

# ETERNITY: NUMBERS - Silver Ratio ( $\delta_s$ )

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## 1 Introduction

This document provides an understanding of only an irrational number called Silver Ratio ( $\delta_s$ ). An irrational number is not a rational number, it is not possible to express an irrational number as a quotient of two integers [3].

### 1.1 History

Silver Ratio is studied from the time of Greek knowledge, which discusses the fundamental characteristics of the number system. Though it is not used by normal people intentionally. Silver ratio is the limiting of consecutive of infinite sequence of integers, The silver ratio is presented in a Greek symbol ( $\delta_s$ ).

### 1.2 Mathematical Definition

The value of silver ration is 2.4142135623 [1]. A ratio of the sequential sum of smaller number and twice of the larger number, which will produce an infinite sequence and the ration between smaller and larger number will be always same [6]. This can be presented in mathematical equation:-

$$\frac{2a + b}{a} = \frac{b}{a} = \delta_s$$

It will be easier to understand if it can be compared with Fibonacci number. In Fibonacci, the smaller and larger number are added to get the next one. Example:-

1, 1, 2, 3, 5, 8, 13, ..

For silver ratio, the smaller and twice of the larger number are added to get the next one. Example:-

1, 2, 5, 12, 29, 70, ..

Then the latest number is divided by the previous larger number.

## 2 Interview

I interviewed a undergraduate student from Dhaka University - Bangladesh with math background. Her name is Esrat Jahan Tonni. As she is currently studying, she might need to use irrational numbers in her undergraduate career. The interview questions and the answers are given below:-

### 2.1 Question and Answer

Q1: How long you are in math domain?

Ans: Almost 3 years.

Q2: How often you use calculator?

Ans: Very often. I mean almost daily.

Q3: What type of device you use to calculate complex equation.

Ans: It is obviously scientific calculator for me but for advance user there are many software available.

Q4: Can you tell me some of the tools name?

Ans: No. I can't remember the names but if you search it in Google you will find some.

Q5: Do you know about the irrational number?

Ans: Yes. I Know,

Q6: Do you know about Silver Ratio?

Ans: I am not sure. I think, I know about it but never used it. But have some basic idea about it.

Q7: Can you explain me what you know about it?

Ans: I am not sure. But so far I can remember i will try to give you a basic idea of it. Hopefully you heard the name of Fibonacci number. It is related to Golden Ratio. Same as there is Silver ratio. It has some difference with golden ratio. It actually describe the twice of the larger number added with the previous number and ratio with previous number.

Q8: Do you know the value of silver ratio?

Ans: It is something one plus square root of two. I forget the value. It will be something 2.414 and more

Q9: What do you think are the applications of the Golden Ratio in mathematics?

Ans: The silver ratio is used mostly in the Geometry to create designs that are in proportions. It is not used as such in Mathematics directly but even the ratio of consecutive numbers in Pell sequence are close to the silver ratio.

Q10: What are the other places where it can help?

Ans: It is usually used in the geometrical calculation. As well as the architect and engineers use this to make shapes calculation. It may be used in art and design, some time it helps in surgery to measure some points.

Q11: Can you give me some example where it can be implemented

Ans: To make a perfect square it helps. Octagon is another example where you need this.

Q12: Do your scientific calculator support silver ratio?

Ans: Never used that. so I am not sure about it. it may be or may not be.

Q13: Would you like to include Irrational constants like silver ratio and others in the calculator?

Ans: Yes. Why not may be in future i need them.

Q14: Do you have any other suggestion that can help me to find your necessity in a scientific calculator?

Ans: Currently, I don't need any other feature in my calculator. but who knows if can come up with something new it may help others.

