Time Series Macroeconomic Analysis

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Introduction:

The following project consists of 3 parts.

In part 1, we replicate the results from the Stock and Watson textbook Introduction to Econometrics 4th Edition global edition (table 17.3). Namely, we produce different forecasts of cumulative GDP Growth using different models.

In part 2, we produce the forecast of Cumulative GDP Growth using random walk model, and then compare it to the models from part 1.

Finally in part 3, we introduce a new variable – 10-year Government Bond Yield- to improve forecast.

Data Set-Up:

We download the dataset with 14 variables and 232 observations: one per each quarter from 1960 – 2017:

- column 0: date
- column1 GDPC1 = Real GDP
- column 2: GDP_Change = first difference of GDPC1
- column 3: LN_GDP = logarithm of GDPC1
- column 4: GDP_Growth = first difference of LN_GDP. These are the regressors in each equation columns5-12 GDP_Growth_hQ_Ann = the h-quarter cumulative growth in GDP at an annual rate
- column 13 = RSPREAD is the term spread variable

Then we split the dataset:

From 1981-01-31 to 2002-09-30 – training data for models.

From 2002-10-01 to 2017-12-31 – testing data for pseudo out of sample predictions.

The regressor variable is GDP_Growth which is calculated by taking first difference of the logged RGDP.

The dependent variable is the h-quarter cumulative growth in GDP at an annual rate:

$$GDPH = \frac{400 \times \ln\frac{GDP_t}{GDP_{t-h}}}{h},$$

And the term spread – RSPREAD.

We want to make a pseudo out of sample forecast for different forecast horizons – h. However, instead of using **iterative approach**, where we forecast one step ahead and then use this prediction as the input for the next step, we will use **direct forecasting** method which does not rely on previous predictions but rather produces forecasts directly.

Part 1:

How do regression matrices look like here?

To forecast the h-quarter cumulative GDP growth:

We start with first 2 observations of GDP growth; they are lags that in the model will correspond to h+2-nd observation of h-quarter cumulative GDP growth. Then we cut h rows from the end of the regressor matrix as these observations do not correspond to any in-sample h-quarter cumulative GDP growth values.

With the dependent variable, we will start at h+2-nd observation and all the way to the end.

By doing this, the dimensions of the matrices match, and the model runs correctly.

We run 6 models and get the following table:

Table 1: Results of Regressions

	GDP_Growth_1Q_Ann	GDP_Growth_4Q_Ann	GDP_Growth_8Q_Ann
GDPGR_t-h, GDPGR_t-h-1	2.295	1.947	2.170
GDPGRt-h, GDPGRt-h-1, TSpread_t-h, TSpread_t-h-1	2.339	2.017	2.167

Part 2:

The **random walk model** is one of the most basic models we can estimate; it is often used as a benchmark for other models. Below is equation for the model:

$$Y_t = Y_{t-1} + \varepsilon_t$$

I.e. prediction for cumulative GDP tomorrow is cumulative GDP today. The first cumulative GDP prediction for all the 3 models is the last observation of the training dataset. Then every forecast for tomorrow is cumulative GDP today. Finally, we receive the following RMSE for forecast horizons:

	$\boldsymbol{h = 1}$	$\boldsymbol{h = 4}$	$\boldsymbol{h = 8}$
$\boldsymbol{RMSE}$	2.51	0.82	0.44

The table above reveals several important insights into the data:

- **Performance of the Random Walk Model for h=4 and h=8**. The random walk model for h=4 and h=8 performs significantly better than the AR models we previously ran. This indicates that AR model fails to capture some patterns in h=4 and h=8.
- **Performance of the Random Walk Model for h=1.** Unlike h=4 and h=8 , for small horizon the random walk model seems to perform poorly compared to AR models. This suggests that h=1 shows a different pattern than others.

