MATLAB Assignment 3

Due Tuesday November 15 at 11:59 PM EDT (Maryland time) on Gradescope

Instructions:
On ELMS, see the file MATLAB_basics.pdf to learn how to get MATLAB and do some basic commands first. You may work with up to two other people (**groups of three total**). If you choose to work together, you may simply submit **one copy, and everyone will be receiving the same grade**. **Make sure to include all names when submitting to Gradescope!**

Submitting: To get an idea of what you should be submitting, you can first download the file `Example_Matlab_File.m` in the Files section. Open it in Matlab. Then at the top of the program, click on the PUBLISH tab. Click on the Publish button, and it should output an html file with all the code/output. This format is what your Matlab assignment should look like. When you are done with the actual Matlab project, click the Publish button, save this as a PDF, and **upload this to Gradescope.** There is a tab on ELMS that links you to Gradescope. **Remember to separate each problem by a section using the double percent signs.** Even if you have the correct code, **if there is no output, you will NOT receive full credit!**

(separate problems by using double percent signs as shown in the example file!!!!)

1. Use `format rat`. Define

$$A = \begin{bmatrix} 3 & -8 & 22 & -8 & 2 \\ 6 & -8 & -2 & -8 & 44 \\ -3 & 3 & 15 & 3 & -27 \\ -1 & 1 & 6 & 1 & -9 \\ 0 & 5 & -7 & 5 & 25 \\ 4 & 2 & -3 & 2 & 66 \end{bmatrix}.$$

   (a) Use the `rref` command and then determine a basis for the column space and the kernel for matrix A. You can use *disp* or *fprintf* to show your answer. For simplicity, you may express the vectors using parentheses like it is done in class.

   (b) Suppose A is now the **augmented matrix** of a system of equations. What is the solution to the system? If it is unique, state it. If there are infinite solutions, describe it in parametric form. If there are no solutions, explain why.

2. Use `format rat`. Define

$$A = \begin{bmatrix} 5 & -3 & 6 & 7 \\ 0 & 0 & 0 & 1 \\ 7 & -5 & 3 & -7 \\ 4 & 0 & 3 & -7 \end{bmatrix}.$$

(a) Find a basis for the row space of A. Use *disp* or *fprintf*, and **express the vectors as row vectors using brackets** e.g. $\{[3\ \ 4\ \ 6\ \ 1], [1\ \ 3\ \ 5\ \ 7]\}$

(b) Find a basis for the column space of A. Use *disp* or *fprintf*. For simplicity, you may express the vectors using parentheses like it is done in class.

(c) Does $\dim(\text{row}(A)) = \dim(\text{col}(A))$? Simply state yes or no.

(d) Does the set $\text{row}(A) = \text{col}(A)$? Use *disp* or *fprintf* to **briefly explain.**

3. Use `format rat`. Consider the set

$$\mathcal{B} = \{(4, 1, -5, 5), (2, 6, 4, -7), (4, 3, 0, 1), (-8, 6, 7, 7))\}$$

Let $\mathbf{v} = (2, -30, 13, -10)$.

(a) Use appropriate Matlab commands to find $\mathbf{u}$ where $[\mathbf{u}]_{\mathcal{B}} = \mathbf{v}$.

(b) Use appropriate Matlab commands to find $\mathbf{w}$ where $\mathbf{w} = [\mathbf{v}]_{\mathcal{B}}$.

Make sure to show your computations, and use *disp* or *fprintf* to briefly explain what you are doing.

4. Use `format rat`. Consider the polynomials (**read carefully**)

$$f_1(x) = 7 - 3x + x^2 + 7x^3 + 2x^4$$

$$f_2(x) = 9 - 3x - 9x^2 - 5x^3 - 6x^4$$

$$f_3(x) = 1 - x + 3x^2 + 4x^3 + 3x^4$$

$$f_4(x) = 5 - 3x - x^2 + x^4$$

that lie in $\mathbb{P}_4$. Let

$$W = \text{Span}(\{f_1(x), f_2(x), f_3(x), f_4(x)\}).$$

(a) Denote each of the 4 vectors $\mathbf{v}_i = [f_i(x)]_{\mathcal{B}}$ to be the coordinate vector of $f_i(x)$ relative to the basis $\mathcal{B} = \{1, x, x^2, x^3, x^4\}$ in $\mathbb{P}_4$. Define these as **column** vectors in Matlab as `v1, v2,`...

(b) Define $A =$ [`v1  v2  v3`...] (matrix whose i-th column is the coordinate vector $\mathbf{v}_i$).

(c) Use appropriate commands with the matrix above to help find a basis `S` for W. Use *disp* or *fprintf* to explicit show what the basis is as polynomials.

(d) Is the set $\{f_1(x), f_2(x), f_3(x), f_4(x)\}$ linearly independent or linearly dependent? If it is linearly dependent, find an explicit dependent relationship. If it is linearly indepedent, use *disp* or *fprintf* to justify your answer. If you write out a combination, you can simply use, for example $f_1(x)$, and not rewrite the entire polynomial out.

