Vysoké Učení Technické v Brne

Elektronika pre informačné technológie

Semestrálny projekt

2019/2020

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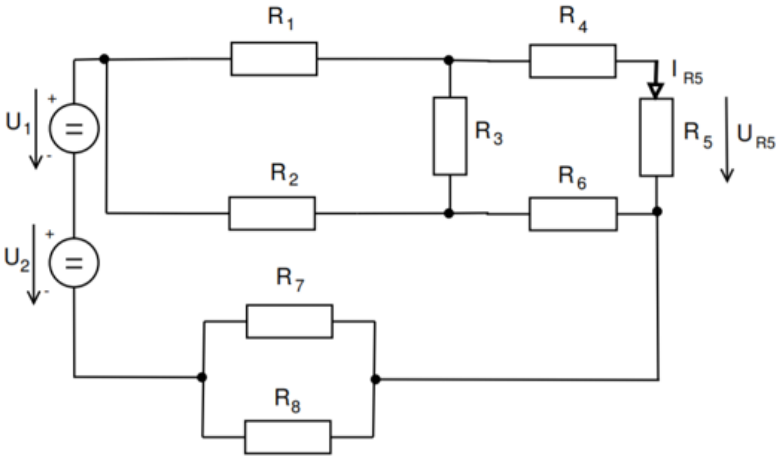
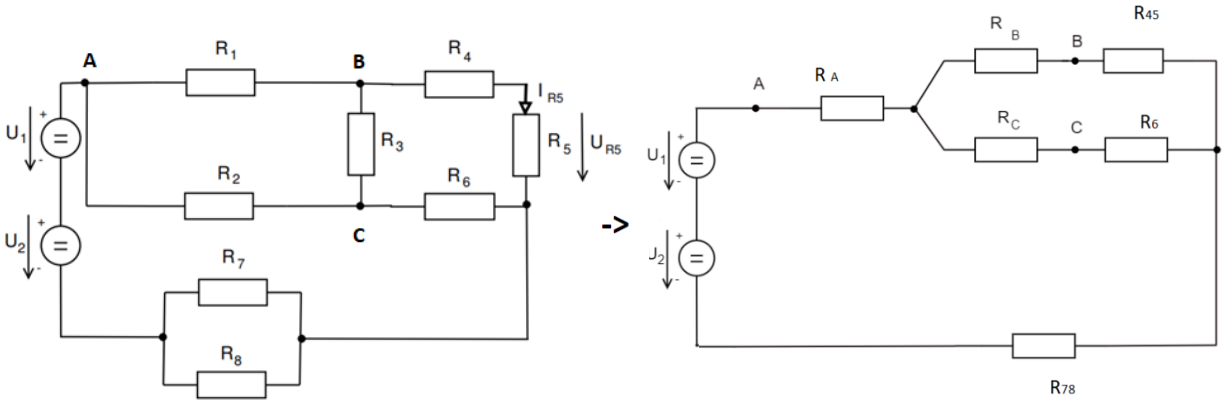
1 **(2 body)**
Stanovte napětí U_{R5} a proud I_{R5}. Použijte metodu postupného zjednodušování obvodu.

sk.	U_1 [V]	U_2 [V]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]	R_5 [Ω]	R_6 [Ω]	R_7 [Ω]	R_8 [Ω]
E	115	55	485	660	100	340	575	815	255	225



Príklad riešime metódou postupného zjednodušovania obvodov.

Využijeme transfiguráciu trojuholník -> hviezda (R_1, R_2 a R_3 na odpory R_A, R_B a R_C



Rezistory R_4 a R_5 sú zapojené sériovo. Rezistory R_7 a R_8 sú zapojené paralelne.

$$R_A = \frac{R_1 R_2}{R_1+R_2+R_3} = \frac{485 * 660}{485+660+100} = \frac{21340}{83}\ \Omega$$

$$R_B = \frac{R_1 R_3}{R_1+R_2+R_3} = \frac{485*100}{485+660+100} = \frac{9700}{249}\Omega$$

$$R_C = \frac{R_2 R_3}{R_1+R_2+R_3} = \frac{660*100}{485+660+100} = \frac{4400}{83}\Omega$$

$R45 = R4 + R5 = 915\ \Omega$

$R78 = \frac{R_7 R_8}{R_7 + R_8} = \frac{255*225}{255+225} = \frac{3825}{32}\ \Omega$

