ME-421

FLUID MACHINERY

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2.1 Dynamic Action of Fluid

When a stream of fluid enters a machine, it generally follows a specific direction. However, in order to alter its velocity, either in magnitude or direction, a force must be applied to the fluid. This force, exerted by the motion of the fluid, is referred to as dynamic force. The power of the machine is determined by the dynamic force generated by the flowing fluid, which arises due to the change in momentum.

Momentum can exist in linear or angular form, with angular momentum being the moment of linear momentum. The force is the rate of change of linear momentum, while torque is the rate of change of angular momentum. According to Newton's second law, the rate of change of momentum is proportional to the applied force and occurs in the direction of the force. Specifically, if the resultant external force in the x-direction is F_x, the mass of the fluid is m, the velocity of the fluid is v_x, and the change in velocity over time dt is dv_x, then:

The change in momentum $= m dv_x$,
And the rate of change of momentum $= m\frac{dv_x}{dt}$

$$F_x = m\frac{dv_x}{dt} \tag{1}$$

eq^n (1) is knows as linear momentum eq^n.
This eq^n may be wriiten as -

$$F_x dt = m dv_x \tag{2}$$

This eq^n is known as impulse momentum eq^n.

For a control volume with fluid entering in uniform velocity v_{x_1}, and leaving after time t within uniform velocity v_{x_2}, then according to eq^n (2),

$$F_x = \frac{m}{t}(v_{x_2} - v_{x_1}) \tag{3}$$

Again,

$$\frac{m}{t} = \rho Q$$

$$\Rightarrow F_x = \rho Q(v_{x_2} - v_{x_1}) \tag{4}$$

Dynamic force exerted by fluid jet on stationary flat plate -

2.1.1 Plate normal to jet:

A fluid jet is issued from a nozzle and strike a flat plate with a velocity v. The plate is held stationary at perpendicular to the centerline of the jet. Let,

$$Q \longrightarrow \text{Volumetric flow rate}$$
$$\rho Q \longrightarrow \text{Mass flow rate}$$

Dynamic force on the fluid by the plate:
Applying eq^n 4,

$$F_x = \rho Q(v_{x_2} - v_{x_1})$$

$$\Rightarrow -F_x = \rho Q(0 - v)$$

$$\Rightarrow F_x = \rho Q v \tag{5}$$
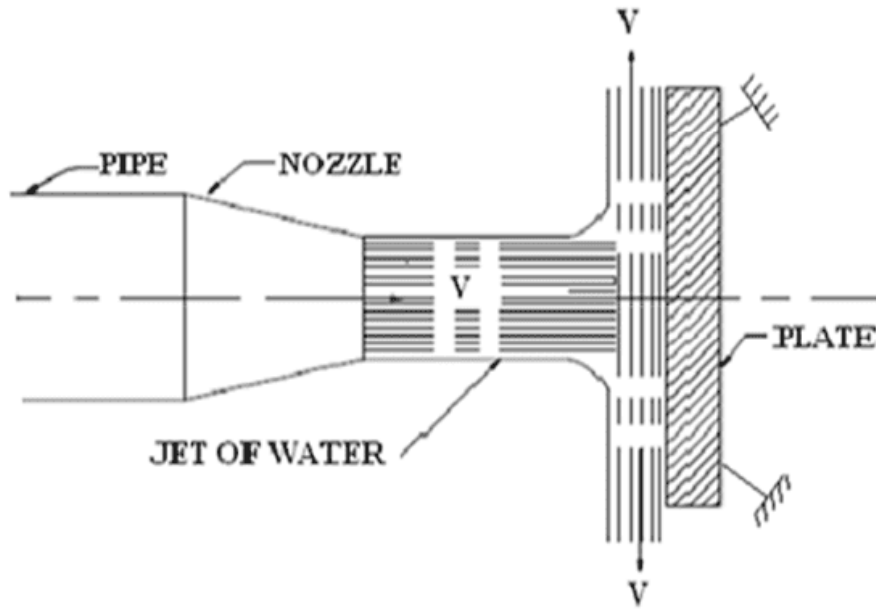
$$\Rightarrow F_x = \frac{\gamma}{g} Q v \tag{6}$$

Figure 1: Plate normal to jet.

If a is the area of jet,

$$F_x = \frac{\gamma}{g}avv$$

$$\Rightarrow F_x = \frac{\gamma}{g}av^2 \tag{7}$$

2.1.2 Inclined Plate
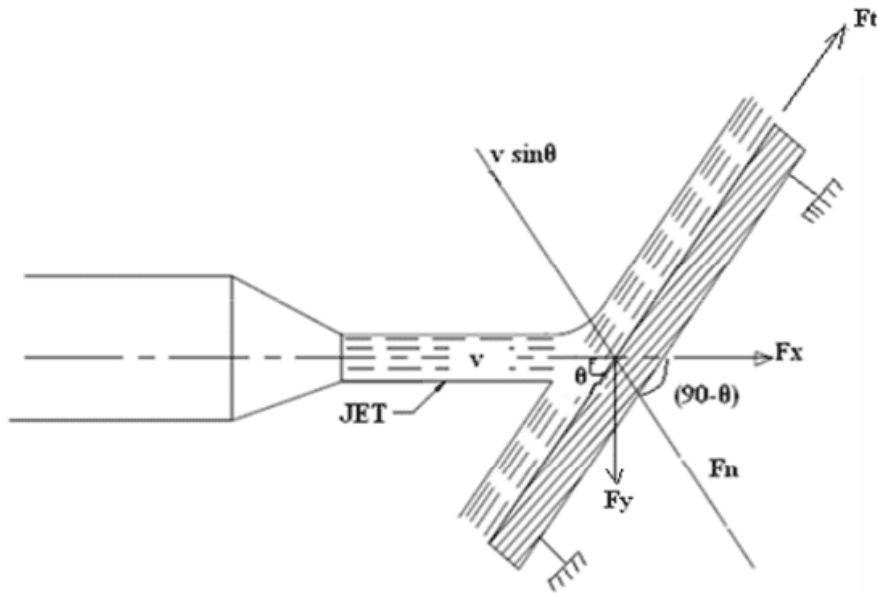

Figure 2: Plate inclined to jet.

$$F = F_n = \rho Qv sin\theta$$

Again,

$$F_x = F sin\theta$$

$$= (\rho Qv sin\theta)sin\theta$$

$$F_x = \rho Q v sin^2\theta \tag{8}$$

And,

$$F_y = F cos\theta$$
$$\Rightarrow F_y = \rho Q v sin\theta cos\theta \tag{9}$$

Determine of division of flows: Let F_s (F_t in fig. 2) be the force along the inclined surface of plate and Q_1 and Q_2 are quantities of flow along the surface. As there is no change in elevation of pressure before and after impact, the magnitude velocity leaving the plate will remain the same.