## 2.2 Interview Analysis

Though the interviewee is not expert in the area of the irrational numbers, She has decent idea about the silver ratio. As she has a math background, she gave a lots of insights about the silver ratio. She is 4th Year student and completed 3 years in math domain. As a math student she has to use calculator almost everyday and she uses scientific calculator for the complex equations. From the interview, it is clear that the silver ratio is used in calculation of geometrical shapes. Also the architect, designer, engineers and Sometimes doctors (Plastic Surgery) uses this. Also stated that in regular scientific calculator the irrational numbers are not available. Then she describe about the silver ratio . which has the value of 4142135623. Its convergent are square triangular numbers, Pell numbers and octagons. According to her it will be good if the irrational numbers are included in the scientific calculator. According to him the calculation can be done here in the scientific calculators but they needs extra effort. But inclusion of these can make few peoples life easier.

## 3 Persona

The persona is given below:

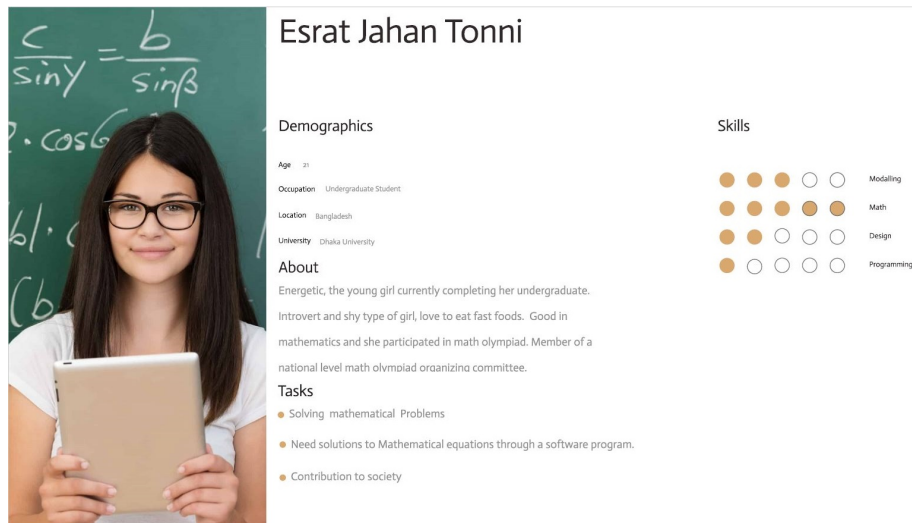


Figure 1: Persona based on the analysis of interview

## 4 Problem Domain Model

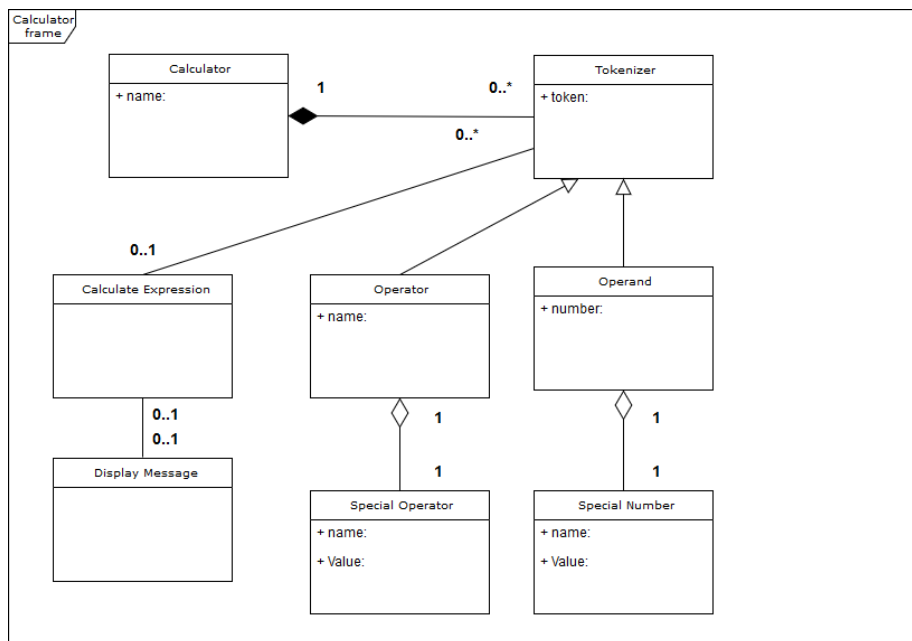


Figure 2: Class diagram for calculator with special number and operator

## 5 Use case Model

### 5.1 Use Case Diagram

Actor of this system will be an user. The user will give inputs of operator and operands. User can sometimes use special operator. but it is optional to use. Same for the operand input field can have any number. but if the user wants can have special number like Silver Ratio. With the Operator and operand it makes an mathematical expression. Calculate the expression and display the result.

