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Chapter No. 22
Basic Statistics

EX: 22.3:

5. The sizes of shoe sold at a store on a 50% off price are listed. Calculate A.M., G.M., H.M., median, Q_1, Q_3 and modal shoe size sold that day.

Shoe size	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5
Number of pairs sold	2	5	15	30	60	40	23	11	4	1

A.M:

$$\therefore \overline{x} = \frac{\sum fx}{\sum f}$$

X	F	Fx
5	2	10
5.5	5	27.5
6	15	90
6.5	30	195
7	60	420
7.5	40	300
8	23	184
8.5	11	93.5
9	4	36
9.5	1	9.25
	$\sum f = 191$	$\sum fx = 1365.25$

$$\therefore \overline{x} = \frac{\sum fx}{\sum f} = \frac{1365.25}{191}$$

$$\overline{x} = 7.1479$$

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G.M:

$$\therefore G.M = anti\log\left[\frac{\sum f\log x}{\sum f}\right]$$

x	f	Log(x)	F log(x)
5	2	0.698	2 x 0.698 =1.397
5.5	5	0.740	5 x 0.740 =3.701
6	15	0.778	15 x 0.778 =11.672
6.5	30	0.813	30 x 0.813 =24.387
7	60	0.845	60 x 0.845 =50.705
7.5	40	0.875	40 x 0.875 =35.0
8	23	0.903	23 x 0.903 =20.771
8.5	11	0.929	11 x 0.929 =10.223
9	4	0.954	4 x 0.954 =3.816
9.5	1	0.977	1 x 0.977 =0.977
	$\sum f = 191$		$\sum f\log x = 162.649$

$$\therefore Mode = L + \left(\frac{(f_M - f_{M-1})\times \mathrm{h}}{2f_M - f_{M-1} - f_{M+1}}\right)$$

$$Mode = 34.5 + \frac{(35-18)\times 5}{2(35)-18-17}$$

$$Mode = 36.929$$

H.M:

$$\therefore H.M = \frac{\sum f}{\sum\left(\frac{f}{x}\right)}$$

$$H.M = \frac{2+5+15+30+60+40+23+11+4+1}{\frac{2}{5}+\frac{5}{5.5}+\frac{15}{6}+\frac{30}{6.5}+\frac{60}{7}+\frac{40}{7.5}+\frac{23}{8}+\frac{11}{8.5}+\frac{4}{9}+\frac{1}{9.5}}$$

$$H.M = \frac{191}{27.048}$$

$$H.M = 7.061$$

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Median:

x	f	C.F
5	2	2
5.5	5	2+5=7
6	15	7+15=22
6.5	30	22+30=52
7	60	52+60=112
7.5	40	112+40=152
8	23	152+23=175
8.5	11	175+11=186
9	4	186+4=190
9.5	1	190+1=191
	$\sum f = 191$	

Here n = 191 (odd), So

$$\therefore m = \left(\frac{n+1}{2}\right)^{th} observation$$

$$m = \left(\frac{191+1}{2}\right)^{th} = \left(\frac{192}{2}\right)^{th} observation$$

$$m = 96^{th} observation$$

$$Median = 7$$

Quartiles:

Q_1:

For first quartile: $\therefore Q_1 = \left(\frac{n+1}{4}\right)^{th}$

Note: We are using $\therefore Q_1 = \left(\frac{n+1}{4}\right)^{th}$ because $\left(\frac{n}{2}\right)$ is not an integer.

$Q_1 = \left(\frac{192+1}{4}\right) = 48$, as 48 is an integer so,

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$$\therefore Q_1 = \frac{1}{2}\left[\left(\frac{n+1}{4}\right)^{th} + \left(\frac{n+1}{4}+1\right)^{th}\right] observation$$

$$Q_1 = \frac{1}{2}\left[\left(\frac{191+1}{4}\right)^{th} + \left(\frac{191+1}{4}+1\right)^{th}\right] observation$$

$$Q_1 = \frac{1}{2}\left[\left(\frac{192}{4}\right)^{th} + \left(\frac{192+4}{4}\right)^{th}\right] observation$$

$$Q_1 = \frac{1}{2}\left[(48)^{th} + (49)^{th}\right] observation$$

, Both 48 and 49 lies in C.F = 52 whose value of x is 6.5. so,

$$Q_1 = \frac{1}{2}\left[(6.5)+(6.5)\right]$$

$$Q_1 = \frac{13}{2}$$

$$Q_1 = 6.5$$

Q_3:

Third quartile = $Q_1 = \left(\frac{3n+3}{4}\right)$

$$Q_3 = \left(\frac{3(191)+3}{4}\right) = \left(\frac{576}{4}\right) = 144$$

, where 144 is an integer so,

$$\therefore Q_3 = \frac{1}{2}\left[\left(\frac{3n+3}{4}\right)^{th} + \left(\frac{3n+3}{4}+1\right)^{th}\right] observation$$

$$Q_3 = \frac{1}{2}\left[\left(\frac{3(191)+3}{4}\right)^{th} + \left(\frac{3(191)+3}{4}+1\right)^{th}\right] observation$$

$$Q_3 = \frac{1}{2}\left[\left(\frac{576}{4}\right)^{th} + \left(\frac{576+4}{4}\right)^{th}\right] observation$$

$$Q_3 = \frac{1}{2}\left[(144)^{th} + (145)^{th}\right] observation$$

, Both 144 and 145 lies in C.F = 152 whose value of x is 7.5. so,

$$Q_3 = \frac{1}{2}\left[(7.5)+(7.5)\right] = \frac{15}{2}$$

$$Q_3 = 7.5$$

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Mode:
7 has the highest frequency value therefore,

Mode = 7.

7. The profits earned by a company for a period of last 50 days are summarized below. Find the A.M. profit using shortcut and coding methods with
(a). A = 9000, h = 2000 (b). A = 11000, h = 2000

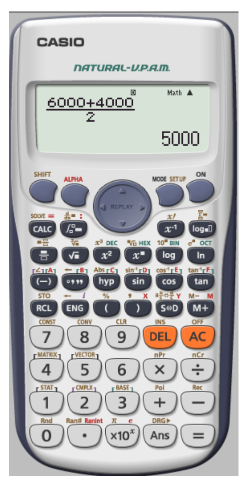
Profits (Rs.)	4000-6000	6000-8000	8000-10000	10000-12000	12000-14000
Number of days	5	7	11	21	6

(a). A=9000, h=2000.

Class limit	Class mark(x)	Frequency (f)	D=x-A	U= $\frac{x-A}{h}$	Df	uf
			D = x-9000	U = $\frac{x-9000}{2000}$		
4000-6000	5000	5	-4000	-2	-20000	-10
6000-8000	7000	7	-2000	-1	-14000	-7
8000-10000	9000	11	0	0	0	0
10000-12000	11000	21	2000	1	42000	21
12000-14000	13000	6	4000	2	24000	12
		$\sum f = 50$			$\sum Df = 32000$	$\sum uf = 16$

By Shortcut Method	By Coding Method
$\therefore \bar{x} = A + \frac{\sum Df}{\sum f}$	$\therefore \bar{x} = A + h\frac{\sum uf}{\sum f}$
$\bar{x} = 9000 + \frac{32000}{50}$	$\bar{x} = 9000 + (2000)\frac{16}{50}$
$\bar{x} = 9640$	$\bar{x} = 9640$

To Find Class mark:



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(b). A=11000, h=2000.

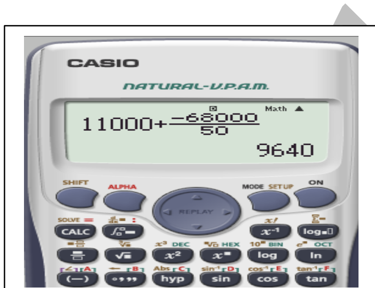
Class limit	Class mark(x)	Frequency (f)	D=x-A	U= $\frac{x-A}{h}$	Df	uf
			D = x-11000	U = $\frac{x-11000}{2000}$		
4000-6000	5000	5	-6000	-3	-30000	-15
6000-8000	7000	7	-4000	-2	-28000	-14
8000-10000	9000	11	-2000	-1	-22000	-11
10000-12000	11000	21	0	0	0	0
12000-14000	13000	6	2000	1	12000	6
		$\sum f=50$			$\sum Df=-68000$	$\sum uf=-34$

By Shortcut Method

$$\therefore \bar{x}=A+\frac{\sum Df}{\sum f}$$

$$\bar{x}=11000+\frac{-68000}{50}$$

$$\bar{x}=9640$$

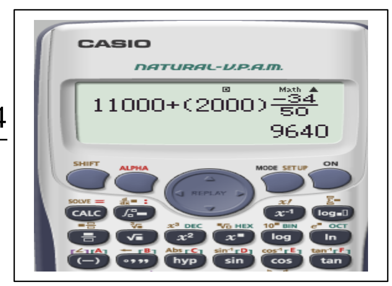


By Coding Method

$$\therefore \bar{x}=A+h\frac{\sum uf}{\sum f}$$

$$\bar{x}=11000+(2000)\frac{-34}{50}$$

$$\bar{x}=9640$$



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8. The marks obtained by students in a subject (out of 50) are given in the following grouped table. Find A.M., G.M. (using direct and logarithmic methods), H.M., median and mode.