since no force is exerted on the fluid by the plate in "S" direction, then,

$$F_s = 0 = \rho Q V cos\theta \tag{10}$$

Again,

$$\rho Q_1 v - \rho Q_2 v = 0 \tag{11}$$

From eq^n (10) & (11),

$$\rho Q v cos\theta = \rho Q_1 v - \rho Q_2 v$$
$$\Rightarrow Q cos\theta = Q_1 - Q_2 \tag{12}$$

From continuity eq^n,

$$Q_1 + Q_2 = Q \tag{13}$$

From eq^n (12) & (13),

$$Q_1 = \frac{1}{2}Q(1 + cos\theta)$$
$$Q_2 = \frac{1}{2}Q(1 - cos\theta) \tag{14}$$

3 Lecture 3

Problem

A jet of water with a velocity of 35 m/s strikes a plat inclined of 30°, cross sectional area if jet 25 cm^2. Find the force exerted by the jet on the plate. Calculate the components of force and find the ratio in which the discharge gets divided after striking the plate.

Solution:

Given,

$$\text{velocity, } v = 35 \, \text{m/s}$$
$$\text{angle, } \theta = 30°$$
$$\text{cross-sectional area, } a = 25 \, \text{cm}^2$$

$$\text{Volumetric flow rate, } Q = a \times v$$
$$= 0.0025 \times 35 \, \text{m}^3/\text{s}$$
$$= 0.0875 \, \text{m}^3/\text{s}$$

$$\text{Force, } F = \rho Q v \sin \theta$$
$$= 1000 \times 0.0875 \times 35 \times \sin(30°) = 1531.25 \, \text{N}$$

$$F_x = F \sin \theta = 765.6 \, \text{N}$$
$$F_y = F \cos \theta = 1326.1 \, \text{N}$$

$$Q_1 = \frac{Q}{2}(1 + \cos \theta) = 0.0816 \, \text{m}^3/\text{s}$$
$$Q_2 = \frac{Q}{2}(1 - \cos \theta) = 0.00586 \, \text{m}^3/\text{s}$$
$$\therefore \frac{Q_1}{Q_2} = 13.92$$

3.1 Thrust on moving flat plate normal to the direction of jet
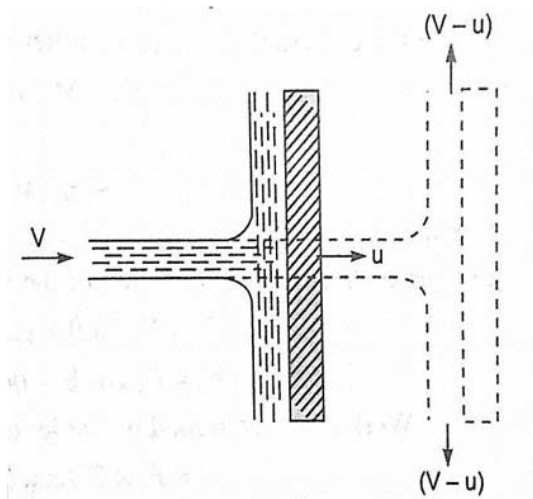

Figure 3: Thrust on moving flat plate normal to the direction of jet

Let, the flat plate moves with a velocity u in the direction of the jet and the velocity of jet is v. The effective velocity with which the jet strike the plate $= v - u$. The mass of the fluid striking the plate per second $= \rho a(v - u)$, where a is the area of the jet.

Thrust exerted on the plate in the direction of the jet is,

$$F = \rho a(v - u)[(v - u) - 0]$$

$$F = \rho a(v - u)^2 \tag{15}$$

$$\text{work done per second} = F \times u = \rho a(v - u)^2 \times u \tag{16}$$

However, this is not practically feasible, because the distance between the nozzle at the plate is go on increasing. If a series of plates were so arranged that each plate appeared successively before the jet in the same position and always moving with a velocity u to the direction of the jet. Then mass of the fluid striking the plate $= \rho av$.

*note : [The whole flow of the nozzle is utilized by the plate.]
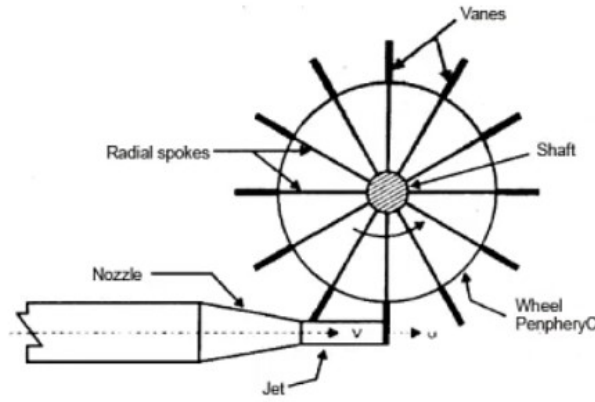
Figure 4: Thrust on Successive moving plat normal to the direction of jet

The thrust on the plate,

$$F = \rho av[(v - u) - 0]$$
$$= \rho av(v - u)$$

Work done per second $= F \times u$

$$= \rho av(v - u)u \tag{17}$$

Now, the input power $=$ K.E. of the jet

$$= \frac{1}{2}mv^2$$

$$= \frac{1}{2}\rho av \times v^2$$

$$\therefore \text{Input power} = \frac{1}{2}\rho av^3 \tag{18}$$

The efficiency of the the wheel,

$$\eta = \frac{\rho av(v - u)u}{\frac{1}{2}\rho av^3}$$

$$\eta = \frac{2u(v - u)}{v^2} \tag{19}$$

✓Generally u is changed, v is not changed significantly.

For a given jet velocity, efficiency will be maximum, if,

6

$$\frac{d\eta}{du} = 0$$

$$\frac{d}{du}\left[\frac{2u(v-u)}{v^2}\right] = 0$$

$$2v - 4u = 0$$

$$u = \frac{v}{2}$$

For the maximum efficiency of wheel, the peripheral speed of the wheel is equal to half of the jet velocity.

