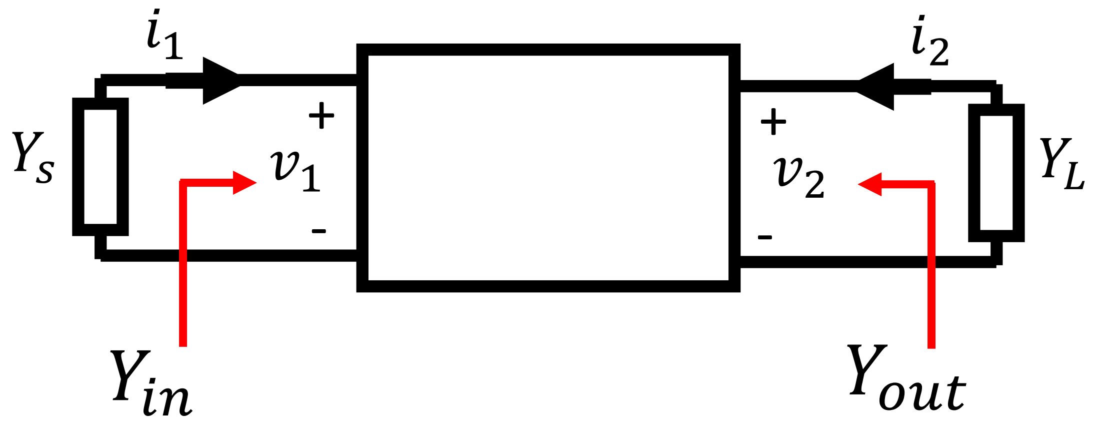
Stability – part I
two-port conditions

Objective of this lecture

- Stability conditions of a two-port
- Example

Input / output admittance



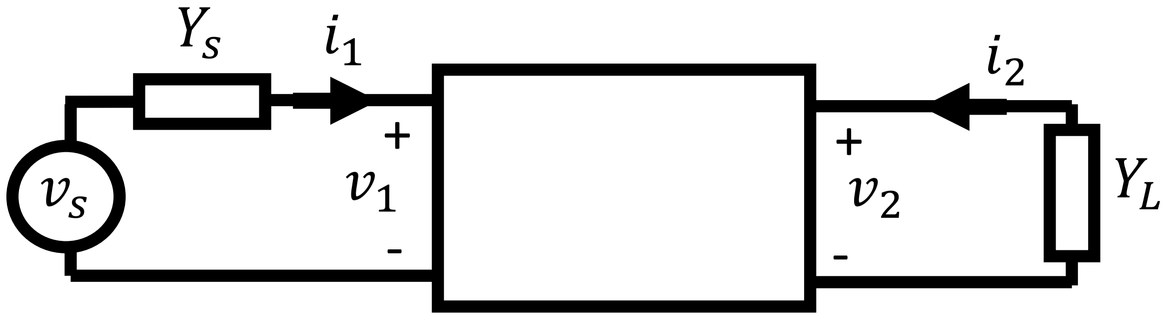
$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$Y_{in} = \frac{i_1}{v_1} = y_{11} + y_{12}\frac{v_2}{v_1} = y_{11} - \frac{y_{21}y_{12}}{(y_{22} + Y_L)}$$

$$Y_{out} = \frac{i_2}{v_2} = y_{22} + y_{21}\frac{v_1}{v_2} = y_{22} - \frac{y_{21}y_{12}}{(y_{11} + Y_S)}$$

Note that in unilateral case $y_{12} = 0$ and thus $Y_{in} = y_{11}, Y_{out} = y_{22}$

(loaded) voltage gain A_v and loop gain T



$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$A_v = \frac{v_2}{v_s} = \frac{v_2}{v_1}\frac{v_1}{v_s} = A_{v,unloaded}\,\frac{Y_s}{Y_{in}+Y_s}$$

$$A_v = \frac{-Y_s y_{21}}{(Y_{in}+Y_s)(y_{22}+Y_L)} = \frac{-Y_s y_{21}}{(y_{11}+Y_s)(y_{22}+Y_L) - y_{21}y_{12}}$$

Define: $A_{v_{unilateral}} = \frac{-Y_s y_{21}}{(y_{11}+Y_s)(y_{22}+Y_L)}$ then $A_v = \frac{A_{v,unilateral}}{1+T}$; $T = \frac{-y_{12}y_{21}}{(y_{11}+Y_s)(y_{22}+Y_L)}$

T is known as the loop gain

Stability condition

A two-port is terminated on both sides $i_1 = -Y_s v_1,\ i_2 = -Y_L v_2$

$$\begin{pmatrix} y_{11} + Y_s & y_{12} \\ y_{21} & y_{22} + Y_L \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

To become unstable: determinant of the matrix must be zero (at a certain frequency):

$$\boxed{\begin{gathered} 1 + T = 0 \\ \mathrm{T} = \frac{-y_{21} y_{12}}{(y_{11} + Y_s)(y_{22} + Y_L)} \end{gathered}}$$

T is the loop gain

Stability boundary

$$1 + T = 0$$

$$\mathrm{T} = \frac{-y_{21}y_{12}}{(y_{11}+Y_s)(y_{22}+Y_L)}$$

- Stability boundary implies $T = -1$, which is exactly the Nyquist criteria.

- $Y_S + Y_{in} = 0$ with $\quad Y_{in} = y_{11} - \dfrac{y_{21}y_{12}}{(y_{22}+Y_L)}$

- $Y_L + Y_{out} = 0$ with $\quad Y_{out} = y_{22} - \dfrac{y_{21}y_{12}}{(y_{11}+Y_S)}$

Stability condition input / output admittance

- The terminated network becomes unstable if

$$Re(Y_S + Y_{in}) = Re(Y_S) + Re(Y_{in}) = 0$$

- and / or

$$Re(Y_L + Y_{out}) = Re(Y_L) + Re(Y_{out}) = 0$$

- As we normally terminate with real (hence passive) admittances, $Re(Y_S) > 0$, $Re(Y_L) > 0$, we arrive at the stability condition. To become unstable:

$$Re(Y_{in}) < 0, \text{ and/or } Re(Y_{out}) < 0$$

Stability conditions using scatter parameters

Unstable if $|\Gamma_{\text{in}}|>1$ and/or $|\Gamma_{\text{out}}|>1$

$$|\Gamma_{in}| \quad = \left| S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} \right| > 1$$

$$|\Gamma_{out}| \quad = \left| S_{22} + \frac{S_{21}S_{12}\Gamma_s}{1 - S_{11}\Gamma_s} \right| > 1$$

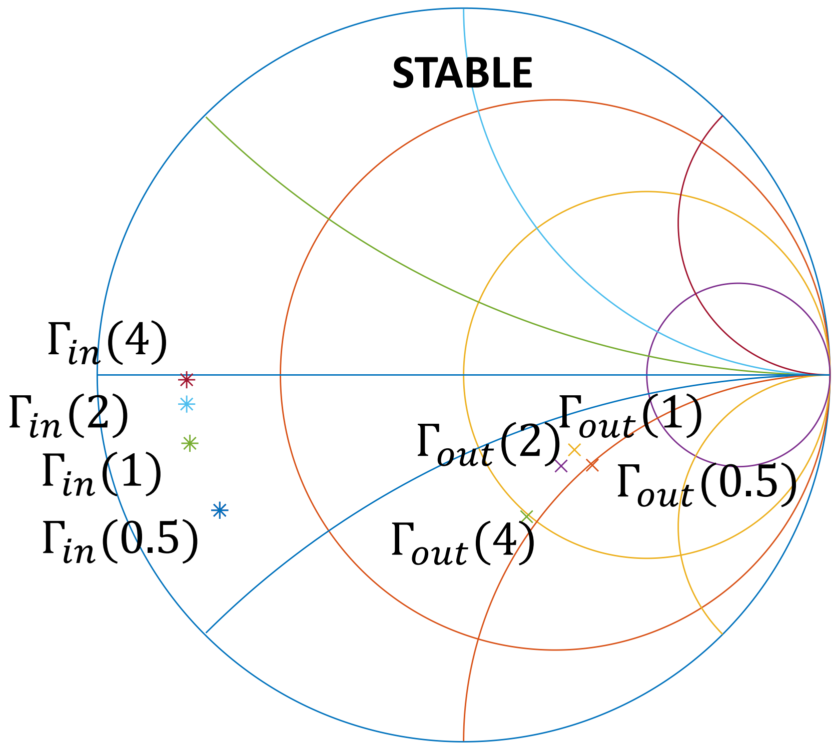
Unilateral case: $|\Gamma_{in}| = |s_{11}|>1$, $|\Gamma_{out}| = |s_{22}|>1$

This implies that $|s_{11}|$ and/or $|s_{22}|$ are <u>outside</u> the unity circle of the Smith chart. That again reflects impedances which have a <u>negative real part</u> and cannot therefore be passive.

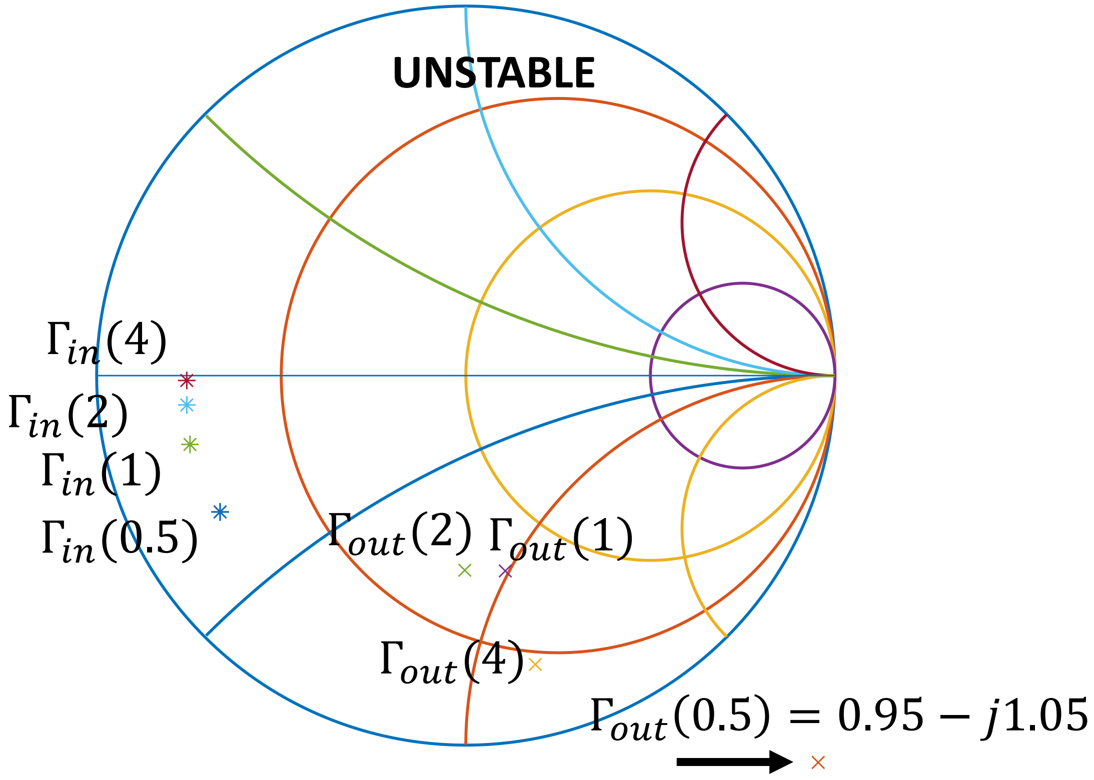
Example

F(GHz)	S11	S12	S21	S22
.5	0.761∠-151	0.025∠31	11.84∠102	0.429∠-35°
1	0.770 ∠-166°	0.029∠35	6.11∠ 89	0.365∠-34
2	0.760 ∠-174°	0.040∠44	3.06∠ 74	0.364∠-43
4	0.756 ∠-179°	0.064∠48	1.53∠ 53	0.423∠-66

source:50Ω load: 50Ω → $\Gamma_s = 0, \Gamma_l = 0$



Source: 1+j10Ω load: 50Ω → $\Gamma_s = -0.9 + j0.37, \Gamma_l = 0$



Summary

- Criteria for two-port stability
- Stability is frequency and source/load termination dependent

Microwave Engineering and Antennas

Stability – part II
stability circles

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Stability – part II
stability circles

Objective of this lecture

- Explain the concept of stability circles
- Example to show how to use these circles
- RF stabilization technique plus example

Stability conditions using scatter parameters

- Stability conditions are dependent on source and load (and frequency)

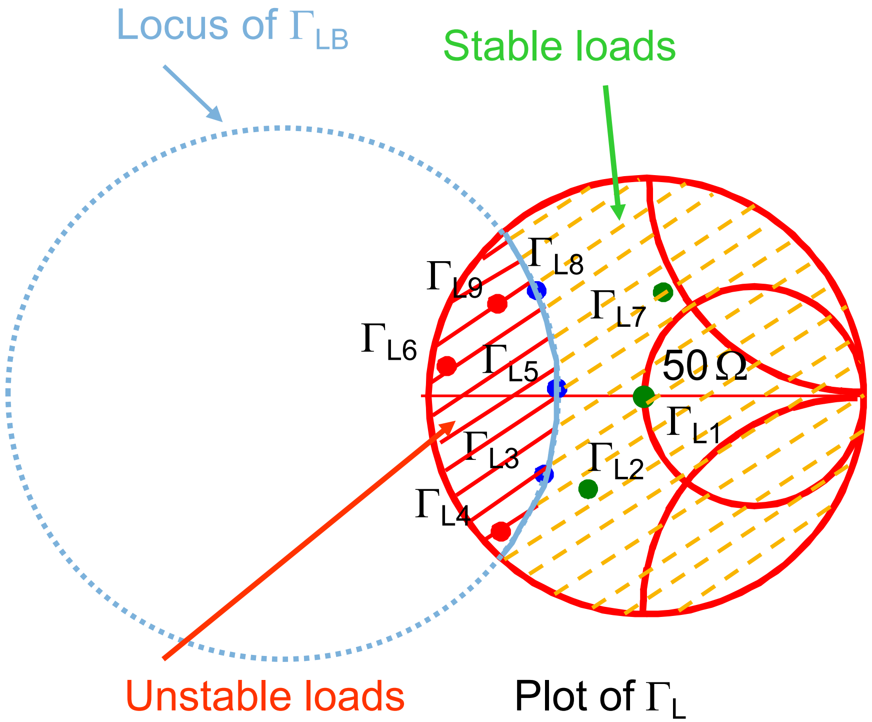
Unstable when: $|\Gamma_{in}|>1$ and/or $|\Gamma_{out}|>1$

$$|\Gamma_{in}| \quad = \left| S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} \right| > 1$$

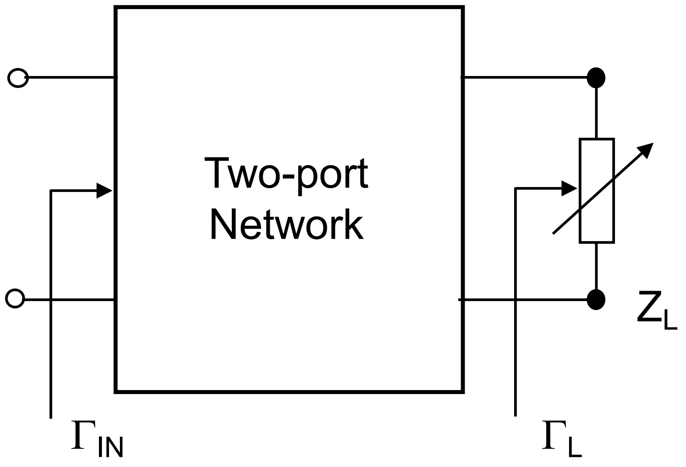
$$|\Gamma_{out}| \quad = \left| S_{22} + \frac{S_{21}S_{12}\Gamma_s}{1 - S_{11}\Gamma_s} \right| > 1$$

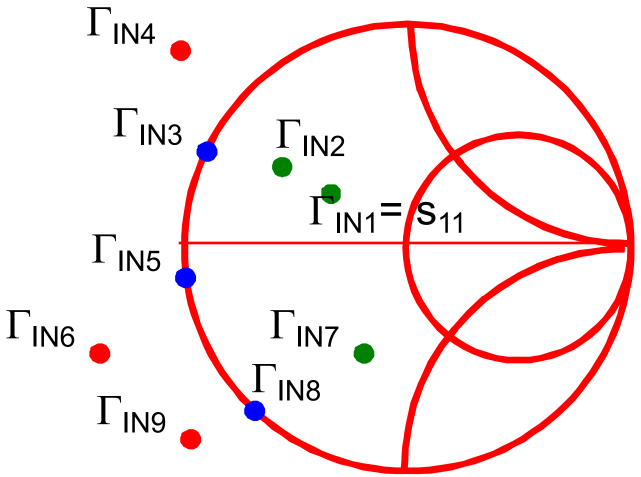
Which load creates instability? Load-plane

- If we set $|\Gamma_{in}| = 1$, then we establish a boundary in the Γ_L −plane that separates stable from unstable regions. The load should create the instability.
- Load terminations Γ_{LB} resulting in $|\Gamma_{in}| = \left|S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1-S_{22}\Gamma_L}\right| = 1$ yields a circle in the Smith chart.