To answer the questions:

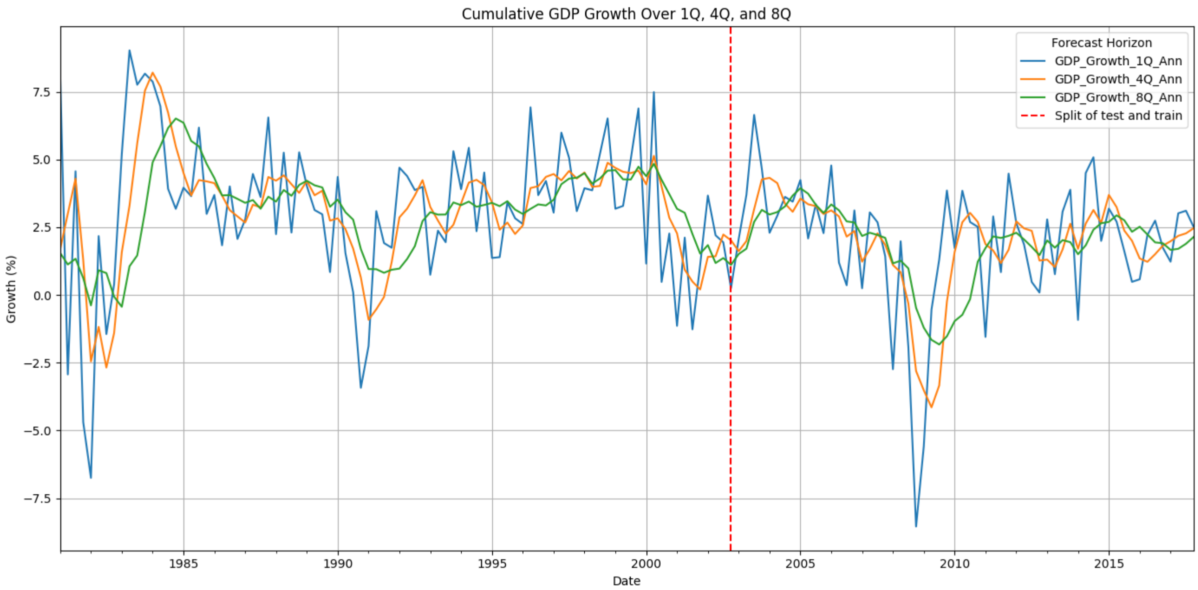
- **"Why does Random Walk perform better than AR?"**
- **"Why does Random Walk perform worse for h=1?"**

Below, I have included a table of summary statistics for the entire dataset along with time series plots for better visual understanding.

Table 2: Summary Statistics for the entire dataset

	Mean	Variance	Skewness	Kurtosis
GDP_Growth_1Q_Ann	2.99	**10.89**	-0.27	1.61
GDP_Growth_4Q_Ann	2.98	4.95	-0.44	0.64
GDP_Growth_8Q_Ann	3.02	3.05	-0.29	-0.15

Graph 1: Comparison of Cumulative GDP Growth Variables



Why Random Walk predicts better than AR?

The better performance of the random walk might be due several reasons:

- Firstly, the cumulative GDP growth for higher h is a relatively stable series, meaning that a simple model can perform well. Note that for the forecast period the h=4 and h=8 series fluctuate about the mean, except the 2008 financial crisis.
- Secondly, the aforementioned 2008 financial crisis introduces the outliers in the dataset. Generally, the random walk model handles outliers better than AR models, which are prone to being influenced by extreme values due to their reliance on previous data.
- Finally, the points above are reinforced by the better performance of AR model for h=1. Note that unlike h=4 and h=8, h=1 series are more volatile, representing a more complex relationship that AR model captures better than RW. Furthermore, as the training data for h=1 model was more volatile, the AR model handled 2008 financial crisis better.

Additionally, the higher volatility of h=1 series is seen both on the graph and the summary table.
- In addition, it could be that the data follows a random walk.

Important note about stationarity: If the data would be non-stationary then we would have a sufficient explanation to why random walk performs better. However, Augmented Dickey-Fuller (ADF) Test suggests that non-stationarity is not a problem here; for all the variables the non-stationarity hypothesis is rejected at 5% significance level.

Why the Random Walk for h=1 does much worse?

Several factors contribute to the poorer performance of the Random Walk model for h=1:

- Fluctuations in GDP growth smoothing out over time, raising random walk performance if with the increase in forecast horizon.
- Higher variance of h=1 variable. From the table above, we may note that 1-quarter cumulative GDP growth has much higher variance than 4-quarter and 8-quarter cumulative GDP growth. This reinforces the aforementioned idea of GDP growth trends smoothing out over time
- Furthermore, the relatively high kurtosis of 1.61 suggests a peaked distribution while 8-quarter cumulative GDP growth has a negative kurtosis (flatter distribution). In other words, the 1-quarter cumulative GDP growth has more outliers than the others.
- Aforementioned, potentially more complex relationship of h=1 series that is poorly captured by the random walk.