The max efficiency is given by -

$$\eta_{max} = \frac{2\frac{v}{2}\left(v - \frac{v}{2}\right)}{v^2} = \frac{1}{2} = 50\%$$

3.2   Fluid Jet (on curved plate)

(a) stationary plate

Velocity of jet at inlet in x-direction $= v_1 cos\alpha_1$
Velocity of jet at outlet in x-direction $= v_2 cos\alpha_2$

Force exerted on the plate,
$$F_x = \rho Q(v_1 cos\alpha_1 - v_2 cos\alpha_2) \tag{20}$$
here, $Q = av$

If the curvature of plate at outlet such that outlet angle α_2 is more than 90°, then the second term of the eq^n (20) will be negligible. Hence in order to get more force, the curvature of the plate should be such that the angle α_2 is obtuse.
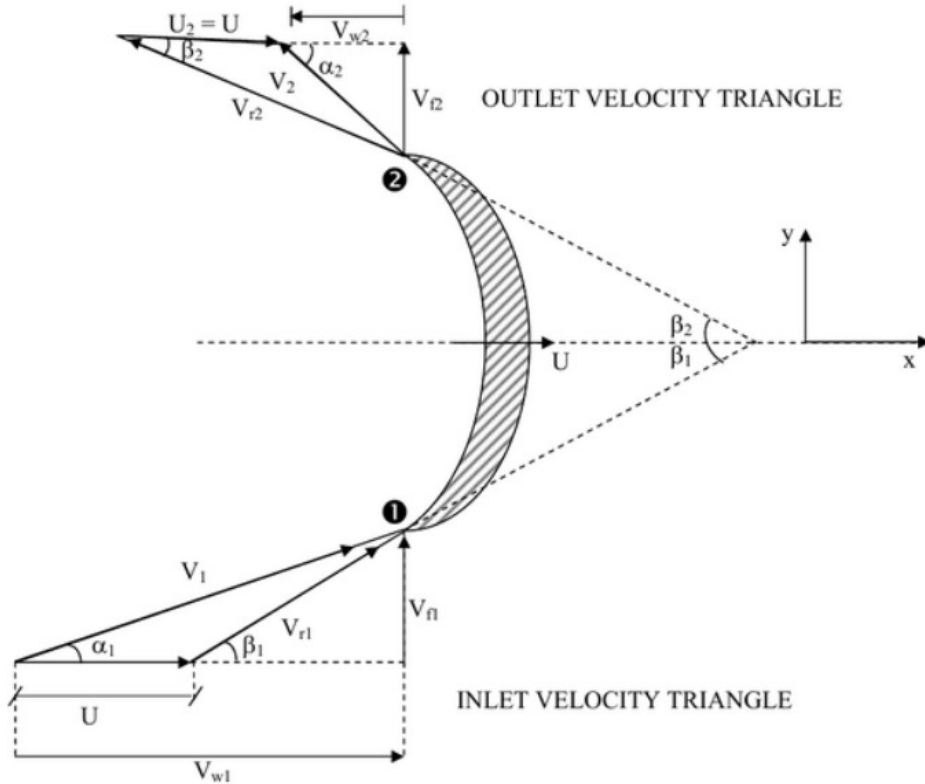
Single Moving Plate / Curved Vane

Figure 5: Single moving plate / curved vane

- $v_1, v_2 \rightarrow$ Absolite velocities of the jet at inlet and outlet respectively

- $u_1, u_2 \rightarrow$ peripheral velocity of the vane at inlet and outlet respectively

- $v_{r1}, v_{r2} \rightarrow$ relative velocity in inlet and outlet respectively.

$$
\begin{aligned}
u + v_r &= v \\
\Rightarrow v_r &= v - u
\end{aligned}
$$

v_w, v_f is the components of v

- $v_{f1}, v_{f2} \rightarrow$ velocity of flow at inlet and outlet respectively. alternatively, components of absolute velocity normal to the direction of motion.

- $v_{w1}, v_{w2} \rightarrow$ velocity of whirl at the inlet and outlet respectively. alternatively, components of absolute velocity in the direction of motion.

4 Lecture 4: Curved Vane

<div align="right">Date: 17/06/2023</div>

4.1 Analysis of Curved Vane

The jet enters the vane without shock if the relative v_{r_1} makes an angle β_1 with the direction of motion (on x-axis). The jet glides over the vane and leaves with a velocity v_{r_2}. If the vane is smooth, then $v_{r_1} = v_{r_2}$. the jet will leave the vane without shock if the relative velocity v_{r_1} makes an angle in β_2.

The mass flow rate of fluid $= \rho Q = \rho a v_r$, where a $\rightarrow$ area of jet.

- Note: if there is only single vane, then relative velocity v_r have to consider, otherwise total velocity v will be counted.

Force on the vane in the direction of motion , if $\alpha_2 < 90°$,

$$
F_x = \rho a v_{r_1} \left[v_{w_1} - (-v_{w_2}) \right]
$$

$$
F_x = \rho a v_{r_1} \left(v_{w_1} + v_{w_2} \right) \tag{21}
$$

if $\alpha_2 > 90°$,

$$
F_x = \rho a v_{r_1} \left(v_{w_1} - v_{w_2} \right) \tag{22}
$$

if $\alpha_2 = 90°$,

$$
F_x = \rho a v_{r_1} \left(v_{w_1} \right) \tag{23}
$$

This is the general expression of the thrust -

$$
F_x = \rho a v_{r_1} \left[v_{w_1} \pm v_{w_2} \right]
$$

if $u_1 = u_2 = u$, work done per second $= F_x \times u$

$$
= \rho a v_{r_1} \left[v_{w_1} \pm v_{w_2} \right] \times u \tag{24}
$$

Also work done can be determined for the changing kinetic energy. Thus the work done per second

$$
\text{Thus the work done per second} = \frac{1}{2} \rho a v_{r_1} \left[v_1^2 - v_2^2 \right] \tag{25}
$$

From the equation (24) & (25),

$$
2(v_{w_1} \pm v_{w_2})u = v_1^2 - v_2^2 \tag{26}
$$

Input energy of jet at inlet $= \frac{1}{2} \rho a v_1 \times v_1^2$
The hydraulic efficiency,

$$
\eta = \frac{\rho a v_{r_1} \left[v_{w_1} \pm v_{w_2} \right] u}{\frac{1}{2} \rho a v_1 \times v_1^2}
$$

$$
\eta = \frac{2 u v_{r_1} \left[v_{w_1} \pm v_{w_2} \right]}{v_1^3} \tag{27}
$$

5   Lecture 5: Series of Vanes

If a series of vanes is fixed radially to the rim of a wheel, there is always one vane or another facing the jet. Thus the entire fluid is utilized. This arrangement of vanes is used in radial flow turbines.