5. Use `format short`. We first show the set of vectors

$$\mathcal{B} = \{1, \cos(x), \cos^2(x), \cos^3(x), \cos^4(x)\}$$

is linearly independent over the vector space of real-valued functions. That is, we want to show the equation

$$a_0(1) + a_1\cos(x) + a_2\cos^2(x) + a_3\cos^3(x) + a_4\cos^4(x) = \hat{0} \tag{1}$$

is true only when $a_0 = a_1 = a_2 = a_3 = a_4 = 0$ (where $\hat{0}$ denotes the zero function that maps everything to 0). Note that (1) must hold for all values of x. If the set of functions was linearly dependent, then for any dependent relation, say $3 - 4\cos(x) + 12\cos^2(x) - 2\cos^3(x) + 2\cos^4(x) = \hat{0}$ **(this is not a true equation)**, we should still observe linear dependency for explicit values of x e.g. if we plugged in $x = \pi/23$.

(a) By subbing in a value for x in (1), we get a linear equation in terms of variables a_0, a_1, a_2, a_3, and a_4. Sub in $x = 0.1, 0.2, 0.3, 0.4, 0.5$ to create a linear system of 5 equations and 5 unknowns. Define its coefficient matrix by A in Matlab.

(b) Observe the system has a non-trivial solution if and only if (1) would have a non-trivial solution. Use appropriate Matlab commands to conclude the system only has a trivial solution, thus implying $\mathcal{B}$ is a linearly independent set of vectors.

(c) Let $\mathcal{C} = \{1, \cos(x), \cos(2x), \cos(3x), \cos(4x)\}$. We have the following identities:

$$\cos(2x) = -1 + 2\cos^2(x)$$

$$\cos(3x) = -3\cos(x) + 4\cos^3(x)$$

$$\cos(4x) = 1 - 8\cos^2(x) + 8\cos^4(x).$$

With the help of the identities, determine the $\mathcal{B}$-coordinate vectors for the 5 vectors in $\mathcal{C}$. Define these as **column** vectors in Matlab as `u1, u2,`...

(d) Using part (c), define an appropriate matrix B and use Matlab commands to determine if $\mathcal{C}$ is a linearly independent set or not. Use *disp* or *fprintf* to briefly explain your answer.

(e) If $D = \text{Span}(\mathcal{B})$, explain why $\mathcal{C}$ forms a basis for D.

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Problem 1

```
format rat;
A = [3 -8 22 -8 2;
     6 -8 -2 -8 44;
     -3 3 15 3 -27;
     -1 1 6 1 -9;
     0 5 -7 5 25;
     4 2 -3 2 66];
% Part a
rref(A)
disp("Basis of Col(A): {(3, 6, -3, -1, 0, 4), (-8, -8, 3, 1, 5, 2), ")
disp("(22, -2, 15, 6, -7, -3)}")
disp("Basis of Ker(A): {(0, -1, 0, 1, 0), (-14, -5, 0, 0, 1)}")
% Part b
disp("There are infinite solutions, as the fourth column (x4) is a free")
disp("variable")
disp("Parametric Vector Form: x4(0, -1, 0, 1, 0) + (-14, -5, 0, 0, 1)")
```

```
ans =

       1              0              0              0             14
       0              1              0              1              5
       0              0              1              0              0
       0              0              0              0              0
       0              0              0              0              0
       0              0              0              0              0

Basis of Col(A): {(3, 6, -3, -1, 0, 4), (-8, -8, 3, 1, 5, 2),
(22, -2, 15, 6, -7, -3)}
Basis of Ker(A): {(0, -1, 0, 1, 0), (-14, -5, 0, 0, 1)}
There are infinite solutions, as the fourth column (x4) is a free
variable
Parametric Vector Form: x4(0, -1, 0, 1, 0) + (-14, -5, 0, 0, 1)
```

Problem 2

```
format rat;

A = [5 -3 6 7,
     0 0 0 1,
     7 -5 3 -7,
     4 0 3 -7];
rref(A)
```

```
%part a
disp("Basis of Row Space: {[1 0 0 0], [0 1 0 0], [0 0 1 0], [0 0 0 1]}")
%part b
disp("Basis of col space: {(5, 0, 7, 4), (-3, 0, -5, 0), (6, 0, 3, 3), ")
disp("(7, 1, -7, -7)}")
%part c
disp("Yes dim(col(A)) = dim(row(A))")
%part d
disp("No row(A) does not equal col(A) as row(A) represents the standard")
disp("basis while col(A) is just the original matrix.")
```

```
ans =

       1             0             0             0
       0             1             0             0
       0             0             1             0
       0             0             0             1

Basis of Row Space: {[1 0 0 0], [0 1 0 0], [0 0 1 0], [0 0 0 1]}
Basis of col space: {(5, 0, 7, 4), (-3, 0, -5, 0), (6, 0, 3, 3),
(7, 1, -7, -7)}
Yes dim(col(A)) = dim(row(A))
No row(A) does not equal col(A) as row(A) represents the standard
basis while col(A) is just the original matrix.
```

