1

Assignment #01

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CE-44-B

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Question NO.4

Gaussian filters are use to smooth or blur data by giving higher weight to Central samples and less to neighbours.

Applications.

1. Computer vision: Noise reduction, preprocessing befor edge detection
2. Image Processing: Image smoothing, scale space represention
3. Medical imaging: MRI and CT noise suppression
4. Signal processing: Low pass filtering
5. Machine learning: Data smoothing before feature extraction

(Question NO: 3)

Feature	Pixel	Voxel	Texel
Space	2D image space	3D Volumetric Space	UV texture Space
Coordinates	(x, y)	(x, y, z)	(u, v)
Typical data	Color (RGB, RGBA)	Density, Opacity or Value	Color, normal or displacement
Stores	8-bit or 16-bit integers	32-bit floats or 12-bit scalars	Compressed RGBA textures
Used in Typical data	Monitors, Digital photos	Medical imaging (CT, MRI), physics	3D graphics, game engines
Use case	Displaying an image on a screen	Representing a tumor in a 3D scan	Wrapping a brick texture on a wall.

Question NO. 1) Bicubic Interpolation using matrix product form

Bicubic interpolation is used to estimate the intensity value at a sub-pixel location by fitting a smooth cubic surface over a 4×4 neighbourhood

1. Bicubic interpolation model

$$I(x,y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} x^i y^j$$

2. Numerical example.

$$P = \begin{bmatrix} 52 & 55 & 61 & 66 \\ 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 \\ 67 & 61 & 68 & 104 \end{bmatrix}$$

Values correspond to integer points $(x,y) \in \{-1,0,1,2\} \times \{-1,0,1,2\}$

3. Matrix formulation of bicubic interpolation

$$I(x,y) = \begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix} A \begin{bmatrix} 1 \\ y \\ y^2 \\ y^3 \end{bmatrix}$$

5. Bicubic Basis matrix

$$M = \frac{1}{2}\begin{bmatrix} 0 & 2 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 2 & -5 & 4 & -1 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

6. Computing the coefficient matrix

$$A = MPM^T$$

$$MP = \begin{bmatrix} 70 & 61 & 64 & 73 \\ 5.5 & 2 & -3 & 12 \\ -13.5 & -6 & -4.5 & 17.5 \\ 6 & 2 & 6.5 & -5 \end{bmatrix}$$

Multiply by M^T

$$A = MPM^T = \begin{bmatrix} 70 & 61 & 64 & 73 \\ 5.5 & 2 & -3 & 12 \\ -13.5 & -6 & -4.5 & 17.5 \\ 6 & 2 & 6.5 & -5 \end{bmatrix} \frac{1}{2}\begin{bmatrix} 0 & -1 & 2 & -1 \\ 2 & 0 & -5 & 3 \\ 0 & 1 & 4 & -3 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

Using matlab for evaluation

$$A = \begin{bmatrix} 61 & -2 & -3 & 1 \\ 4 & -1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

7. Interpolation at a sub-pixel point

Point = $(x, y) = (0.5, 0.75)$

1. Construct coordinate vectors

$$X = [1 \quad 0.5 \quad 0.25 \quad 0.125]$$

$$Y = \begin{bmatrix} 1 \\ 0.75 \\ 0.5625 \\ 0.421875 \end{bmatrix}$$

Multiply XA

$$XA = [62.25 \quad -2.75 \quad -2.75 \quad 1]$$

Final Interpolated fee value

$$I(0.5, 0.75) = [62.25 \quad -2.75 \quad -2.75 \quad 1] \begin{bmatrix} 1 \\ 0.75 \\ 0.5625 \\ 0.421875 \end{bmatrix}$$

$$\boxed{I(0.5, 0.75) = 60.82}$$

QNo4) Catalouge your findining for low light enhancement using histogram manipulation by presenting a toy numerical example.

Low light images have
- → Poor contrast
- → Dark regions with low visibility
- → Loss of Structural and texture details

Direct histogram equalization may
- → Applify Noise
- → Over-brighten shawdows
- → Reduce image quality

2:-

Dummy low-light image:

$$I = \begin{bmatrix} 12 & 15 & 18 & 20 & 22 & 23 & 21 \\ 14 & 16 & 19 & 21 & 23 & 26 & 22 \\ 13 & 15 & 18 & 20 & 22 & 24 & 21 \\ 12 & 14 & 17 & 19 & 21 & 23 & 20 \\ 10 & 12 & 15 & 17 & 19 & 21 & 18 \\ 9 & 11 & 14 & 16 & 18 & 20 & 17 \\ 8 & 10 & 13 & 15 & 17 & 19 & 16 \\ 7 & 9 & 12 & 14 & 16 & 18 & 15 \end{bmatrix}$$

