18/01/2021 Experiment - 1

Second Order System

Theoretical Calculations:

For underdamped systems:

$$\frac{C(s)}{R(s)} = \frac{625}{s^2+25s+625}$$

$$\frac{C(s)}{R(s)} = \frac{1225}{s^2+35s+1225}$$

$\omega_n^2 = 1225$, $2\xi\omega_n = 35$

$\omega_n = 35$ $\xi = 0.5$

For step input. $R(s) = \frac{1}{s}$

$$c(t) = 1 - \frac{e^{-1.25t}}{\sqrt{1-\xi^2}} \sin\left[\omega_d t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\text{delay time} = \frac{1+0.7\xi}{\omega_n} = \frac{1+(0.7)(0.5)}{35} = 0.038 \text{ sec}$$

$$\text{Peak time} = \frac{\pi}{\omega_d} = \frac{\pi}{35\sqrt{1-(0.5)^2}} = 0.1036 \text{ sec}$$

$$\text{Max peak overshoot } M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = 0.163$$

$$\text{Settling time } T_s = \frac{4}{\xi\omega_n} = 0.228 \text{ sec}$$

rise time $T_r = \frac{\pi - 0}{\omega_d} = \frac{\pi - \left(\cos^{-1}\left(\xi \times \frac{\pi}{180}\right)\right)}{\omega_n\sqrt{1-\xi^2}}$

$= 0.0878 \text{ sec}$

For Critically damped:

$\xi = 1, \omega_n = 35$

for step input $R(s) = \frac{1}{s}$

$$c(t) = 1 - (1 + \omega_n t)e^{-\omega_n t}$$

delay time $t_d = \frac{1 + 0.7\xi}{\omega_n} = 0.0485 \text{ sec}$

Settling time $t_s = \frac{4}{\xi\omega_n} = 0.114 \text{ sec}$

For over damped:

$\xi = 1.75, \omega_n = 35$

for step input $R(s) = \frac{1}{s}$

$$c(t) = A + Be^{-k_1 t} + Ce^{-k_2 t}$$

delay time $t_d = \frac{1 + 0.7\xi}{\omega_n} = 0.0635 \text{ sec}$

Settling time $t_s = \frac{4}{\xi\omega_n} = 0.0653 \text{ sec}$

For undamped system:

$$\xi = 0, \omega_n = 3.5$$

for step input $R(s) = \frac{1}{s}$

$$c(t) = 1 - \cos\omega_n t.$$