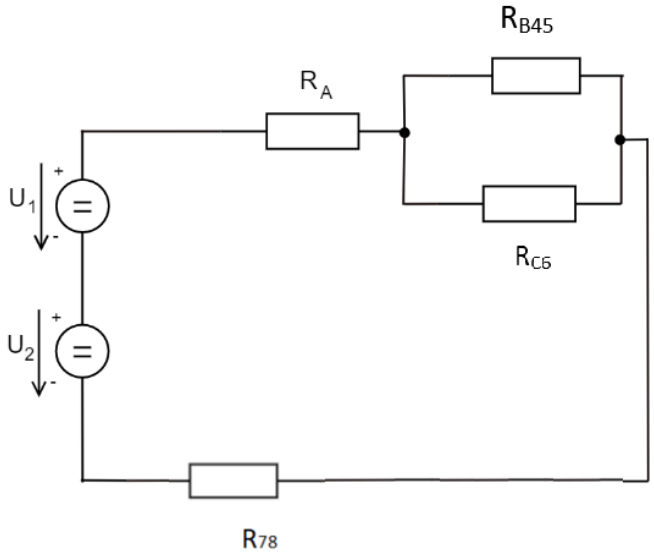
Rezistory R_B a R_{45} sú zapojené sériovo rovnako ako rezistory R_C a R_6

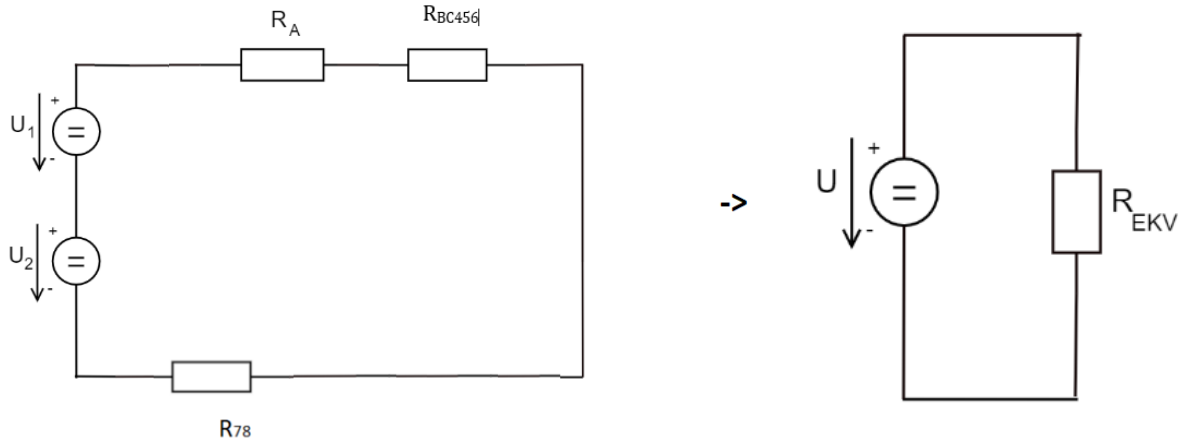
$R_{B45} = R_B + R_{45} = \frac{9700}{249} + 915 = 953.9558\ \Omega$

$RC6 = RC + R6 = \frac{4400}{83} + 815 = 868.0120\ \Omega$

Rezistory R_{B45} a R_{C6} sú zapojené paralelne.

$R_{BC456} = \frac{R_{B45} R_C}{R_{B45} + R_c} = \frac{953.9558 * 868.0120}{953.9558 + 868.0120} = 454.4784\ \Omega$





Rezistory R_A, R_{BC456} a R_{78} sú zapojené sériovo

$$R_{EKV} = R_A + R_{BC456} + R_{78} = \frac{21340}{83} + 454.4784 + \frac{3825}{32} = 831.1181\ \Omega$$

$$U = U_1 + U_2 = 170\ V$$

Celkový prúd I: vypočítame pomocou Ohmového zákonu

$$I = \frac{U}{R_{EKV}} = \frac{170}{831.1181} = 0.2045\ A$$

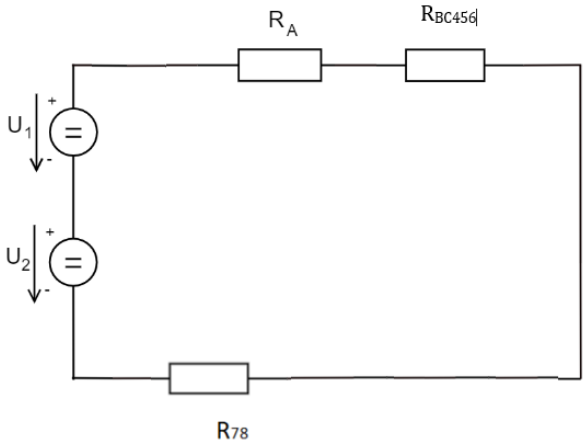
Pomocou II. K. Z. vypočítame napätia na rezistoroch R_A, R_{BC456} a R_{78}

