ME-421

FLUID MACHINERY

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Contents

1 Lecture 01: Introduction

Date: 04/06/2023

Booklist

- Hydraulic Mechanics **Govind Rao**
- Hydraulic Machines Through worked out problems **Published by BUET**

Fluid Machines

The working principle of certain machinery where a fluid is employed to do work.

Components

- Liquid Fluid [1]
  - Pumps
    - **Rotodynamics** : Axial flow pump, centrifugal pump etc
    - **Positive Displacement** : Reciprocating, gear, screw pump etc
  - Turbines
    - **Impulse** : Felton wheel (high head)
    - **Reaction** :
      - Radial Flow
      - Mix Flow
      - Axial Flow
- Gaseous Material
  - Fans
  - Blowers
  - Compressors
  - Fluid Coupling
  - Torque Converter

In the case of -

Turbines Energy is extracted from the fluid to produce torque on a rotating shaft.

Pumps Pump is a device to convert mechanical energy into hydraulic energy.

Pumps

Positive Displacement Type

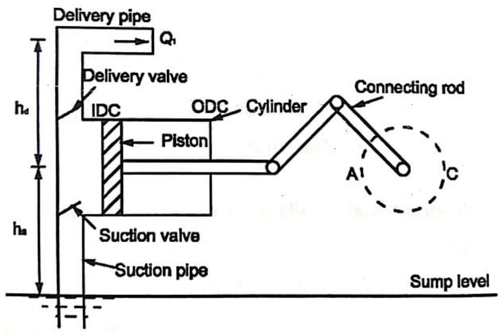
Usually consists of one or more chambers which are alternately filled with liquid to be pumped and then emptied again. The rate of discharge depends on the speed of rotation. It takes care relatively small volume of liquid.
Example - **reciprocating pump, gear pump, screw pump** etc

Rotodynamics Pump

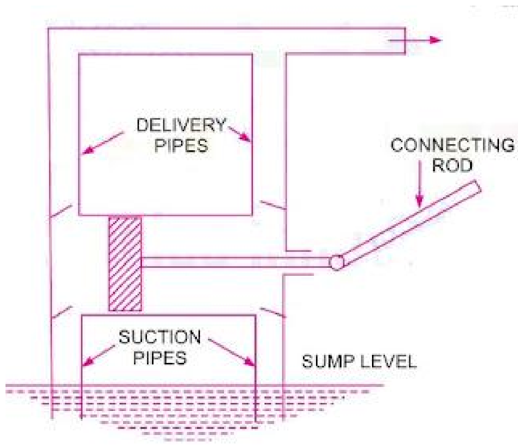
In the case of a roto dynamic pump, a rotating element called **impeller** imparts energy to the liquid and there is a pressure rise.
Example - **centrifugal pump, axial flow pump** etc

[1] Pump and turbine built together to transmit power smoothly.

Reciprocating Pump



(a) Single Acting Pump



(b) Double Acting Pump

Figure 1: Types of Reciprocating Pump

Charateristics

- Reciprocating pump is a positive displacement which is driven by power from an external source and consists of a cylinder in which a piston or plunger is blloked backwards and forwards
- The movement of the piston or plunger creates alternating vacuum pressure and positive pressure inside the cylinder by means of which water is rised.
- If the water acts one side of pistons only, the pump is single acting. If the water acts on both side of the piston, it will suck and deliver during one stroke. such a pump is known as double acting pump.
- The reciprocating pump is generally used for producing very high pressure.

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2 Lecture 2: Reciprocating Pump

Date: 11/06/2023

Schamatic diagram of a Reciprocating Pump:

[Note: Non-return valve, check valve, foot valve - all are same.]

Main Components:

- A piston and a cylinder
- Suction & delivery valve
- Suction & delivery pipes
- Crank & connecting rod

Applications:

The reciprocating pump is best suited for relatively small capacities and high heads. The reciprocating pump is used for -

- Oil drilling operations
- Pneumatic pressure systems
- Feeding small boilers condensate return
- Light oil pumping

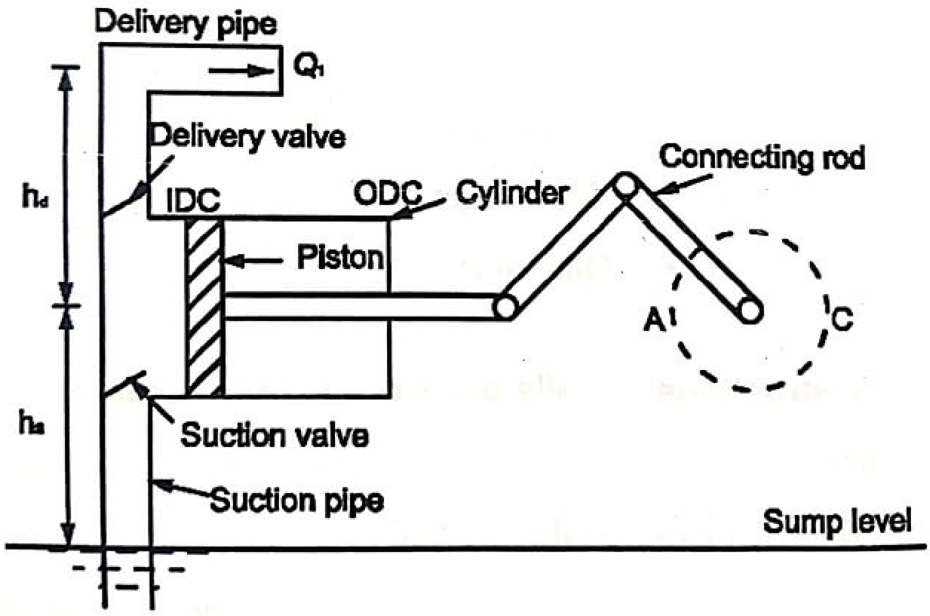

Figure 2: Schamatic diagram of a Reciprocating Pump

Operation Principle:

- For a reciprocating pump as crank rotate for piston p moves backwards and forwards with the cylinder c. The piston moves to the right during the suction stroke, which causes vaccum in the cylinder.
- The atmospheric pressure under sump (reservoir) water surface forces the water up the suction pipe.
- The suction valve a is opened and water enters into the cylinder. The delivery valve b remains closed.
- During the return stroke of the piston, the water pressure closes the suction valve and opens the delivery valve b. Water is then forced up the delivery pipe and raised to the required height or pressure.
- For a single acting pump, the theoretical volume of water raise per revolution is equal to the stroke volume of the cylinder and twice this volume is, if the pump is double acting.

