**Observations from simulations**

* All four classes of cellular automata (CA) proposed by Wolfram are observed in our model.
  + Class 1: uniform end states. Corresponds to full activation / deactivation.
  + Class 2: periodic end states. Observed structures with period 3 and 4. Specific patterns may give other periods (e.g. period 5).
  + Class 3: chaotic. Due to limited number of states, will eventually terminate or become oscillatory, but not observable on simulation timescale.
  + Class 4: transient chaos to ordered state. Observed a few examples of this, on the edge between parameters giving rise to chaotic (class 3) and … (class …) states. Wikipedia: "For larger cellular automaton rule space, it is shown that class 4 rules are located between the class 1 and class 3 rules.[[66]](https://en.wikipedia.org/wiki/Cellular_automaton#cite_note-66)”. Is this also true in our system?
* “Trailing effect”: the dynamics of one type of molecule trails behind that of the other by exactly one time step. Mathematically, for all and for all . This is due to the one of the molecules having only a single input which depends on the state of the other molecule. When all interactions are present, e.g. , the system can become chaotic and not settle down to any periodic motion (Q: is it a necessary requirement for chaos?).
* Travelling waves. Some states are deceiving and may appear to be of a low period, while in reality they constitute travelling waves (because of periodic boundary conditions they will eventually end up in the same initial state, but in practical terms we do not think of them as periodic).
* Initiating lattices which are highly organized might reveal more ‘exotic’ dynamics, such as period 5 motion.

List of observed phenomena for 2 signalling molecules

1. Activation / deactivation. Cell expression increases or decreases over time until equilibrium reached. Also possible for multiple channels.
   1. Can be complete (with end state all cells ON/OFF) or partial.
   2. For certain interactions, we find oscillations in the process.
2. Autonomy. Not observed => try to derive for 1 cell.
3. Periodic end states.
   1. Period 2. Common when interactions between genes are of same type (both activation or both repression).
   2. Period 3. Typically obtained for certain parameter ranges between two regions of period 4 oscillations.
   3. Period 4. Standard case for interactions of different type (activation & repression).
   4. Period 5. Rare, but may occur when starting with a particular initial configuration, such as a single block of ON cells of a certain size.
   5. Periodic and stable. One of the types can oscillate periodically, while the other remains constant. Find more examples of this (also with higher periods)?
   6. Is it possible to have the two types oscillate with different frequencies?
4. Travelling waves. They are also time-periodic in a system with periodic boundary conditions.
   1. Single travelling pattern (e.g. stripe).
5. Quasi-periodic travelling wave.
   1. Observed a sequence that repeats every 10 time steps, where the pattern after 10 time steps is the same as the initial configuration shifted over space.
6. Chaotic state (double check).
   1. System does not settle down after a long time. No repeating states found (check by periodicity test).
   2. Also: termination after a large number of time steps.
7. Long-lived non-periodic transient state ending in periodic state.
8. More?