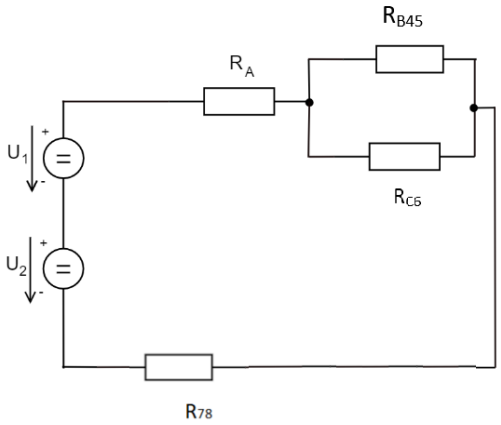
$$U = U_{RA} + U_{RBC456} + U_{R78}$$

$$U_{RA} = I * R_A = 0.2045 * \frac{21340}{83} = 52.5786\ V$$

$$U_{RBC456} = I * R_{BC456} = 0.2045 * 454.4784 = 92.9408\ V$$

$$U_{R78} = I * R_{78} = 0.2045 * \frac{3825}{32} = 24.4441\ V$$



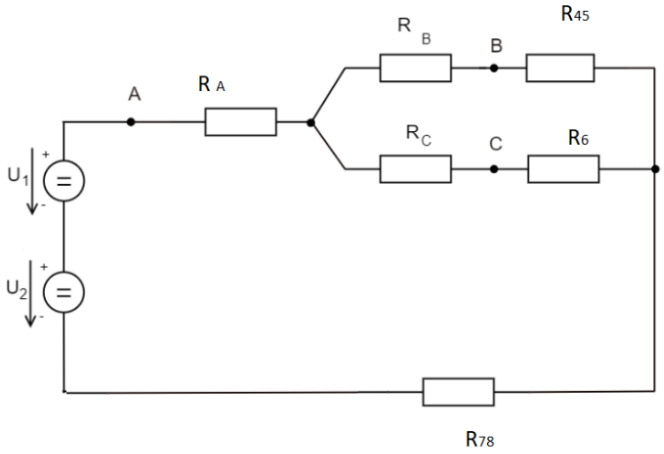


Pomocou I. K. Z. vypočítame prúdy na rezistoroch R_{B45} a R_{C6}

$I = I_{RB45} + I_{C6}$

$I_{RB45} = \frac{U_{RBC456}}{R_{B45}} = \frac{92.9408}{953.9958} = 0.097426\ A$

$I_{RC6} = \frac{U_{RBC456}}{R_{C6}} = \frac{92.9408}{868.0120} = 0.1071\ A$

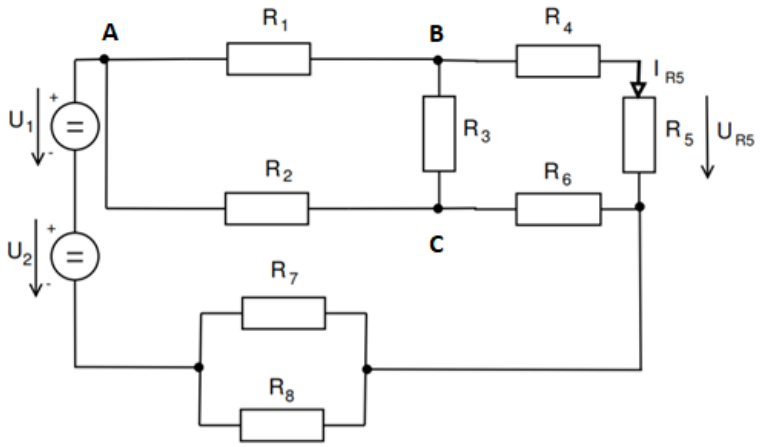


Pomocou Ohmového zákona vypočítame napätia U_{RB}, U_{R45} a prúd I_{R45}

$U_{RB} = I_{RB45} * R_B = 0.097426 * \frac{9700}{249} = 3.7953\ V$

$U_{R45} = I_{RB45} * R_{45} = 0.097426 * 915 = 89.14479\ V$

$I_{R45} = \frac{U_{R45}}{R_{45}} = \frac{89.1448}{915} = 0.097426\ A$



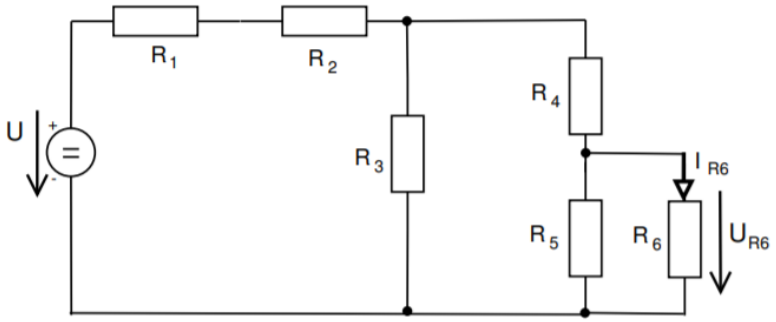
Pomocou Ohmového zákona vypočítame prúd I_{R5} a napätie U_{R5}.