Two types of radial flow turbine:

1. Inward flow turbine : If jet enters outer periphery

2. Outward flow turbine : If jet enters inner periphery

Let R_1 and R_2 be the radii of the wheel at the inlet and outlet.
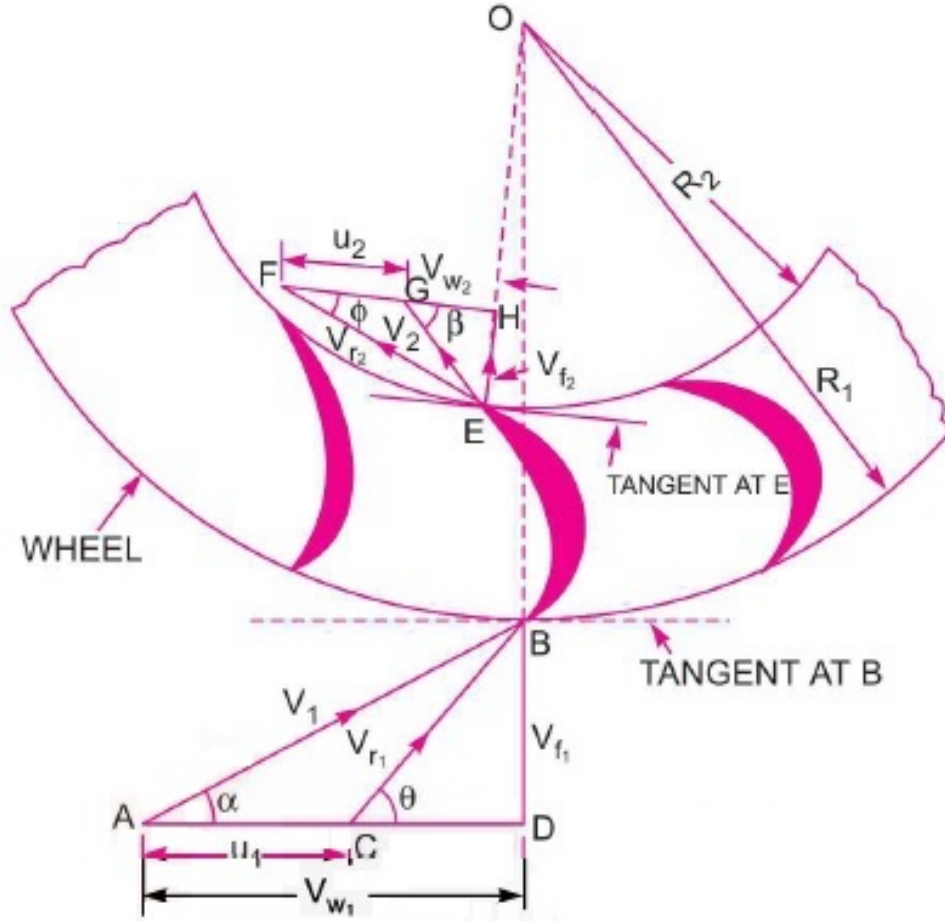
Figure 6: Series of vanes

Let, ω is the angular velocity of the wheel,

$$u_1 = \omega R_1 = \frac{2\pi N}{60} R_1$$

$$u_2 = \omega R_2 = \frac{2\pi N}{60} R_2$$

here, N = RPM

Let m be the mass flow rate striking the vane. The momentum in the tangential direction at the inlet $= m v_w$

The moment of momentum about the center at inlet $= m v_{w_1} R_1$

The moment of momentum about the center at outlet $= -m v_{w_2} R_2$

$\therefore$ Torque on the wheel,

$$T = m v_{w_1} R_1 - (-m v_{w_2} R_2)$$

$$T = m \left(v_{w_1} R_1 + v_{w_2} R_2 \right) \tag{28}$$

$$\text{Work done per second} = T \times \omega$$
$$= m\left(v_{w_1}R_1 + v_{w_2}R_2\right) \times \omega$$

$$\text{work done per second} = m\left(v_{w_1}u_1 + v_{w_2}u_2\right) \tag{29}$$

Here, $\beta < 90°$

If $\beta > 90°$,
$$\text{work done per second} = m\left(v_{w_1}u_1 - v_{w_2}u_2\right) \tag{30}$$

The general expression,
$$\text{work done per second} = m\left(v_{w_1}u_1 \pm v_{w_2}u_2\right) \tag{31}$$

This equation is known as Euler Pump Turbine equation.

Hydraulic Efficiency,
$$\eta_h = \frac{m\left(v_{w_1}u_1 \pm v_{w_2}u_2\right)}{\frac{1}{2}mv_1^2} = \frac{2\left(v_{w_1}u_1 \pm v_{w_2}u_2\right)}{v_1^2} \tag{32}$$

Problem 01

A jet of water having a velocity of 30 ms^{-1} impinging for a series of vanes mounted on the periphery of a wheel moving with a peripheral velocity of 15 ms^{-1}. The aboslute velocity is directed at an angle of 30° to the direction of motion of vanes. The relative velocity at outlet is 95% of the value at inlet. The absolute velocity at exit is normal to the direction of the motion of the vane. Assuming smooth entry, find the vanes angle at inlet and outlet and the hydraulic efficiency of vane.

Solution:

given,
$v_1 = 30$ m/s
$\alpha = 30°$
$u_1 = 15$ m/s

$v_{r_2} = 0.95 \times v_{r_1}$
$\beta = 90°$ [normal]
so, $v_{w_2} = 0, v_2 = v_{f_2}$
✓ have to add figure in exam

$v_{w_1} = v_1 \cos 30$
$\quad = 30 \cos 30$
$\quad = 25.98\,m/s$

$v_{r_2} = 0.95 \times v_{r_1}$
$\quad = 17.67\,m/s$

$\eta_h = \dfrac{2v_{w_1}u_1}{v_1^2}$

$\eta_h = \dfrac{2 \times 25.98 \times 15}{30^2}$
$\quad = 0.866$
$\quad = 86.7\%$

$\tan\theta = \dfrac{v_{f_1}}{v_{w_1} - u_1}$
$\theta = 53.8$

Assuminng $u_1 = u_2$,

$\cos\phi = \dfrac{v_{f_2}}{u_2}$

$v_{r_1} = \sqrt{[v_{w_1} - u_1]^2 + (v_{f_1})^2}$
$\quad = 18.59\,m/s$

$\phi = \cos^{-1}\left(\dfrac{15}{17.66}\right)$
$\quad = 27.53$

Problem 02

A jet of water 5 cm in diameter inpinges on a curved vane and is deflected through an angle of 175°. The vane moves in the same direction at that of the jet with a velocity of 35 m/s. The rate of flow is 170 l/s. Determine the compounds forces, horse power developed by the vane and water efficiency. Neglect friction. Solve this problem, if instead of one vane, there are a series of vanes fixed to a wheel.