Plot of Γ_L

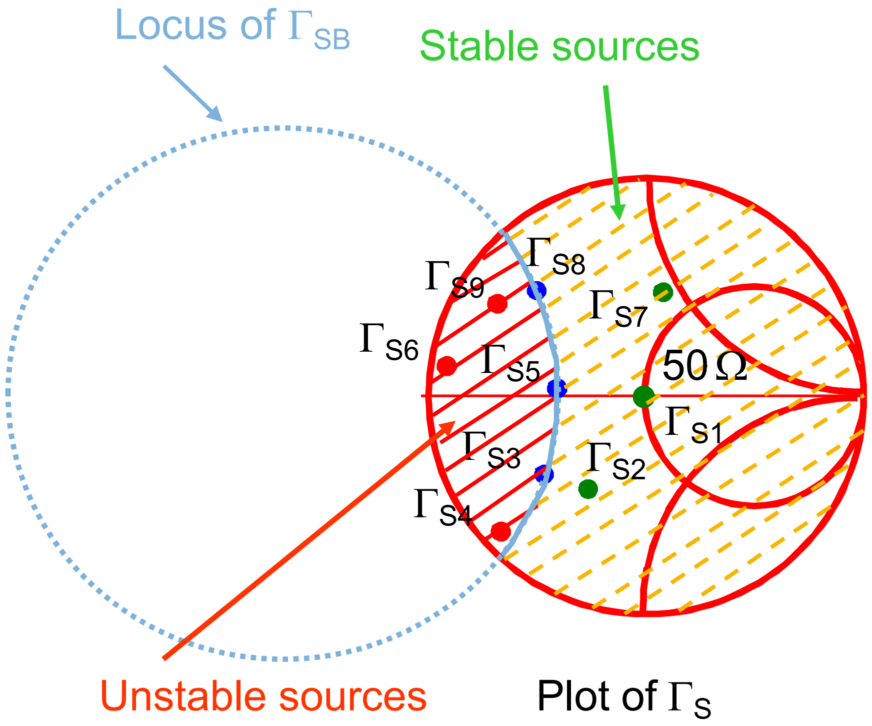


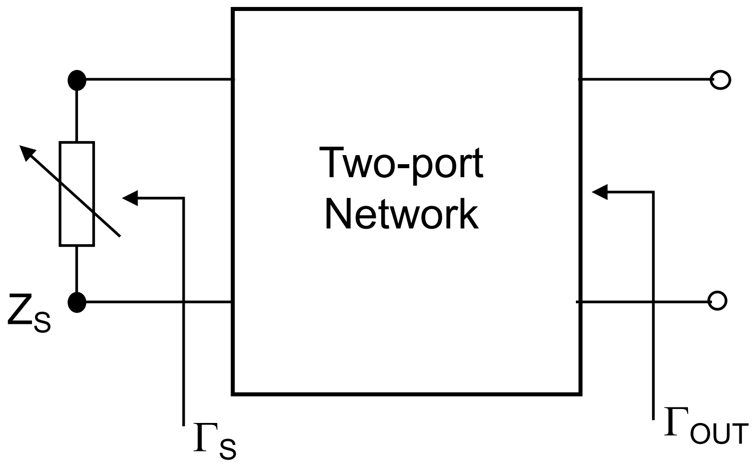


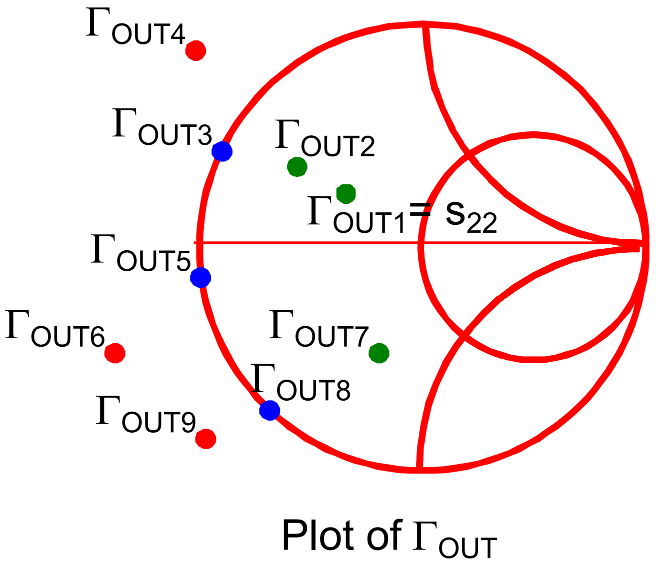
Plot of Γ_{IN}

Which source creates instability? Source-plane

- If we set $|\Gamma_{out}| = 1$, then we establish a boundary in the Γ_S –plane that separates stable from unstable regions. The source should create the instability.
- Source terminations Γ_{SB} resulting in $|\Gamma_{out}| = \left|S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1-S_{11}\Gamma_S}\right| = 1$ yields a circle in the Smith chart.

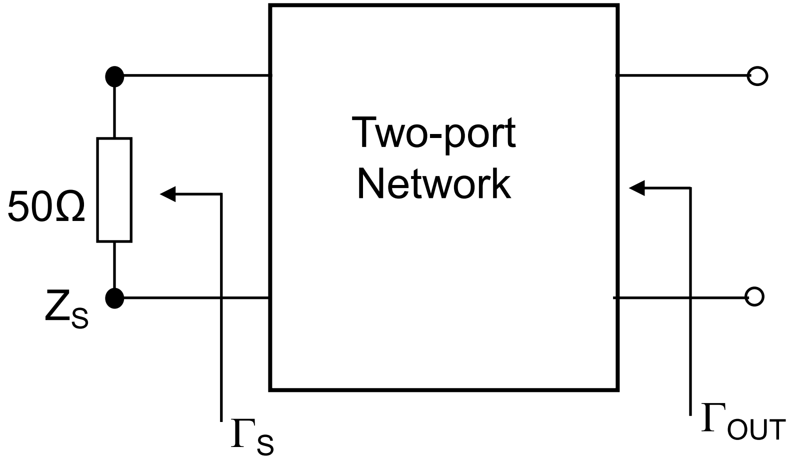




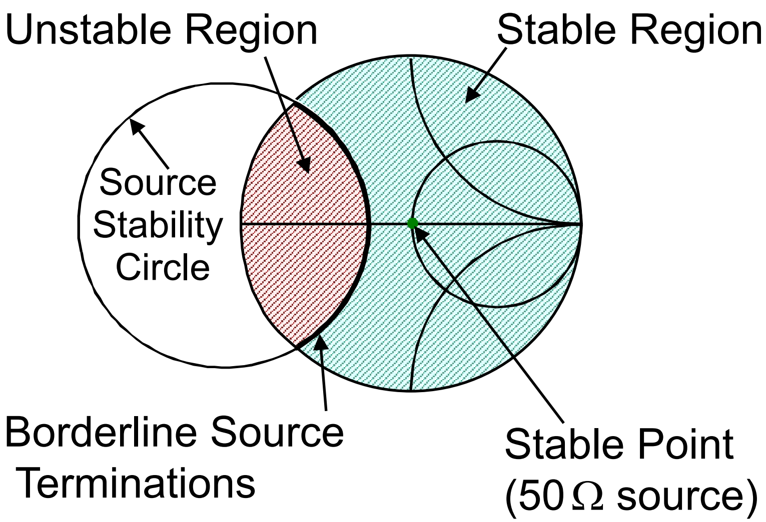


Which region represents stable sources?

- A simple test to find the stable side of the source stability circle is to start with a 50Ω source.

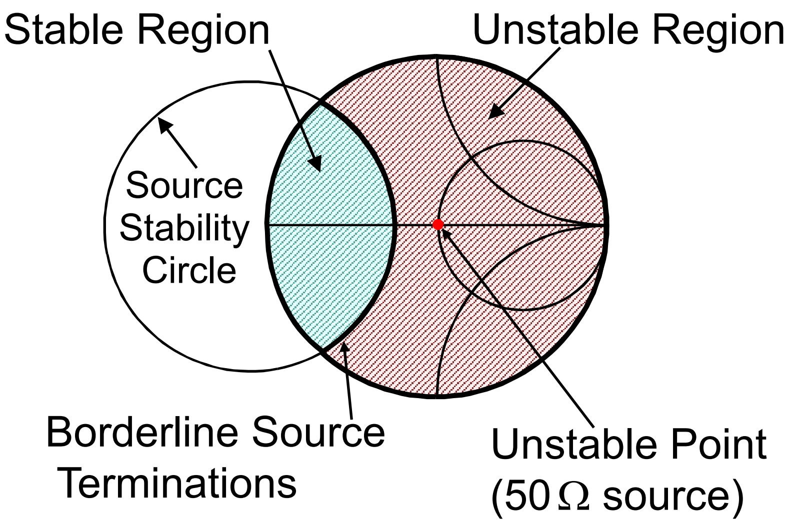


$$|\Gamma_{out}| = \left|S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1 - S_{11}\Gamma_S}\right|$$



$$|\Gamma_{out}| = \left|S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1-S_{11}\Gamma_S}\right| < 1$$

$$|s_{22}| < 1$$

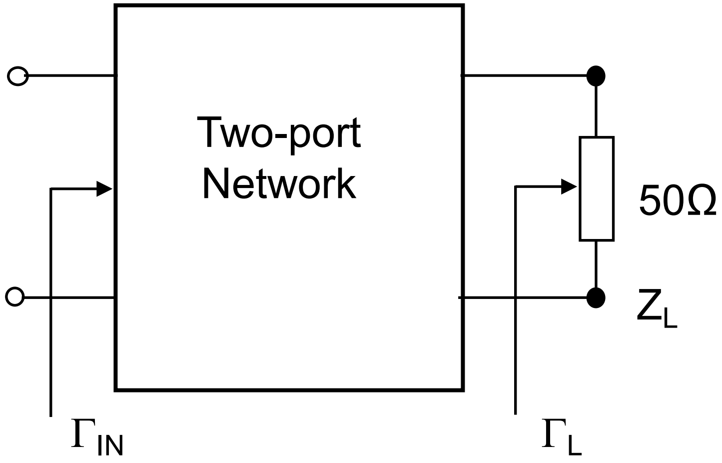


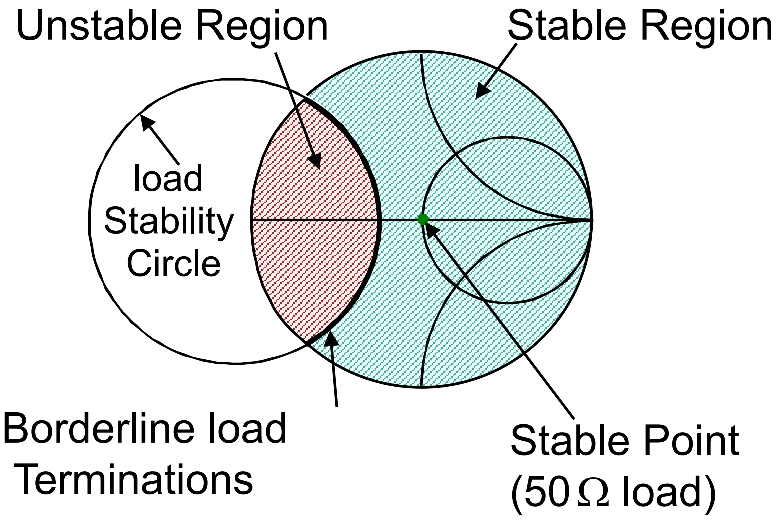
$$|\Gamma_{out}| = \left|S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1-S_{11}\Gamma_S}\right| > 1$$

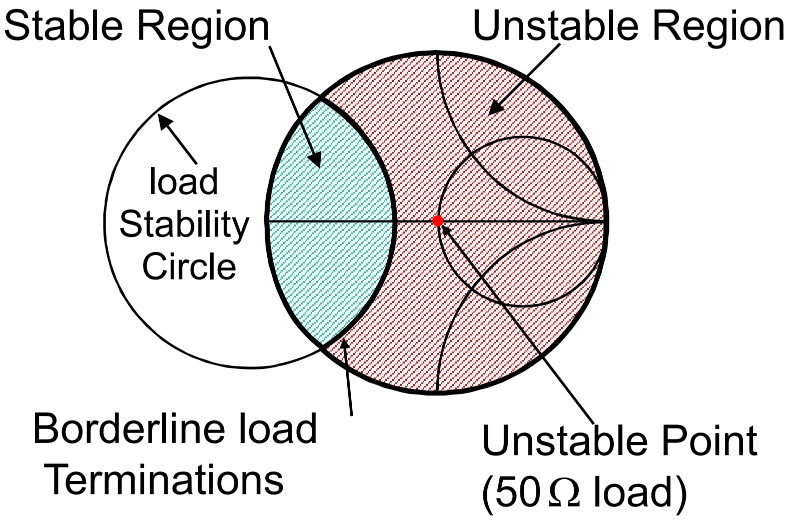
$$|s_{22}| > 1$$

Which region represents stable loads?

- A simple test to find the stable side of the load stability circle is to start with a 50Ω load.







