MATLAB Assignment 1

Due Monday September 26 at 11:59 PM EDT (Maryland time) on Gradescope

Instructions:
On ELMS, see the file MATLAB_basics.pdf to learn how to get MATLAB and do some basic commands first. You may work with up to two other people (**groups of three total**). If you choose to work together, you may simply submit **one copy, and everyone will be receiving the same grade**. **Make sure to include all names when submitting to Gradescope!**

Submitting: To get an idea of what you should be submitting, you can first download the file `Example_Matlab_File.m` in the Files section. Open it in Matlab. Then at the top of the program, click on the PUBLISH tab. Click on the Publish button, and it should output an html file with all the code/output. This format is what your Matlab assignment should look like. When you are done with the actual Matlab project, click the Publish button, save this as a PDF, and **upload this to Gradescope.** There is a tab on ELMS that links you to Gradescope. **Remember to separate each problem by a section using the double percent signs.** Even if you have the correct code, **if there is no output, you will NOT receive full credit!**

(separate problems by using double percent signs as shown in the example file!!!!)

1. For this problem, we will keep everything as rational numbers. Copy the following first

```
format rat
```

Consider the following system of equations

$$\begin{aligned} x_1 - 2x_2 + x_3 + x_4 &= 2 \\ -2x_1 + 5x_2 - 3x_3 + x_4 &= 0 \\ x_1 - x_2 + 3x_3 + 4x_4 &= 4 \\ x_1 + 2x_2 + 3x_3 + 13x_4 &= 3 \end{aligned}$$

(a) Define the augmented matrix for the system in Matlab, and denote it by A.

(b) Apply elementary row operations (see item (10) in MATLAB_basics.pdf) to put the matrix in **ROW REDUCED ECHELON FORM**. Do NOT suppress the output for each operation you do.

(c) Redefine your matrix A as in part (a), and now simply use the `rref` command on the matrix A. **From hereon, we will assume the usage of the command without showing the row operations in Matlab**.

(d) Use the *disp* or *fprintf* command to state what the solution of the system is. If there is no solution, explain why. If there is a unique solution, state what it is. If there are infinite solutions, find the general solution. **You may need to LINE BREAK if your explanation is long. If the grader cannot read what you wrote because the text was chopped off, you won't get full credit!**

2. (Make sure to use double percent signs to make problem 2 a separate section!) We show that the vector $\begin{bmatrix} a \\ b \end{bmatrix}$ lies in the span of $S = \left\{ \begin{bmatrix} 132 \\ 328 \end{bmatrix}, \begin{bmatrix} 500 \\ 400 \end{bmatrix} \right\}$.

   (a) We must treat a and b as symbolic variables. Therefore begin by copying the following

```
syms a b
```

   (b) Create an appropriate matrix and applying the `rref` command that would help find the linear combination.

   (c) Use the *disp* or *fprintf* command to explicitly state what the weights/coefficients c_1 and c_2 are in the linear combination in part (b).

   (d) Consider the set of two vectors $T = \left\{ \begin{bmatrix} 132 \\ 328 \end{bmatrix}, \mathbf{w} \right\}$. Find an explicit vector (with numbers) $\mathbf{w}$ that is NOT $\begin{bmatrix} 132 \\ 328 \end{bmatrix}$ or the zero vector, so that NOT every vector $\begin{bmatrix} a \\ b \end{bmatrix}$ lies in the span of T. **Explain why your choice of w works, and provide an explicit vector that won't lie in the span of T.**

   You may need to LINE BREAK if your explanation is long. If the grader cannot read what you wrote because the text was chopped off, you won't get full credit! This holds for all parts that you must explain. This is the last warning!

3. (a) Define a matrix A that would help determine if the set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\} = \left\{ \begin{bmatrix} 1 \\ 5 \\ 6 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 5 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} ,3 \\ 3 \\ -7 \\ 0 \end{bmatrix} \right\}$ is linearly independent or linearly dependent.

   (b) Use the `rref` command and then use *disp* or *fprintf* to **CLEARLY EXPLAIN** if the set is linearly independent or dependent. If you say something like "there's infinite solutions" you will get very little credit! Be very clear what you are saying! (Look back at how linear independence is defined!)
   If the set is linearly dependent, use *disp* or *fprintf* to express $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, and $\mathbf{v}_4$ as a non-trivial linear combination of the zero vector.