$F_x = ?, F_y = ?, HP = ?, \eta_h = ?$

Solution:

Given,

d = 5 cm

u = 35 m/s

Q = 170 l/s = 0.17 m^3/s

$$a = \frac{\pi}{4}[0.05]^2 = 0.196 \times 10^{-2}/, m^2$$

$$v = \frac{Q}{a} = \frac{0.17}{0.196 \times 10^{-2}} = 86.58/, m/s$$

v = u + v_r

$v_r = 86.58 - 35 = 51.58$ m/s

Neglecting friction,

$$v_{r_1} = v_r = 51.58\, m/s$$
$$v_{r_1}\cos\phi = 51.38\, m/s$$
$$\therefore v_{w_1} = v_{r_1}\cos\phi - u_1 = 51.38 - 35 = 16.38\, m/s\,[u = u_1]$$

$$F_x = \rho a v_r\,(v_w + v_{w_1}) = 10427.87N$$
$$F_y = \rho a v_r \times v_{f_1}$$
$$HP = \frac{F_x u}{746}$$
$$\eta_h = \frac{F_x u}{\frac{1}{2}mv^2} = \frac{10427.87 \times 35}{\frac{1}{2} \times 1000 \times 0.17 \times 86.58^2} = 57.3\%$$

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6 Lecture 6: Hydraulic Turbines

Hydraulic turbines are the machines which converts the hydraulic energy into mechanical energy. In other words, hydraulic turbines are prime movers which run with hydraulic energy. The mechanical energy produced by a hydraulic turbine can be connected into electric energy by coupling the turbine to an electric generator.

Classfications of Turbines

May be classified according to the following criteria:

1. Hydraulic action

2. Direction of fuel of water

3. Direction of the shaft

4. Head

5. Specific Speed

Prime Mover

Prime movers refer to the mechanical devices or systems that convert hydraulic energy (the energy of flowing or falling water) into mechanical energy (rotational motion). These prime movers are responsible for driving the turbine rotor and generating power. In the case of hydraulic turbines, the most common type of prime mover is an electric generator or alternator. The turbine is connected to the rotor of the generator, and as the turbine rotates, it transfers its mechanical energy to the generator, which then converts it into electrical energy. The prime mover can also be a mechanical drive system, such as a gearbox or a shaft, which is connected to various industrial machines or equipment. In such cases, the mechanical energy produced by the turbine is directly transmitted to the driven machinery to perform specific tasks. The choice of prime mover depends on the application and the desired output. In most large-scale hydropower plants, electric generators are used as prime movers to produce electrical power that can be transmitted over long distances and distributed to consumers. In other industrial applications, mechanical prime movers may be employed for specific operations within a facility.

The specific speed of a hydraulic turbine is a dimensionless parameter that characterizes the geometric and hydraulic performance of the turbine. Specific speed (Ns) is defined as the speed at which a geometrically similar turbine would operate to produce unit power (1 horsepower or 1 kilowatt) under a unit head (1 foot or 1 meter).

Hydraulic Action

1. **Impulse Turbine:** Here the available head is converted into kinetic energy by passing the water through a nozzles. Ex - Pelton wheel, girard turbine, banki turbine etc.

2. **Reaction Turbine:** A part of the total head is connected in kinetic energy and the rest remains in the form of pressure energy. Example of reaction turbines - kaplan turbine, propeller turbine, etc.

Direction of flow of water

1. Tangent flow turbine : The water strikes the runner in the direction of tangent to the wheel. Ex - pelton wheel.

2. radial flow turbine: may be classified as inward flow and outward flow. Ex - francis turbine (old) (inward flow)

3. Axial Flow : Kaplan and propeller turbine

4. Mixed Flow Turbine : Francis turbine

Direction of the shaft

1. Vertical : shaft vertical, runner horizontal

2. Horizontal : shaft horizontal, runner vertical

Head

1. High Speed Turbine: Net head is 150m - 2000 m. Ex - pelton wheel turbine

2. Medium Head Turbine: Head from 30 m - 150 m. Ex - Francis Turbine

3. Tow Head Turbine: Head ¡ 30 m. Ex - Kaplan and propeller. (kaptai)

Specific speed

1. Low Specific Speed : speed ¡ 60. Ex - pelton wheel

2. Medium Specific Speed : 60 ¡ speed ¡ 400. Ex - Francis turbine.

3. High Specific Speed: speed ¿ 400. Ex - Kaplan Turbine.

Difference between axial and radial hydraulic turbine

- Fluid Flow Direction:

  Axial Hydraulic Turbine: In an axial hydraulic turbine, the water flow is parallel to the axis of rotation. The water enters the turbine axially and passes through the rotor blades in the same axial direction.

  Radial Hydraulic Turbine: In a radial hydraulic turbine, the water flow is radial, moving inward or outward from the center of the turbine. The water enters the turbine radially and flows through the rotor blades in a radial direction.

- Blade Orientation and Configuration:

  Axial Hydraulic Turbine: Axial turbines have rotor blades arranged in a propeller-like configuration, similar to the blades of a ship's propeller. The blades are oriented parallel to the axis of rotation and typically have a streamlined shape to efficiently capture the water's energy.

Radial Hydraulic Turbine: Radial turbines have rotor blades arranged in a radial pattern, perpendicular to the axis of rotation. The blades extend radially from the center of the turbine and are typically straight or slightly curved. They are designed to effectively capture the water flow in a radial direction.

- Applications:

Axial Hydraulic Turbine: Axial turbines are commonly used in high-flow, low-head applications, such as in large-scale hydropower plants. They are suitable for situations where a large volume of water needs to be harnessed efficiently.

Radial Hydraulic Turbine: Radial turbines are typically employed in low-flow, high-head applications. They are often used in small-scale hydropower installations, such as run-of-river projects or micro-hydropower systems.

- Efficiency and Performance:

Axial Hydraulic Turbine: Axial turbines are known for their high efficiency at high flow rates. They can handle a significant volume of water and are well-suited for sites with a substantial water supply.

Radial Hydraulic Turbine: Radial turbines are efficient in applications where there is a significant pressure drop. They are designed to generate power under high head conditions, where water is available at a relatively low flow rate.

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7 Lecture 7: Pelton Wheel

Date: 08/07/2023

Mainly 2 types of turbine:

1. Impulse Turbine

2. Reaction Turbine

✓Force always acts perpendicular to the wall.

Pelton Wheel

The whole hydraulic energy is converted into kinetic energy.

Major Components

a) Nozzle With Control Mechanism

The jet velocity remains the same and only the discharge changes.

b) Buckets & Runner

A pelton wheel is fitted with buckets havingthe shape of double hemispherical cup. The angle at the outlet teeth varies from 10° to 20° and the jet deflects 170°-160°.

The advantage of double cup bucket is that because of symmetric, the axial thrust on the shaft is zero.