Coefficient of Discharge, C_d:

It is the ratio of actual volume of water discharge to the volume swept by the piston.

$$C_d = \frac{\text{actual discharge per stroke}}{\text{volume swept per stroke}}$$

Slip :

Slip is the difference between actual discharge and theoretical discharge.

$$Slip = Q_t - Q_a$$

where, $Q_t \rightarrow$ Theoretical Discharge
and $Q_a \rightarrow$ Actual Discharge

$$\text{percentage slip} = \frac{Q_t - Q_a}{Q_t} \times 100$$

Negative Slip:

In case of a reciprocating pump with long suction pipe, short delivery pipe and running at high speed, inertia force in the suction pipe becomes large as compared to the pressure force on the outside of delivery valve. This opens the delivery valve even before the piston has completed its suction stroke. Some of the water is pushed into the delivery pipe before the delivery stroke is actually commenced. The actual discharge will be more than the theoretical discharge and slip will be negative. The coefficient of discharge will be greater than 1.

Problem 01:

The actual discharge of a single acting reciprocating pump is 0.02 m^3/s, when running at 55 rpm. The length of the stroke is 500 mm and diameter of the piston is 250 mm. For a total static heads of 16 m, calculate the percentage slip, coefficient of discharge and power required to drive the pump.

Solution:

Given data :

Actual Discharge, Q_a	$= 0.02\ m^3/sec$
Speed of the pump, N	= 55 rpm
Stroke Length, L	= 500 mm
Diameter of piston, d	= 250 mm
Total static head, H_{st}	= 16 m

Find - (a) Percentage Slip, (b) Coeff. of discharge, (c) Power required to drive the pump.

$$\begin{aligned}\text{Cross sectional area of piston, A} &= \frac{\pi}{4} \times d^2 \\ &= \frac{\pi}{4} \times (0.25)^2\, m^2 \\ &= 0.0491\, m^2\end{aligned}$$

$$\begin{aligned}\text{Theoretical Discharge, } Q_t &= \frac{L \times A \times N}{60} \\ &= \frac{0.5 \times 0.0491 \times 55}{60}\, m^3/sec \\ &= 0.0225\, m^3/sec\end{aligned}$$

$$\begin{aligned}\text{Percentage slip } &= \frac{Q_t - Q_a}{Q_t} \times 100 \\ &= \frac{0.0225 - 0.02}{0.0225} \times 100 \\ &= 11.10\%\end{aligned}$$

$$\begin{aligned}\text{Coefficient of Discharge, } C_d &= \frac{Q_a}{Q_t} \\ &= \frac{0.02}{0.0225} \\ &= 0.89\end{aligned}$$

$$\begin{aligned}\text{Power required to drive the pump } &= Q_t \times \gamma \times H_{st} \\ &= 0.0225 \times 9800 \times 16\, watt \\ &= 3.53\, kW\end{aligned}$$

3 Lecture 3: Flow Rate and Work Done

Date: 18/06/2023

let,
r = crank radius
L = Length of stroke = 2r
A = Cross Sectional Area of Piston
N = RPM
H_s = Suction Head
H_d = Delivery Head
γ = Specific weight of water

Volume of water supplied in one stroke = AL
Theoretical flow rate per second, $Q = \frac{LAN}{60}$
For a double acting pump discharge = $\frac{2LAN}{60}$
W = weight flow rate = $Q\gamma$
Total height lifted, H = $H_s + H_d$
Theoretical power required to drive the pump = $Q\gamma\,(H_s + H_d) = WH$
The actual power required will be greater than the theoretical power due to friction, leakage etc.

Indicator diagram of a reciprocating pump:

Indicator diagram may be defined as the graphical representation of pressure head in the cylinder and the volume swept by piston for one complete revolution.