$$U_{R5} = I_{R45} * R_5 = 0.097426 * 575 = 56.0165 \text{ V}$$

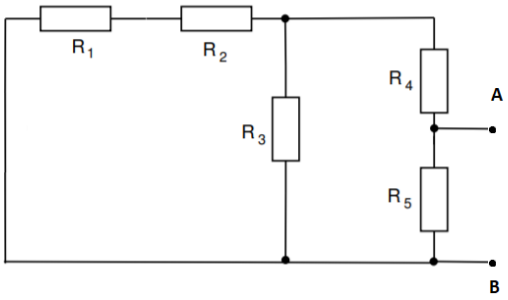
$$I_{R5} = \frac{U_{R5}}{R_5} = \frac{56.0185}{575} = 0.9742 \text{ A}$$

2 **(1 bod)**
Stanovte napětí U_{R6} a proud I_{R6}. Použijte metodu Théveninovy věty.

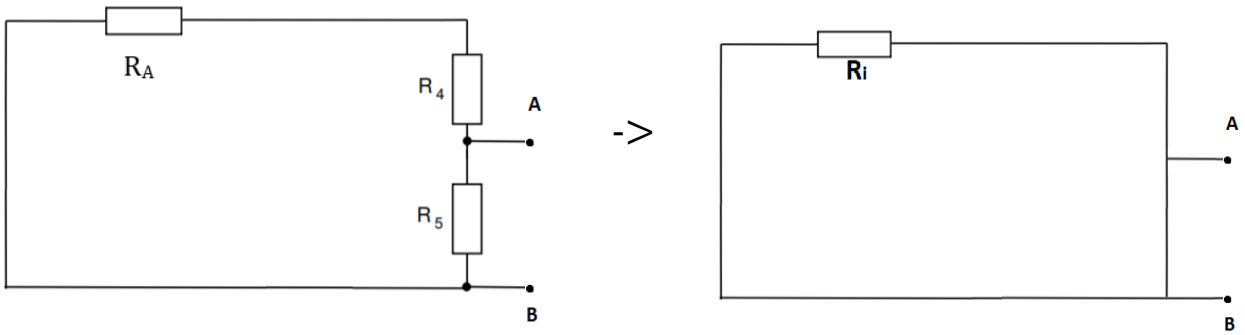
sk.	U [V]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]	R_5 [Ω]	R_6 [Ω]
C	200	70	220	630	240	450	300



Vypočítame R_i (spočítame odpor medzi bodmi A,B)



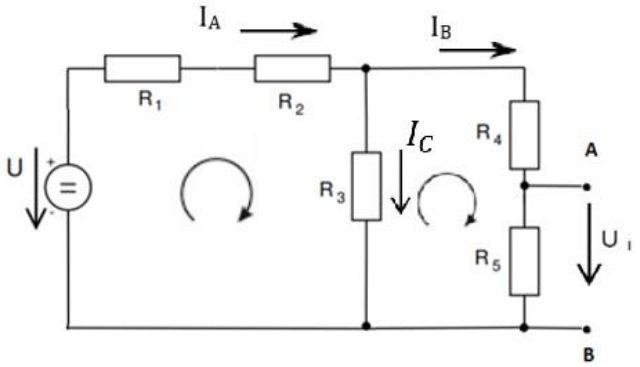
Prekreslíme obvod bez R_6, napäťový zdroj skratujeme.



$R_{12} = R_1 + R_2 = 70 + 220 = 290\ \Omega$

$R_A = \frac{R_{12} * R_3}{R_{12} + R_3} = \frac{290*630}{290+630} = \frac{9135}{46}\ \Omega$

$R_i = \frac{(R_A+R_4) * R_5}{R_A+R_4+R_5} = 222.1101\ \Omega$



Pomocou K.Z určíme I_B

$I_A - I_C - I_B = 0$

$U_{R1}+U_{R2}+U_{R3} = U$

$U_{R4}+U_{R5} - U_{R3} = 0$

1.) $I_A R_1 + I_A R_2 + R_3(I_A - I_B) = U$

2.) $I_B R_4 + I_B R_5 - R_3 (I_A - I_B) = 0$

Z 1.) rovnice si vyjadríme I_A a dosadíme do druhej rovnice

$$I_A = \frac{U + I_B * R_3}{R_1 + R_2 + R_3}$$

$$I_B = \frac{R_3 U}{(R_1 + R_2 + R_3)(R_3 + R_4 + R_5) - R_3 * R_3}$$

$$I_B = \frac{200*630}{(70+220+630)*(630+240+450)-630*630} = \frac{126\,000}{817\,500} = \frac{84}{545}\ A$$

