MATLAB Assignment 4

Due Tuesday, December 6 at 11:59 PM EDT (Maryland time) on Gradescope.

Instructions:

On ELMS, see the file MATLAB_basics.pdf to learn how to get MATLAB and do some basic commands first. You may work with up to two other people (**groups of three total**). If you choose to work together, you may simply submit **one copy, and everyone will be receiving the same grade**. **Make sure to include all names when submitting to Gradescope!**

Submitting: To get an idea of what you should be submitting, you can first download the file `Example_Matlab_File.m` in the Files section. Open it in Matlab. Then at the top of the program, click on the PUBLISH tab. Click on the Publish button, and it should output an html file with all the code/output. This format is what your Matlab assignment should look like. When you are done with the actual Matlab project, click the Publish button, save this as a PDF, and **upload this to Gradescope.** There is a tab on ELMS that links you to Gradescope. **Remember to separate each problem by a section using the double percent signs.** Even if you have the correct code, **if there is no output, you will NOT receive full credit!**

(separate problems by using double percent signs as shown in the example file!!!!)

1. **Use the `short` format for this problem.**

   (a) Input the following matrix in Matlab: $A = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 1 & 5 & 1 & 3 \\ 0 & 2 & -3 & 5 \\ 1 & 2 & 1 & 0 \end{bmatrix}$.

   (b) Compute `[P,D]=eig(A)`, which creates a diagonal matrix D whose diagonal entries are the eigenvalues of A, and P is a matrix whose columns correspond to eigenvectors of the eigenvalues.

   (c) Suppose you are only given the information of the matrix D above. Use **a result from class** to determine whether or not A is diagonalizable. Use *disp* or *fprintf* to briefly explain your answer.

2. **Use the `rat` format for this problem.** We look at an example of an inner product that is not the dot product. Consider the inner product over $\mathbb{P}_n$ where

$$\langle f(x), g(x) \rangle = \int_0^1 f(x)g(x)\,dx.$$

   (a) Look up the `int` (integral) command, and compute the inner product of $f(x) = 2x^3 - 4x^2 + x - 2$ and $g(x) = x^5 - 3x + 1$. (make sure to put an asterisk when multiplying with x).

(b) Are the two functions $f(x)$ and $g(x)$ orthogonal? Show your work, and use *disp* or *fprintf* to briefly explain your answer.

(c) Create 2 polynomials explicitly $h_1(x)$, $h_2(x)$, **NEITHER OF WHICH IS A CONSTANT POLYNOMIAL i.e. it must be degree 1 or higher** that ARE orthogonal. Show/justify they are orthogonal using Matlab. *Hint: Don't over-complicate the polynomials. Try finding a simple polynomial that satisfies the condition, and then try to write it as a product.*

3. **Use `short` format.**

(a) Define the vectors $(2, 3, -3, -6)$, $(6, -1, 4, 1)$, $(0, 5, -3, 6)$, $(-4, 5, -2, 4)$ as **column vectors**. Label them `u1`, `u2`, `u3`, `u4`.

(b) Define `A=[u1 u2 u3 u4]`

(c) We now apply the Gram-Schmidt process to the vectors. Input

```
v1=u1
v2=u2-dot(u2,v1)/dot(v1,v1)*v1  %continue defining v3 and
    v4 similarly
```

Continue the process and define `v3` and `v4` by the Gram-Schmidt process.

(d) From the vectors in the previous part, create an **orthonormal basis** by dividing by the magnitude. You can use the `norm` command to help. Denote these vectors as `w1`, `w2`,...

(e) Define `Q=[w1 w2 w3 w4]`.

(f) Define $R = Q^T A$, and check that $A = QR$.

(g) Input

```
[Q1,R1] = qr(A,0)
```

to find a QR-factorization immediately. Observe your matrices are slightly different from our computation! This is because an orthonormal basis is not unique (think about how the first vector you defined could have been any of the 4).

4. **Use `rat` format.**

(a) Define $A = \begin{bmatrix} 3 & 6 & -7 \\ 4 & -4 & 1 \\ 7 & -6 & 3 \end{bmatrix}$, and let it be the matrix representation of a linear transformation.

(b) Suppose $\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix} \right\}$. Define the three vectors in $\mathcal{B}$ by `v1`, `v2`, `v3` in Matlab (as column vectors).

(c) Use the previous parts to help you find the $\mathcal{B}$-matrix for the transformation. Denote this matrix by `C`.