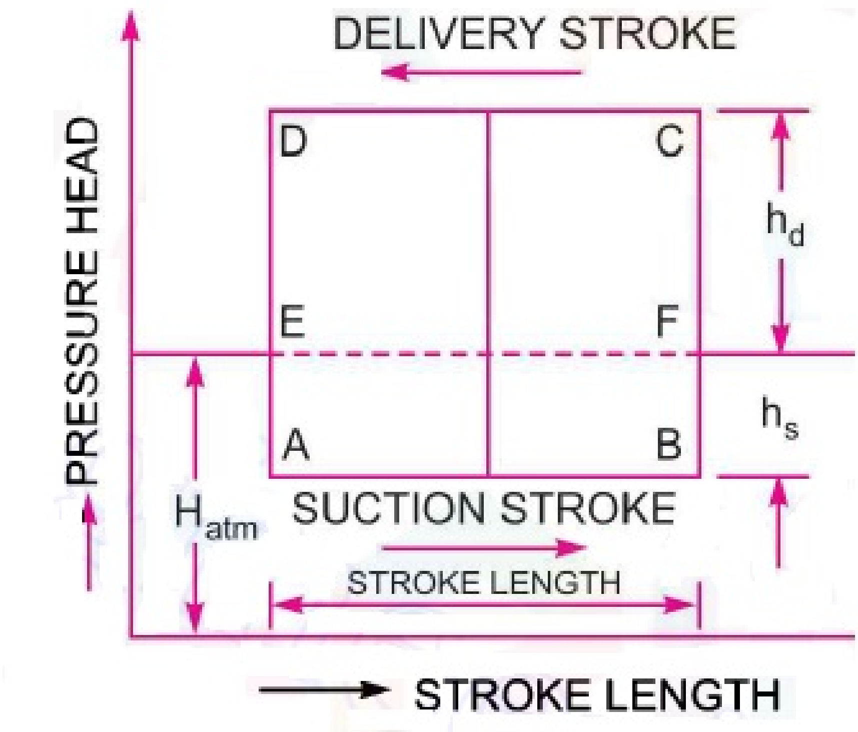

Figure 3: Indicator diagram of a reciprocating pump.

In the above figure, shows the theoretical indicator diagram of a reciprocating pump under an ideal conditions (without friction and leakage). The stroke length and pressure head inside the cylinder are represented by x and y axis in the diagram respectively.

In the diagram,
H_d = Delivery Head
H_s = Suction Head
Line ef = Represents atmospheric pressure
Line ab = Represents pressure inside the cylinder during suction stroke
Line cd = Represents pressure inside the cylinder during delivery stroke

x-axis = absolute zero pressure

area $abfe$ = Work done by the piston dring suction stroke
stroke
area $cdef$ = work done by the piston during delivery

Variation of Pressure due to acceleration of piston

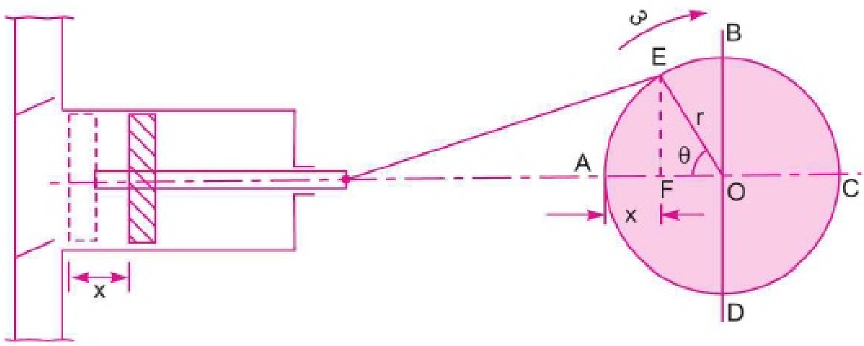

Figure 4: Variation of Pressure due to acceleration of piston

Due to reciprocating motion of the piston, there will be acceleration at the beginning and retardation at the end of each stroke. For this reason, the inertia of water will cause a variation in the pressure in the cylinder.

Assumption

- Length of the connecting rod is very large
- The rotation of the crank is uniform
- The piston makes simple harmonic motion

Nomenclature

A = Cross sectional area of piston
a = Cross sectional area of pipe
ω = Angular velocity of rotating crank
r = crank radius
l = Total length of pipe
γ = specific weight of water
v = velocity of water in a pipe
V = velocity of piston

Derivation of Equation

Let, adter time t, the crank makes an angle θ with horizontal. Therefore,

$$\theta = \omega \times t$$

Displacement of piston in time t, $x = r - r\cos\theta = r - r\cos(\omega t)$
Velocity of piston in time t, $v = \frac{dx}{dt} = r\omega \sin(\omega t)$
acceleration of piston in time t, $f = \frac{dv}{dt} = r\omega^2 \cos(\omega t)$
From continuity equation,
Cross sectional area of pipe × velocity of water in pipe = Velocity of pison × cross sectional area of piston
i.e., $av = AV$
or, $v = \frac{A}{a}V = \frac{A}{a}r\omega \sin\omega t = \frac{A}{a}r\omega \sin\theta$
Acceleration of water in pipe = $\frac{dv}{dt} = \frac{A}{a}r\omega^2 \cos\omega t = \frac{A}{a}\omega^2 r\cos\theta$
weight of water in pipe = γal
mass of water in pipe = $\frac{\gamma al}{g}$
let, P_a = intensity of pressure due to acceleration of water in pipe
from newton's second law of motion,
Force = Mass × Acceleration

$$P_a \times a = \frac{\gamma al}{g}\frac{A}{a}\omega^2 r\cos\theta$$

$$P_a = \frac{\gamma l}{g}\frac{A}{a}\omega^2 r\cos\theta \tag{1}$$

Let, H_a = acceleration pressure head $=\frac{\text{intensity of pressure}}{\text{specific weight of water}} = \frac{P_a}{\gamma}$
Dividing the equation (1) by γ we have,

$$\frac{P_a}{\gamma} = \frac{l}{g}\frac{A}{a}\omega^2 r\cos\theta$$

$$H_a = \frac{l}{g}\frac{A}{a}\omega^2 r\cos\theta \tag{2}$$

From equation (2) it is found that pressure head due to acceleration vary with the angle θ.
At the beginning of stroke, $\theta = 0, cos\theta = 1$

$$\therefore H_a = \frac{l}{g}\frac{A}{a}\omega^2 r$$

At the middle of stroke, $\theta = 90, cos\theta = 0$

$$\therefore H_a = 0$$

At the end of stroke, $\theta = 180, cos\theta = -1$

$$\therefore H_a = -\frac{l}{g}\frac{A}{a}\omega^2 r$$

✓ here (-ve) means retardation.