Pokiaľ poznáme prúd I_B tak môžme dopočítať U_i a následne aj I_{R6} a U_{R6}

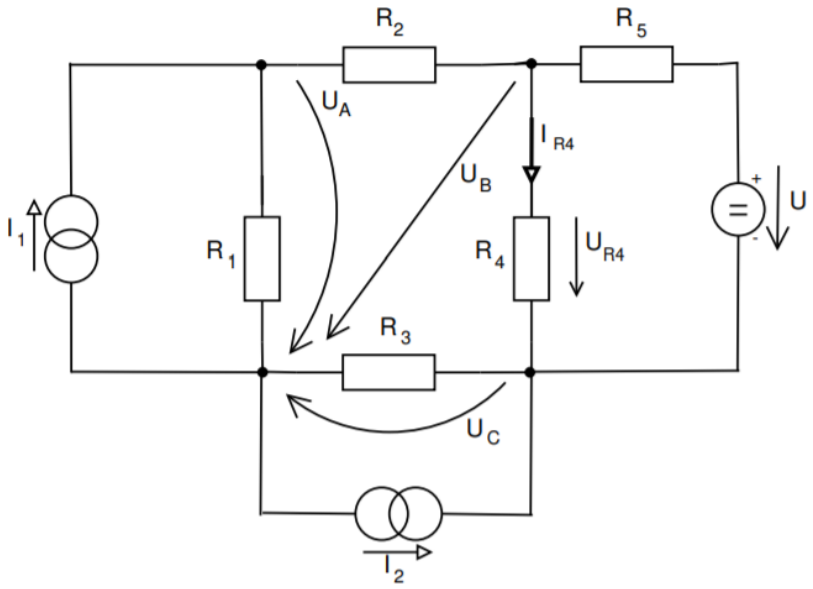
$U_i = I_B * R_5 = \frac{84}{545} * 450 = 69.3578\ V$

$I_{R6} = \frac{U_i}{R_{i+}R_6} = \frac{69.3578}{222.1101+ 300} = 0.1328\ A$

$U_{R6} = R_6 * I_{R6} = 300 * 0.1328 = 39.8524\ V$

3 **(2 body)**
Stanovte napětí U_{R4} a proud I_{R4}. Použijte metodu uzlových napětí (U_A, U_B, U_C).

sk.	U [V]	I_1 [A]	I_2 [A]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]	R_5 [Ω]
C	110	0.85	0.75	44	31	56	20	30

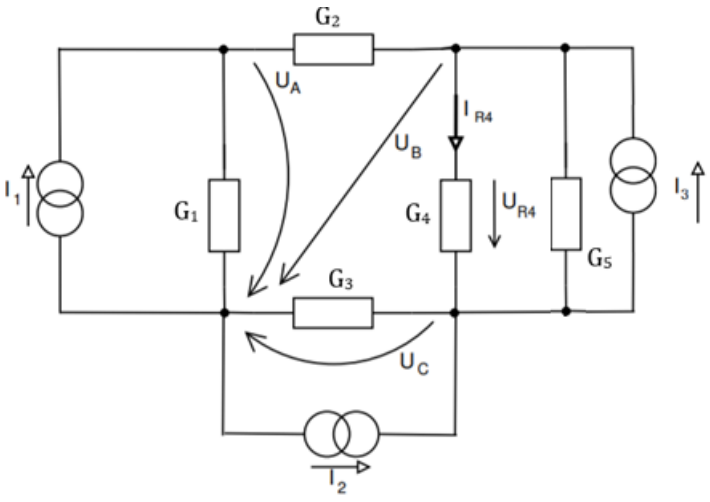


Prevedieme napäťový zdroj na prúdový. Odpory prevedieme na vodivosti. ($G = \frac{1}{R}$)

$G_1 = \frac{1}{44}$ S $\quad G_2 = \frac{1}{31}$ S $\quad G_3 = \frac{1}{56}$ S $\quad G_4 = \frac{1}{20}$ S $\quad G_5 = \frac{1}{30}$ S

Pomocou Ohmovho zákona si vypočítame I_3

$I_3 = \frac{U}{R_5} = \frac{110}{30}$ A



1.) $-I_1 + G_1U_A + G_2\,(U_A - U_B) = 0$

2.) $-I_3 - G_2\,(U_A - U_B) + G_4\,(U_B - U_C) + G_5\,(U_B - U_C) = 0$

3.) $-I_2 - G_4\,(U_B - U_C) - G_5\,(U_B - U_C) + G_3U_C + I_3 = 0$

1.) $U_A\,(G_1 + G_2) + U_B\,(-G_2) = I_1$

2.) $U_A\,(-G_2) + U_B\,(G_2 + G_4 + G_5) + U_C\,(-G_4 - G_5) = I_3$

3.) $U_B\,(-G_4 - G_5) + U_C\,(G_3 + G_4 + G_5) = I_2 - I_3$

$$\begin{vmatrix} G_1 + G_2 & -G_2 & 0 \\ -G_2 & G_2 + G_4 + G_5 & -G_4 - G_5 \\ 0 & -G_4 - G_5 & G_3 + G_4 + G_5 \end{vmatrix} * \begin{bmatrix} U_A \\ U_B \\ U_C \end{bmatrix} = \begin{bmatrix} I_1 \\ I_3 \\ I_2 - I_3 \end{bmatrix}$$