Marks	25-29	30-34	35-39	40-44	45-49
Number of students	9	18	35	17	5

A.M(Direct Method):

$$\therefore \bar{x} = A + \frac{\sum Df}{\sum f}$$

Class limit	Class mark (x)	Frequency	D = x-A Taking A = 37	DF
25-29	27	9	27-37 = -10	-90
30-34	32	18	32-37 = -5	-90
35-39	37	35	37-37 = 0	0
40-44	42	17	42-37 = 5	85
45-49	47	5	47-37 = 10	50
		$\sum f = 84$		$\sum Df = -45$

$$\therefore \bar{x} = A + \frac{\sum Df}{\sum f}$$

$$\bar{x} = 37 + \frac{-45}{84}$$

$$\bar{x} = 36.464$$

Way to find class mark:

G.M(Logarithmic Method):

$$\therefore G.M = anti\log\left[\frac{\sum f \log x}{\sum f}\right]$$

Class limit	Class mark	Frequency	Log(x)	F log(x)
25-29	27	9	1.431	12.882
30-34	32	18	1.505	27.092
35-39	37	35	1.568	54.887
40-44	42	17	1.623	27.595
45-49	47	5	1.672	8.360
		$\sum f = 84$		$\sum f \log x = 130.816$

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$$\therefore G.M = anti\log\left[\frac{\sum f\log x}{\sum f}\right]$$

$$G.M = anti\log\left[\frac{130.816}{84}\right]$$

$$G.M = anti\log = (1.55)$$

$$G.M = 36.08$$

H.M:

Class limit	Class mark	Frequency	f/x
25-29	27	9	0.3333
30-34	32	18	0.5625
35-39	37	35	0.9459
40-44	42	17	0.4047
45-49	47	5	1063
		$\sum f = 84$	$\sum \frac{f}{x} = 2.3527$

$$H.M = \frac{\sum f}{\sum\left(\frac{f}{x}\right)}$$

$$H.M = \frac{84}{2.3527}$$

$$H.M = 35.70$$

Median:

Class limit	Class Boundary	Frequency	C.F
25-29	24.5-29.5	9	9
30-34	29.5-34.5	18	9+18=27
35-39	34.5-39.5	35	27+35=62
40-44	39.5-44.5	17	62+17=79
45-49	44.5-49.5	5	79+5=84
		$\sum f = 84$	

$$\therefore Median = m = L + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

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Note: Read this very carefully.

Where:

n = $\sum f = 84$

f is the highest frequency in the table.

L is the lower class boundary of the highest frequency class row.

h = Upper class boundary – lower class boundary (of the highest frequency class row).

C = C.F of the previous class.

So According to the data in the data of median:

Highest frequency is 35. So f = 35.

L = 34.5.

h = 39.5 – 34.5 = 5.

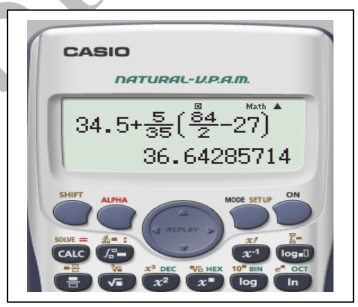
c = 27

So,

$$\therefore Median = m = L + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

$$m = 34.5 + \frac{5}{35}\left(\frac{84}{2} - 27\right)$$

$$m = 36.642$$



Mode:

Class limit	Class Boundary	Frequency
25-29	24.5-29.5	9
30-34	29.5-34.5	18(f_{M-1})
35-39	34.5-39.5	35(f_M)
40-44	39.5-44.5	17 (f_{M+1})
45-49	44.5-49.5	5
		$\sum f = 84$

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$$\therefore Mode = L + \left(\frac{(f_M - f_{M-1}) \times h}{2f_M - f_{M-1} - f_{M+1}} \right)$$

Note: (Read this very carefully)

Where:

n = $\sum f = 84$

f is the highest frequency in the table.