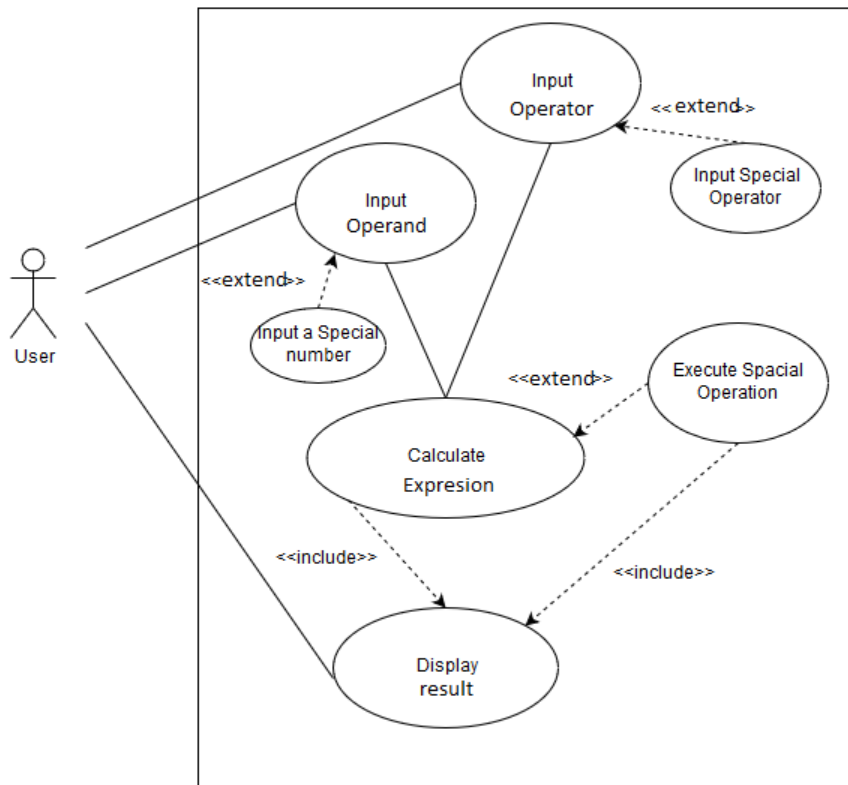


Figure 3: Use case diagram for calculator with special number and operator

### 5.2 Activity Diagram

Here it will work like a regular calculator with an extra feature of an operand silver ratio number and operator for getting a value from a number using the

silver ratio content. If silver ratio is used as a operand it will work like a double number. An example is given below :-