$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} \right|$$

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1$$

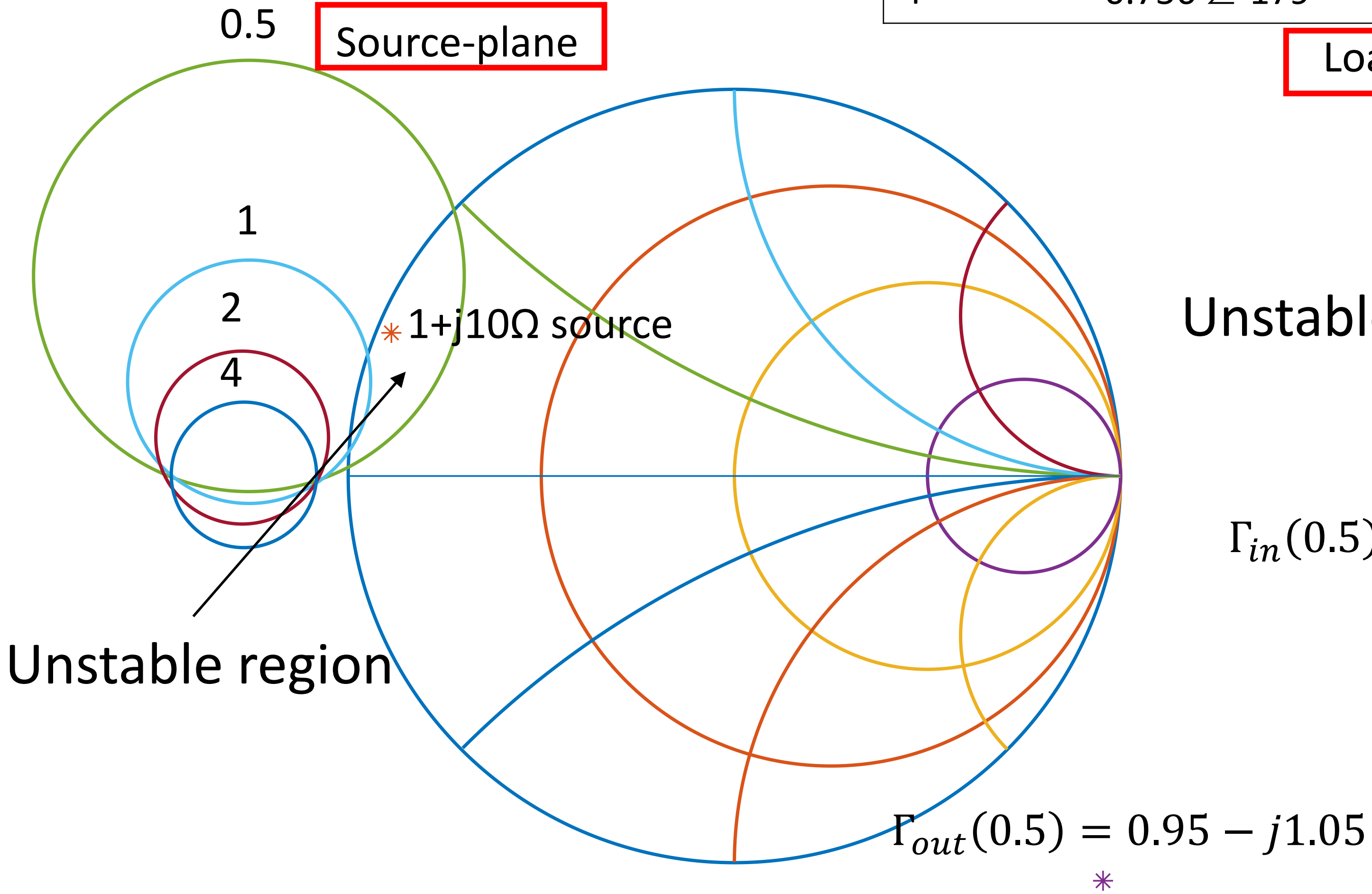
$$|s_{11}| < 1$$

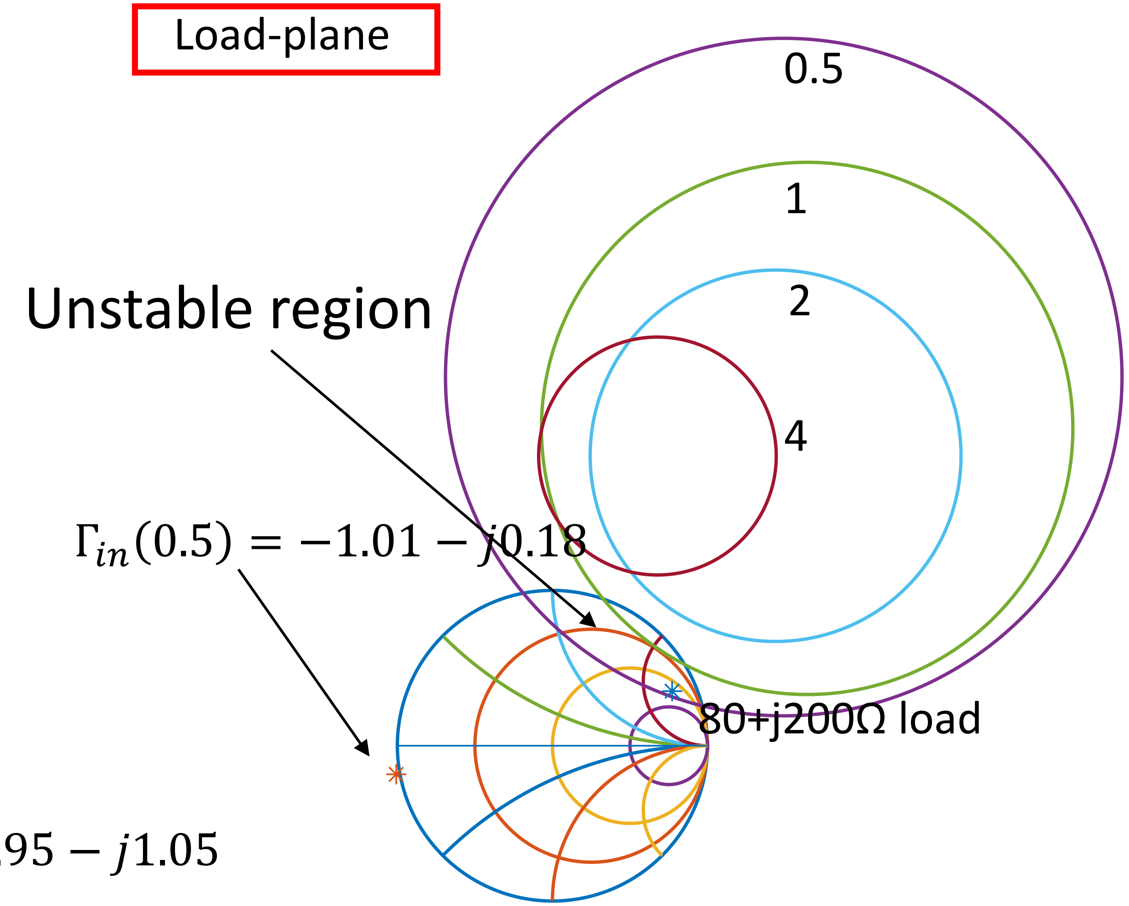
$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} \right| > 1$$

$$|s_{11}| > 1$$

Our example

F(GHz)	S11	S12	S21	S22
.5	0.761∠-151	0.025∠31	11.84∠102	0.429∠-35°
1	0.770 ∠-166°	0.029∠35	6.11∠ 89	0.365∠-34
2	0.760 ∠-174°	0.040∠44	3.06∠ 74	0.364∠-43
4	0.756 ∠-179°	0.064∠48	1.53∠ 53	0.423∠-66





Rollet's stability criteria (1962)

- A two-port is unconditional stable if and only if

$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1 - S_{11}\Gamma_S} \right| < 1$$ ← **Passive output impedance,** $\Re(Z_{out}) > 0$

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1$$ ← **Passive input impedance,** $\Re(Z_{in}) > 0$

$|\Gamma_S| < 1$ ← **Passive source**

$|\Gamma_L| < 1$ ← **Passive load**

Rollet's stability criteria (1962, IRE)

Requirements for two - port stability

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1$$

$$B = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 > 0$$

$$|\Delta| = |S_{11}S_{22} - S_{21}S_{12}| < 1$$

$$\mu = \frac{1 - |S_{22}|^2}{|S_{11} - \Delta S_{22}^*| + |S_{12}S_{21}|} > 1$$

Our example

F(GHz)	S11	S12	S21	S22	k	|Δ|	
.5	0.761∠-151	0.025∠31	11.84∠102	0.429∠-35°	0.482	0.221	Unstable
1	0.770 ∠-166°	0.029∠35	6.11∠ 89	0.365∠-34	0.857	0.173	Unstable
2	0.760 ∠-174°	0.040∠44	3.06∠ 74	0.364∠-43	1.31	0.174	Stable
4	0.756 ∠-179°	0.064∠48	1.53∠ 53	0.423∠-66	1.535	0.226	Stable

Requirements for two-port stability

$$K = \frac{1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1$$

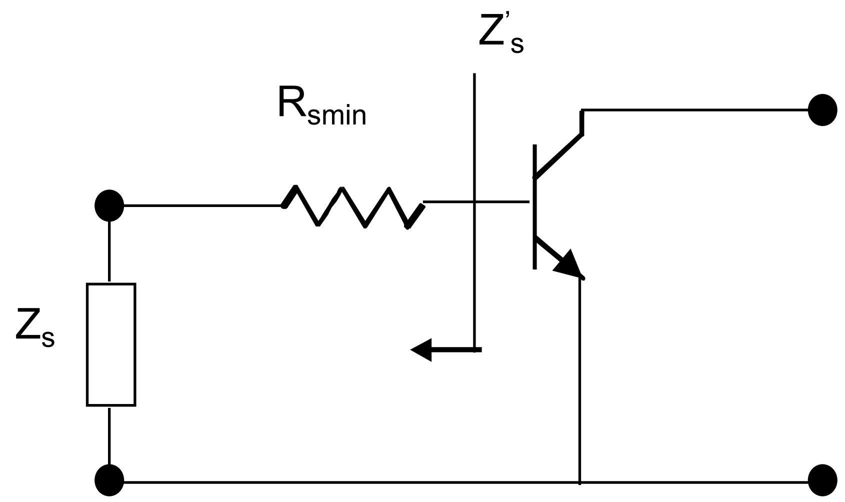
$$B = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 > 0$$

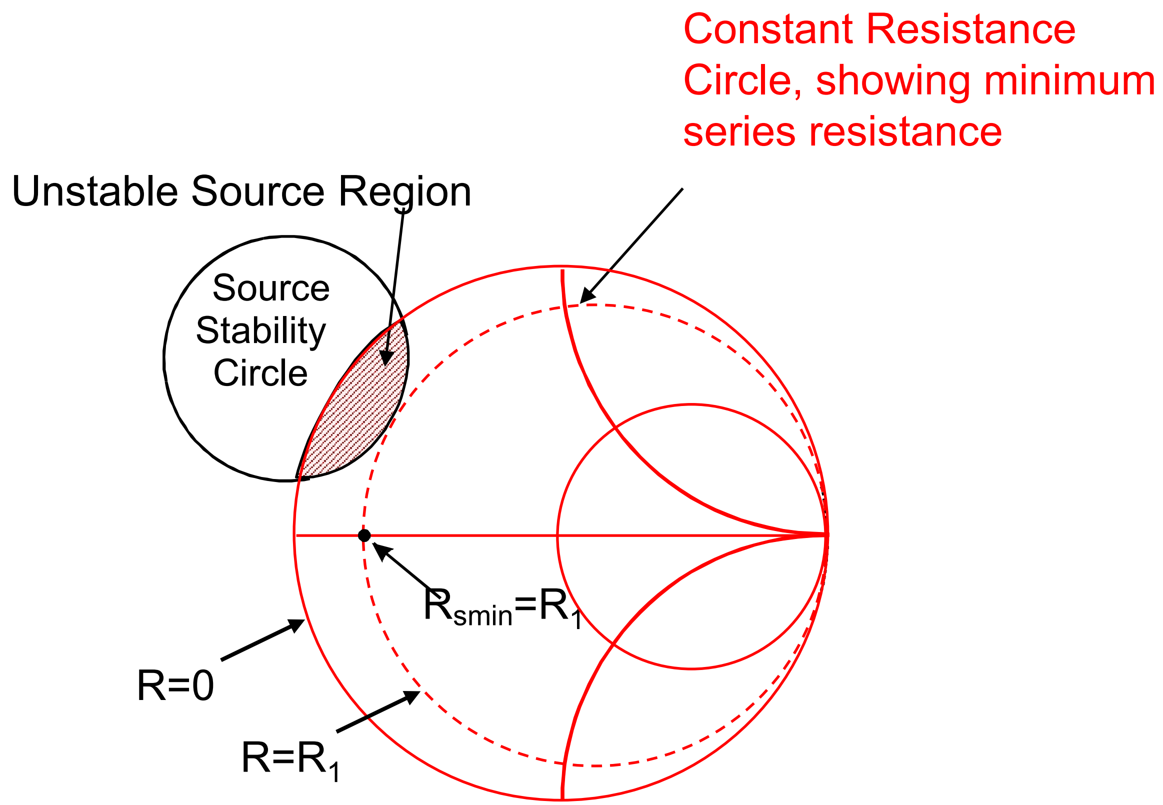
$$|\Delta| = |S_{11}S_{22} - S_{21}S_{12}| < 1$$

$$\mu = \frac{1-|S_{22}|^2}{|S_{11} - \Delta S_{22}^*| + |S_{12}S_{21}|} > 1$$

RF stabilization techniques

- Stabilization is achieved by adding a lossy element to the input or output port.