Casing

- To prevent splashing

- To lead the water to tail race

- To act as a safeguard against accident

Hydraulic Brake

The hydraulic brake consists of a small nozzle which directs a jet on the back of the bucket, in the opposite direction which causes braking.
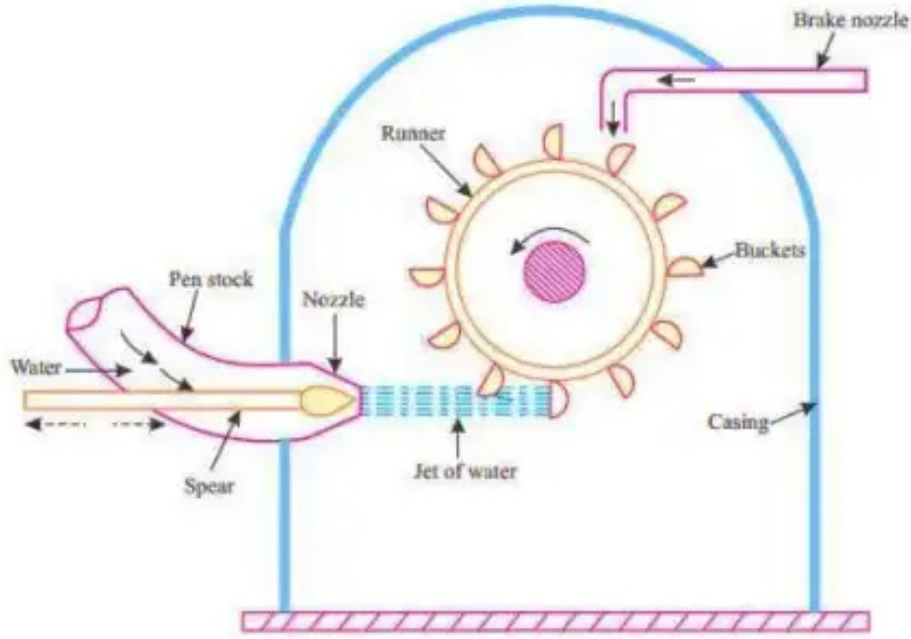
Figure 7: Pelton wheel components

Deflector

The sudden closing of the nozzle may cause excessive water hammer. To avoid water hammer, the nozzle is closed slowly by a moveable steel plate known as deflector.

Why Pelton wheel deflection angle is not set for 180°

Although, deflection angle 180° will give the best thrust, it is not set in pelton wheel. When water flows through the Pelton wheel, it strikes the buckets or cups, causing the wheel to rotate. The design of the buckets is crucial for efficient energy transfer. The water jet enters the bucket at a specific angle, known as the jet's deflection angle.

   If the deflection angle were set to 180 degrees, the water jet would hit the bucket and then reverse its direction completely, flowing back in the opposite direction. This would result in a loss of kinetic energy and disrupt the smooth operation of the wheel.

   To maximize the energy transfer, the deflection angle is typically set to less than 180 degrees. Ideally, the water jet should leave the bucket with minimal residual velocity, meaning that most of its kinetic energy has been transferred to the wheel. This is achieved by setting the deflection angle at a value that allows the water jet to leave the bucket at an optimal angle, usually around 160 to 170 degrees.

Expression of work done

Effective head, H = Gross head - h_f
$h_f \rightarrow$ loss of head in the penstock

   torque,
$$T = \rho a v \left[v_w R - (v_{w1} R_1) \right] = \rho a v \left(v_w R + v_{w1} R_1 \right)$$

$$W = T \times \omega = \rho a v \left(v_w R + v_{w1} R_1 \right) \times \omega$$

Now, $R\omega = u, R_1\omega_1 = u_1$
$$W = \rho a v \left(v_w u + v_{w1} u_1 \right)$$

In general,
$$W = \rho a v \left(v_w u \pm v_{w1} u_1 \right) \tag{33}$$

Figure 8: Force analysis on pelton wheel

Let, $u = u_1$

$$
\begin{aligned}
W &= \rho a v \left(v_w + v_{w1}\right) \times u \\
&= \rho a v \left(v + v_{r1} \cos \phi - u\right) \times u \\
&= \rho a v \left(v - u + K v_r \cos \phi\right) \times u \\
&= \rho a v \left[(v - u) + K(v - u) \cos \phi\right] \times u \\
&= \rho a v (v - u)(1 + K \cos \phi) \times u
\end{aligned}
$$

$$W = \rho a v (v - u)(1 + K \cos \phi) \times u \tag{34}$$

Neglecting losses in nozzle,

$$\text{Input power} = \frac{1}{2} \rho a v \times v^2 \tag{35}$$

Hydraulic efficiency,

$$
\begin{aligned}
\eta_h &= \frac{\text{work done}}{\text{input power}} \\
&= \frac{\rho a v (v - u)(1 + K \cos \phi) \times u}{\frac{1}{2} \rho a v \times v^2} \\
\eta_h &= \frac{2(v - u)(1 + K \cos \phi) \times u}{v^2}
\end{aligned} \tag{36}
$$

8 Lecture 8: Working proportions of Pelton Wheel

η_h will be maximum if,

$$\frac{d\eta_h}{du} = 0$$

$$\frac{d}{du}\left\{2(v - u)(1 + K \cos \phi)u\right\} = 0$$

$$v - 2u = 0$$

$$u = \frac{v}{2}$$

now,

$$(\eta_h)_{max} = \frac{2(v - \frac{v}{2})(1 + K\cos\phi) \times \frac{v}{2}}{v^2}$$

$$= \frac{\frac{v^2}{2}(1 + K\cos\phi)}{v^2}$$

$$= \frac{1 + K\cos\phi}{2}$$

If K=1, i.e. the vanes are smooth,

$$\eta_h = \frac{1 + \cos\phi}{2}$$

The η_h is less than unity, because K is always less than unity and $\phi \neq 0$

The work done by a pelton wheel may also be expressed as the difference of kinetic energy at the inlet and outlet. This work done $= \frac{1}{2}\rho av\left(v^2 - v_1^2\right)$

$$\eta_h = \frac{\frac{1}{2}\rho av\left(v^2 - v_1^2\right)}{\frac{1}{2}\rho avv^2} = \frac{v^2 - v_1^2}{v^2} \tag{37}$$

The mechanical efficiency (η_m) is defined as the ratio of the power obtained at the shaft (S.P.) to the power developed by the runner. Thus,

$$\eta_m = \frac{S.P.}{P} \tag{38}$$

The overall efficiency,

$$\eta_o = \frac{S.P}{\text{Power supplied ti the turbine}(P_t)} = \frac{S.P}{P} \times \frac{P}{P_t} = \eta_h \times \eta_m \tag{39}$$

The working proportions of pelton wheel

1. Velocity of Jet:
$$V_{theo.} = \sqrt{2gH}$$

   Here, H = net head. Now actual velocity,
$$V = C_v\sqrt{2gH} \tag{40}$$
$$C_v = 0.97 \sim 0.99$$

2. Theoretical Power (P_t)
$$P_t = \gamma QH \tag{41}$$

$$\boxed{\tfrac{1}{2}mv^2 = \tfrac{1}{2}m \times 2gH = mgH = \rho QgH = \gamma QH}$$