$$2 + \delta_s = 4.4142$$

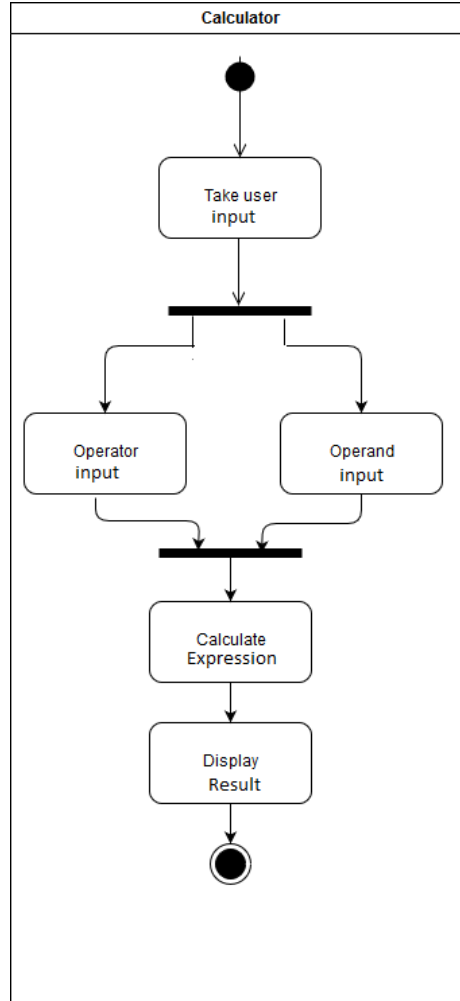


Figure 4: Activity diagram for calculator with special number and operator

if it uses like an operator than it will give the value of the another number which ratio is  $\delta_s$ . If one value is 2 than the silver ratio operand will give the another value (0.82842712474619 )based on the silver ratio equation. the equation is give below: -

$$\frac{2a+b}{a} = \delta_s$$

$$\Rightarrow 2a + b = a * \delta_s$$

$$\Rightarrow b = (a * \delta_s) - 2a$$

## 6 User Stories

### 6.1 Global Constraints

- The information about silver ratio is available in the internet. But the application of this ratio is not clear. This irrational number is used in many geometrical calculation but the use of that is not available in internet.
- Geometric interpretation of the silver ratio is not clearly mentioned [1].

### 6.2 Local Constraints

Interviewee doesn't apply silver ratio in his career. So he exactly doesn't know how it helps. But he has a conceptual idea about the ratio.

### 6.3 User Stories (US)

The concept of the type of user stories are taken from online search[5, 2]. And the user story presented here are based on interview and use case model from this document.

US 1 - Input the digital numbers			
<b>Story ID</b>	US1	<b>Priority</b>	HIGH
<b>Description</b>	As an user, I want click able 0 to 9 numbers in calculator	<b>Acceptance Test</b>	<ul style="list-style-type: none"> <li>• Click 2 twice and click on any operator number. it will save 22 in a variable.</li> </ul>
<b>Estimate</b>	0.5 d	<b>Constrains</b>	Number has to be in this range $9999999999 > x > -9999999999$
<b>Acceptance Criteria</b>	When i click them they will store it in a variable until i click any operator or a equal sign.	<b>Implemented</b>	Yes

US 2 - Addition of two numbers			
<b>Story ID</b>	US2	<b>Priority</b>	HIGH
<b>Description</b>	As an user, I want add two numbers so that I can see what is the total number	<b>Acceptance Test</b>	<ul style="list-style-type: none"> <li>Given two numbers need to be added and have to return the total result. Example: 14 add 7 will produce the result 21. 6 add 4 will produce the result 10.</li> </ul>
<b>Estimate</b>	0.2 d	<b>Constrains</b>	Addition of
<b>Acceptance Criteria</b>	<ul style="list-style-type: none"> <li>Click on 1 button</li> <li>Click on 4 button</li> <li>Click on + button</li> <li>Click on 7 button</li> <li>Click on = button</li> <li>It will give you 21 as a result</li> </ul>	<b>Implemented</b>	Yes



US 3 - Subtraction of two numbers			
<b>Story ID</b>	US3	<b>Priority</b>	HIGH
<b>Description</b>	As an user, I want subtract two numbers so that I can see what is the result	<b>Acceptance Test</b>	<ul style="list-style-type: none"> <li>Given two numbers need to be subtracted and have to return the rest. Example: 7 subtracted from 14 will produce the result 7. 6 subtracted from 10 will produce the result 4.</li> </ul>
<b>Estimate</b>	0.1 d	<b>Constrains</b>	If the number which will be subtracted is bigger than actual number it will produce minus result
<b>Acceptance Criteria</b>	<ul style="list-style-type: none"> <li>Click on 1 button</li> <li>Click on 4 button</li> <li>Click on - button</li> <li>Click on 7 button</li> <li>Click on = button</li> <li>It will give you 7 as a result</li> </ul>	<b>Implement- ed</b>	Yes