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4 Lecture 4: Indicator diagram & Air Vessel

Date: 09/07/2023

Problem

The and stroke of a double acting pump are 250mm and 300 mm respectively. The pump discharge is 0.038 m^3/s of water at 50 rpm. Through a portal static head of 12m. Find the ship and actual power required to drive the pump with 80% efficiency.

Solution

Given Data

Bore diameter, d = 350 mm
Stroke length, L = 300 mm
Speed, N = 50 rpm

Total Static Head, H_{st} = 12 m
Actial discharge, Q_a = 0.038 m^3/s
Pump efficiency, η = 80%

Find:
- Slip of the pump
- Power Required to drive the pump

Cross sectional area of cylinder, $A = \frac{\pi}{4}d^2 = 0.096m^2$
Theoretical discharge, $Q_t = \frac{2LAN}{60} = 0.048m^3/s$
Slip of the pump $= Q_t - Q_a = 0.048 - 0.038 = 0.01m^3/s$

Power Required to drive the pump, $P = \gamma Q_t H_{st} = 5.65kW$
Efficiency,

$$\eta = \frac{\text{theoretical power}}{\text{actual power}}$$

$$\Rightarrow 0.8 = \frac{5.65}{\text{actual power}}$$

$$\Rightarrow \text{actual power} = 7.063kW$$

(ans:)

Important Note

Actual power will be more than theoretical power due to friction, weight, auxiliary back flow and others.

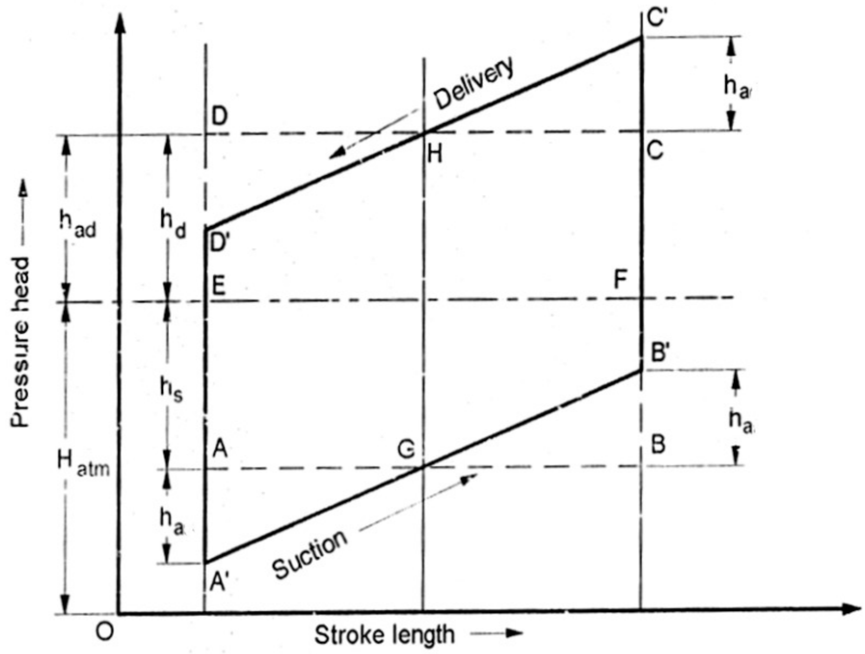
Separation

It has been experimentally found that when the vaccum pressure head $(H_s + H_a)$ reaches 7.9 m of water or 2.4 m absolute. The continuity of flow will stop because of water will commenced vaporized and the separation of flow will take place, therefore to avoid separation,

$$H_s + H_a = 7.9m$$

- If water temperature is higher, it can not be pumped by creating vaccum. Rather it will be vaporized.

Indicator Diagram with Acceleration Head



Indicator diagram with acceleration head

1. The acceleration head, H_a is added to the vaccum head at the beginning of suction stroke and subtracted at the end.
2. In the delivery stroke, the acceleration head, H_a is added at the beginning of the delivery strooke and subtracted at the end.
3. The inertia of water doesn't affect the net work done, but only causes a variation of pressure in the cylinder.

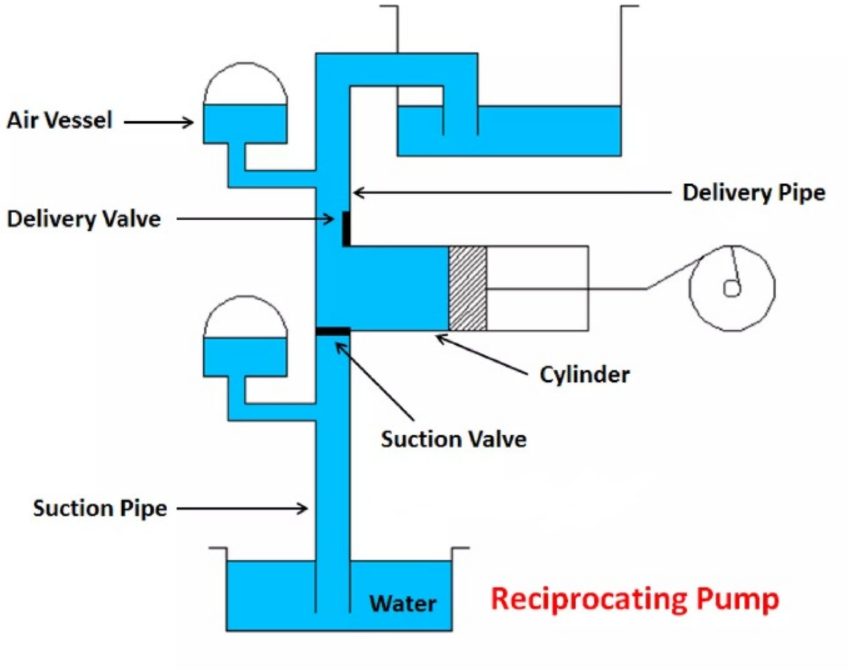
Air Vessel in a Reciprocating Pump

Definition

An air vessel is a cast iron chamber which has an openning at the base through which water can flow.