<u>Series</u> resistive stabilization at the input port shown on the impedance Smith Chart

Our example

F(GHz)	S11	S12	S21	S22
.5	0.761∠-151	0.025∠31	11.84∠102	0.429∠-35°
1	0.770 ∠-166°	0.029∠35	6.11∠ 89	0.365∠-34
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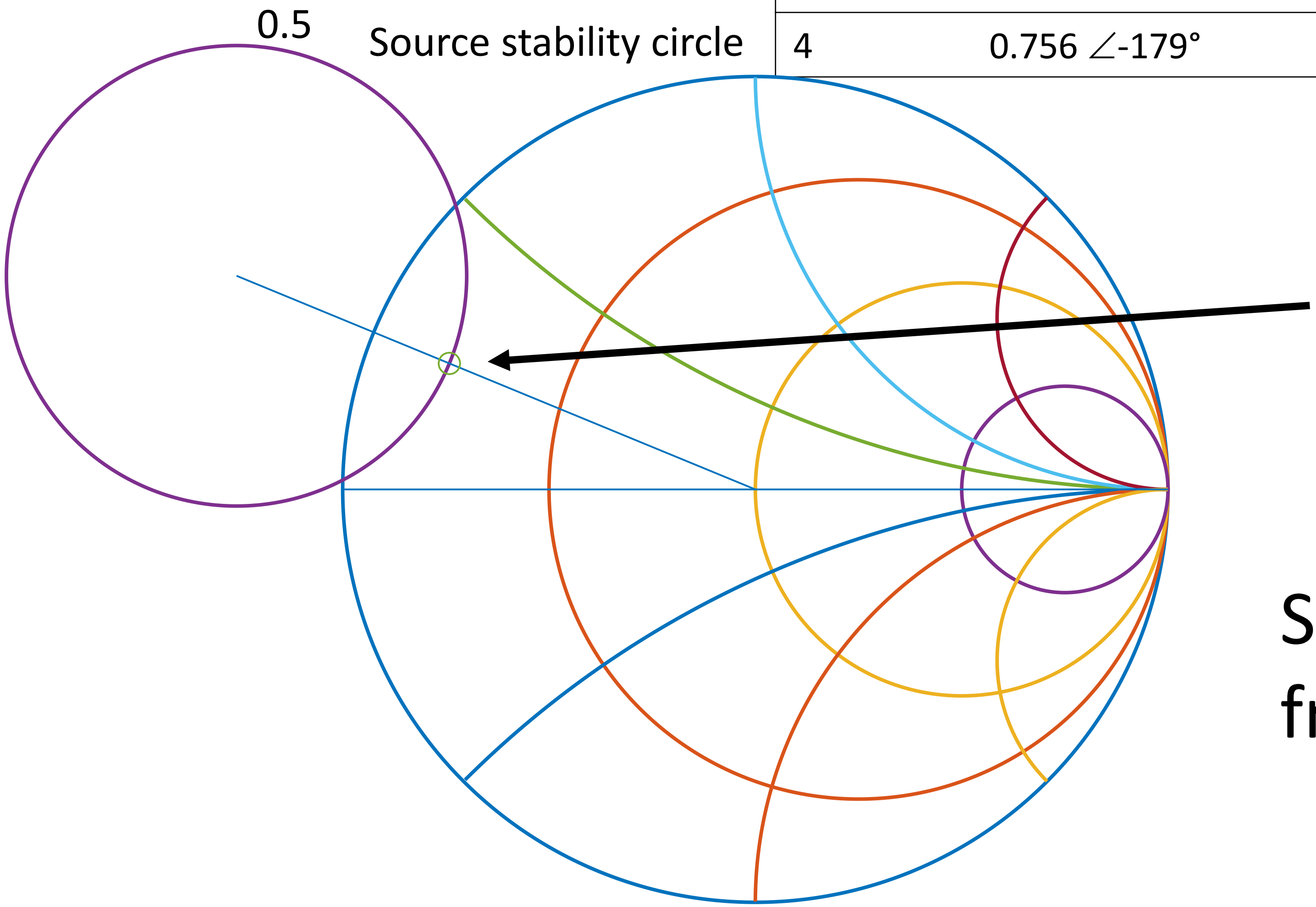


$$\Gamma_{stable}(0.5) = -0.74 + j0.305$$

$$Z_{stable}(0.5) = 5.7 + j9.7$$

Series resistance to stabilize from 500MHz onwards: 5.7Ω

Summary

- Stability circles for source and load
- RF stabilization
- Examples

Microwave Engineering and Antennas

Impedance matching
concept, example

Domine Leenaerts, Professor

Department of Electrical Engineering

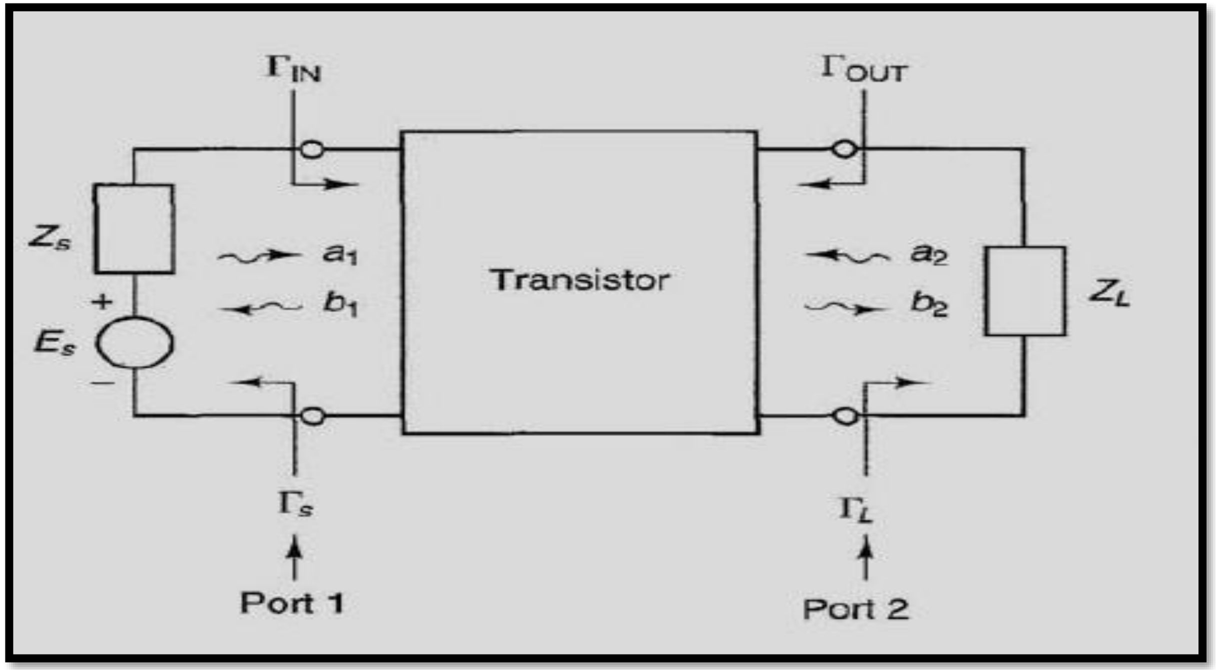
Center for Wireless Technologies Eindhoven

Impedance matching

Objective of this lecture

- Discuss concept of L-matching
- Provide an example for simultaneous conjugate matching

Remember input/output reflection coefficients



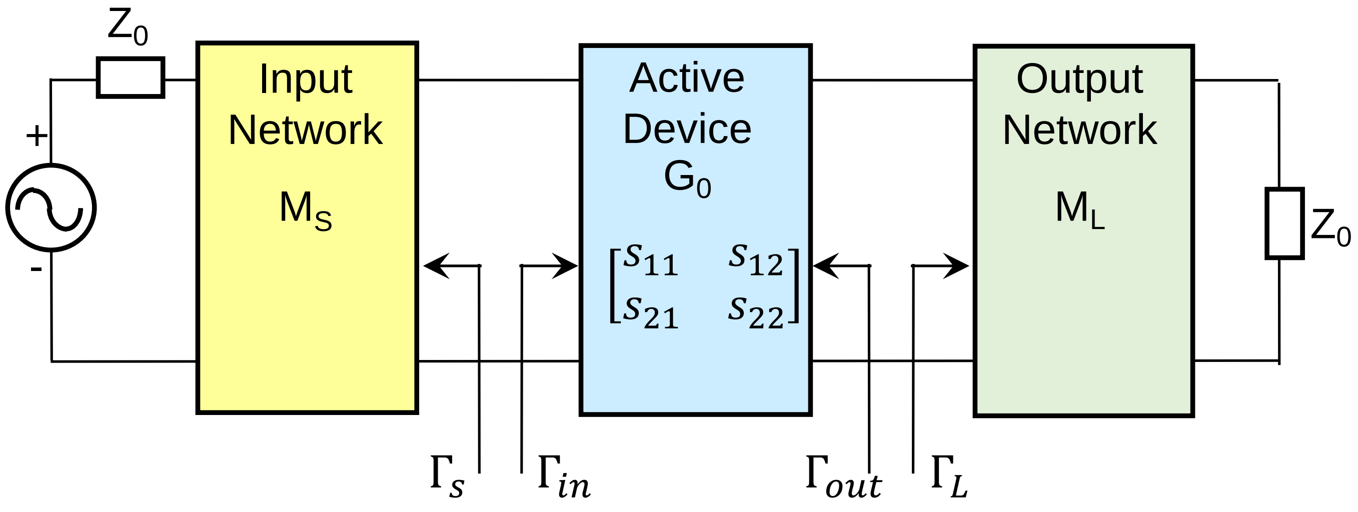
$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}}{\frac{1}{\Gamma_L} - S_{22}}$$

$$\Gamma_{out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12}S_{21}}{\frac{1}{\Gamma_S} - S_{11}}$$

Simultaneous conjugate matching

$$G_{max}(\Gamma_S, \Gamma_L, S) = \frac{P_{av,out}}{P_{del,in}} = \frac{|S_{21}|}{|S_{12}|}\left(K - \sqrt{K^2 - 1}\right), \mathrm{K} \geq 1$$

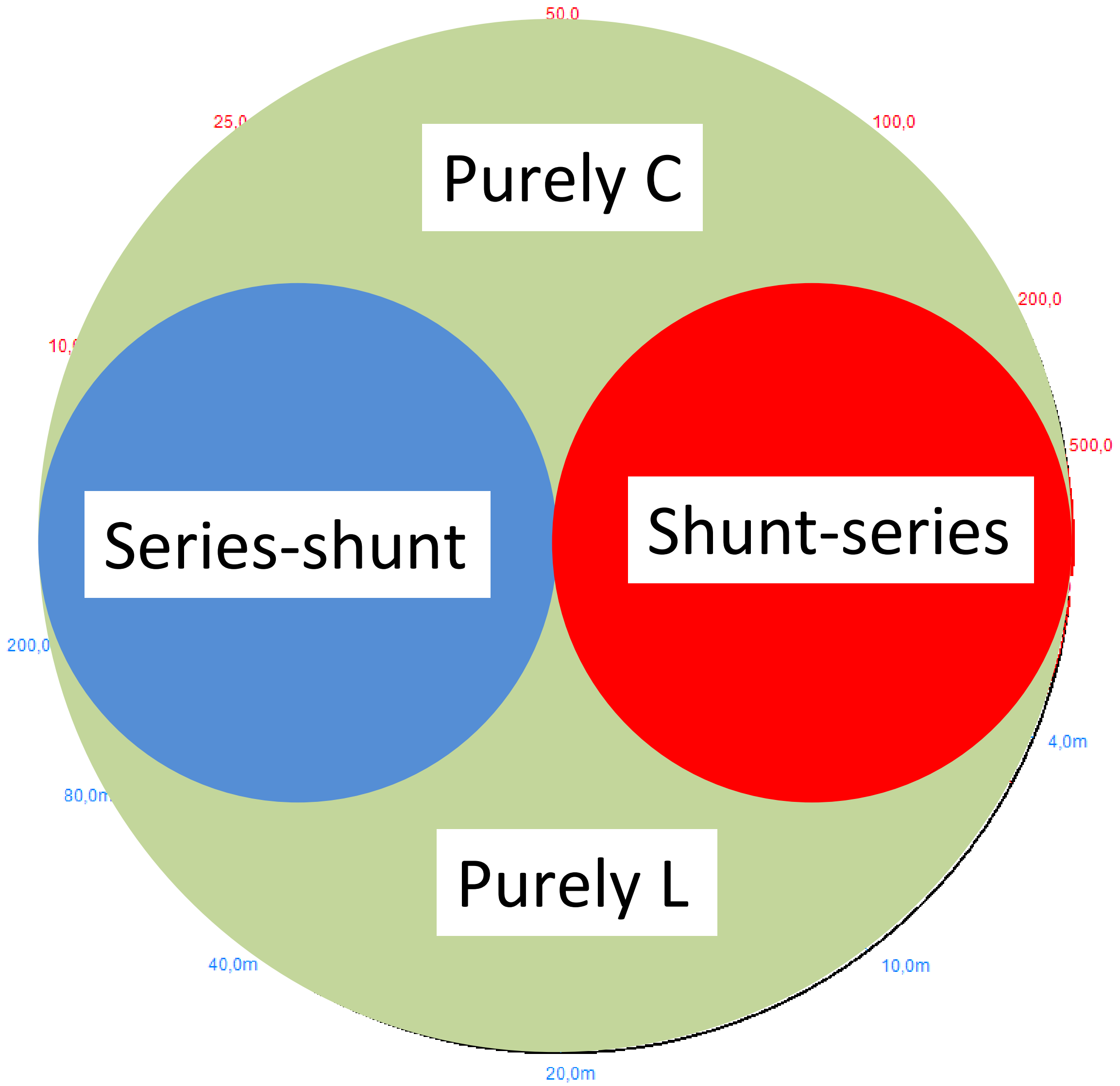
K = Rollet stability factor



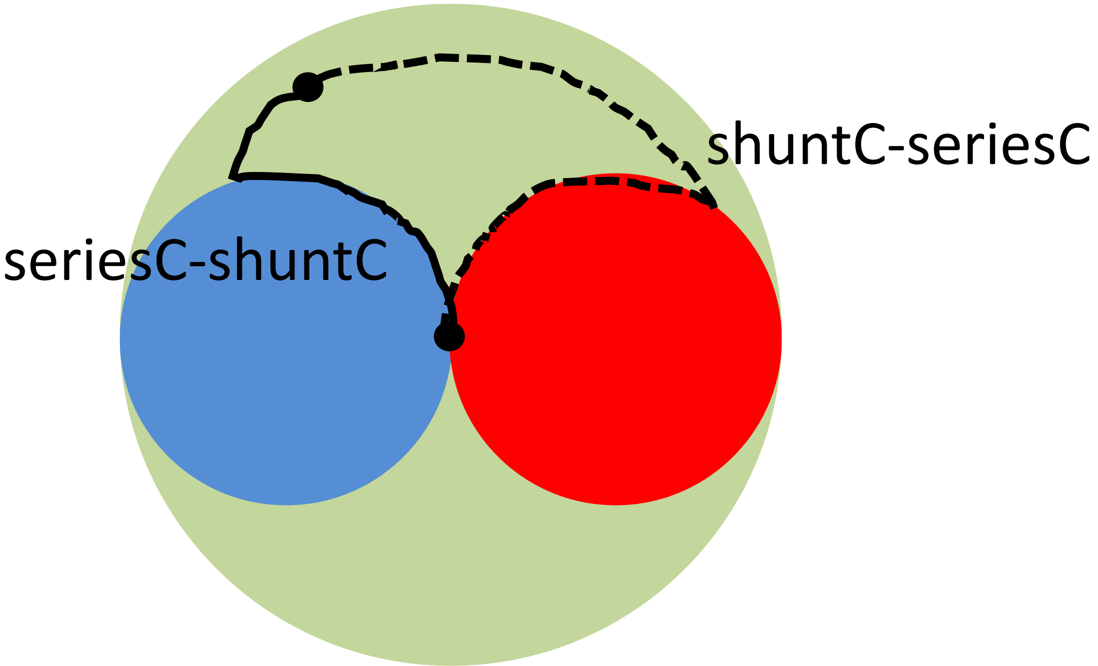
$\Gamma_{in} = \Gamma_S^*$ $\Gamma_{out} = \Gamma_L^*$

How do we find the proper matching networks?