US 4 - Multiplication of two numbers			
<b>Story ID</b>	US4	<b>Priority</b>	HIGH
<b>Description</b>	As an user, I want multiply two numbers so that I can see what is the result	<b>Acceptance Test</b>	<ul style="list-style-type: none"> <li>Given two numbers need to be multiplied and have to return the result. Example: 14 multiplied with 7 will produce the result 98. 6 multiplied with 10 will produce the result 60.</li> </ul>
<b>Estimate</b>	0.1 d	<b>Constrains</b>	If the number which will be multiplied is less than zero it will produce a minus number and less than 1 but greater than zero will reduce the number.
<b>Acceptance Criteria</b>	<ul style="list-style-type: none"> <li>Click on 1 button</li> <li>Click on 4 button</li> <li>Click on * button</li> <li>Click on 7 button</li> <li>Click on = button</li> <li>It will give you 98 as a result</li> </ul>	<b>Implemented</b>	Yes

US 5 - Division of two numbers			
<b>Story ID</b>	US5	<b>Priority</b>	HIGH
<b>Description</b>	As an user, I want divide two numbers so that I can see what is the division result	<b>Acceptance Test</b>	<ul style="list-style-type: none"> <li>Given two numbers need to be divided and have to return the result. Example: 14 divided by 7 will produce the result 2. 16 divided by 2 will produce the result 8.</li> </ul>
<b>Estimate</b>	0.1 d	<b>Constrains</b>	No number can be divided by zero.
<b>Acceptance Criteria</b>	<ul style="list-style-type: none"> <li>Click on 1 button</li> <li>Click on 4 button</li> <li>Click on / button</li> <li>Click on 7 button</li> <li>Click on = button</li> <li>It will give you 2 as a result</li> </ul>	<b>Implemented</b>	Yes

US 6 - Toggle a number			
<b>Story ID</b>	US6	<b>Priority</b>	Low
<b>Description</b>	As an user, I want toggle the sign of the number so that i can get opposite value	<b>Acceptance Test</b>	<ul style="list-style-type: none"> <li>Given number need to be opposite sign number. Example: 4 will be -4. -6 will be 6.</li> </ul>
<b>Estimate</b>	0.1 d	<b>Constrains</b>	
<b>Acceptance Criteria</b>	<ul style="list-style-type: none"> <li>Click on 4 button</li> <li>Click on +/- button</li> <li>It will give you -4 as a result</li> </ul>	<b>Implemented</b>	Yes

US 7 - Equal operation			
<b>Story ID</b>	US7	<b>Priority</b>	High
<b>Description</b>	As an user, I want every calculation result after clicking in the equal button	<b>Acceptance Test</b>	<ul style="list-style-type: none"> <li>• For US2 to US5 all the use case result will be displayed if the user press equal button.</li> </ul>
<b>Estimate</b>	0.1 d	<b>Constrains</b>	If there is no operation occurred it will show the current value in display
<b>Acceptance Criteria</b>	<ul style="list-style-type: none"> <li>• Click on 1 button</li> <li>• Click on 4 button</li> <li>• Click on + button</li> <li>• Click on 7 button</li> <li>• Click on = button</li> <li>• It will give you 21 as a result</li> </ul>	<b>Implemented</b>	Yes

US 8 - Silver ratio as a operand			
<b>Story ID</b>	US8	<b>Priority</b>	High
<b>Description</b>	As an user, I want the button of silver ratio as number. So that i can use it as a number for arithmetic operation.	<b>Acceptance Test</b>	<ul style="list-style-type: none"> <li>If you click the button it will be equivalent to 2.4142135623. So all the arithmetic operation will occur with this operand. Example : <math>5 + (\delta_s)</math>  <math>= 7.4142135623</math>,  <math>(\delta_s) - 1.4142135623</math>  <math>= 1</math></li> </ul>
<b>Estimate</b>	0.1 d	<b>Constrains</b>	can not be divided by zero.
<b>Acceptance Criteria</b>	<ul style="list-style-type: none"> <li>Click on <math>(\delta_s)</math> button</li> <li>It will give you 2.4142135623 as a value</li> </ul>	<b>Implemented</b>	Yes