Pixel values => $\underbrace{7-26}_{\text{low-light condition}}$

① Gamma (Power-law) transformation

$$s = c\,r^{\gamma}$$

$r = \frac{I}{255}$ → normalised intensity

$c = 255$ → scalling

$\gamma = 0.5$ → brightens dark pixels

1. $I = 12$

$r = \frac{12}{255} = 0.047$, $s = 255\sqrt{0.047} = 55$

2. $I = 20$

$r = \frac{20}{255} = 0.078 \Rightarrow s = 71$

Using matlab to solve all

② Gamma corrected image.

$$I_1 = \begin{bmatrix} 55 & 62 & 68 & 71 & 74 & 80 & 78 & 72 \\ 59 & 64 & 70 & 73 & 76 & 82 & 80 & 75 \\ 57 & 62 & 68 & 71 & 74 & 79 & 77 & 72 \\ 55 & 59 & 66 & 70 & 73 & 76 & 74 & 71 \\ 50 & 55 & 62 & 66 & 71 & 73 & 71 & 68 \\ 48 & 53 & 59 & 64 & 70 & 71 & 69 & 66 \\ 45 & 50 & 57 & 62 & 70 & 70 & 68 & 64 \\ 42 & 48 & 55 & 59 & 68 & 68 & 66 & 62 \end{bmatrix}$$

Dark pixels brightened, contrast slightly improved.

5. CLAHE (Local Histogram equalization)

i. Divide into small tiles (4x4 blocks)

ii. Compute local histogram for each

iii. Apply clip limit to avoid noise amplification.

Tile -1 (Top left)

Row(1-4)

$$T_1 = \begin{bmatrix} 55 & 62 & 68 & 71 \\ 59 & 64 & 70 & 73 \\ 57 & 62 & 68 & 71 \\ 55 & 59 & 66 & 70 \end{bmatrix}$$

Histogram

Range	Freq	PDF	CDF
55 - 59	6	0.375	0.375
60 - 64	4	0.25	0.625
65 - 69	4	0.25	0.875
70 - 74	2	0.125	1

PDF = freq/16

Mapping
$s = 255 \cdot CDF$

Range	Output
55-59	96
60-64	159
65-69	223
70-74	255

Tile -2 (Top right)

$$T_2 = \begin{bmatrix} 74 & 80 & 78 & 72 \\ 76 & 82 & 80 & 75 \\ 74 & 79 & 77 & 72 \\ 73 & 76 & 74 & 71 \end{bmatrix}$$

Range	Freq	PDF	CDF
70-74	5	0.3125	0.3125
75-79	6	0.375	0.6875
80-84	5	0.3125	1

Mapping
$s = 2255 \cdot CDF$

Range	Output
70-74	~~8~~ 80
75-79	175
80-84	255

Tile 3 (bottom left)

$$T_3 = \begin{bmatrix} 50 & 55 & 62 & 66 \\ 48 & 53 & 59 & 64 \\ 45 & 50 & 57 & 62 \\ 42 & 48 & 55 & 59 \end{bmatrix}$$

Range	Freq	PDF	CDF
42-49	5	0.3125	0.3125
50-55	6	0.375	0.6875
56-61	3	0.1875	0.875
62-66	2	0.125	1

Mapping

Range	Output
42-49	80
50-55	175
56-61	223
62-66	255

Tile 4 (bottom right)

$$T_4 = \begin{bmatrix} 70 & 73 & 71 & 68 \\ 68 & 71 & 69 & 66 \\ 66 & 70 & 68 & 64 \\ 64 & 68 & 66 & 62 \end{bmatrix}$$

Range	Freq	PDF	CDF
62-65	3	0.1875	0.1875
66-69	7	0.4375	0.625
70-73	6	0.375	1.0000

Mapping

Range	Output
62-65	48
66-69	159
70-73	255

Final Enhanced image

$$I_{CLAHE} = \begin{bmatrix} 96 & 159 & 223 & 255 & 80 & 255 & 255 & 175 \\ 96 & 159 & 255 & 255 & 175 & 255 & 255 & 175 \\ 96 & 159 & 223 & 255 & 80 & 175 & 175 & 175 \\ 96 & 96 & 223 & 255 & 80 & 175 & 80 & 80 \\ 175 & 175 & 255 & 255 & 255 & 255 & 159 & 159 \\ 80 & 175 & 175 & 255 & 159 & 255 & 159 & 159 \\ 80 & 175 & 223 & 255 & 159 & 255 & 159 & 48 \\ 80 & 80 & 175 & 175 & 48 & 159 & 159 & 48 \end{bmatrix}$$

Final image brightens dark regions, enhances local contrast and avoid noise amplification.