3. Angle (θ)
   The angle varies from 10°-20°. The average value of angle is 15°.

4. Diameter of Jet(α)
   The diameter of jet may be obtained if the discharge is low.

$$\frac{\pi}{4}d^2 = \frac{Q}{C_v\sqrt{2gH}}$$

$$d = \left(\frac{4Q}{\pi C_v\sqrt{2gH}}\right)^{\frac{1}{2}} \tag{42}$$

5. Speed Ratio (K_u)
   It is the ratio of the velocity of the wheel at pitch circle (u) to the theoretical velocity of the jet,

$$K_u = \frac{u}{\sqrt{2gH}} \tag{43}$$

K_u varies from $0.43 \sim 0.47$

6. Mean diameter of the wheel (D)

   The diameter of the wheel measured upto the center of the bucket is called the mean diameter.

   $$u = \frac{\pi D N}{60}$$

   $$D = \frac{60u}{\pi N} \tag{44}$$

   D is also known as pitch diameter.

7. Jet Ratio (J_r)

   $$J_r = \frac{D}{d} \tag{45}$$

   J_r varies from 11 to 14.

8. Size of Bucket

   The flowing proportion of the bucket are usually adopted.
   Radial length of the bucket $= (2 \sim 3) \times d$
   Axial width of the bucket $= (3 \sim 5) \times d$
   Depth of the bucket $= (0.8 \sim 1.2) \times d$
   Where, 'd' is the diameter of the jet.

9. Number of Jets

   Ordinarily pelton wheel has a single jet, but to develop more power, it has number of jets. More than 6 jets are not provided.

10. Number of Buckets (z)

    Number of buckets should be as small as possible. So as to keep the frictional losses to a minimum. But number of buckets should also be sufficiently large, so that the jet is always intercepted by one bucket or another. The jet will always be intercepted by the buckets if the angle between two successive buckets is equal to or less than 2θ, such that

    $$\cos\theta = \frac{R + \frac{d}{2}}{r_o}$$

    or,

    $$\sin\theta = \sqrt{1 - \cos^2\theta} = \left[1 - \left(\frac{R + \frac{d}{2}}{r_o}\right)^2\right]^{1/2}$$

    For a small θ, $\sin\theta \approx \theta$

    $$\theta = \left[1 - \left(\frac{R + \frac{d}{2}}{r_o}\right)^2\right]^{1/2}$$

    Therefore, the number of buckets is given by,

    $$z = \frac{2\pi}{2\theta} = \frac{\pi}{\theta}$$

    $$z = \frac{\pi}{\left[1 - \left(\frac{R + \frac{d}{2}}{r_o}\right)^2\right]^{1/2}} \tag{46}$$

    Emperical formula given by Taygun,

    $$z = \frac{D}{2d} + 15$$

    $$z = 0.5 J_r + 15 \tag{47}$$

    Here, $J_r = $ Jet Ratio

9   Lecture 9: Problems on Pelton Wheel

Problem-01

A single jet pelton turbine is required to drive a generator to develop 10000 kw. The available head at the nozzle is 760m. Assuming electric generator efficiency as 95%, pelton wheel efficiency 87%, coefficient of velocity for nozzle as 0.97, mean bucket velocity as 0.46 of jet velocity. Outlet angle of the bucket as 15˚ and the relative velocity of water leaving the bucket as 0.85 of that at inlet. Find: a) Diameter of jet, b) The flow in cumecs, c) The force exerted by the jet on the bucket, d) If the ratio of mean bucket circle diameter to the jet diameter is not to be less than 10, find the synchronous speed for generator at 50 cycles/second and the corresponding mean diameter of the runner.

Solution:

Given, P = 10000 kW, H = 760 m, $\eta_g = 0.95, \eta_h = 0.85, C_v = 0.97$, u = 0.46 v, $\phi = 15 \deg, V_{r1} = 0.85 V_r$, f=50 hz, $\frac{D}{d} > 10$

Available power at the wheel $= \frac{10000}{0.95} = 10526.3$ kW
Now,

$$\eta_h = \frac{10526.3 \times 10^3}{\gamma Q H}$$
$$0.87 = \frac{10526.3 \times 10^3}{9810 \times Q \times 760}$$
$$Q = 1.623\, cumecs$$

Again,

$$Q = \frac{\pi}{4} d^3 \times C_v \times \sqrt{2gH}$$
$$d = 0.132\, m$$

Assuming $D/d = 10$, D = 1.32 m
Now,

$$v_w = v = 0.97 \times \sqrt{2 \times 9.81 \times 760} = 118.5\, m/s$$

$$u = 0.46 \times v = 54.51\, m/s$$

$$v_r = v_w - u = 64\, m/s$$

$$v_{r1} = 0.85 v_r = 54.4\, m/s$$

$$v_{w1} = v_{r1} - v_{r1} \cos \phi = 544 - 52.55 = 2\, m/s$$

$$\text{Force} = \rho Q(v_w - v_{w1}) = 1000 \times 1.623 \times (118.5 - 2)$$

So, Force = 189.1 kN
Again,

$$u = \frac{\pi D N}{60}$$
$$N = 788\, rpm$$

Now,

$$N = \frac{120 \times f}{Pole}$$
$$Pole = 7.614 \approx 8$$

Putting, Pole = 8, $N = \frac{120f}{8} = 750 rpm$

$$u = \frac{\pi D \times 750}{60} \Rightarrow D = 1.388\, m$$

Problem-02

A pelton wheel is developing 11000 kW under a head of 650 m at 600 rpm. The peripheral velocity of wheel is 0.47 times the jet. The coefficient of velocity is 0.97 and overall efficiency is 85%. Find jet diameter, wheel diameter, and number of buckets.

Solution:

P = 11000 kW, H = 650 m, N = 600 rpm, $C_v = 0.97, \eta_o = 0.85$

$$\eta_o = \frac{P}{\gamma Q H}$$
$$Q = 2.03\, m^3/s$$

again,

$$Q = \frac{\pi}{4} d^2 \times v$$
$$d = 0.1528\, m$$

$$V = C_v \sqrt{2gH} = 110.67\, m/s$$
$$u = 0.47v = 52.02\, m/s$$
$$u = \frac{\pi D N}{60} \Rightarrow D = 1.66\, m$$

Now,

$$z = \frac{D}{2d} + 15 = 21$$

Problem-03

A pelton wheel is rotating at a speed of 650 rpm. The diameter of the jet is 82 mm and the velocity of jet is 111 m/s. The ratio of the bucket speed to jet speed is 0.47. The bucket deflects the jet through an angle of 170˚. Neglecting friction, find -
a) D, b) Power, c) KE per unit weight of exit water

Solution:

Follow Quamrul sir book: 2.15

10 Lecture 10: Radial Flow Turbine

Francis Turbine

1. Radially inward flow reaction turbine

2. Radially outward flow reaction turbine

3. Mixed flow reaction turbine

Francis turbine is designed by an eminent hydraulic engineer J.B. Francis in 1849. It is an inward flow reaction turbine. If the head is less than 150 m, pelton wheel is so slow and unweildy that they are unsuitable for power generation. In a francis turbine, only a part of net head is transformed into kinetic energy at inlet. At the rest of head remains in the form of pressure energy. There is a difference of pressure between the entrance and exit of runner. The difference of pressure is known as the reaction pressure. This pressire is responsible for running of the reaction turbine.