Characteristics

1. Volume of air vessel is 5-9 times of pump displacement volume.
2. The vessel is filled with compressed air with opening at it's base.
3. The vessels are fitted in suction and delivery pipe close to the cylinder.



Air Vessel in a Reciprocating Pump

5 Lecture 5: Problems on Reciprocating Pump

Date: 16/07/2023

Problem-01

A double acting reciprocating pump has a 500 mm diameter and stroke of 500 mm respectively. The pump is required to deliver 0.1 m^3/s water at a head of 100 m. Frictional losses are estimated to be 1 m in suction pipe and 19m in delivery pipe. Velocity of water in delivery pipe is 1 m/s. Overall efficiency is 85% and slip is 3%. Determine the speed of the pump and power required for the pump.

Solution:

Given data:

Cylinder diameter, $d = 500$ mm
Stroke length, $L = 500$ mm
Actual discharge, $Q_a = 0.1$ m^3/s
Static head, $H_s + H_d = 100$ m
Frictional losses in suction pipe, $h_{fs} = 1$ m
Frictional losses in delivery pipe, $h_{fd} = 19$ m
Velocity of water in delivery pipe, $v_d = 1$ m/s
Overall efficiency, $\eta_o = 85\%$
Percentage slip $= 3\%$

Slip $= \frac{Q_t - Q_a}{Q_t} \times 100\% \Rightarrow Q_t = 0.1031\, m^3/s$
Theoretical discharge, $Q_t = \frac{2LAN}{60} \Rightarrow N = 31.51\, rpm$

Total head against which the pump has to work,
$H = H_s + H_d + h_{fs} + h_{fd} + \frac{v^2}{2g} = 120.05\, m$
Overall efficiency,$\eta_o = \frac{\text{Theoretical power}}{\text{actual power}}$
Actual power = 142.8 kW

Problem-02

A single acting reciprocating pump running at 45 rpm as a piston of diameter 150 mm and stroke length of 200 mm. The suction pipe diameter is 150 mm and it's length is 20 m. Determine the acceleration head at the beginning of suction stroke.

Solution:

Given data:

Piston diameter, d $= 150mm$
stroke length, L $= 200mm$
suction pipe diameter,$d_s = 150mm$
suction pipe length ,$l_s = 20m$
Speed of pump,$N = 45rpm$

At the beginning of suction stroke, $\theta = 0, \cos\theta = 1$
Angular velocity, $\omega = \frac{2\pi N}{60} = 5.3014$ rad/s
Crank radius, r $= \frac{L}{2} = 100mm$
Cross sectional area of piston, A $= \frac{\pi}{4}(0.15)^2$
Cross sectional area of pipe, a $= \frac{\pi}{4}(0.15)^2$

Acceleration head,

$$H_a = \frac{l_s}{g}\frac{A}{a}\omega^2 r\cos\theta = 4.52\,m$$

Problem-03

Single acting reciprocating pump as a bore of 500 mm and a stroke of 500 mm respectively. The pump delivers 0.11 m^3/s water against a head of 100 m. The head loss due to friction in suction and delivery pipe are 2 m and 14 m respectively. The velocity of water in pipe is 1.5 m/s. If the pump efficiency is 92% and slip is 5%. Calculate the speed of the pump and power required to drive the pump.

Solution:

Given data:

Cylinder diameter, $d = 500$ mm
Stroke length, $L = 500$ mm
Actual discharge, $Q_a = 0.11$ m^3/s
Head, $H = 100$ m
Frictional losses in suction pipe, $h_{fs} = 2$ m
Frictional losses in delivery pipe, $h_{fd} = 14$ m
Velocity of water in delivery pipe, $v_d = 1.5$ m/s
Overall efficiency, $\eta_o = 92\%$
Percentage slip = 5%

Slip = $\frac{Q_t - Q_a}{Q_t} \times 100\% \Rightarrow Q_t = 0.116\,m^3/s$
Theoretical discharge, $Q_t = \frac{2LAN}{60} \Rightarrow N = 70.89\,rpm$
Total head,

$H_t = H + h_{fs} + h_{fd} + \frac{v^2}{2g} = 116.11\,m$

Overall efficiency,$\eta_o = \frac{\text{Theoretical power}}{\text{actual power}}$
Actual power = 143.47 kW

Problem-04

Single acting reciprocating pump running at 45 rpm as a piston of diameter of 150 mm and the stroke length of 200 mm. The suction pipe is 150 mm diameter and of length 20 m. Determine the maximum possible suction head to avoid separation.

[Formula: $H_s + H_a = 7.9$]

Solution:

Given data:

Piston diameter, d = $150mm$
stroke length, L = $200mm$
suction pipe diameter,$d_s = 150mm$
suction pipe length ,$l_s = 20m$
Speed of pump,$N = 45rpm$

Crank radius, r = $\frac{L}{2} = 100mm$
Cross sectional area of piston, A = $\frac{\pi}{4}(0.15)^2$
Cross sectional area of pipe, a = $\frac{\pi}{4}(0.15)^2$
Acceleration head,

$$H_a = \frac{l_s}{g}\frac{A}{a}\omega^2 r\cos\theta = 4.527\,m$$

To avoid separation,

$$H_s + H_a = 7.9\,m$$

$$\Rightarrow H_s = 3.37\,m$$

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6 Lecture 06: Air Vessel

Date: 23/07/2023

Functions of Air Vessel

1. Reduces the possibility of sepration and cavitation
2. Allows the pump to run at a higher speed.
3. Large amount of power is set due to the reduction of acceleration head.
4. The discharge is uniform.