L is the lower class boundary of the highest frequency class row.

h = Upper class boundary – lower class boundary (of the highest frequency class row).

f_M **= f (the highest frequency in the table).**

f_{M-1} **is the frequency of the previous class.**

f_{M+1} **is the frequency of the next class.**

So According to the data in the data of Mode:

Highest frequency is 35. So f = 35.

L = 34.5.

h = 39.5 – 34.5 = 5.

f_M **= 35**

f_{M-1} **= 18**

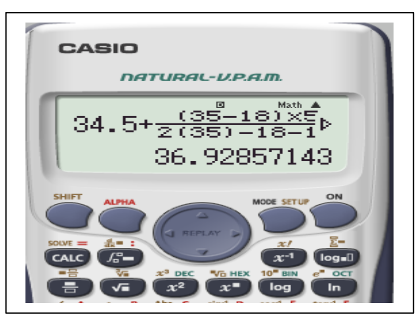
$f_{M+1} = 17$

So,

$$\therefore Mode = L + \left(\frac{(f_M - f_{M-1}) \times h}{2f_M - f_{M-1} - f_{M+1}} \right)$$

$$Mode = 34.5 + \frac{(35-18) \times 5}{2(35) - 18 - 17}$$

$$Mode = 36.929$$



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9. The following data show number of devices resulting in observed values in appropriate ranges. Find A.M., G.M., H.M., median, quartiles and mode.

Class limits	10.5-10.9	11.0-11.4	11.5-11.9	12.0-12.4	12.5-12.9
Frequencies	2	7	10	12	8

A.M:

$$\therefore \bar{x} = A + \frac{\sum Df}{\sum f}$$

Class limit	Class mark (x)	Frequency	D = x-A Taking A = 11.7	DF
10.5 – 10.9	10.7	2	10.7-11.7 = -1	-2
11.0 – 11.4	11.2	7	11.2-11.7 = -0.5	-3.5
11.5 – 11.9	11.7	10	11.7-11.7 = 0	0
12.0 -12.4	12.5	12	12.5-11.7 = 0.8	9.6
12.5 – 12.9	12.7	8	12.7-11.7 = 1	8
		$\sum f = 39$		$\sum Df = 12.1$

$$\therefore \bar{x} = A + \frac{\sum Df}{\sum f}$$

$$\bar{x} = 11.7 + \frac{12.1}{39}$$

$$\bar{x} = 12.0$$

G.M(Logarithmic Method):

$$\therefore G.M = anti\log\left[\frac{\sum f \log x}{\sum f}\right]$$

Class limit	Class mark	Frequency	Log(x)	F log(x)
10.5 – 10.9	10.7	2	1.0293	2.0586
11.0 – 11.4	11.2	7	1.0492	7.3444
11.5 – 11.9	11.7	10	1.068	10.68
12.0 -12.4	12.5	12	1.0969	13.1628
12.5 – 12.9	12.7	8	1.1033	8.8264
		$\sum f = 39$		$\sum f \log x = 42.0722$

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$$\therefore G.M = anti\log\left[\frac{\sum f\log x}{\sum f}\right]$$

$$G.M = anti\log\left[\frac{42.0722}{39}\right]$$

$$G.M = anti\log = (1.078)$$

$$G.M = 11.988$$

H.M:

Class limit	Class mark	Frequency	f/x
10.5 – 10.9	10.7	2	0.1869
11.0 – 11.4	11.2	7	0.625
11.5 – 11.9	11.7	10	0.8547
12.0 -12.4	12.5	12	0.96
12.5 – 12.9	12.7	8	0.6299
		$\sum f = 39$	$\sum \frac{f}{x} = 3.2562$

$$H.M = \frac{\sum f}{\sum\left(\frac{f}{x}\right)}$$

$$H.M = \frac{39}{3.2565}$$

$$H.M = 11.976$$

Median:

Class limit	Class mark	Class Boundary	Frequency	C.F
10.5 – 10.9	10.7	10.45 – 10.95	2	2
11.0 – 11.4	11.2	10.95 – 11.45	7	2+7 = 9
11.5 – 11.9	11.7	11.45 – 11.95	10	9+10 = 19
12.0 -12.4	12.5	11.95 – 12.45	12	19+12 = 31
12.5 – 12.9	12.7	12.45 – 12.95	8	31+8 = 39
			$\sum f = 39$	

$$\therefore Median = m = L + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

Where:

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n = $\sum f = 39$

f is the highest frequency in the table.