4. Let

$$S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\} = \left\{ \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ -9 \\ -10 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 18 \end{bmatrix} \right\}.$$

   (a) Define an appropriate matrix and applying the `rref` command to determine if $\mathbf{w} = \begin{bmatrix} -25 \\ 93 \\ 130 \end{bmatrix}$ can be expressed as a linear combination of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$

(b) If no such combination exists, briefly explain why. Otherwise, **if there is a unique** linear combination, find it and use *disp* or *fprintf* to state what it is. **If there is more than one way** to write the linear combination, use *disp* or *fprintf* to **show TWO different linear combinations of** w.

5. Let

$$S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\} = \left\{ \begin{bmatrix} -7 \\ 13 \\ -22 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -3 \\ 1 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} -4 \\ 22 \\ -15 \\ -8 \end{bmatrix} \right\}.$$

(a) Define an appropriate matrix and applying the `rref` command to determine if the vector $\mathbf{v}_1$ can be expressed as a linear combination of $\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$. Make sure to show the output of the RREF. **Then simply state yes or no.**

(b) Define an appropriate matrix and applying the `rref` command to determine if the vector $\mathbf{v}_2$ can be expressed as a linear combination of $\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_4$. Make sure to show the output of the RREF. **Then simply state yes or no.**

(c) Define an appropriate matrix and applying the `rref` command to determine if the vector $\mathbf{v}_3$ can be expressed as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4$. Make sure to show the output of the RREF. **Then simply state yes or no.**

(d) Define an appropriate matrix and applying the `rref` command to determine if the vector $\mathbf{v}_4$ can be expressed as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$. Make sure to show the output of the RREF. **Then simply state yes or no.**

(e) From the previous parts, **use a result from class** to deduce whether the set of vectors is linearly independent or linearly dependent. You can use *disp* or *fprintf* to explain your reasoning.

(f) Suppose we only tested if $\mathbf{v}_3$ can be expressed as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4$. Can we conclude that S is linear independent or dependent? From your answer in part (e), does this contradict the theorem from class? You can use *disp* or *fprintf* to explain your reasoning.

There is a short 1 minute video (Gradescope_tutorial.mp4) in the Files section on how to upload to Gradescope. If you are working with other people, MAKE SURE TO ADD THE GROUP MEMBERS WHEN YOU UPLOAD THE FILE. There should be click on a Group Members button for you to list the people in your group that will count as 1 submission. Also make sure to MARK YOUR PAGES AS SHOWN IN THE VIDEO.

Contents


```
% ------------------------------------------------------------------
%  Generated by MATLAB on 22-Sep-2022 19:22:52
%  MATLAB version: 9.13.0.2049777 (R2022b)
% ------------------------------------------------------------------
```

