Robotics Mini Project
Kinematic Analysis of a Robot Arm

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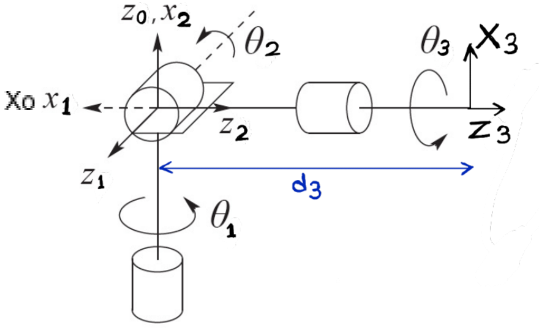
Introduction

This robot arm consists of 3 Degrees of Freedom. This analysis contains the procedure of creating a Denavit-Hartenberg (DH) table, calculating the forward kinematics, deriving the inverse kinematics, and finding the manipulator Jacobian for a pick and place task.

1. Denavit-Hartenberg (DH) Table



- Link length (a) , Link twist (alpha) , Link offset (d) , Joint angle (theta)

Link	**Link Length(a)**	**Link twist(alpha)**	**Link Offset(d)**	**Joint angle(theta)**
1	0	-90	0	θ_1*
2	0	90	0	θ_2*
3	0	0	d_3 (238.84mm)	θ_3*

2. Forward Kinematics

$$A_i = \begin{bmatrix} C_\theta & -S_\theta C_\alpha & S_\theta S_\alpha & aC_\theta \\ S_\theta & C_\theta C_\alpha & -C_\theta S_\alpha & aS_\theta \\ 0 & S_\alpha & C_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation matrices

$$A_1 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

End effector is,

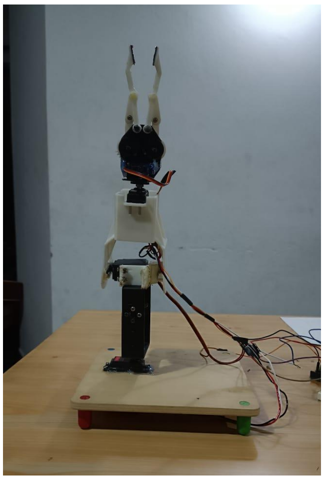
$$T_3^0 = \begin{bmatrix} C_1C_2C_3 - S_1S_3 & -C_1C_2S_3 - S_1C_3 & C_1S_2 & C_1S_2d_3 \\ S_1C_2C_3 + C_1S_3 & -S_1C_2S_3 + C_1C_3 & S_1S_2 & S_1S_2d_3 \\ -S_2C_3 & S_2S_3 & C_2 & C_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

To testing the forward kinematics,

- Test 01 - When $\theta_1 = 0$, $\theta_2 = 0$, $\theta_3 = 0$:

  $C_1 = 1$, $C_2 = 1$, $C_3 = 1$, $S_1 = 0$, $S_2 = 0$, $S_3 = 0$

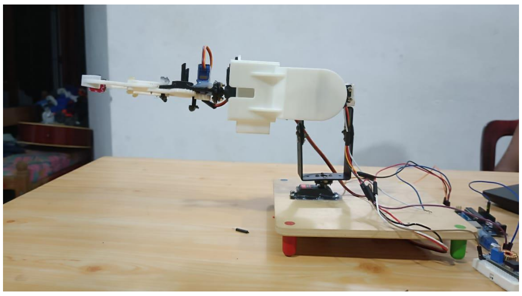
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



- Test 02 - When $\theta_1 = 90$, $\theta_2 = -90$, $\theta_3 = 0$:

  $C_1 = 0$, $C_2 = 0$, $C_3 = 1$, $S_1 = 1$, $S_2 = -1$, $S_3 = 0$

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & d_3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



From these results we ensure that our homogenous matrix is correct.

3. Inverse Kinematics

We only focus only on determining the joint angles required to **position** the end-effector at a given location.

x,y,z = end effector position

C_1 =cos θ_1 , S_1= sin θ_1 , t_1= tan θ_1

C_2 =cos θ_2 , S_2= sin θ_2

$$C_1S_2 = \frac{x}{d}$$

$$S_1S_2 = \frac{y}{d}$$

$$t_1 = \frac{y}{x} \quad \text{(equation 01)}$$

$$C_2 = \frac{z}{d} \quad \text{(equation 02)}$$

If we only consider position,from equation 01 and 02 , we can find θ_1 and θ_2.

If we also consider the inverse kinematics for orientation , From the below equation we can find θ_3.

$$t_3 = -\frac{h_{23}}{h_{13}}$$

4. Manipulator Jacobian

Since all the joints are revolute the Jacobian matrix,

$$J_i = \begin{bmatrix} z_{i-1}^0 \times \left(t_n^0 - t_{i-1}^0\right) \\ z_{i-1}^0 \end{bmatrix}$$

$$t_3^0 = \begin{bmatrix} 0 \\ 0 \\ d \end{bmatrix} \quad t_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad t_1^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad t_2^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad z_0^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_1^0 = \begin{bmatrix} -S_1 \\ C_1 \\ 0 \end{bmatrix} \quad z_2^0 = \begin{bmatrix} C_1S_2 \\ S_1S_2 \\ C_2 \end{bmatrix}$$

$$J = \begin{bmatrix} 0 & C_1d & S_1S_2d \\ 0 & S_1d & C_1S_2d \\ 0 & 0 & 0 \\ 0 & -S_1 & C_1S_2 \\ 0 & C_1 & S_1S_2 \\ 1 & 0 & C_2 \end{bmatrix}$$

5. Pick and Place Task

- The initial and final positions of the object to be picked and placed:
  - Initial position - x=0 , y=180 , z=-110
  - Final position - x= -250 , y=40 , z=100

- The joint angles and the end effector position for the initial configuration:

  θ_1 $=\tan^{-1}$ (180/0) = 90 , θ_2= $\cos^{-1}$(-110/238.84) = 117 , θ_3 =0

- The joint angles and the end effector position for the final configuration:

  θ_1 = $\tan^{-1}$ (40/-250) = 170 , θ_2=$\cos^{-1}$(100/238.84) = 115 , θ_3 =180