Part 3:

We download and import another macroeconomic variable from FRED to see whether we can improve the forecasts.

The variable of choice is - Interest Rates: Long-Term Government Bond Yields: 10-Year – imported from FRED.

Why Long-Term Government Bond Rate?

Government bond interest rates can be a useful indicator of **economic health** and **monetary policy**. Specifically, this variable - **10-year bond yield -** is often considered a benchmark interest rate, representing the cost of long-term borrowing for the U.S. government. Generally, the bond rate may be considered as an indicator of a belief in the future of an economy. A higher interest rate could signal an expectation of a large economic growth, while a lower interest rate may suggest a weaker economic prospects.

This is especially relevant given that we are estimating AR models for the economic growth; we would expect the lags of interest rate to be statistically significant. Furthermore, as for the direct forecast we do not predict the in-between values, the model relies heavily on long-term trends. For this reason, the **10-year yield** is especially relevant to the analysis.

Now we will run 3 models with lags of GDP_Growth and lags of a 10-year-yield. We would expect to not get a significant improvement on 1-quarter predictions; however, for h=4 and h=8 there may be an improvement.

Below are the results of the models: (We will call 10-year yield - LTGB)

Table 3: Results of 3 models with Long-Term Government Bond Yield

	LTGB lag_1 significant?	LTGB lag_2 significant?	R-squared	Adjusted R-squared	RMSE of forecast	RMSE without LTGB
$h = 1$	No	No	0.264	0.227	2.40	2.30
$h = 4$	No	Yes	0.273	0.235	1.97	1.95
$h = 8$	No	Yes	0.118	0.070	**1.81**	2.17

Results:

From the table, we observe that the LTGB (Long-Term Government Bond) lags were statistically insignificant in most cases. Furthermore, they reduce the adjusted R-squared suggesting a problem of **overfitting**. However, the forecast accuracy for higher horizons(particularly for h=8) improved significantly. This raises an important question:

Why would statistically insignificant variable improve forecasts accuracy?

This situation might be happening due to several reasons:

Long-term predicting power: The 10 year yield is a long-term indicator of a general belief about the future of an economy. While its influence may be irrelevant in the short term (like h=1, h=4), in the longer periods (specifically, when h=8) it becomes crucial for the model. In other words, the LGTB lags might not have an immediate effect on GDP growth, however, they might capture a meaningful long-term relationships that are valuable for forecasting.

Omitted Variable Bias: The 10 year yield of the government bond represents the general belief of people in the economy. Although the variable appears to be insignificant, it captures many important underlying factors that other variables do not. In this case, the yield indirectly accounts for various economic forces that affect GDP growth, improving the forecast.

Overfitting: Overfitting is generally considered harmful for a model; it hurts the in-sample performance of a model. However, in this case it could be that the overfitting helps capture some underlying pattern in the data that improve forecasts.

Conclusion:

The AR models we have derived do not appear to produce reliable forecasts. Most of the AR models (except for h=1) seem to be overshadowed by the random walk model. Furthermore, the model described in part (c) could not outperform the random walk either. However, the variable added in the model does appear to improve forecast accuracy for some cases.

It is possible that omitted variable bias (OVB) is present, meaning that the models may be missing variables that are essential for achieving a good fit.

There could be a potential non-linearity in the data or an interaction term.

Additionally, it may be that there is a contemporaneous effect in the model that we did not account for.

High volatility of the data could also lead to poor predictions.

Finally, it is plausible that this stationary series does not follow an AR model, but rather a random walk or a random walk with drift. It would be reasonable to run a Dickey-Fuller test to see if the data follows a random walk with a drift.

To sum up, the entire analysis suggests that the series behaves more like a **random walk**.

Bibliography:

Federal Reserve Bank of St. Louis. (n.d.). *Long-Term Government Bond Yields: 10-Year: Main (Including Benchmark) for United States*. FRED, Federal Reserve Bank of St. Louis. https://fred.stlouisfed.org/series/IRLTLT01USQ156N