Problem 3

```
format rat;
% part a
B = [4 2 4 -8;
     1 6 3 6;
     -5 4 0 7;
     5 -7 1 7]
v = [2; -30; 13; -10];
disp("What I do here is just put the coefficents/values of vector v")
disp("and multiply them in order according to the matrix B, which")
disp("gets you the u below")
u = [80; -199; -200; 163]
%part b
BsolutionV = [4 2 4 -8 2;
     1 6 3 6 -30;
     -5 4 0 7 13;
     5 -7 1 7 -10]
rref(BsolutionV)
disp("I set up the matrix as an augmented matrix where v is the solution")
disp("which will allow us to get the answer for the coefficients of the")
disp("vectors in B that produce the vector v, our final answer is the W")
disp("below: ")
W = [-958/91; -1783/273; 1231/118; -305/158]
```

```
B =

       4             2             4            -8
       1             6             3             6
```

```
    -5             4             0             7
     5            -7             1             7
```

```
What I do here is just put the coefficents/values of vector v
and multiply them in order according to the matrix B, which
gets you the u below

u =

      80
    -199
    -200
     163


BsolutionV =

       4             2             4            -8             2
       1             6             3             6           -30
      -5             4             0             7            13
       5            -7             1             7           -10


ans =

       1             0             0             0          -958/91
       0             1             0             0         -1783/273
       0             0             1             0          1231/118
       0             0             0             1          -305/158

I set up the matrix as an augmented matrix where v is the solution
which will allow us to get the answer for the coefficients of the
vectors in B that produce the vector v, our final answer is the W
below:

W =

    -958/91
   -1783/273
    1231/118
    -305/158
```

Problem 4

```
format rat;
%part a
v1 = [7; -3; 1; 7; 2];
v2 = [9; -3; -9; -5; -6];
v3 = [1, -1, 3, 4, 3];
v4 = [5; -3; -1; 0; 1];
%part b
A = [7 9 1 5;
     -3 -3 -1 -3;
     1 -9 3 -1;
     7 -5 4 0;
     2 -6 3 1]
```

```
%part c
rref(A)
disp("Basis is {(1, x, x^2, -1 + x + 3x^2)}")
%part d
disp("It is linearly dependent because there are free variables.")
disp("v4 can be written as a linear combination of v1, v2, v3")
```

```
A =

     7     9     1     5
    -3    -3    -1    -3
     1    -9     3    -1
     7    -5     4     0
     2    -6     3     1


ans =

     1     0     0    -1
     0     1     0     1
     0     0     1     3
     0     0     0     0
     0     0     0     0

Basis is {(1, x, x^2, -1 + x + 3x^2)}
It is linearly dependent because there are free variables.
v4 can be written as a linear combination of v1, v2, v3
```

Problem 5

```
format short;
%part a
A = [1 cos(0.1) (cos(0.1))^2 (cos(0.1))^3 (cos(0.1))^4;
     1 cos(0.2) (cos(0.2))^2 (cos(0.2))^3 (cos(0.2))^4;
     1 cos(0.3) (cos(0.3))^2 (cos(0.3))^3 (cos(0.3))^4;
     1 cos(0.4) (cos(0.4))^2 (cos(0.4))^3 (cos(0.4))^4;
     1 cos(0.5) (cos(0.5))^2 (cos(0.5))^3 (cos(0.5))^4];
%part b
rref(A)
disp("As we can see there is a pivot in every row and column")
disp("and there are no free variables. This means that one vector")
disp("cannot be written as a linear combination of another, thus")
disp("meaning that we only have a trivial solution.")
%part c
u1 = [1; 0; -1; 0; 1];
u2 = [0; 1; 0; -3; 0];
u3 = [0; 0; 2; 0; -8];
u4 = [0; 0; 0; 4; 0];
u5 = [0; 0; 0; 0; 8];
%part d
B = [1 0 0 0 0;
     0 1 0 0 0;
     -1 0 2 0 0;
     0 -3 0 4 0;
     1 0 -8 0 8]
```

```
rref(B)
disp("Yes, C is linearly independent because it has a pivot/row in")
disp("every column/no free variables and one vector is not a linear")
disp("combination of another.")
%part e
disp("C is a basis for D because the rref of C is the standard basis")
disp("with a pivot in every row/column, meaning it must be a basis, it")
disp("is also the same matrix in rref as B.")
```

```
ans =

     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     1     0
     0     0     0     0     1

As we can see there is a pivot in every row and column
and there are no free variables. This means that one vector
cannot be written as a linear combination of another, thus
meaning that we only have a trivial solution.

B =

     1     0     0     0     0
     0     1     0     0     0
    -1     0     2     0     0
     0    -3     0     4     0
     1     0    -8     0     8


ans =

     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     1     0
     0     0     0     0     1

Yes, C is linearly independent because it has a pivot/row in
every column/no free variables and one vector is not a linear
combination of another.
C is a basis for D because the rref of C is the standard basis
with a pivot in every row/column, meaning it must be a basis, it
is also the same matrix in rref as B.
```

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