Working of Air Vessel

During Delivery Stroke:

The discharge beyond air vessel is more or less uniform.

1^{ST} half of delivery stroke	**Piston moves with acceleration. Velocity of water is more than average velocity. The excess flow of water flows into the air vessel and compressing the air inside the vessel.**
2^{ND} half of delivery stroke	**Piston moves with retardation, velocity of water is less than average velocity. Water flows from the air vessel into the delivery pipe.**

During Suction Stroke:

1^{ST} half of suction stroke	**Piston moves with acceleration. Water flows on the suction pipe into the air vessel.**
2^{ND} half of suction stroke	**Piston moves with retardation, water flows from air vessel to the cylinder.**

The discharge beyond air vessel is more or less uniform.

End of lecture with a class test!!

7 Lecture 7: Work Done against Friction in Pipe

Date: 06/08/2023

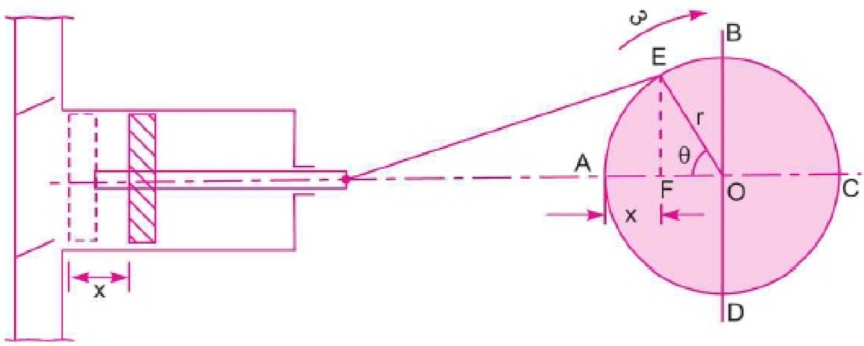

Figure 5: Work Done against Friction in Pipe

Assumptions

- Rotation of crank is uniform
- Connecting rod is very large
- The piston has simple harmonic motion

Nomenclature

- r = crank radius
- l = Length of pipe
- d = diameter of pipe
- A = cross sectional area of piston
- a = cross sectional area of pipe

- h_f = frictional head loss in pipe
- ω = angular velocity of rotating crank
- f = friction factor for the pipe
- v = velocity of water in the pipe
- Q = discharge of the pump

Derivation of Equation

Let, θ = angular distance of crank from zero position at time t = ωt
Displacement of piston, $x = r - r\cos\theta = r - r\cos(\omega t)$
Velocity of piston, $v = \frac{dx}{dt} = r\omega\sin(\omega t)$
Flow of water in pipe = Flow of water in cylinder

$$\Rightarrow a \times v = A \times V$$
$$v = \frac{A}{a} \times v$$
$$v = \frac{A}{a} \times r\omega\sin(\omega t)$$
$$v = \frac{A}{a} \times r\omega\sin\theta$$

Head loss due to friction,

$$h_f = \frac{flv^2}{2gd} = \frac{fl}{2gd}\left(\frac{A}{a} \times r\omega\sin\theta\right)^2 \quad (3)$$

From eqn (3),
i) At the beginning of stroke, $\theta = 0°$, $\sin\theta = 0$, $h_f = 0$
ii) At the end of stroke, $\theta = 180°$, $\sin\theta = 0$, $h_f = 0$
iii) At the middle of stroke, $\theta = 90°$, $\sin\theta = 1$, $h_f = \frac{fl}{2gd}\left(\frac{A}{a} \times r\omega\right)^2$, which is the eqn of a parabola.

During Suction Stroke

- At the beginning, $h_f = 0$,
  pressure head = $H_{atm} - (H_s + H_a)$
- At the middle, $H_a = 0$,
  pressure head = $H_{atm} - (H_s + h_f)$
- At the end, $h_f = 0$,
  pressure head = $H_{atm} - (H_s - H_a)$

During Delivery Stroke

- At the beginning, $h_f = 0$,
  pressure head = $H_{atm} + (H_d + H_a)$
- At the middle, $H_a = 0$,
  pressure head = $H_{atm} - (H_d + h_f)$
- At the end, $h_f = 0$,
  pressure head = $H_{atm} - (H_d - H_a)$

Modified indicator diagram

The combined indicator diagram in shown in the above figure:

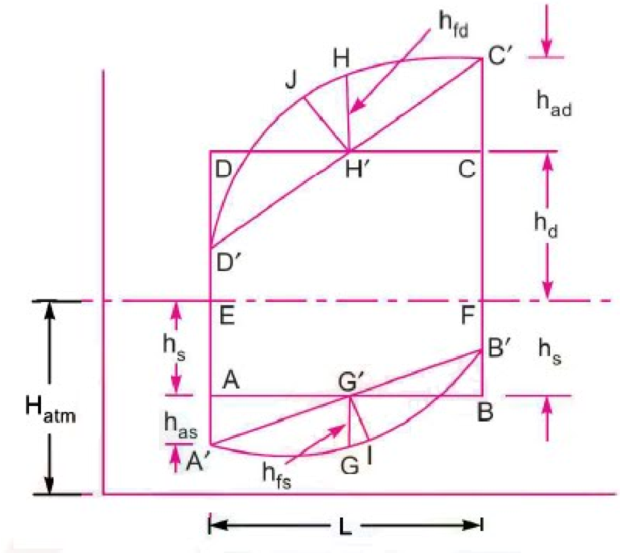

Figure 6: Modified indicator diagram

Let,
h_{fs} = head loss in suction pipe
h_{fd} = head loss in delivery pipe
d_s = diameter of suction pipe
d_d = diameter of delivery pipe
l_s = length of suction pipe
l_d = length of delivery pipe
W = Weight of water pumped per second.

