MATLAB Assignment 2

Due Tuesday, October 18 at 11:59 PM EDT (Maryland time) on Gradescope

Instructions:

On ELMS, see the file MATLAB_basics.pdf to learn how to get MATLAB and do some basic commands first. You may work with up to two other people (**groups of three total**). If you choose to work together, you may simply submit **one copy, and everyone will be receiving the same grade**. **Make sure to include all names when submitting to Gradescope!**

Submitting: To get an idea of what you should be submitting, you can first download the file `Example_Matlab_File.m` in the Files section. Open it in Matlab. Then at the top of the program, click on the PUBLISH tab. Click on the Publish button, and it should output an html file with all the code/output. This format is what your Matlab assignment should look like. When you are done with the actual Matlab project, click the Publish button, save this as a PDF, and **upload this to Gradescope.** There is a tab on ELMS that links you to Gradescope. **Remember to separate each problem by a section using the double percent signs.** Even if you have the correct code, **if there is no output, you will NOT receive full credit!**

(separate problems by using double percent signs as shown in the example file!!!!)

1. Use `format short`.
   (a) Define a matrix A that would rotate a point in $\mathbb{R}^2$ counterclockwise about the origin by an angle $\theta = \pi/3$. Use this to transform the vector $\mathbf{v} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$.
   (b) Define a 2×2 matrix B that would expand the x-coordinate of a point by scaling it 8 units. Use this to transform the vector $\mathbf{v} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$.
   (c) If T_1 and T_2 denotes the transformations above, respectively, find the matrix representation of the transformations that first rotates, then scales, and find the matrix representation of the transformations that first scales then rotates. Equivalently, compute the composition of the transformations $T_1 \circ T_2$ and $T_2 \circ T_1$ i.e. find AB and BA.
   (d) What can be said about the commutative property when applying multiple transformations to a point? You can use *disp* or *fprintf* to briefly explain this.

2. Use `format rat`.

   Consider the following system of equations

$$\begin{aligned} 7x_1 + 4x_2 - 5x_3 &= 4 \\ 10x_1 + 4x_2 &= -8 \\ -5x_1 - 6x_2 + x_3 &= 3. \end{aligned}$$

(a) Define the matrix A to be the coefficient matrix, and use `inv` to find its inverse.

(b) Define an appropriate matrix, and use matrix multiplication with part (a) to find the unique solution to the system.

(c) We now find the inverse using the algorithm from class. Construct the augmented matrix with the identity matrix by copying the following

```
X=[A eye(3)];
```

Compute the RREF of X, and denote this matrix by Y.

(d) Use appropriate commands to extract the correct columns from Y that would represent the inverse of A (see item 12 in the MATLAB_basics.pdf). Denote this matrix by *inverse*.

3. Use `format rat`.

(a) Define the following matrix:

$$A = \begin{bmatrix} 6 & 4 & 2 & 2 & 4 \\ 2 & 2 & -3 & 2 & 1 \\ -1 & -5 & 1 & 1 & -3 \\ -4 & -6 & 1 & 0 & -5 \\ 4 & 2 & 7 & 4 & 0 \end{bmatrix}$$

(b) Use Matlab to compute `det`(A).

(c) Knowing the previous part, **without using Matlab**, compute `det`(A^{-1}). Use *disp* or *fprintf* to briefly explain how you concluded this.

(d) Now suppose

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 30 & 2 & 0 & 0 & 0 \\ 4 & 2 & -2 & 0 & 0 \\ 3 & 7 & 7 & 2 & 0 \\ 7 & -3 & -8 & 1 & 1 \end{bmatrix}.$$

Determine $\det\left(A^{-2}B^2\right)$ **without the use of Matlab.** Use *disp* or *fprintf* CLEARLY explain what properties you applied.

4. (a) Create 3×3 matrices A and B **such that neither is the zero matrix** such that (they are NOT equal)

$$(AB)^2 \neq A^2B^2.$$

Compute $(AB)^2$ in one line of code first, then A^2B^2 in another line.

(b) Repeat part (a) now with matrices C and D such that (they ARE equal)

$$(CD)^2 = C^2D^2.$$

(c) If A and B are 2×2 matrices, is it possible for A and B to be both invertible, but $A + B$ to be NOT invertible? If it is impossible, briefly explain why. If it IS possible, provide explicit examples of matrices, and **justify your reasoning using determinants.**

5. (Use `format rat`) Let

$$A = \begin{bmatrix} 2x+1 & x & x+2 \\ x+2 & -2x & -x-3 \\ 2x-1 & x & 3x \end{bmatrix}.$$

(a) Use the `syms` command to symbolically define the variable x.

(b) Look up how to use the `solve` command for Matlab. Use this, along with the `det` to find all values of x for which A does NOT have an inverse.

(c) For any integer $k > 3$, determine all values of x for which A^k will NOT have an inverse. Use *disp* or *fprintf* to **explain** how you deduced your answer. *Hint: Use properties of determinants.*

Contents


```
%Hashem Wahed
%Section 0132
%Matlab Project 2
```

Problem 1

```
format short;
% part a
A = [cos(pi/3) -sin(pi/3);
     sin(pi/3) cos(pi/3)];
v = [2;
     5];
rotates = A*v

% part b
B = [1/2, 3;
     0, 1];
v = [2;
     5];
scales = B*v

% part c

%first rotate and scale
rotationThenScale = B * rotates

%first scale then rotate
scaleThenRotation = A * scales

T1T2 = A*B
T2T1 = B*A

%part d
disp("The commutative property does not apply here.")
```

```
rotates =

   -3.3301
    4.2321


scales =

    16
```

```
     5


rotationThenScale =

   11.0311
    4.2321


scaleThenRotation =

    3.6699
   16.3564


T1T2 =

    0.2500    0.6340
    0.4330    3.0981


T2T1 =

    2.8481    1.0670
    0.8660    0.5000

The commutative property does not apply here.
```