US 9 - Silver ratio as a operator			
<b>Story ID</b>	US9	<b>Priority</b>	High
<b>Description</b>	As an user, I want to know the number for which silver ratio will be a given number.	<b>Acceptance Test</b>	<ul style="list-style-type: none"> <li>The silver ratio of 2 will be 0.82842712474619.</li> </ul>
<b>Estimate</b>	0.1 d	<b>Constrains</b>	The ratio of zero is not possible.
<b>Acceptance Criteria</b>	<ul style="list-style-type: none"> <li>Click on 2 button</li> <li>Click on <math>(\delta_s)</math> -op button</li> <li>It will give you 0.82842712474619. as a result</li> </ul>	<b>Implemented</b>	Yes

US 10 - Decimal separator			
<b>Story ID</b>	US10	<b>Priority</b>	High
<b>Description</b>	As an user, I want to use decimal number so I want a decimal separator for decimal number input.	<b>Acceptance Test</b>	<ul style="list-style-type: none"> <li>• Need to take decimal input.</li> </ul>
<b>Estimate</b>	0.1 d	<b>Constrains</b>	Decimal separator can be used once in a number.
<b>Acceptance Criteria</b>	<ul style="list-style-type: none"> <li>• Click on 2 button</li> <li>• Click on . button</li> <li>• Click on 2 button</li> <li>• It will give you 2.2 as a value</li> </ul>	<b>Implemented</b>	Yes

US 11 - Square			
<b>Story ID</b>	US11	<b>Priority</b>	Medium
<b>Description</b>	As an user, I want to use the square value of a given number.	<b>Acceptance Test</b>	<ul style="list-style-type: none"> <li>• The value will be the multiplication of it's own. 2 square is 4</li> </ul>
<b>Estimate</b>	0.1 d	<b>Constrains</b>	For Square of non positive number, the sign of the number will be changed. For -2 it will be 4.
<b>Acceptance Criteria</b>	<ul style="list-style-type: none"> <li>• Click on 2 button</li> <li>• Click on x*x button</li> <li>• It will give you 4. as a result</li> </ul>	<b>Implemented</b>	Yes

US 12 - Square Root			
<b>Story ID</b>	US12	<b>Priority</b>	Medium
<b>Description</b>	As an user, I want to use the square root value of a given number.	<b>Acceptance Test</b>	<ul style="list-style-type: none"> <li>The value will be the a number which square is the given number. 4 square root is 2.</li> </ul>
<b>Estimate</b>	0.1 d	<b>Constrains</b>	
<b>Acceptance Criteria</b>	<ul style="list-style-type: none"> <li>Click on 4 button</li> <li>Click on root button</li> <li>It will give you 2. as a result</li> </ul>	<b>Implemented</b>	Yes

## 7 Traceability Matrix

The traceability matrix is realized to ensure that all the requirements defined for the system are tested and passed [7].

ID	REQ ID	REQ TYPE	REQ DESC	REQ SRC	FLAG
US1	1	FN	As an user, I want click able 0 to 9 numbers in calculator	System description from Deliverable 1 (D1) and ref [5]	Passed
US2	2	FN	As an user, I want add two numbers so that I can see what is the total number	Class diagram from D1. as operator	Passed
US3	3	FN	As an user, I want subtract two numbers so that I can see what is the result	Class diagram from D1. as operator	Passed
US4	4	FN	As an user, I want multiply two numbers so that I can see what is the result	Class diagram from D1. as operator	Passed

US5	5	FN	As an user, I want divide two numbers so that I can see what is the division result	Class diagram from D1. as operator	Passed
US6	6	FN	As an user, I want toggle the sign of the number so that i can get opposite value	System description from Deliverable 1 (D1) and ref [2]	Passed
US7	7	FN	As an user, I want every calculation result after clicking in the equal button	REQ ID 2,3,4,5	Passed
US8	8	FN	As an user, I want the button of silver ratio as number. So that i can use it as a number for arithmetic operation.	Use case diagram from D1	Passed
US9	9	FN	As an user, I want to know the number for which silver ratio will be a given number.	Use case diagram from D1	Passed
US10	10	FN	As an user, I want to use decimal number so I want a decimal separator for decimal number input	System description from Deliverable 1 (D1) and ref [5]	Passed
US11	11	FN	As an user, I want to use the square value of a given number. The value will be the multiplication of it's own. 2 square is 4	Use case from D1	Passed
US12	12	FN	As an user, I want to use the square root value of a given number.	Use case from D1	Passed