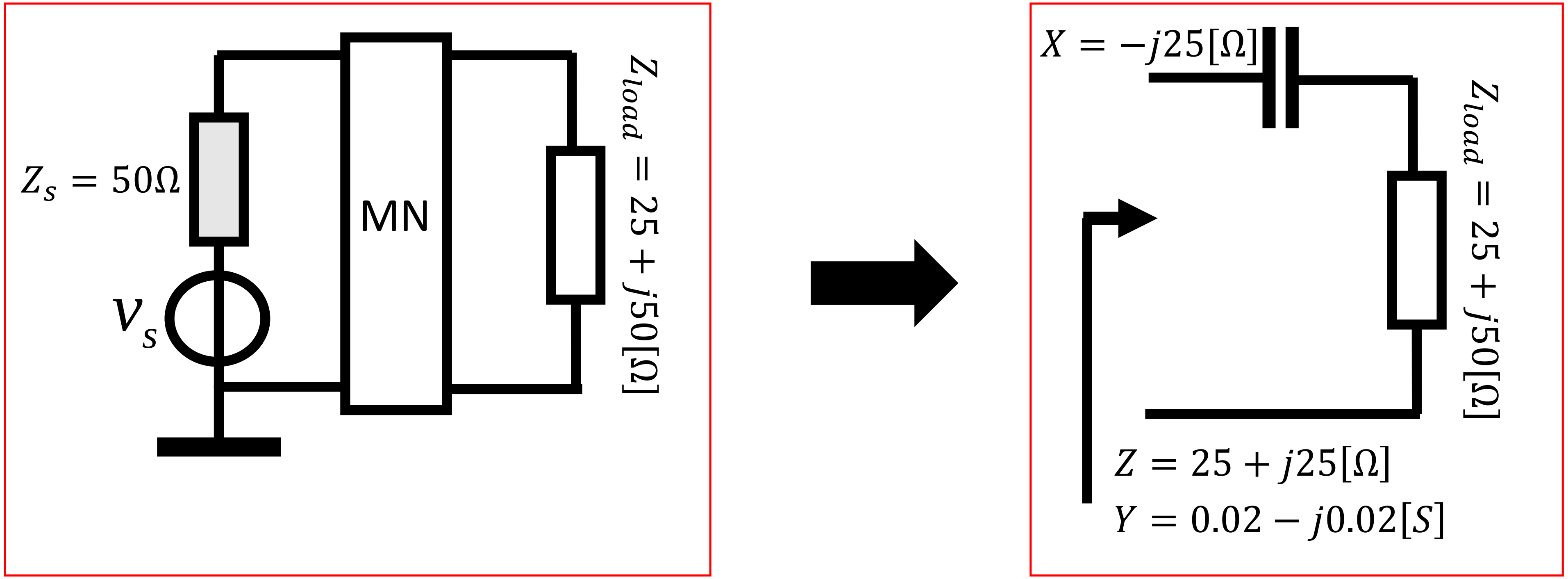
L-match strategy



- From any point inside the Smith chart the center can be reached in two transformation steps
- There are always two solutions

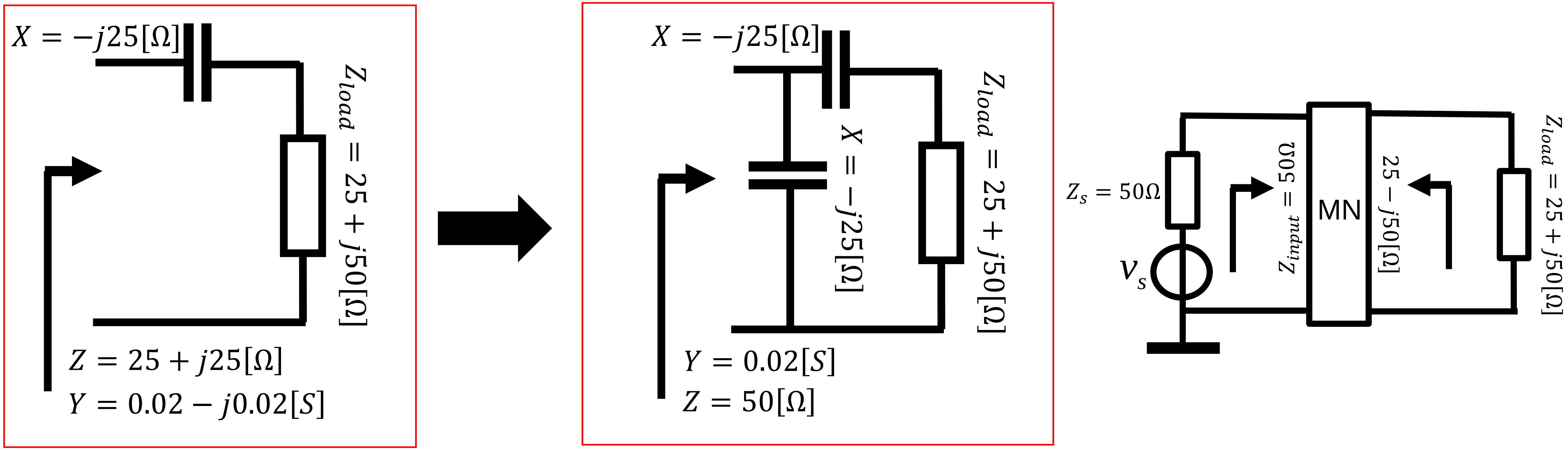


L-match: 2 reactive elements to match



Step 1: find impedance to bring real part of “input admittance” back to 0.02 [S]

L-match: 2 reactive elements to match

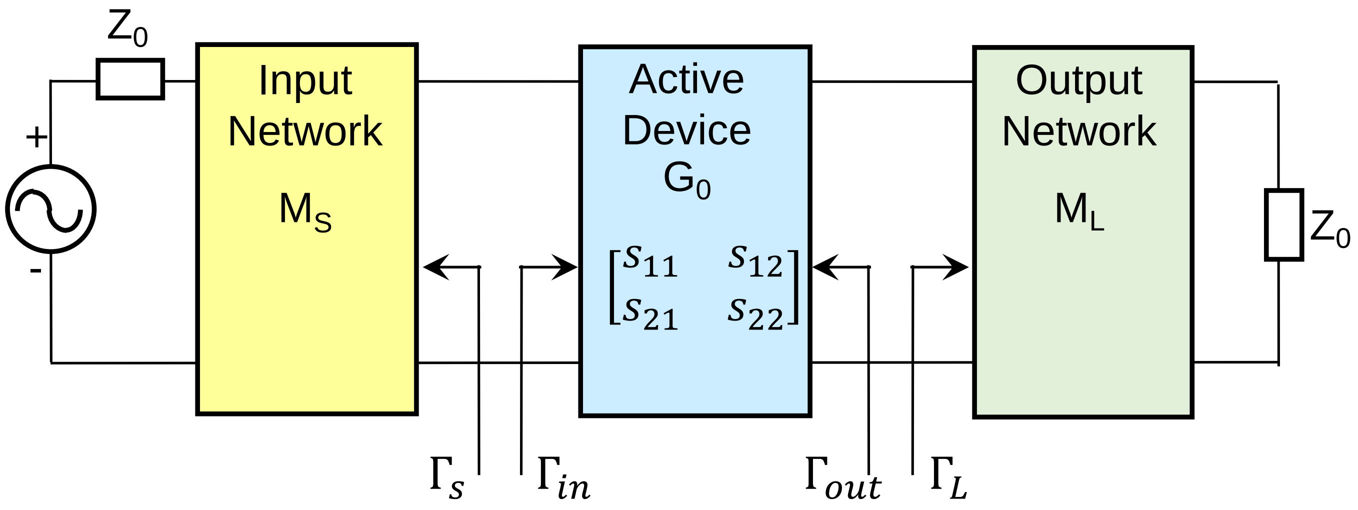


Step 2: find impedance to eliminate remaining reactive part

An example

- An amplifier has the following s-parameter values at 2GHz:

$$S_{11} = 0.61\angle 165^o,\ S_{21} = 3.72\angle 59^o,\ S_{12} = 0.05\angle 42^o,\ S_{22} = 0.45\angle -48^o$$



$$Z_{in} = 5.12 + j7.54[\Omega]$$

$$Z_{out} = 33.68 - j91.48[\Omega]$$

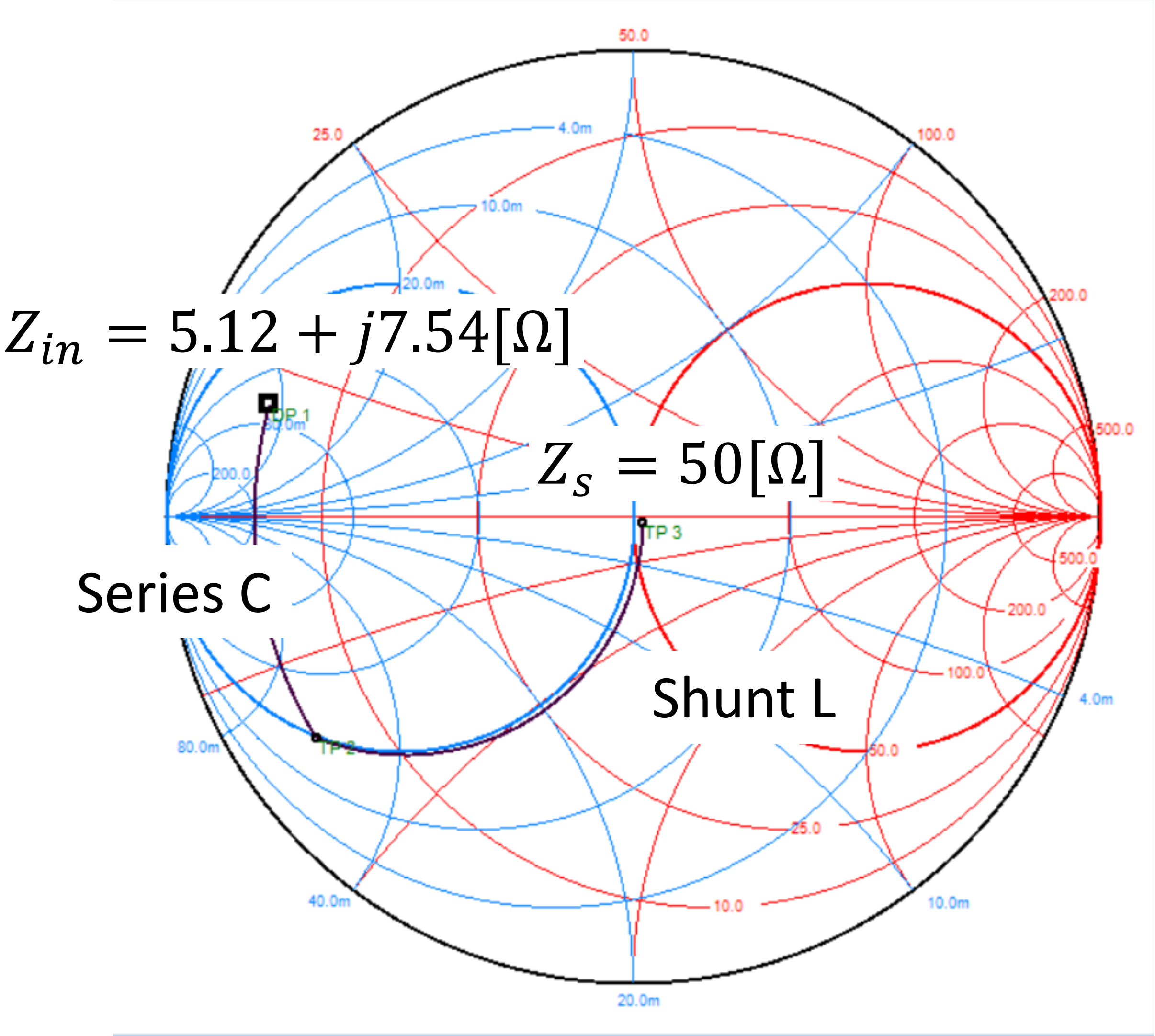
Simultaneous conjugate matching

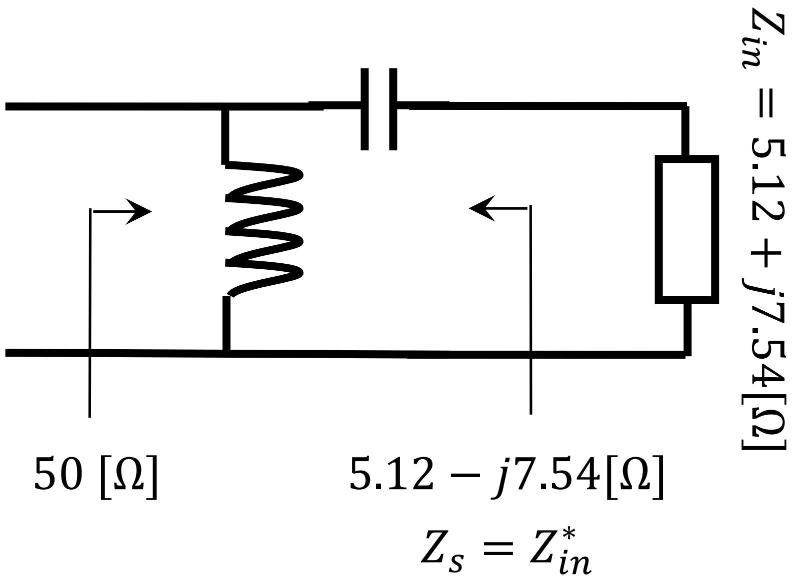
$$Z_s = Z_{in}^* \quad Z_L = Z_{out}^*$$