Major Components

1. Scroll casing

2. Guide vane

3. Runner

4. Draft tube

5. Penstock

Scroll Casing:

The purpose of casing is to distribute water over the guide vanes and prevent the formation of eddies. For small head, it is made of concrete. For higher head, it is made of steel or cast iron.

Guide Vanes:

Guide vanes are of the shape of airfoil and spaced evenly around the periphery of runner. Each guide vane can be rotated about it's pivot center. The incharge can be varied depending upon the load. The passage between the adjacent guide vanes can be varies. The vanes are made of cast steel.

Runner:

In runner, the vanes are in between 16 to 24. They are feeded between two circular plates. They are made of cast steel for smaller unit and made of stainless steel in larger units. They are keyed to the shaft. The shape of vanes is such that, the water enters the runner radially, at the outer periphery and leaves in axial direction at the inner periphery.

Draft Tube:

The draft tube is a closed conduit which takes the water coming out from the runner exit and discharges it to the tail race.

Main Purposes:

- A negative head is created at runner exit by a draft tube

- The negative head created is approximately is equal to the vertical height of the tube. Thus the turbine may be installed above the tail race without any loss of effective head.

- Because of gradual increase in its cross sectional area, a large portion of the kinetic energy of the water at the runner exit is converted into useful pressure energy. Thus the draft tube acts as a pressure recuperating (recovering) and increase the effective head.

Work Done

$$P = m \left(v_w u \pm v_{w1} u_1 \right) \qquad (48)$$

If $v_{w1} = 0$, then

$$P = m \left[v_w u \right] \qquad (49)$$

$$\text{Input Power of turbine} = \frac{1}{2} m v^2 = \frac{1}{2} m.2gH = mgH \qquad (50)$$

$$\text{Hydraulic Efficiency, } \eta_h = \frac{m v_w u}{mgH} = \frac{v_w u}{gH} \qquad (51)$$

If the $v_w 1 \neq 0$, then the hydraulic efficiency,

$$\eta_h = \frac{v_w u \pm v_{w1} u_1}{gH} \qquad (52)$$

Generally, η_h varies from $0.85 \approx 0.90$.

$$\eta_m (\text{Mechanical efficiency}) = \frac{S.P.}{\text{Power Developed}}$$

$$\eta_o (\text{Overall efficieny}) = \frac{S.P}{mgH}$$

$$= \frac{S.P.}{\text{Power dev.}} \times \frac{\text{Power Dev.}}{mgH}$$

$$= \eta_m \times \eta_h$$

11 Lecture 11: Radial Flow Turbine

The weight of water may be obtained from the flow area and the velocity of flow.

Let, N = The number of vanes
B, D & t = width, diameter and thickness of the vane at inlet
B_1, D_1 & t_1 = width, diameter and thickness of the vane at outlet

The area of flow $= \pi DB - NBt$
$= (\pi D - Nt)B$
$= K\pi DB$ [where, K = Vane thickness factor; k¡1]

"K" is the vane thickness factor. It is always less than unity.

For example - If 10% area is occupied by vanes. Then, K = 0.9.

$$\text{Discharge} = \text{Area of flow} \times \text{Flow velocity}$$
$$\Rightarrow Q = K\pi DBV_f$$
$$D \rightarrow inlet$$
$$D_1 \rightarrow outlet$$

The mass of water striking per second,
$$m = \rho Q = \rho K\pi DBV_f$$

Similarly, at the exit,
$$m_1 = \rho K_1 \pi D_1 B_1 V_{f1}$$

If velocity of flow is constant, $m = m_1$

Working Proportions of a Francis Turbine

1. Ration of width to diameter :
$$n = \frac{B}{D}$$
   Varies from $0.1 \sim 0.45$

2. Flow ratio (ϕ):
$$\phi = \frac{V_f}{\sqrt{2gH}}$$
   varies from $0.15 \sim 0.30$

3. Speed ratio, k_u:
$$k_u = \frac{u}{\sqrt{2gH}}$$
   varies from $0.6 \sim 0.9$

Design of Runner

1. Assume the suitable values of $\eta_o, \eta_h, n, k_u, \phi$
2.
$$u = \frac{\pi DN}{60}$$

Problem-01:

Water enters an inward flow reaction turbine at an angle of 22° to the tangent to the outer rim and leaves the turbine radially. The speed of the wheel is 300 rpm and the velocity of flow is constant at 3 m/s. Find the necessary angles of the blade when the innder and outer diameters of the turbine are 30 cm and 60 cm respectively. If width of wheel at inlet is 15 cm, calculate horse power developed. Thickness of blade may be neglected.

Solution:

Given,
$$\alpha = 22$$
$$\beta = 90$$
$$N = 300\,rpm$$
$$V_f = 3\,m/s$$
$$D = 0.6\,m$$
$$D_1 = 0.3\,m$$
$$B = 0.15\,m$$

[Vecocity diagram is mendatory at exam!!]

$$u = \frac{\pi DN}{60} = \frac{\pi \times 0.6 \times 300}{60} = 9.42\,m/s$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.3 \times 300}{60} = 4.71\,m/s$$

$$V = \frac{V_f}{\sin\alpha} = \frac{3}{\sin 22} = 8.01\,m/s$$

$$V_w = V\cos\alpha = 8.01 \times \cos 22 = 7.42\,m/s$$

Here, $u > v_w$

$$\tan\theta = \frac{V_f}{u - V_w} = \frac{3}{9.42 - 7.42}$$
$$\Rightarrow \theta = 56.4$$

$$\tan\phi = \frac{V_{f_1}}{u_1} = \frac{3}{4.71}$$
$$\Rightarrow \phi = 32.5$$

Here, $V_{f1} = v$ and $V_{w1} = 0$

$$Q = \pi DBV_f = \pi \times 0.6 \times 0.15 \times 3 = 0.85\,m^3/s$$

$$hp = \frac{\rho QV_w u}{746}$$
$$= \frac{1000 \times 0.85 \times 7.42 \times 9.42}{746}$$
$$= 79.65\,hp$$