$$\begin{vmatrix} \frac{75}{1364} & -\frac{1}{31} & 0 \\ -\frac{1}{31} & \frac{43}{372} & -\frac{1}{12} \\ 0 & -\frac{1}{12} & \frac{17}{168} \end{vmatrix} * \begin{bmatrix} U_A \\ U_B \\ U_C \end{bmatrix} = \begin{bmatrix} \frac{17}{20} \\ \frac{110}{30} \\ -\frac{35}{12} \end{bmatrix}$$

$$\text{DET.} = \begin{vmatrix} \frac{75}{1364} & -\frac{1}{31} & 0 \\ -\frac{1}{31} & \frac{43}{372} & -\frac{1}{12} \\ 0 & -\frac{1}{12} & \frac{17}{168} \end{vmatrix} = \frac{18275}{28414848} - \frac{25}{65472} - \frac{17}{161448} = \frac{13}{83328}$$

$$U_B = \begin{vmatrix} \frac{75}{1364} & \frac{17}{20} & 0 \\ -\frac{1}{31} & \frac{110}{30} & -\frac{1}{12} \\ 0 & -\frac{35}{12} & \frac{17}{168} \end{vmatrix} = \frac{\frac{22483}{2291520}}{\text{DET.}} = \frac{44966}{715} \text{ V}$$

$$U_C = \begin{vmatrix} \frac{75}{1364} & -\frac{1}{31} & \frac{17}{20} \\ -\frac{1}{31} & \frac{43}{372} & \frac{110}{30} \\ 0 & -\frac{1}{12} & -\frac{35}{12} \end{vmatrix} = \frac{\frac{391}{109120}}{\text{DET.}} = \frac{16422}{715} \text{ V}$$

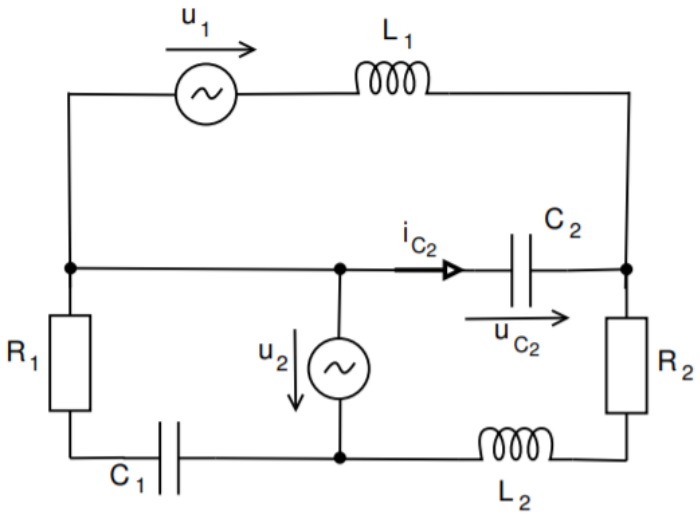
$$U_{R4} = U_B - U_C = \frac{44966}{715} - \frac{16422}{715} = 39.9217 \text{ V}$$

$$I_{R4} = \frac{U_{R4}}{R_4} = 1.9961 \text{ A}$$

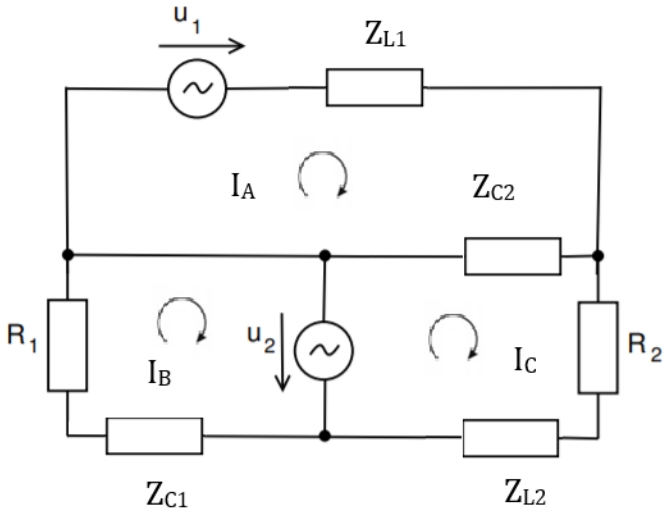
4 **(2 body)**
Pro napájecí napětí platí: $u_1 = U_1 \cdot \sin(2\pi ft)$, $u_2 = U_2 \cdot \sin(2\pi ft)$.
Ve vztahu pro napětí $u_{C_2} = U_{C_2} \cdot \sin(2\pi ft + \varphi_{C_2})$ určete $|U_{C_2}|$ a φ_{C_2}. Použijte metodu smyčkových proudů.