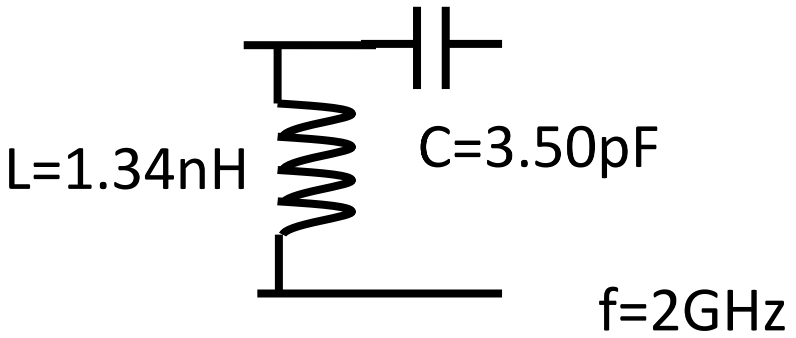
$$\Gamma_S = 0.82\angle -163^o$$

$$\Gamma_L = 0.75\angle\ 52{,}6^o$$

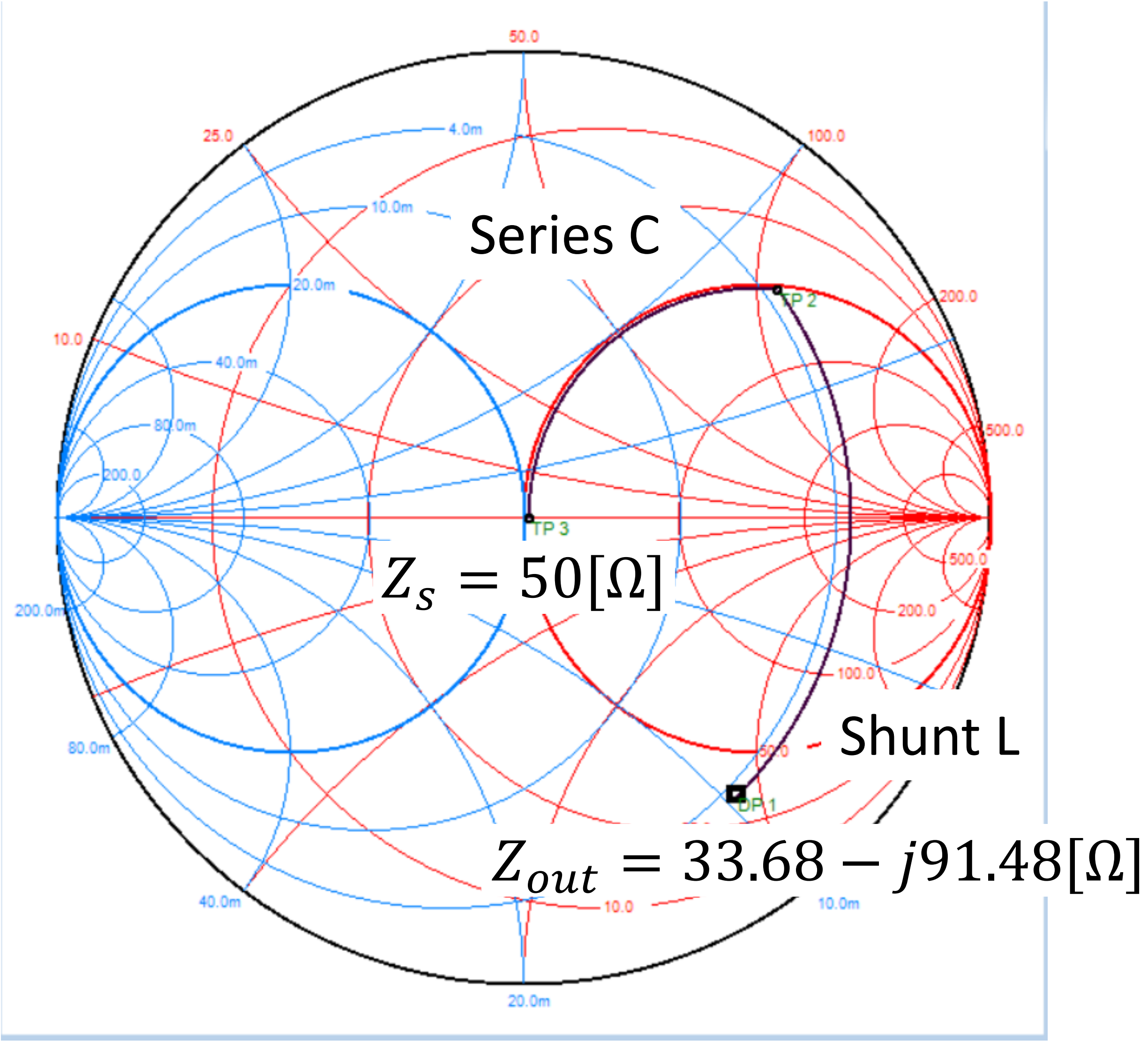
Input matching network M_S

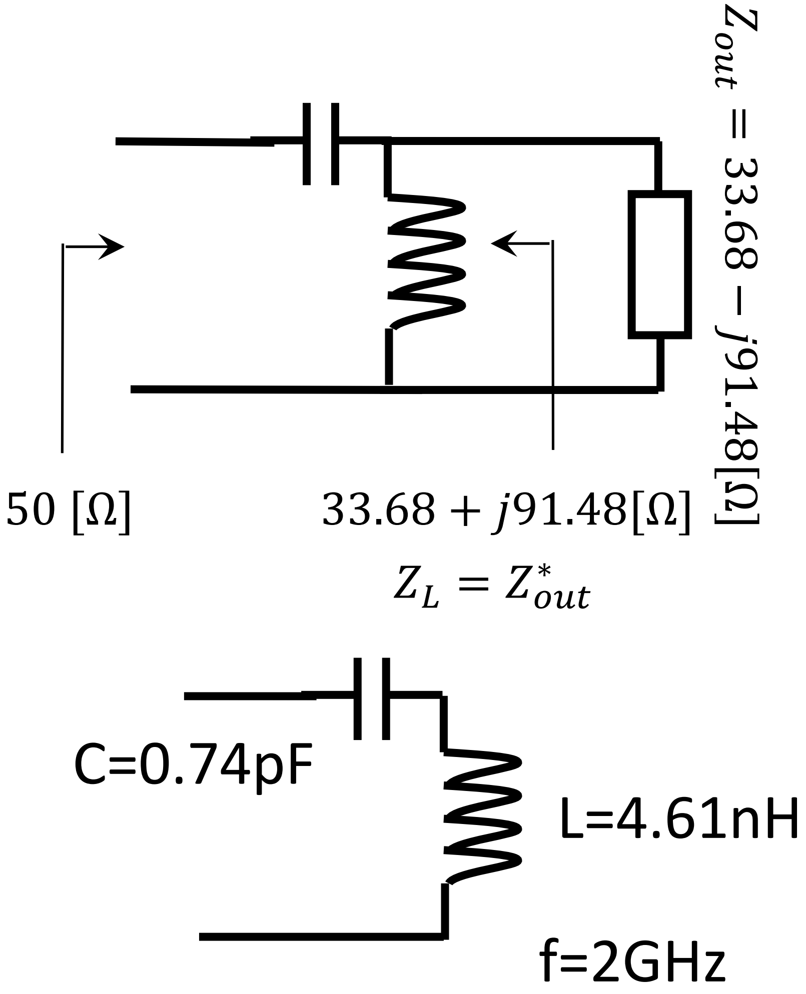




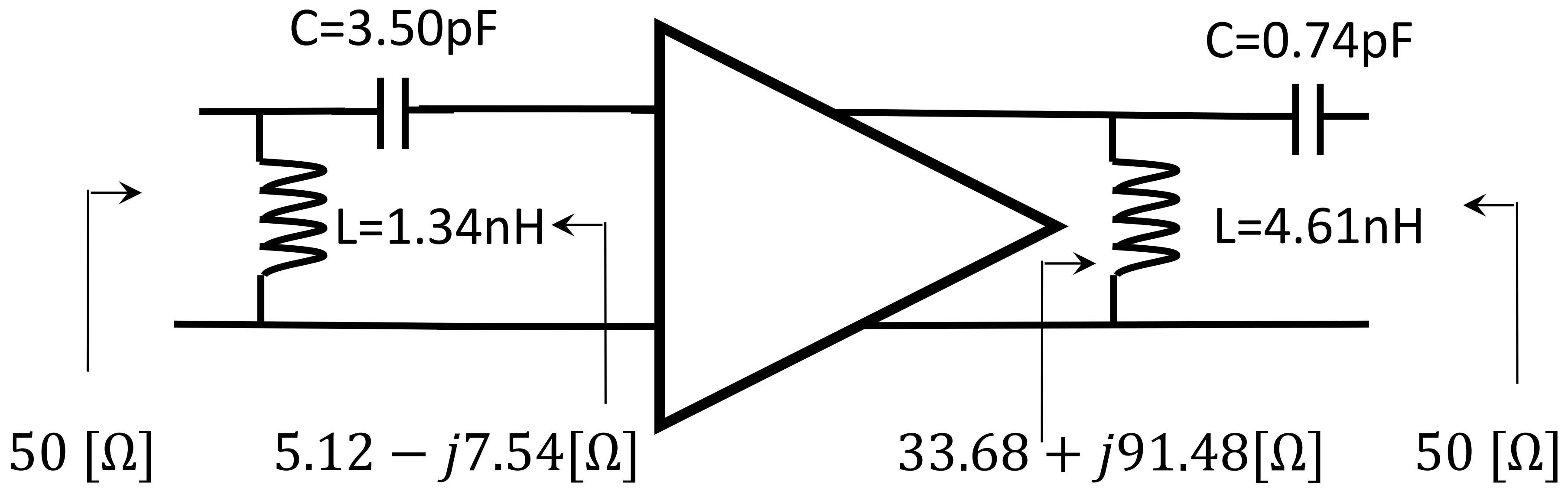


Output matching network M_L





Simultaneous power matched for 2GHz



$$G_{max} = 41.5dB$$

$$K = 1.17$$

Summary

- L-matching using two reactive components
- Simultaneous power matching

Microwave Engineering and Antennas

Constant-gain circles

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Department of Electrical Engineering

Center for Wireless Technologies Eindhoven

Constant-gain circles

Objective of this lecture

- Discuss available gain circle
- Discuss unilateral constant-gain circles
- Provide an example

Circles in the Γ-plane

- Take as example the expression for G_{av} as a function of reflection coefficients and scatter parameters:

$$G_{av}(\varGamma_S, S) = \frac{P_{av,output}}{P_{av,s}} = \frac{1 - |\varGamma_S|^2}{|1 - S_{11}\varGamma_S|^2} |S_{21}|^2 \frac{1}{1 - |\varGamma_{out}|^2}$$

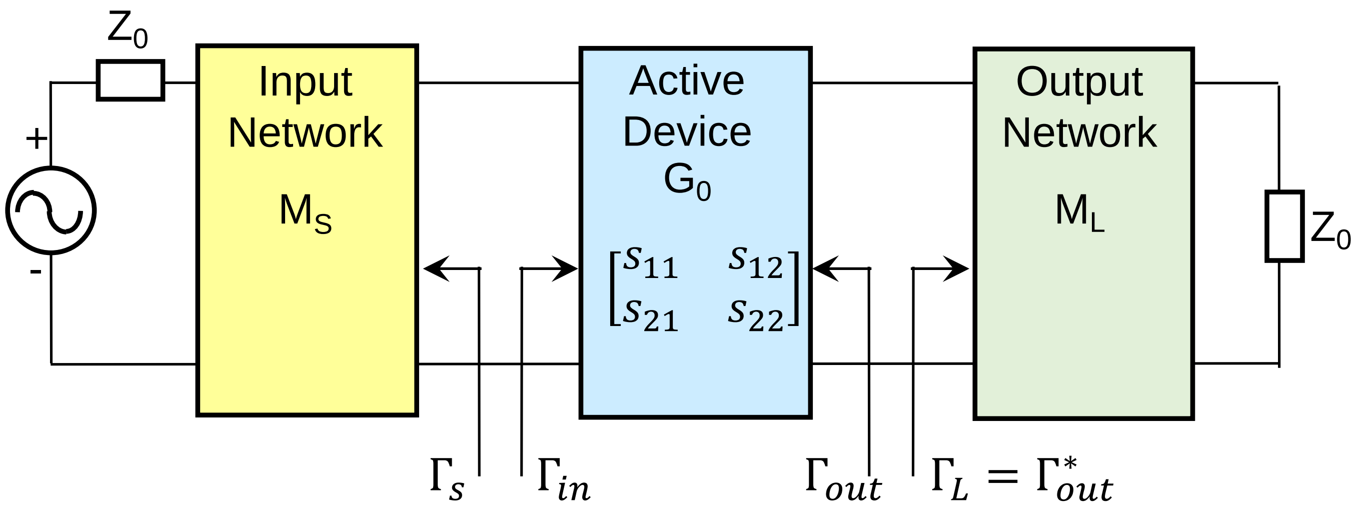
- A certain G_{av} can be realized with many combinations of Γ_S and Γ_{out}
- The locus of such points in the Γ-plane is typically a circle of the form

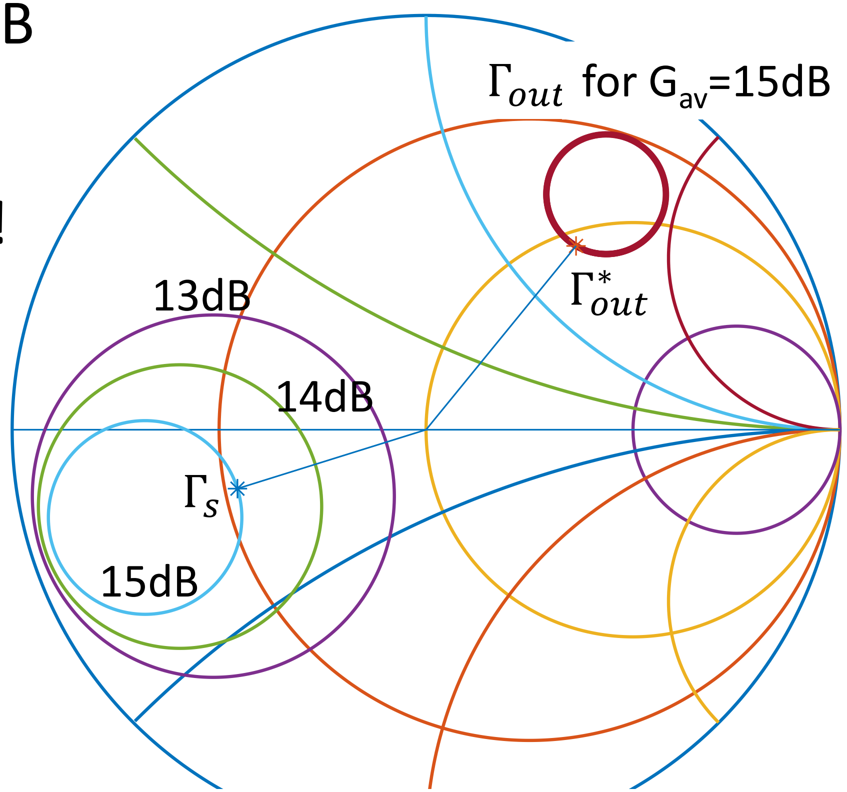
$$|\Gamma - C| = r$$

- with C the center and r the radius of the circle and dependent on the realized gain

Available gain circles

- An amplifier has the following s-parameter values:
  - $S_{11} = 0.61\angle 165^o$, $S_{21} = 3.72\angle 59^o$, $S_{12} = 0.05\angle 42^o$, $S_{22} = 0.45\angle{-48}^o$
- We can plot the Γ_s, Γ_{out} circles for G_{av} = 13, 14 and 15dB
- Note that G_{max} is equal to 16.2dB
- Note that for G_{av}, the output is matched, not the input!





Available power gain circles

Unilateral transducer Gain

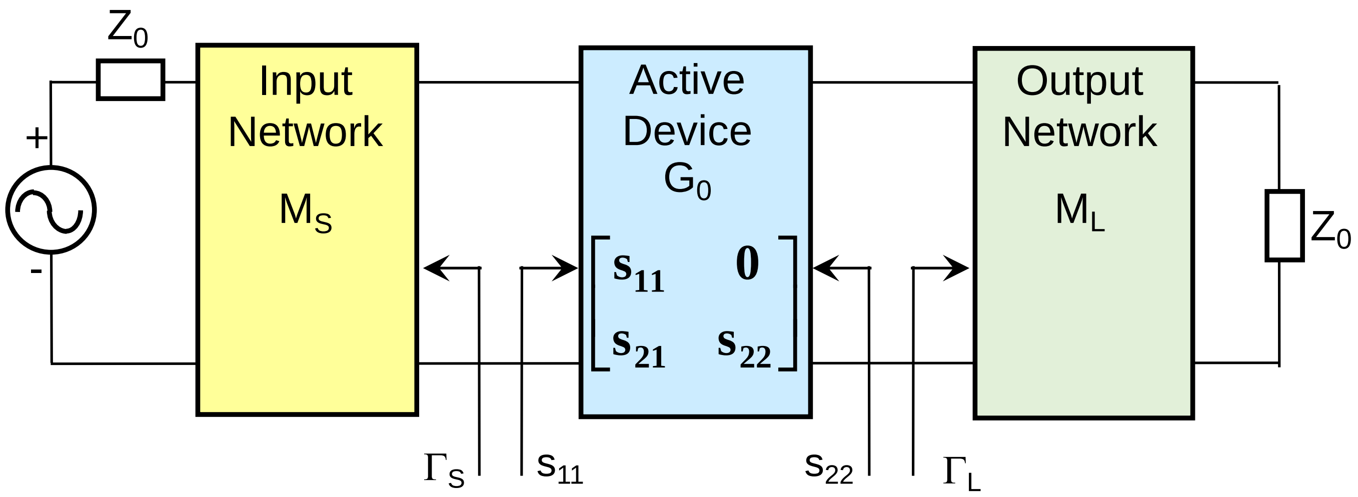
- In the unilateral case, the reverse transmission coefficient is zero ($s_{12} = 0$), yielding for the unilateral transducer (power) gain $G_{T,u}$

$$G_{T,u}(S) = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} |S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = M_S \, |S_{21}|^2 M_L$$

- Maximum gain improvement is observed when one port is conjugately matched. For instance, if the input port is matched, we have

$$M_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} \xrightarrow{\text{yields}} M_{S,max}\Big|_{\Gamma_S = s_{11}^*} = \frac{1}{1-|S_{11}|^2}$$

Unilateral transducer Gain



$$G_{T,u}(\Gamma_S, S) = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = M_S\,|S_{21}|^2 M_L = M_S G_O M_L$$

- The “gain” $|S_{21}|^2$ is defined based on 50Ω termination. As s_{11} and s_{22} are mostly not 50Ω, power loss occurs. Hence the difference between $G_{T,u}(\Gamma_S, S)$ and $|S_{21}|^2$. By optimizing M_S and M_L we do not improve gain, we reduce the losses!