Now, mean ordinate of a parabola = $\frac{2}{3}\times$ maximum ordinate
Mean ordinate of suction pipe parabola = $\frac{2}{3}h_{fs}$

$$= \frac{2}{3}\frac{fl_s}{2gd_s}\left(\frac{A}{a_s}r\omega\right)^2$$

Total work done during suction stroke
= area $EFB'A'$
= area $EFBA$ + area $A'GB'$

Total work done during delivery stroke
= area $EFC'D'$
= area $EFCD$ + area $C'HD'$

Now, work done against friction during suction stroke = area of parabola $A'GB'$

Work done against friction per second during suction stroke,

$$= \frac{2}{3} h_{fs} W$$
$$= \frac{2}{3} \frac{fl_s}{2gd_s} \left(\frac{A}{a_s} r\omega\right)^2 W$$

Work done against friction per second during delivery stroke,

$$= \frac{2}{3} h_{fd} W$$
$$= \frac{2}{3} \frac{fl_d}{2gd_d} \left(\frac{A}{a_d} r\omega\right)^2 W$$

Total work done per second per revolution of crank of a single acting pump

$$= \left(H_s + H_d + \frac{2}{3} h_{fs} + \frac{2}{3} h_{fd}\right) W$$

8 Lecture 08: Work Saved by Fitting Air Vessel

Date: 07/08/2023

Considering suction and delivery strokes, let us consider a reciprocating pump filled with air vessels on suction and delivery pipes.

Nomenclature

Let,
L = Length of stroke
A = Cross sectional area of piston
N = Speed of crank in rpm
ω = Angular speed of crank in rad/sec
v = Velocity of water in the pipe
W = Weight of water pumped per second
l = Length of suction/delivery pipe
a = Cross sectional area of pipe
r = Radius of crank
Q = Volume flow rate
f = Friction factor for pipes
d = Diameter of pipe

Derivation of Equations

For Single Acting Pump:

Work done per second in overcoming pipe friction without air vessels,

$$W_1 = W \times \frac{2}{3} \frac{fl}{2gd} \left(\frac{A}{a} r\omega\right)^2$$

Area of parabola = $\frac{2}{3} \times$ maximum ordinate

By fitting air vessel the average velocity of water beyond air vessel will be uniform.
Now,

$$v = \frac{Q}{a} = \frac{LAN}{60a}$$
$$v = \frac{2rA}{60a} \times \frac{60\omega}{2\pi}$$
$$V = \frac{A}{a} \frac{r\omega}{\pi}$$

Work done per second against friction with air vessels,

$$W_2 = W \times \frac{flv^2}{2gd} = W \times \frac{fl}{2gd} \times \left(\frac{A}{a} \frac{r\omega}{\pi}\right)^2$$

$\therefore$ Work saved per second by fitting air vessels,

$$W_1 - W_2 = \frac{Wfl}{2gd} \left(\frac{A}{a} r\omega\right)^2 \left(\frac{2}{3} - \frac{1}{\pi^2}\right)$$

Now, percentage of work saved per second,

$$\frac{W_1 - W_2}{W_1} \times 100 = \frac{\left(\frac{2}{3} - \frac{1}{\pi^2}\right)}{\left(\frac{2}{3}\right)} \times 100$$
$$= 84.8\%$$

For Double Acting Pump:

Work done per second in overcoming pipe friction without air vessels,

$$W_1 = W \times \frac{2}{3} \frac{fl}{2gd} \left(\frac{A}{a} r\omega\right)^2$$

Area of parabola = $\frac{2}{3} \times$ maximum ordinate
Velocity of water in the pipe with air vessel,

$$v = \frac{Q}{a} = \frac{2LAN}{60a}$$
$$v = \frac{2 \times 2rA}{60a} \times \frac{60\omega}{2\pi}$$
$$V = \frac{A}{a} \frac{2r\omega}{\pi}$$

Work done per second in overcoming friction with air vessels,

$$W_2 = W \times \frac{flv^2}{2gd} = W \times \frac{fl}{2gd} \times \left(\frac{2A}{a} \frac{r\omega}{\pi}\right)^2$$

$\therefore$ Work saved per second by fitting air vessels,

$$W_1 - W_2 = \frac{Wfl}{2gd} \left(\frac{A}{a} r\omega\right)^2 \left(\frac{2}{3} - \frac{4}{\pi^2}\right)$$

Now, percentage of work saved per second by fitting air vessels,

$$\frac{W_1 - W_2}{W_1} \times 100 = \frac{\left(\frac{2}{3} - \frac{4}{\pi^2}\right)}{\left(\frac{2}{3}\right)} \times 100$$
$$= 39.2\%$$

These two proves are important for term final

- Reaction Turbine: Head less, Water more
- Impulse Turbine: Head more, Water less, velocity high

Turbine Performance

Considering the following Characteristics different types of turbines can be compared:

- Specific speed, N_s
- Unit power, P_u
- Unit speed, N_u
- Unit discharge, Q_u

8.0.1 Unit Power, P_u

Power produced by turbine working under a unit head (1m) is known as unit power. The overall efficiency remain uneffective.

Derivation of equation:
Let,
P_u = Power developed by turbine under unit head
H = Working head
P = power produced by turbine when head is H.
Q = Discharge through the turbine.
η_o = Overall efficiency
D = Diameter of runner
b = Width of runner

Power produced by turbine:

$$P = Q\gamma H\eta_o$$
$$P \propto QH$$

Again Discharge,

$$Q = \pi DbV_f$$
$$\therefore Q \propto V_f$$

From the above equation,

$$P \propto V_f H$$
$$\Rightarrow P \propto \sqrt{H}H\,[\because V_f \propto \sqrt{H}]$$
$$\Rightarrow P \propto H^{3/2}$$
$$\Rightarrow = KH^{3/2}$$

$$P = KH^{3/2} \text{ , where K is a constant} \tag{4}$$

From the definition of unit power,

$$P = K = P_u$$

Putting K = P_u in eqn (4), we have,

$$P = P_u H^{3/2}$$

$$P_u = \frac{P}{H^{3/2}}$$

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9 Lecture 09: Unit Speed

Date: 13/08/2023

Unit Speed, N_u
It is the speed of a turbine when working under unit head (1m). Overall efficiency in this case remains same.