## 8 Implementation

Based on a set of user stories a calculator is build by java programming language. The calculator is having basic calculator operation as well as it has some special operation based on the user stories and use cases. Some operation based on silver ratio is in the calculation. All the user stories are implemented in this project. The logical concept is taken from another project [4].

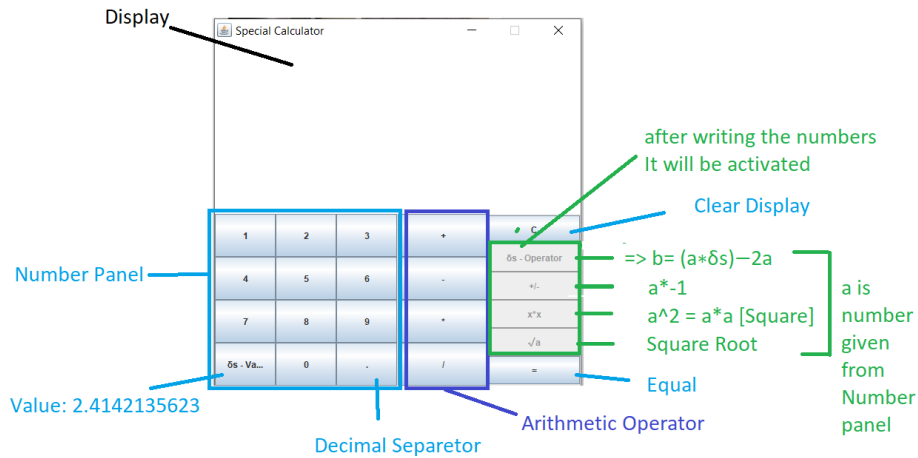


Figure 5: Screen Shot of the implemented calculator.

The project itself contains the java files along with jar file of this application. The JUnit test case file is also added with the java file named "CalculatorTest.java". The project link is also given in the online version section in this report along with the link of digital copy of this report.

## 9 Online Version

**Report:** <https://github.com/Hasib-rafi1/SRS-Silver-Ratio>

**Project:** <https://github.com/Hasib-rafi1/srs-project-calculator>

## 10 Conclusion

In this document, the user stories for the calculator system have been written to capture basic requirement of a calculator system along with silver ratio. This user stories are implemented by java program where all the acceptance test are passed. This document contains both the D1 and D2.

## References

- [1] John D. Cook. *The silver ratio*. 2009. URL: <https://www.johndcook.com/blog/2009/05/20/the-silver-ratio/> (visited on 05/20/2009).
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- [3] P. Kamthan. “PROJECT DESCRIPTION”. University Lecture. 2019.
- [4] mansya prime Kydon Chantzaridis Achintha Gunasekara. *Simple-Java-Calculator*. 2019. URL: <https://github.com/pH-7/Simple-Java-Calculator> (visited on 07/06/2019).
- [5] Quincy Larson. *JavaScript Calculator project with testable user stories - Guinea Pigs needed*. 2016. URL: <https://www.freecodecamp.org/forum/t/javascript-calculator-project-with-testable-user-stories-guinea-pigs-needed/58941> (visited on 11/24/2016).
- [6] Tony Padilla. *The Silver Ratio - Numberphile*. Youtube. 2018. URL: <https://www.youtube.com/watch?v=7lRgeTmxnlG> (visited on 05/11/2018).
- [7] Prashanthi Ramesh. *Calculator Stories*. 2012. URL: <https://github.com/PrashanthiRamesh/SOEN-6481-Project> (visited on 04/07/2019).