Pozn: Pomocné směry šipek napájecích zdrojů platí pro speciální časový okamžik ($t = \frac{\pi}{2\omega}$).

sk.	U_1 [V]	U_2 [V]	R_1 [Ω]	R_2 [Ω]	L_1 [mH]	L_2 [mH]	C_1 [μF]	C_2 [μF]	f [Hz]
E	50	30	14	13	130	60	100	65	90



Určíme si všetky slučky v obvode



Vypočítame uhlovú frekvenciu a impedancie jednotlivých článkov obvodu

$\omega = f * 2\pi = 90 * 2\pi = 180\ \pi$

$Z_{L1} = j\,\omega L_1 = j * 180\ \pi * 0.13 = \frac{114}{5}\ \pi j\Omega = 71.6283 i\Omega$

$Z_{L2} = j\,\omega L_2 = j * 180\ \pi * 0.06 = \frac{54}{5}\ \pi j\Omega = 33.9292 i\Omega$

$Z_{C1} = -\frac{j}{\omega C_1} = -\frac{j}{180\pi * 10^{-4}} = -17.6839\ i\Omega$

$Z_{C2} = -\frac{j}{\omega C_1} = -\frac{j}{180\pi * 6.5 * 10^{-5}} = -27.2060\ i\Omega$

Zostavíme rovnice:

$I_A: u_1 + I_A(Z_{L1}) + Z_{C2}\,(I_A - I_C) = 0$

$I_B: u_2 + I_B\,(Z_{C1}) + I_B\,(R_1) = 0$

$I_C: -u_2 + Z_{C2}\,(I_C - I_A) + I_C\,(R_2) + I_C(Z_{L2}) = 0$

$I_A\,(Z_{L1} + Z_{C2}) + I_C\,(-Z_{C2}) = -u_1$

$I_B\,(Z_{C1} + R_1) = -u_2$

$I_C\,(Z_{C2} + R_2 + Z_{L2}) + I_A(-Z_{C2}) = u_2$

$$\text{DET.} = \begin{vmatrix} Z_{L1} + Z_{C2} & 0 & -Z_{C2} \\ 0 & Z_{C1} + R_1 & 0 \\ -Z_{C2} & 0 & Z_{C2} + R_2 + Z_{L2} \end{vmatrix} = \begin{vmatrix} I_A \\ I_B \\ I_C \end{vmatrix} * \begin{vmatrix} -50 \\ -30 \\ 30 \end{vmatrix}$$

$$\text{DET.} = \begin{vmatrix} 6.7232j & 0 & 27.2060j \\ 0 & 14 - 17.6839j & 0 \\ 27.2060j & 0 & -27.2060j + 13 + 33.9292j \end{vmatrix} =$$

(6.7232j)*(14 – 17.6839j)*(-27.2060j+13+33.9292j) – (27.2060j)*(14 – 17.6839j)*(27.2060j) = 11275.1114 - 11066.0696j

$$\mathrm{DET_A} = \begin{vmatrix} -50 & 0 & 27.2060\mathrm{j} \\ -30 & 14 - 17.6839\mathrm{j} & 0 \\ 30 & 0 & -27.2060\mathrm{j} + 13 + 33.9292\mathrm{j} \end{vmatrix}$$

$$\mathrm{DET_C} = \begin{vmatrix} 6.7232\mathrm{j} & 0 & -50 \\ 0 & 14 - 17.6839\mathrm{j} & -30 \\ 27.2060\mathrm{j} & 0 & 30 \end{vmatrix}$$

$i_A = \frac{DET_A}{DET} = \frac{-29477.8653 - 4638.225j}{11275.1114 - 11066.0696j} =$ -1.12602 -1.51651j A

$i_C = \frac{DET_C}{DET} = \frac{27622.181 + 21867.944j}{11275.1114 - 11066.0696j} =$ 0.278+2.21259j A

Okamžité napätie na kondenzátore C_2 :

$$U_{C2} = Z_{C2} * (I_C - I_A)$$

$$U_{C2} = -27.2060i * (0.278 + 2.2126j + 1.12602 + 1.5165j)$$
$$= 101.4539 - 38.1978j$$

Modul okamžitého napätia na kondenzátore C_2 :

$$|U_{C2}| = \sqrt{Re(U_{C2})^2 + lm(U_{C2})^2} = \sqrt{(101.4539)^2 + (-38.1978)^2} = 108.406\ V$$

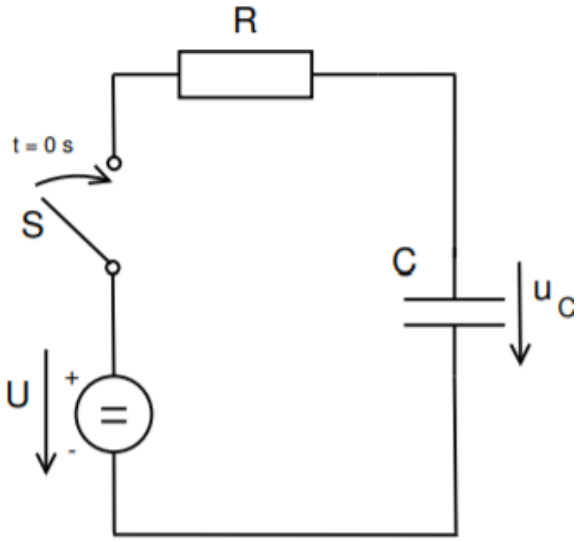
$$\varphi_{C2} = \arctan\left(\frac{Im}{Re}\right) = \arctan\left(\frac{-38.1978}{101.4539}\right)$$