(d) Compute $C \begin{bmatrix} 17 \\ 17 \\ 17 \end{bmatrix}$. Denote this by `values`. Use *disp* or *fprintf* to briefly explain what these numbers mean in terms of the mapping, linear combinations, and basis.

(e) Verify your explanation in the previous part makes sense by computing out the linear combination and mapped vector.

5. **Use `rat` format.** Let

$$S_1 = \{(5, -2, 1, -5, 0), (-3, 1, 6, 4, 2), (-6, 1, 0, 4, 2)\}$$

$$S_2 = \{(2, 3, -4, 3, 1), (-14, -8, 10, -8, -8), (-3, 2, -3, 2, -2)\}.$$

(a) Use appropriate Matlab commands (show your work!) to determine whether S_1 or S_2 is linearly independent. Use *disp* or *fprintf* to state which set is linearly independent. You do not need to explain why.

(b) Let W denote the SPAN of the set chosen in the previous part. Thus, W is a subspace of $\mathbb{R}^5$. Denote the vectors in your chosen set as `u1`,`u2`,`u3`.

(c) Use the Gram-Schmidt process to find an orthogonal basis for W. Denote the vectors by `v1`,`v2`,`v3`.

(d) Define $\mathbf{y} = (7, -9, 0, 3, 2)$ (as a column vector).

(e) Express y as a sum of two vectors, one in W, denoted by `z1`, and the other in $W^{\perp}$, denoted by `z2`.

(f) What point in W is closest to $\mathbf{y}$?

Contents

Problem 1

```
format short;
%part a
A = [2 0 1 0;
     1 5 1 3;
     0 2 -3 5;
     1 2 1 0];
%part b
[P,D] = eig(A)
%part c
disp("With a result from class, we determined that a matrix which is n*n")
disp("and has n distinct eigenvectors, it must be diagonizable. Thus, this")
disp("matrix is diagonizable.")
```

```
P =

   -0.0763    0.1608   -0.2901    0.9287
   -0.8730    0.0273   -0.3537   -0.3566
   -0.3523   -0.9687    0.7326   -0.0661
   -0.3285    0.1873    0.5040    0.0775


D =

    6.6197         0         0         0
         0   -4.0229         0         0
         0         0   -0.5256         0
         0         0         0    1.9288

With a result from class, we determined that a matrix which is n*n
and has n distinct eigenvectors, it must be diagonizable. Thus, this
matrix is diagonizable.
```

Problem 2

```
format rat;
%part a
syms x;
fxgx = (2*x^3 - 4*x^2 + x - 2)*(x^5 -3*x + 1);
answer = int(fxgx, x, 0, 1)
%part b
disp("No they are not orthogonal, the integral should be equal to 0, ")
disp("but it is equal to 629/630, not 0.")
```

```
%part c
h1x = x*sqrt(3) + 1
h2x = x*sqrt(3) - 1
answer = int(h1x*h2x, x, 0, 1)
disp("Thus the answer is 0, and it is orthogonal.")
```

```
answer =

629/630

No they are not orthogonal, the integral should be equal to 0,
but it is equal to 629/630, not 0.

h1x =

3^(1/2)*x + 1


h2x =

3^(1/2)*x - 1


answer =

0

Thus the answer is 0, and it is orthogonal.
```