Problem 1

```
A = [1 -2 1 1 2;
    -2 5 -3 1 0;
     1 -1 3 4 4;
     1 2 3 13 3];
% Creating a sole pivot in column 1
A(2, :) = 2*A(1, :) + A(2, :)
A(3, :) = -1*A(1, :) + A(3, :)
A(4, :) = -1*A(1, :) + A(4, :)
% Creating a sole pivot in column 2
A(1, :) = 2 *A(2, :) + A(1, :)
A(3, :) = -1*A(2, :) + A(3, :)
A(4, :) = -4*A(2, :) + A(4, :)
% Cleaning up the rest
A(3, :) = 1/3 * A(3, :)
A(2, :) = A(3, :) + A(2, :)
A(1, :) = A(3, :) + A(1, :)
A(4, :) = -6 * A(3, :) + A(4, :)
A(4, :) = -1/11 * A(4, :)
A(3, :) = 2/3 * A(4, :) + A(3, :)
A(2, :) = -10/3 * A(4, :) + A(2, :)
A(1, :) = -28/3 * A(4, :) + A(1, :)
% Redeclare the matrix, but use rref this time instead
A = [1 -2 1 1 2;
    -2 5 -3 1 0;
     1 -1 3 4 4;
     1 2 3 13 3]

rref(A)

disp("There are no solutions because if there were 0 would equal")
disp("1, but that is clearly false.")
```

```
A =

     1    -2     1     1     2
     0     1    -1     3     4
     1    -1     3     4     4
```

```
     1     2     3    13     3


A =

     1    -2     1     1     2
     0     1    -1     3     4
     0     1     2     3     2
     1     2     3    13     3


A =

     1    -2     1     1     2
     0     1    -1     3     4
     0     1     2     3     2
     0     4     2    12     1


A =

     1     0    -1     7    10
     0     1    -1     3     4
     0     1     2     3     2
     0     4     2    12     1


A =

     1     0    -1     7    10
     0     1    -1     3     4
     0     0     3     0    -2
     0     4     2    12     1


A =

     1     0    -1     7    10
     0     1    -1     3     4
     0     0     3     0    -2
     0     0     6     0   -15


A =

    1.0000         0   -1.0000    7.0000   10.0000
         0    1.0000   -1.0000    3.0000    4.0000
         0         0    1.0000         0   -0.6667
         0         0    6.0000         0  -15.0000


A =

    1.0000         0   -1.0000    7.0000   10.0000
         0    1.0000         0    3.0000    3.3333
         0         0    1.0000         0   -0.6667
         0         0    6.0000         0  -15.0000
```

```
A =

    1.0000         0         0    7.0000    9.3333
         0    1.0000         0    3.0000    3.3333
         0         0    1.0000         0   -0.6667
         0         0    6.0000         0  -15.0000


A =

    1.0000         0         0    7.0000    9.3333
         0    1.0000         0    3.0000    3.3333
         0         0    1.0000         0   -0.6667
         0         0         0         0  -11.0000


A =

    1.0000         0         0    7.0000    9.3333
         0    1.0000         0    3.0000    3.3333
         0         0    1.0000         0   -0.6667
         0         0         0         0    1.0000


A =

    1.0000         0         0    7.0000    9.3333
         0    1.0000         0    3.0000    3.3333
         0         0    1.0000         0         0
         0         0         0         0    1.0000


A =

    1.0000         0         0    7.0000    9.3333
         0    1.0000         0    3.0000         0
         0         0    1.0000         0         0
         0         0         0         0    1.0000


A =

     1     0     0     7     0
     0     1     0     3     0
     0     0     1     0     0
     0     0     0     0     1


A =

     1    -2     1     1     2
    -2     5    -3     1     0
     1    -1     3     4     4
     1     2     3    13     3


ans =
```

```
     1     0     0     7     0
     0     1     0     3     0
     0     0     1     0     0
     0     0     0     0     1
```

```
There are no solutions because if there were 0 would equal
1, but that is clearly false.
```

Problem 2

```
syms a b
A = [132 500 a; 328 400 b]
rref(A)
%ask about weights
disp("The weights are (5b/1112 - a/278) and (41*a/13900 - 33*b/27800")

w = [1 2.5625];
disp("This w works because then it will have no solutions. To cancel")
disp("out 328 from 132, you need to multiply it by -2.5625, if you")
disp("cancel out both 328 and 2.5625, 0 will have to equal b, and so")
disp("not EVERY vector will work for a and b, an explicit")
disp("vector a b that will not span T is 1 2 where a = 1 and b = 2")
disp("again, applying row operations will have us get no solutions.")

A =[132 1 1; 500 2.5626 2]
rref(A)
```

```
A =

[132, 500, a]
[328, 400, b]


ans =

[1, 0,          (5*b)/1112 - a/278]
[0, 1, (41*a)/13900 - (33*b)/27800]

The weights are (5b/1112 - a/278) and (41*a/13900 - 33*b/27800
This w works because then it will have no solutions. To cancel
out 328 from 132, you need to multiply it by -2.5625, if you
cancel out both 328 and 2.5625, 0 will have to equal b, and so
not EVERY vector will work for a and b, an explicit
vector a b that will not span T is 1 2 where a = 1 and b = 2
again, applying row operations will have us get no solutions.

A =

  132.0000    1.0000    1.0000
  500.0000    2.5626    2.0000


ans =
```

```
    1.0000         0   -0.0035
         0    1.0000    1.4592
```