Derivation of Equation

Let,
N_u = Speed of turbine when working under unit head
N = Speed of turbine when working under head H
D = Diameter of turbine runner
U = Peripheral velocity of runner.

Now, peripheral velocity of runner,

$$U = \frac{\pi DN}{60}$$
$$\therefore D \propto \frac{U}{N}$$

But,

$$U \propto \sqrt{H}$$

so,

$$\therefore D \propto \frac{\sqrt{H}}{N}$$

For a given turbine D is constant. So,

$$N \propto \sqrt{H}$$

$$N = K_2\sqrt{H} \text{ , where } K_2 \text{ is a constant} \tag{5}$$

From the definition of unit speed, H = 1m and N = N_u.

$$N_u = K_2\sqrt{1}$$
$$\therefore K_2 = N_u$$

Putting $K_2 = N_u$ in eqn (5), we have,

$$N = N_u\sqrt{H}$$

$$\therefore N_u = \frac{N}{\sqrt{H}}$$

Unit Discharge, Q_u

It is the discharge of a turbine when working under unit head (1m). Overall efficiency in this case remains same.

Derivation of Equation

Let,
Q_u = Discharge of turbine when working under unit head
Q = Discharge of turbine when working under head H
D = Diameter of turbine runner
b = Width of runner
V_f = Velocity of flow through runner
H = Head of water

Now, discharge through the turbine,

$$Q = \pi D b V_f$$
$$\therefore Q \propto V_f$$

But,

$$V_f \propto \sqrt{H}$$

So,

$$\therefore Q \propto \sqrt{H}$$

$$Q = k_3\sqrt{H} \text{ , where } k_3 \text{ is a constant} \tag{6}$$

From the definition of unit discharge, H = 1m and Q = Q_u.

$$Q_u = k_3\sqrt{1}$$
$$\therefore k_3 = Q_u$$

Putting $k_3 = Q_u$ in eqn (6), we have,

$$Q = Q_u\sqrt{H}$$

$$\therefore Q_u = \frac{Q}{\sqrt{H}}$$

Specific Speed, N_s

It is the speed of an imaginary turbine geometrically similar to the actual turbine that under corresponding condition it develops unit power (1 kw) when working under unit head (1 m).

Specific speed is used to determining the type of turbine.

Derivation of Equation

Let,
N_s = Specific speed of turbine
N = Speed of turbine when working under head H in RPM
H = Head of water
P = Power developed in the turbine
D = Diameter of turbine runner
b = Width of runner
Q = Discharge through the runner
U = Peripheral velocity of runner
V = Absolute velocity of the water
V_f = Velocity of flow through runner

Speed ratio is a constant for turbines.
i.e. -

$$Q = \frac{U}{\sqrt{2gH}} = const.$$

$$\therefore U \propto Q\sqrt{2gH}$$
$$\Rightarrow U \propto \sqrt{H}$$

Peripheral velocity of runner,

$$U = \frac{\pi D N}{60}$$
$$\Rightarrow DN \propto U$$
$$\Rightarrow DN \propto \sqrt{H}$$

So, we get,

$$D \propto \frac{\sqrt{H}}{N} \tag{7}$$

The discharge through the turbine,

$$Q = \pi D b V_f \tag{8}$$

But, $b \propto D$

Flow ratio is constant for turbines.

$$\psi = \frac{V_f}{\sqrt{2gH}} = const.$$
$$\Rightarrow V_f \propto \psi\sqrt{2gH}$$
$$\Rightarrow V_f \propto \sqrt{H}$$

Substituting the values of b and V_f in equation (8), we have,

$$\begin{aligned} Q &= \pi \times D \times D \times \sqrt{H} \\ \Rightarrow Q &\propto D\sqrt{H} \\ \Rightarrow Q &\propto D^2\sqrt{H} \end{aligned}$$

Substituting the value of D from equation (7), we have,

$$\begin{aligned} Q &\propto \left(\frac{\sqrt{H}}{N}\right)^2 \sqrt{H} \\ \Rightarrow Q &\propto \frac{H^{3/2}}{N^2} \end{aligned}$$

$$\therefore Q \propto \frac{H^{3/2}}{N^2} \tag{9}$$

Power generated by turbine,

$$\begin{aligned} P &= Q\gamma H \\ \Rightarrow P &\propto QH \end{aligned}$$

Putting the value of Q from equation (9), we have,

$$\begin{aligned} P &\propto \frac{H^{3/2}}{N^2} \times H \\ \Rightarrow P &\propto \frac{H^{5/2}}{N^2} \\ \Rightarrow N^2 &\propto \frac{H^{5/2}}{P} \\ \Rightarrow N &\propto \frac{H^{5/4}}{\sqrt{P}} \end{aligned}$$

$$\therefore N = K\frac{H^{5/4}}{\sqrt{P}} \tag{10}$$

Where, K is a constant.

Now, from the definition of specific speed, H = 1 m, P = 1 kw

From equation (10), N = K = N_s
Putting K = N_s in equation (10), we have,

$$\begin{aligned} N &= N_s\frac{H^{5/4}}{\sqrt{P}} \\ \therefore N_s &= \frac{N\sqrt{P}}{H^{5/4}} \end{aligned}$$