Problem 3

```
format short;
%part a
u1 = [2; 3; -3; -6];
u2 = [6; -1; 4; 1];
u3 = [0; 5; -3; 6];
u4 = [-4; 5; -2; 4];
%part b
A =[u1, u2, u3, u4]
%part c
v1 = u1;
v2 = u2 - (dot(u2, v1)/dot(v1, v1))*v1;
v3 = u3 - (dot(u3, v1)/dot(v1, v1))*v1 - (dot(u3, v2)/dot(v2, v2))*v2;
v4 = u4 - (dot(u4, v1)/dot(v1, v1))*v1 - (dot(u4, v2)/dot(v2, v2))*v2 - (dot(u4, v3)/dot(v3, v3))*v3;
%part d
w1 = v1/norm(v1);
w2 = v2/norm(v2);
w3 = v3/norm(v3);
w4 = v4/norm(v4);
%part e
Q = [w1 w2 w3 w4]
%part f
R = transpose(Q)*A;
Q*R
```

```
disp("Thus, Q*R = A.")
%part g
[Q1, R1] = qr(A, 0)
```

```
A =

     2     6     0    -4
     3    -1     5     5
    -3     4    -3    -2
    -6     1     6     4


Q =

    0.2626    0.8701    0.2439   -0.3385
    0.3939   -0.0737    0.6843    0.6092
   -0.3939    0.4873   -0.3436    0.6995
   -0.7878    0.0095    0.5952   -0.1580


ans =

    2.0000    6.0000    0.0000   -4.0000
    3.0000   -1.0000    5.0000    5.0000
   -3.0000    4.0000   -3.0000   -2.0000
   -6.0000    1.0000    6.0000    4.0000

Thus, Q*R = A.

Q1 =

   -0.2626    0.8701   -0.2439    0.3385
   -0.3939   -0.0737   -0.6843   -0.6092
    0.3939    0.4873    0.3436   -0.6995
    0.7878    0.0095   -0.5952    0.1580


R1 =

   -7.6158    1.1818    1.5757    1.4444
         0    7.2528   -1.7734   -4.7853
         0         0   -8.0232   -5.5137
         0         0         0   -2.3693
```

Problem 4

```
format rat;
%part a
A = [3 6 -7;
    4 -4 1;
    7 -6 3];
%part b
v1 = [3; 4; -3];
v2 = [-1; 1; 0];
```

```
v3 = [2; 6; -1];
B = [v1 v2 v3];
%part c
C = inv(B)*A*B;
%part d
sev = [17; 17; 17];
values = C* sev
disp("The values vector gives us a way to get the mapped vector by multiplying")
disp("v1-v3 and adding them all up. Thus, the mapped vector is: ")
mappedVector1 = (326*v1)+(-1080*v2)+(-128*v3)
%part e
disp("To verify, since we know C = [17, 17, 17], we must find the vector")
disp("which it yields by multiping 17 with v1-v3, and multiplying the ")
disp("resulting vector by A, which yields the mapped vector.")
v110 = 17 * v1;
v221 = 17 * v2;
v313 = 17* v3;
VBaseB = v110 + v221 + v313;
disp("Now, if we multiply A with VBaseB, we get the same mapped vector")
disp("which we got early. ")
MappedVector2 = A*VBaseB
disp("This we get the same mapped vector and this verifies our previous explanation")
```

```
values =

     326
   -1080
    -128

The values vector gives us a way to get the mapped vector by multiplying
v1-v3 and adding them all up. Thus, the mapped vector is:

mappedVector1 =

    1802
    -544
    -850

To verify, since we know C = [17, 17, 17], we must find the vector
which it yields by multiping 17 with v1-v3, and multiplying the
resulting vector by A, which yields the mapped vector.
Now, if we multiply A with VBaseB, we get the same mapped vector
which we got early.

MappedVector2 =

    1802
    -544
    -850

This we get the same mapped vector and this verifies our previous explanation
```

Problem 5

```
format rat;
%part a
s1 = [5; -2; 1; -5; 0];
s2 = [-3; 1; 6; 4; 2];
s3 = [-6; 1; 0; 4; 2];
S1 = [s1 s2 s3];
rref(S1)
disp("S1 is linearly independent")
s4 = [2; 3; -4; 3; 1];
s5 = [-14; -8; 10; -8; -8];
s6 = [-3; 2; -3; 2; -2];
S2 = [s4 s5 s6];
rref(S2);
disp("S2 is linearly dependent")
%part b
u1 = s1;
u2 = s2;
u3 = s3;
%part c
v1 = u1;
v2 = u2 - (dot(u2, v1)/dot(v1, v1))*v1;
v3 = u3 - (dot(u3, v1)/dot(v1, v1)) * v1 - (dot(u3, v2)/dot(v2, v2))*v2;
%part d
y = [7; -9; 0; 3; 2];
z1 = (dot(y, v1)/dot(v1, v1)) * v1 + (dot(y, v2)/dot(v2, v2))*v2 + (dot(y, v3)/dot(v3, v3))*v3;
z2 = y - z1;
%part f
disp("point in W closest to y is represented by z2: ")
z2
```

```
ans =

       1              0              0
       0              1              0
       0              0              1
       0              0              0
       0              0              0

S1 is linearly independent
S2 is linearly dependent
point in W closest to y is represented by z2:

z2 =

     121/35
    -192/25
    -121/70
    1045/169
    1614/875
```