Problem 3

```
A = [1 4 -4 3; 5 -2 5 3; 6 5 1 -7; 4 -4 5 0]
rref(A)
disp("A set of vectors is linearly independent if it only has the trivial solution")
disp("This set has a pivot in all rows of the matrix, meaning that ")
disp("it has no free variables, meaning that it is linearly independent")
```

```
A =

     1     4    -4     3
     5    -2     5     3
     6     5     1    -7
     4    -4     5     0


ans =

     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1

A set of vectors is linearly independent if it only has the trivial solution
```

```
This set has a pivot in all rows of the matrix, meaning that
it has no free variables, meaning that it is linearly independent
```

Problem 4

```
A = [-1 5 3 -25; 3 -9 3 93; 4 -10 18 130]
rref(A)
disp("There is only one solution, therefore there is only one way to")
disp("write the linear combination.")
```

```
A =

    -1     5     3   -25
     3    -9     3    93
     4   -10    18   130


ans =

     1     0     0    40
     0     1     0     3
     0     0     1     0

There is only one solution, therefore there is only one way to
write the linear combination.
```

Problem 5

```
A = [-1 5 -4 -7; 1 5 22 13; -3 5 -15 -22; 1 5 -8 3]
rref(A)
disp("Yes")
A = [-7 5 -4 -1; 13 5 22 1; -22 5 -15 -3; 3 5 -8 1]
rref(A)
disp("Yes")
A = [-7 -1 -4 5; 13 1 22 5; -22 -3 -15 5; 3 1 -8 5]
rref(A)
disp("No")
A = [-7 -1 5 -4; 13 1 5 22; -22 -3 5 -15; 3 1 5 -8]
rref(A)
disp("Yes")
disp("We know that the set is linearly dependant because the set of")
disp("vectors always has a free variable")
disp("A result from class states that a")
disp("set of vectors is linearly dependant if one can be represented as")
disp("a linear combination of the others, in our case, many of the vectors")
disp("can be shown as a linear combination of the others, so it is. ")
disp("If we only tested if v3 can be a linear combination of the ")
disp("rest of the vectors we cannot conclude whether the system is")
disp("linearly dependant or independant because we only tested if one")
disp("vector is a linear combination of the set, we must test them all. ")
disp("This does not contradict the theorem in class because the theorem")
disp("clearly states that ONE of the vectors must be able to be represented")
disp("as a linear combination of the rest, this means it can be v1, v2, or")
disp("v4, not just v3")
```

```
A =

    -1     5    -4    -7
     1     5    22    13
    -3     5   -15   -22
     1     5    -8     3


ans =

    1.0000         0         0    5.6667
         0    1.0000         0         0
         0         0    1.0000    0.3333
         0         0         0         0

Yes

A =

    -7     5    -4    -1
    13     5    22     1
   -22     5   -15    -3
     3     5    -8     1


ans =

    1.0000         0         0    0.1765
         0    1.0000         0         0
         0         0    1.0000   -0.0588
         0         0         0         0

Yes

A =

    -7    -1    -4     5
    13     1    22     5
   -22    -3   -15     5
     3     1    -8     5


ans =

     1     0     3     0
     0     1   -17     0
     0     0     0     1
     0     0     0     0

No

A =

    -7    -1     5    -4
    13     1     5    22
   -22    -3     5   -15
     3     1     5    -8
```

```
ans =

     1     0     0     3
     0     1     0   -17
     0     0     1     0
     0     0     0     0

Yes
We know that the set is linearly dependant because the set of
vectors always has a free variable
A result from class states that a
set of vectors is linearly dependant if one can be represented as
a linear combination of the others, in our case, many of the vectors
can be shown as a linear combination of the others, so it is.
If we only tested if v3 can be a linear combination of the
rest of the vectors we cannot conclude whether the system is
linearly dependant or independant because we only tested if one
vector is a linear combination of the set, we must test them all.
This does not contradict the theorem in class because the theorem
clearly states that ONE of the vectors must be able to be represented
as a linear combination of the rest, this means it can be v1, v2, or
v4, not just v3
```

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