Problem 2

```
format rat;

% part a
A = [7 4 -5;
     10 4 0;
     -5 -6 1];
inverseOfA = inv(A)

% part b

A = [7 4 -5 4;
     10 4 0 -8;
     -5 -6 1 3];
B = [1/47, 13/94, 5/47;
     -5/94 -9/94 -25/94;
     -10/47 11/94 -3/47];
B*A

% part c
X=[A eye(3)];
Y = rref(X)
% part d
inverse = Y(: ,5:7)
```

```
inverseOfA =

      1/47          13/94           5/47
     -5/94          -9/94         -25/94
    -10/47          11/94          -3/47


ans =

       1              0              0            -33/47
       0              1              0            -23/94
       *              0              1            -93/47


Y =

  Columns 1 through 5

       1              0              0            -33/47           1/47
       0              1              0            -23/94          -5/94
       0              0              1            -93/47         -10/47

  Columns 6 through 7

     13/94           5/47
     -9/94         -25/94
     11/94          -3/47


inverse =

      1/47          13/94           5/47
     -5/94          -9/94         -25/94
    -10/47          11/94          -3/47
```

Problem 3

```
format rat;
% part a
A = [6 4 2 2 4;
     2 2 -3 2 1;
     -1 -5 1 1 -3;
     -4 -6 1 0 -5;
     4 2 7 4 0]
% part b
determinantA = det(A)
% part c
disp("According to the properties of determinants, det(A^-1) is ")
disp("simply 1/det(A), thus det(A^-1) is -1/80")
% part d
B = [1 0 0 0 0;
     30 2 0 0 0;
     4 2 -2 0 0;
     3 7 7 2 0;
     7 -3 -8 1 1];
```

```
disp("A^-2B^2 can be split into (A^-1 * A^-1) * (B * B)")
disp("A^-1 equals -1/80 as proven earlier, and B = -8, here, due to")
disp("the matrix being lower traingular, thus we can just multiply the")
disp("diagonals. Combine it all, (-1/80)^2 * (-8)^2 = 64/6400, which is")
disp(" 1/100");
```

```
A =

     6     4     2     2     4
     2     2    -3     2     1
    -1    -5     1     1    -3
    -4    -6     1     0    -5
     4     2     7     4     0


determinantA =

   -80

According to the properties of determinants, det(A^-1) is
simply 1/det(A), thus det(A^-1) is -1/80
A^-2B^2 can be split into (A^-1 * A^-1) * (B * B)
A^-1 equals -1/80 as proven earlier, and B = -8, here, due to
the matrix being lower traingular, thus we can just multiply the
diagonals. Combine it all, (-1/80)^2 * (-8)^2 = 64/6400, which is
 1/100
```

Problem 4

```
%part a
A = [1 5 9;
     2 1 3;
     6 7 1];
B = [1 2 3;
     4 5 6;
     7 8 9];
ABsquared = (A*B)^2
AsquaredBsquared = (A^2)*(B^2)
%part b
A = [1 0 0;
     0 1 0;
     0 0 1];
B = [2 0 0;
     0 0 2;
     0 2 0];
ABsquared = (A*B)^2
AsquaredBsquared = (A^2)*(B^2)
%part c
disp("This statement is completely false. Say we have two matrices, M1")
disp("and M2, such that: ")
M1 = [1 0; 0 1]
M2 = [-1 0; 0 -1];
disp("While M1 and M2 are invertible, M1+M2 is not invertible, we can")
disp("prove this using determinants, as det(M1) and det(M2) != 0, but")
disp("det(M1 + M2) is 0.")
```

```
ABsquared =

   14403          17853          21303
    4758           5907           7056
    7758           9669          11580


AsquaredBsquared =

   10134          12411          14688
    5220           6408           7596
   11436          14076          16716


ABsquared =

       4              0              0
       0              4              0
       0              0              4


AsquaredBsquared =

       4              0              0
       0              4              0
       0              0              4

This statement is completely false. Say we have two matrices, M1
and M2, such that:

M1 =

       1              0
       0              1

While M1 and M2 are invertible, M1+M2 is not invertible, we can
prove this using determinants, as det(M1) and det(M2) != 0, but
det(M1 + M2) is 0.
```

Problem 5

```
format rat;
% part a
syms x;
A = [2*x+1 x x+2;
    x+2 -2*x -x-3;
    2*x-1 x 3*x]
%part b
solve(det(A), x)
disp("A does not have an inverse for 0, -15^(1/2)/5, and 15^(1/2)/5")
%part c
disp("the det(A^k) would not matter neither would it change anything")
disp("because det(A^k) = (det(A))^k. If we are finding out where it")
disp("does not have an inverse, it is the same answer to part b, ")
disp("because (det(A))^k = 0, square root both sides, it is det(A) = 0.")
```

```
A =

[2*x + 1,    x,   x + 2]
[  x + 2, -2*x, - x - 3]
[2*x - 1,    x,     3*x]


ans =

          0
-15^(1/2)/5
 15^(1/2)/5

A does not have an inverse for 0, -15^(1/2)/5, and 15^(1/2)/5
the det(A^k) would not matter neither would it change anything
because det(A^k) = (det(A))^k. If we are finding out where it
does not have an inverse, it is the same answer to part b,
because (det(A))^k = 0, square root both sides, it is det(A) = 0.
```

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