Unilateral constant-gain circle, source

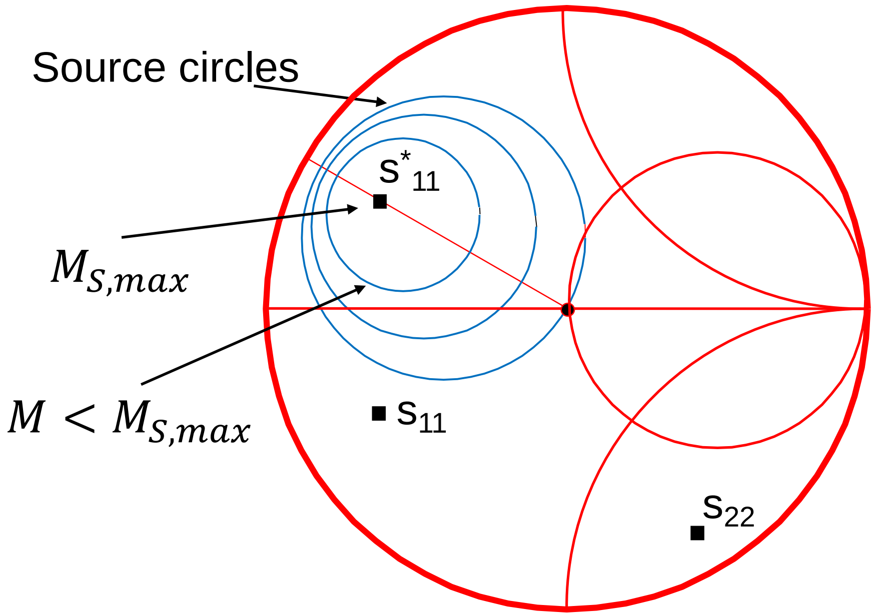
$$M_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}$$

- Equating M_S to a constant, we can solve the unilateral transducer gain for terminations Γ_S, yielding equations of "constant-gain circles" for source terminations:

$$g_S = M_S(1 - |s_{11}|^2)$$ (normalization)

$$C_S = \frac{g_S s_{11}^*}{1 - |s_{11}|^2(1 - g_S)}$$ (center)

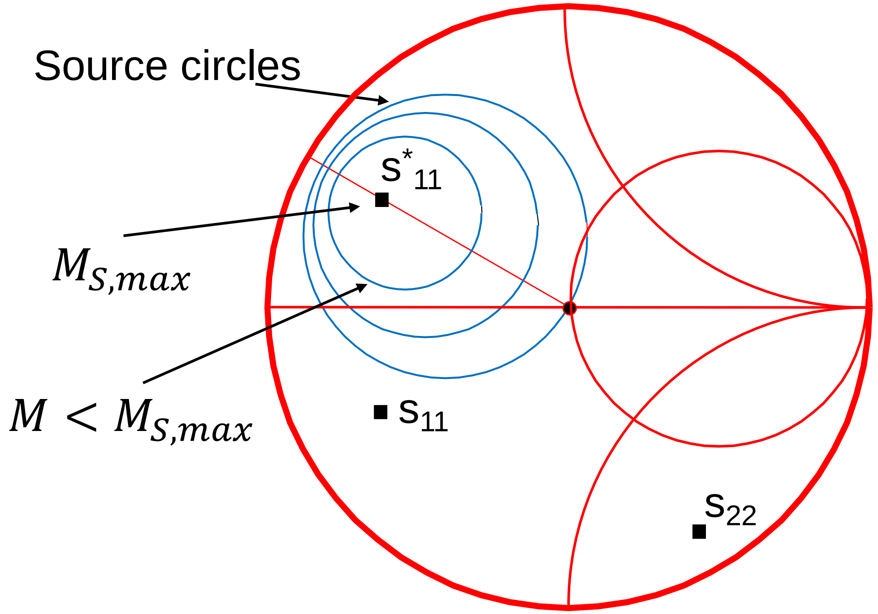
$$r_S = \frac{(1 - |s_{11}|^2)\sqrt{(1 - g_S)}}{1 - |s_{11}|^2(1 - g_S)}$$ (radius)



Unilateral constant-gain circle, source

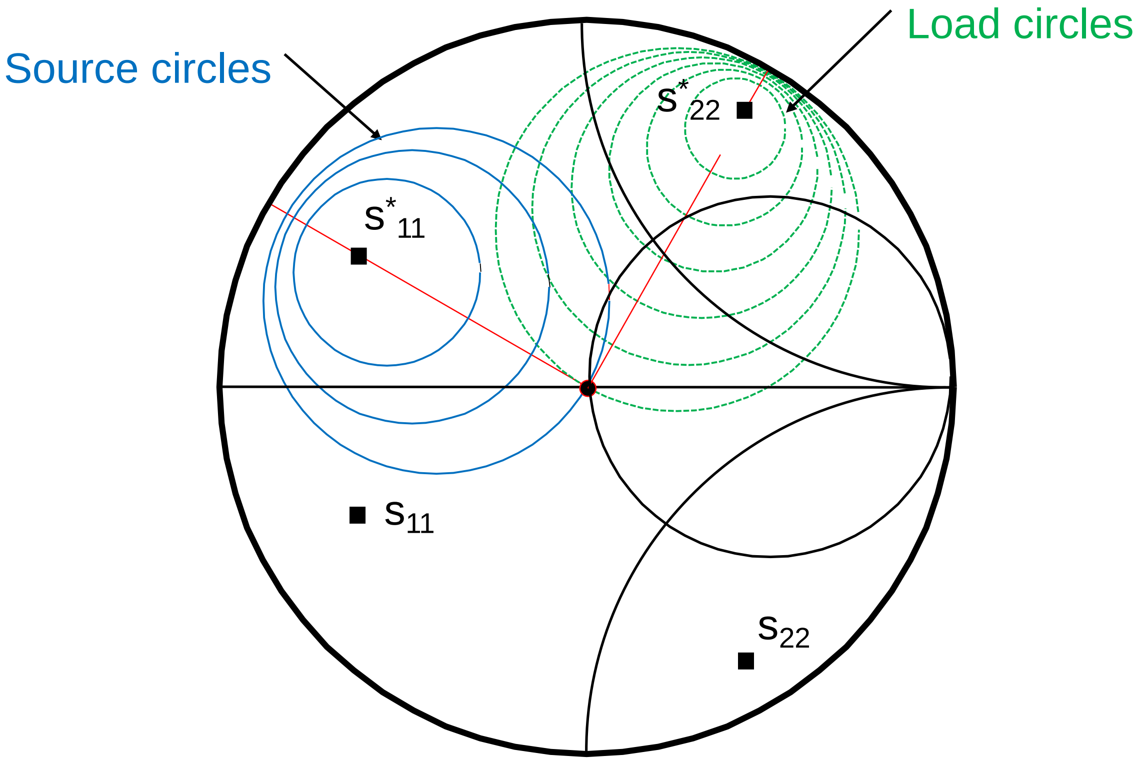
- Some observations:

1. the centers of source plane constant gain circles always lie on a line drawn between the point s_{11}^* and the origin of the source plane.
2. The radius of constant gain circles decreases with increasing gain or reduced losses. The maximum gain is reached for just a single point



Unilateral constant-gain circle

- We can apply the same strategy for the load terminations, simple change $M_s \rightarrow M_L, g_s \rightarrow \; g_L, s_{11} \rightarrow s_{22}, \Gamma_S \rightarrow \Gamma_L$



Example: maximize gain while minimizing NF

- A unilateral amplifier has the following s-parameter values:

$$S_{11} = 0.8\angle 120^o,\ S_{21} = 4\angle 60^o,\ S_{12} = 0,\ S_{22} = 0.2\angle -30^o$$

- For the amplifier also the noise parameters are given:

$$NF_{min} = 1.6dB, r_n = 0.16, \Gamma_{opt} = 0.26\angle -152^o$$

Example: investigate the unilateral gain

- Using

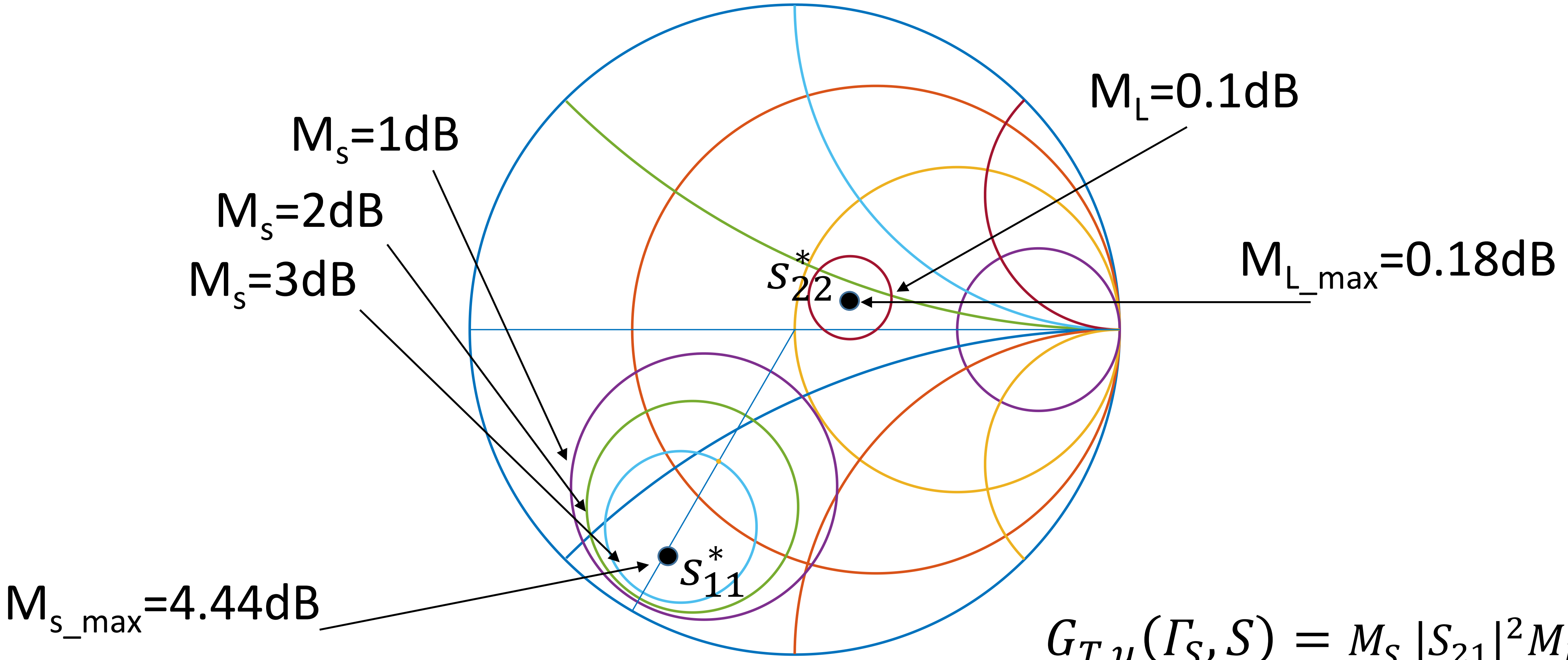
$$G_{T,u}(\Gamma_S, S) = M_S\,|S_{21}|^2 M_L$$

- We find

$$|S_{21}|^2 = 12.04dB\ ,\ M_{S,max} \quad = 4.44dB\ ,\ M_{L,max} \quad = 0.18dB$$

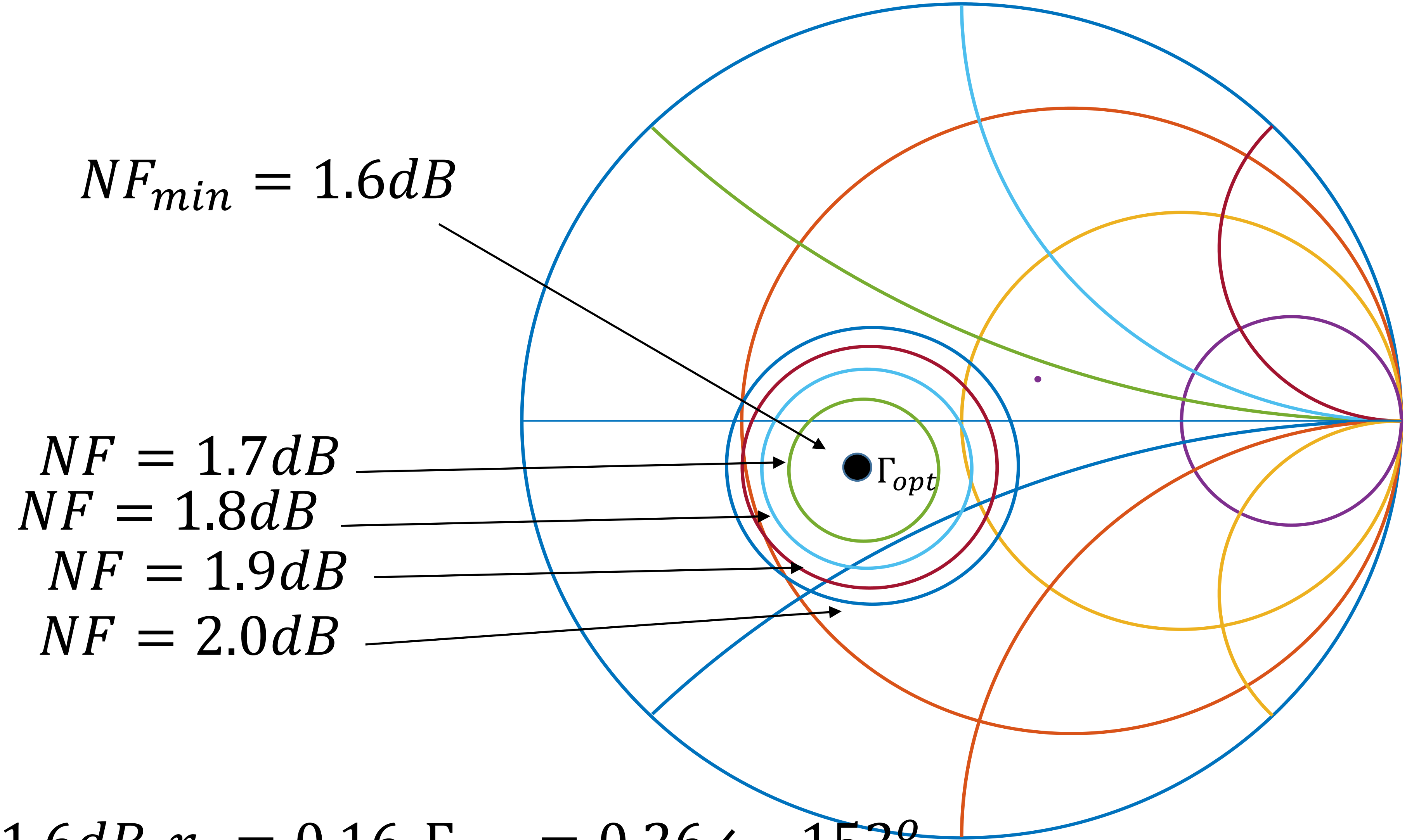
1. The maximum unilateral gain is therefore 4.44+12.04+0.18 = 16.66 dB
2. There is not much loss in the output. In other words, the ouput impedance is close to 50Ω.
3. We would like to keep the matching losses at the input to a minimum, but also have a good noise figure.