L is the lower class boundary of the highest frequency class row.

h = Upper class boundary – lower class boundary (of the highest frequency class row).

C = C.F of the previous class.

So According to the data in the data of median:

Highest frequency is 12. So f = 12.

L = 11.95.

h = 12.45 – 11.95 = 0.5.

c = 19

So,

$$\therefore Median = m = L + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

$$m = 11.95 + \frac{0.5}{12}\left(\frac{39}{2} - 19\right)$$

$$m = 11.97 \cong 12.0$$

Quartiles:

Q_1:

For first quartile: $\therefore Q_1 = \left(\frac{n+1}{4}\right)^{th}$

Note: We are using $\therefore Q_1 = \left(\frac{n+1}{4}\right)^{th}$ because $\left(\frac{n}{2}\right)$ is not an integer.

$Q_1 = \left(\frac{39+1}{4}\right) = 10$, as 10 is an integer so,

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$$\therefore Q_1 = \frac{1}{2}\left[\left(\frac{n+1}{4}\right)^{th} + \left(\frac{n+1}{4}+1\right)^{th}\right] observation$$

$$Q_1 = \frac{1}{2}\left[\left(\frac{39+1}{4}\right)^{th} + \left(\frac{39+1}{4}+1\right)^{th}\right] observation$$

$$Q_1 = \frac{1}{2}\left[\left(\frac{40}{4}\right)^{th} + \left(\frac{40+4}{4}\right)^{th}\right] observation$$

$$Q_1 = \frac{1}{2}\left[(10)^{th} + (11)^{th}\right] observation$$

, Both 10th and 11th lies in C.F = 19 whose value of x is 11.7. so,

$$Q_1 = \frac{1}{2}\left[(11.7)+(11.7)\right]$$

$$Q_1 = \frac{23.4}{2}$$

$$Q_1 = 11.7$$

Q3:

Third quartile = $Q_1 = \left(\frac{3n+3}{4}\right)$

$Q_3 = \left(\frac{3(39)+3}{4}\right) = \left(\frac{120}{4}\right) = 30$, where 30 is an integer so,

$$\therefore Q_3 = \frac{1}{2}\left[\left(\frac{3n+3}{4}\right)^{th} + \left(\frac{3n+3}{4}+1\right)^{th}\right] observation$$

$$Q_3 = \frac{1}{2}\left[\left(\frac{3(39)+3}{4}\right)^{th} + \left(\frac{3(39)+3}{4}+1\right)^{th}\right] observation$$

$$Q_3 = \frac{1}{2}\left[\left(\frac{120}{4}\right)^{th} + \left(\frac{120+4}{4}\right)^{th}\right] observation$$

$$Q_3 = \frac{1}{2}\left[(30)^{th} + (31)^{th}\right] observation$$

, Both 30 and 31 lies in C.F = 31 whose value of x is 12.5. so,

$$Q_3 = \frac{1}{2}\left[(12.5)+(12.5)\right] = \frac{25}{2}$$

$$Q_3 = 12.5$$

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Mode:

Class limit	Class Boundary	Frequency
10.5 – 10.9	10.45 – 10.95	2
11.0 – 11.4	10.95 – 11.45	7
11.5 – 11.9	11.45 – 11.95	10
12.0 -12.4	11.95 – 12.45	12
12.5 – 12.9	12.45 – 12.95	8
		$\sum f = 39$

$$\therefore Mode = L + \left(\frac{(f_M - f_{M-1}) \times \text{h}}{2f_M - f_{M-1} - f_{M+1}} \right)$$

Where:

n = $\sum f = 39$

f is the highest frequency in the table.

L is the lower class boundary of the highest frequency class row.

h = Upper class boundary – lower class boundary (of the highest frequency class row).

f_M = f (the highest frequency in the table).

f_{M-1} is the frequency of the previous class.

f_{M+1} is the frequency of the next class.

So According to the data in the data of Mode:

Highest frequency is 12. So f = 12.

L = 11.95.

h = 12.45 – 11.95 = 0.5.

f_M = 12

f_{M-1} = 10

f_{M+1} = 8

So,

$$\therefore Mode = L + \left(\frac{(f_M - f_{M-1}) \times \text{h}}{2 f_M - f_{M-1} - f_{M+1}} \right)$$

$$Mode = 11.95 + \frac{(12-10) \times 0.5}{2(12) - 10 - 8}$$

$$Mode = 12.117$$

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