Keďže ide o IV. Kvadrant:

$$\varphi_{C2} = -\arctan\left(\frac{-38.1978}{101.4539}\right) = 0.3601 \text{ rad}$$

5 **(2 body)**
V obvodu na obrázku níže v čase $t = 0$[s] sepne spínač S. Sestavte diferenciální rovnici popisující chování obvodu na obrázku, dále ji upravte dosazením hodnot parametrů. Vypočítejte analytické řešení $u_C = f(t)$. Proveďte kontrolu výpočtu dosazením do sestavené diferenciální rovnice.

sk.	U [V]	C [F]	R [Ω]	$u_C(0)$ [V]
D	25	5	25	12



Počiatočná podmienka: $u_c\,(0) = ucp$

Platí: $u_c\,' = \frac{1}{C} * i_c$

II. Kirchhofov zákon:

$$u_r + u_c - U = 0$$

$$R * i + u_c - U = 0$$

$$i = \frac{U - u_c}{R}$$

Vytvoríme diferenciálnu rovnicu prvého rádu.

$$u_c' = \frac{1}{C} * i$$

$$u_c' = \frac{1}{C} * \frac{U - u_c}{R}$$

$$u_c' = \frac{U - u_c}{R * C}$$

$$u_c' = \frac{25 - u_c}{25 * 5}$$

$$u_c' + \frac{1}{125} u_c = \frac{25}{125}$$

Všeobecný tvar: $u_c(t) = k(t)e^{\lambda t}$

Vypočítame λ: $\lambda + \frac{1}{125} = 0 \Rightarrow \lambda = -\frac{1}{125}$

Očakávané riešenie: $u_c(t) = k(t)e^{-\frac{t}{125}}$

Zderivujeme:

$$u_c'(t) = k'(t)e^{-\frac{t}{125}} + k(t)\ (-\frac{1}{125})e^{-\frac{t}{125}}$$

Dosadíme do všeobecnej rovnice:

$$u_c' + (\frac{1}{125})u_c = \frac{25}{125}$$

$$k'(t)e^{-\frac{t}{125}} + k(t)\ (-\frac{1}{125})e^{-\frac{t}{125}} + k(t)\ (\frac{1}{125})e^{-\frac{t}{125}} = \frac{25}{125}$$

$$k'(t)e^{-\frac{t}{125}} = \frac{1}{5}$$

$$k'(t) = \frac{e^{\frac{t}{125}}}{5}$$

$$k(t) = \int \frac{e^{\frac{t}{125}}}{5} dt$$

$$k(t) = 25e^{\frac{t}{125}} + K$$

Výsledok dosadíme do očakávaného riešenia:

$$u_c(t) = k(t)e^{\lambda t}$$

$$u_c(t) = 25 + Ke^{-\frac{t}{125}}$$

$$u_c(0) = 25 + Ke^{-\frac{0}{125}}$$

$$12 = 25 + K$$

$$K = -13$$

$$u_c(t) = 25 - 13e^{-\frac{t}{125}}$$

Skúška:

Pre t = 0 $$u_c(t) = 25 - 13e^{-\frac{t}{125}}$$

$$12 = 12$$

$$u_c' + \frac{1}{125}u_c = \frac{25}{125}$$

$$u_c(t) = 25 - 13e^{-\frac{t}{125}}$$

$$u_c'(t) = \frac{13}{125}e^{-\frac{t}{125}}$$

$$\frac{13}{125}e^{-\frac{t}{125}} + \frac{1}{125} * (25 - 13e^{-\frac{t}{125}}) = \frac{25}{125}$$

$$\frac{13}{125}e^{-\frac{t}{125}} + \frac{25}{125} - \frac{13}{125}e^{-\frac{t}{125}} = \frac{25}{125}$$

$$0 = 0$$

Tabuľka výsledkov

Číslo príkladu	Skupina	Výsledok
1.	E	$U_{R5} = 56.0165$ V $I_{R5} = 0.9742$ A
2.	C	$U_{R6} = 39.8524$ V $I_{R6} = 0.1328$ A
3.	C	$U_{R4} = 39.9217$ V $I_{R4} = 1.9961$ A
4.	E	$\lvert U_{C2} \rvert = 108.406\ V$ $\varphi_{C2} = 0.3601$ rad
5.	D	$u_c(t) = 25 - 13e^{-\frac{t}{125}}$