Example: investigate the unilateral gain



$$G_{T,u}(\Gamma_S, S) = M_S\,|S_{21}|^2 M_L$$

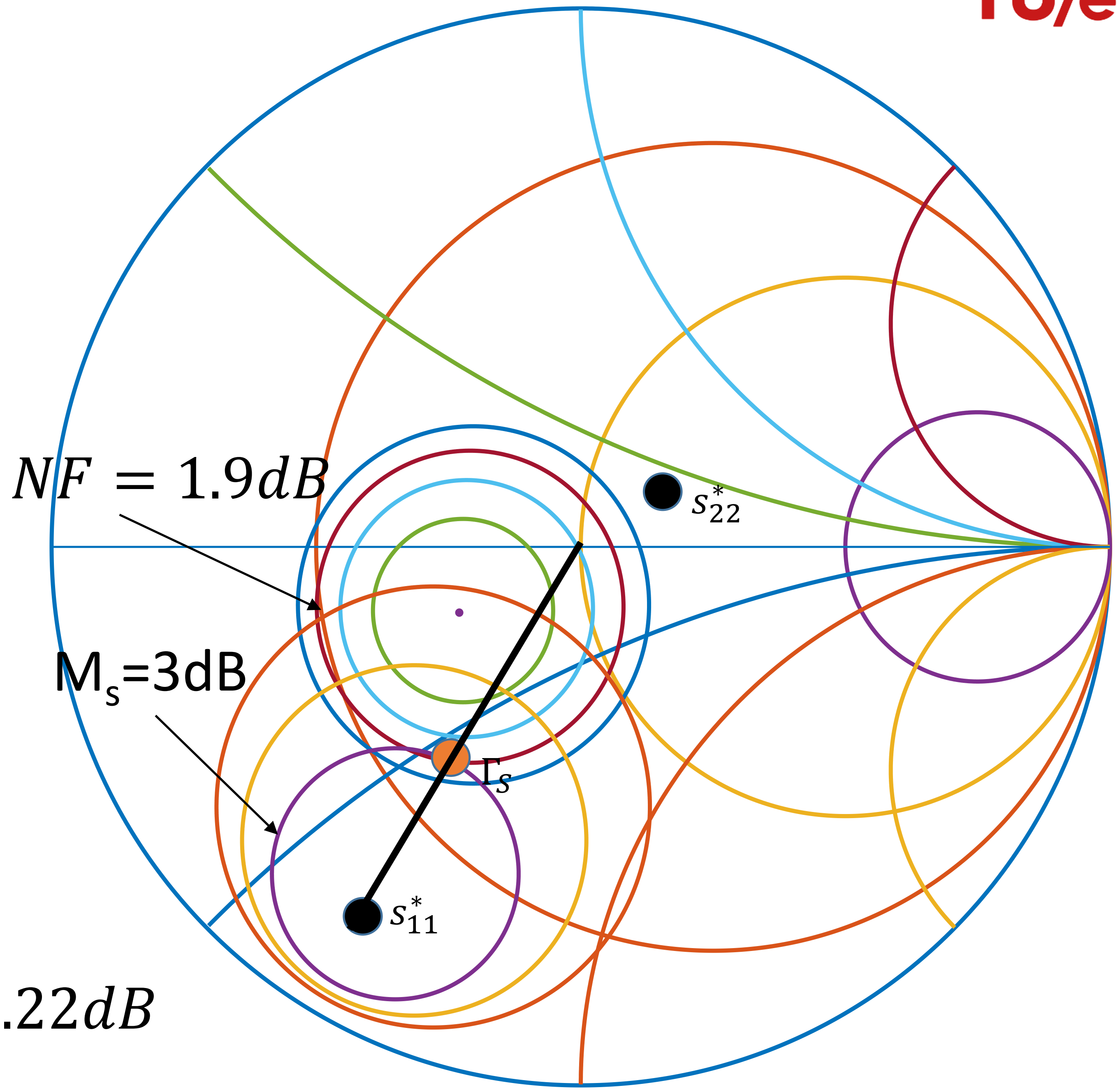
Example: plot the noise circles



$NF_{min} = 1.6dB, r_n = 0.16, \Gamma_{opt} = 0.26\angle -152^o$

Example: choose M_s

Plotting the noise circles and the gain circles in the same Smith chart reveals that $M_S = 3dB$ is a good trade off between maximizing the gain and minimizing the noise figure.



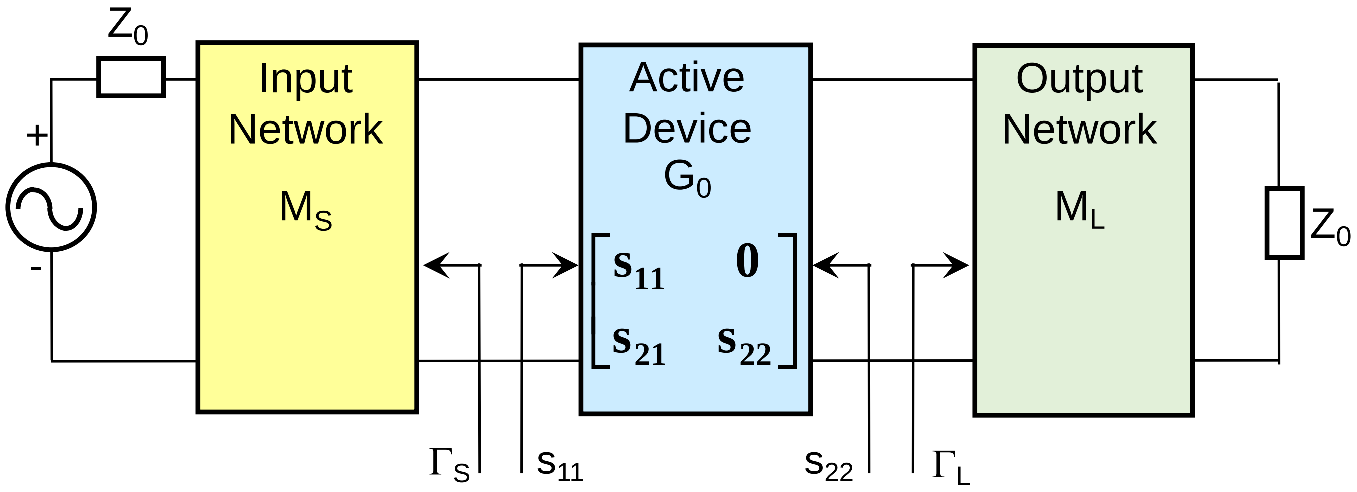
$$G_{T,u}(S) = 3 + 12.04 + 0.18 = 15.22dB$$

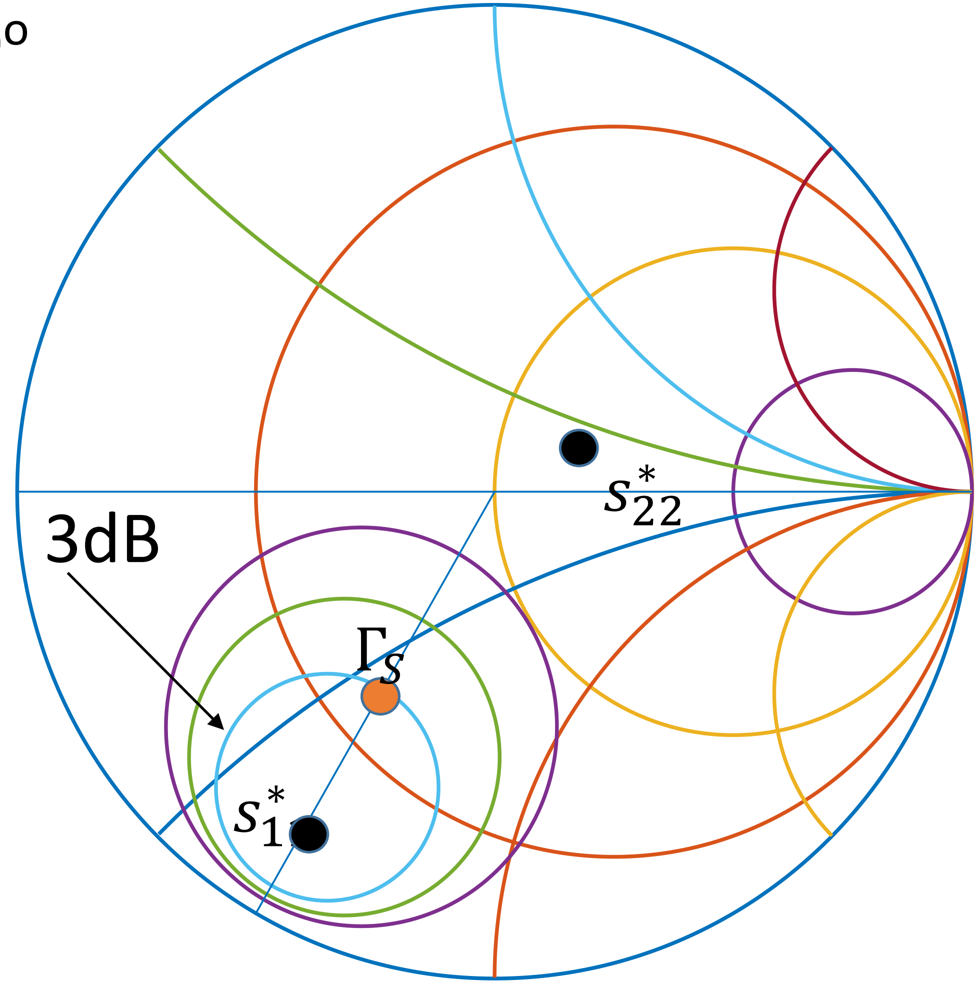
Example: input matching network

- With $M_S = 3dB$, we have $\Gamma_S = 0.468\angle{-120^o}$
- This leads to the source impedance

$$Z_S = 23.15 - j24\ \Omega$$

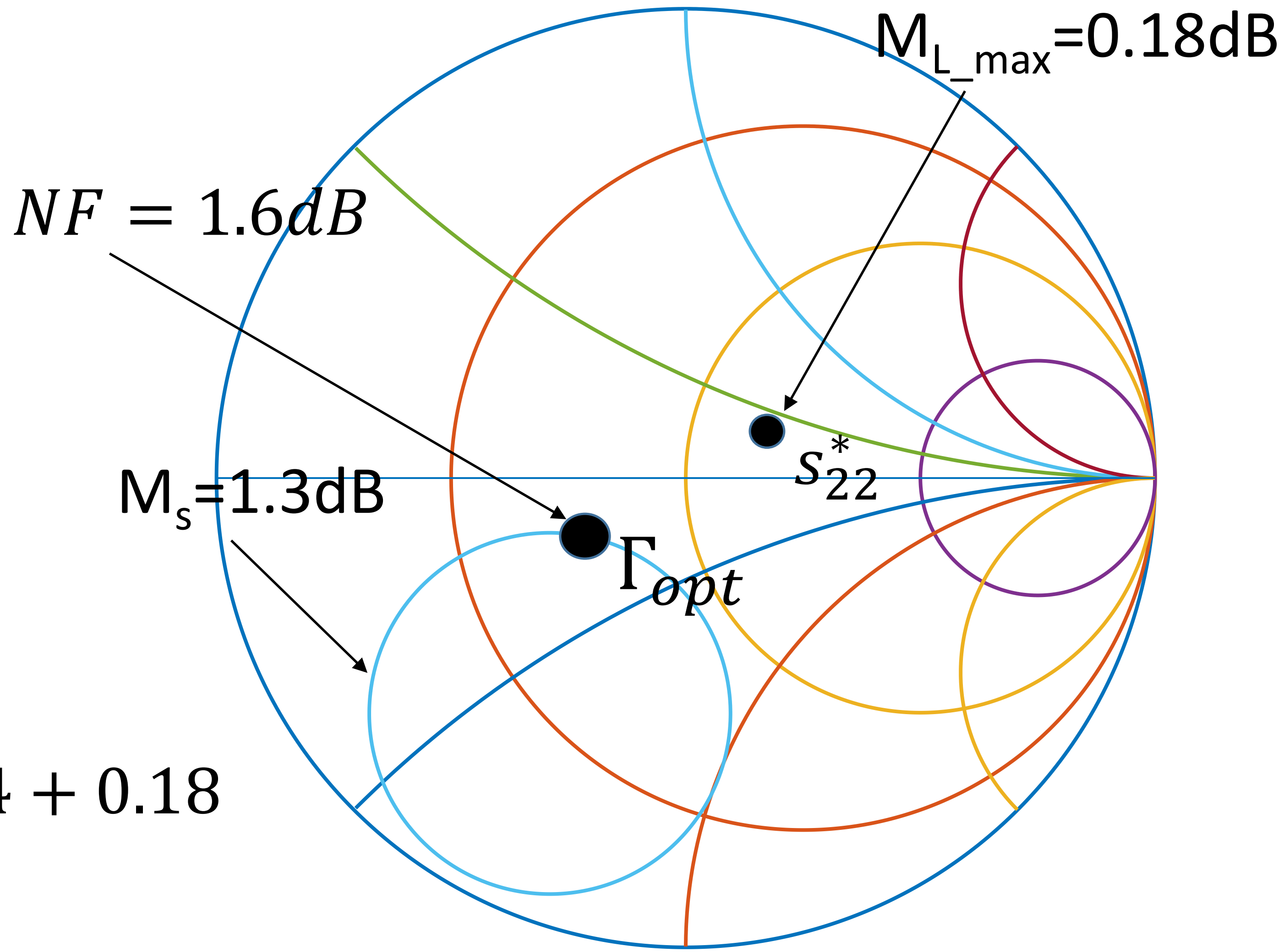
- Which we then need to transform to 50Ω





What about G_{av} at optimum noise match?

- The M_s = 1.3dB intersects with Γ_{opt}.
- For G_{av} the output is conjugately matched,



$$G_{T,u}(S) = G_{av,opt} = 1.3 + 12.04 + 0.18$$
$$= 13.52dB$$

Summary

- Discussed available gain circle
- Discussed unilateral constant-gain circles
- Example to optimize between unilateral gain and noise figure