Aims Public School

Chapter No. 22

Basic Statistics

EX: 22.4:

1. If W = {148, 145, 160, 157, 156, 160, 160, 165}, X = {-2, -2, -2, -2}, then:
   a. Show that sum of all deviations of W about its A.M. is zero.
   b. Show that A.M. = G.M. = H.M. = Median = Mode for X.
   c. Compute A.M. of Y = 3X.
   d. Compute A.M. of Z = 3W-11
   e. Show that H.M. < G.M. < A.M. < Median < Mode for W.

Part A:

$$\therefore \bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{148+145+160+157+156+160+160+165}{8}$$

$$\bar{x} = 156.375$$

$$Deviations \Rightarrow \sum(X-\bar{x}) = (148-156.375)+(145-156.375)+(160-156.375)+(157-156.375)+(156-156.375)+(160-156.375)+(160-156.375)+(165-156.375)$$

$$\sum(X-\bar{x}) = (-8.375)+(-11.375)+(3.625)+(0.625)+(-0.375)+(3.625)+(3.625)+(8.625)$$

$$\sum(X-\bar{x}) = -20.125+20.125$$

$$\sum(X-\bar{x}) = 0$$

Part B:

A.M:

$$\therefore \bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{-2-2-2-2}{4} = \frac{-8}{4} = -2$$

G.M:

$$\therefore G.M = (x_1 \times x_2 \times x_3 \times .....\times x_n)^{\frac{1}{n}}$$

$$G.M = ((-2)\times(-2)\times(-2)\times(-2))^{\frac{1}{4}}$$

$$G.M = -2$$

H.M:

$$\therefore H.M = \frac{n}{\sum\left(\frac{1}{x}\right)}$$

$$H.M = \frac{4}{\left(\frac{1}{-2}\right)+\left(\frac{1}{-2}\right)+\left(\frac{1}{-2}\right)+\left(\frac{1}{-2}\right)}$$

$$H.M = -2$$

For Median:

Here n = 4(Even), So,

$$\therefore m = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} + \left(\frac{n+2}{2}\right)^{th}\right] observation$$

$$m = \frac{1}{2}\left[\left(\frac{4}{2}\right)^{th} + \left(\frac{4+2}{2}\right)^{th}\right] observation$$

$$m = \frac{1}{2}\left[(2)^{nd} + (3)^{rd}\right] observation$$

$$m = \frac{1}{2}\left[(-2)+(-2)\right]$$

$$Median = -2$$

2. Check if X = {7,9,3,3,3,4,1,3,2,2} and Y = {2,1,4,4,4,6,6,5,7,1} are symmetric?

For X:

A.M:

$$\therefore \bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{7+9+3+3+3+4+1+3+2+2}{10} = \frac{37}{10} = 3.7$$

Median:

Ascending order: 1,2,2,3,3,3,3,4,7,9

Here n=10(Even), So,

$$\therefore m = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} + \left(\frac{n+2}{2}\right)^{th}\right] observation$$

$$m = \frac{1}{2}\left[\left(\frac{10}{2}\right)^{th} + \left(\frac{10+2}{2}\right)^{th}\right] observation$$

$$m = \frac{1}{2}\left[(5)^{nd} + (6)^{rd}\right] observation$$

$$m = \frac{1}{2}\left[(3)+(3)\right]$$

$$Median = 3$$

As A.M is not equal to Median, so X is asymmetric.

For Y:

NOTE: Solve it by your self by using the above approach.

3. The required weights (Kg) and price (Rs./Kg) of monthly items required by a family are given. Find weighted mean prices using A.M., G.M. and H.M.

Item	A	B	C	D	E
Price (Rs./Kg)	300	200	600	250	650
Required weight (Kg)	25	5	4	8	3

A.M : $\therefore \bar{x} = \frac{\sum fx}{\sum f}$ **G.M :** $\therefore G.M = anti\log\left(\frac{\sum f \log x}{\sum f}\right)$ **H.M :** $\therefore H.M = \frac{\sum f}{\sum\left(\frac{f}{x}\right)}$

x	f	fx	f/x	log x	f log x
300	25	7500	0.0833	2.477	61.928
200	5	1000	0.025	2.301	11.505
600	4	2400	0.0066	2.778	11.113
250	8	2000	0.032	2.397	19.183
650	3	1950	0.0046	2.812	8.438
	$\sum f = 45$	$\sum fx = 14850$	$\sum \frac{f}{x} = 0.1515$		$\sum f \log x = 112.167$

(NOTE: By using the above formulas complete this question using the given data).

4. Of 50 bricks bought, 21 bricks have mean mass of 24.2Kg, 29 bricks have mean mass of 23.6Kg, Find weighted mean mass of the 50 bricks.

A.M : $\therefore \bar{x} = \frac{\sum fx}{\sum f}$ **G.M :** $\therefore G.M = anti \log\left(\frac{\sum f \log x}{\sum f}\right)$ **H.M :** $\therefore H.M = \frac{\sum f}{\sum\left(\frac{f}{x}\right)}$

x	f	fx	f/x	log x	f log x
24.2	21	508.2	0.8677	1.299	29.06
23.6	29	648.4	1.2288	1.372	39.81
	$\sum f = 50$	$\sum fx = 1192.6$	$\sum \frac{f}{x} = 2.0965$		$\sum f \log x = 68.87$

(NOTE: By using the above formulas complete this question using the given data).

7. Locate and estimate median, quartiles and mode graphically in the data.

Overtime (hrs/week)	25-29	30-34	35-39	40-44	45-49	50-54	55-59
Number of days	5	4	7	11	12	8	1

Median:

Class limit	Class mark	Class Boundary	Frequency	C.F
25 – 29	27	24.5 – 29.5	5	5
30 – 34	32	29.5 – 34.5	4	5+4=9
35 – 39	37	34.5 – 39.5	7	9+7=16
40 – 44	42	39.5 – 44.5	11	16+11=27
45 – 49	47	44.5 – 49.5	12	27+12=39
50 – 54	52	49.5 – 54.5	8	39+8=47
55 - 59	7	54.5 – 59.5	1	47+1=48
			$\sum f = 48$	

$$m = \frac{n}{2} = \frac{48}{2} = 24 observation$$

As 24 is nearest to C.F=27 so the data will be taken from the row of C.F = 27.

$$\therefore m = L + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

$L = 40$

$f = 11$

$h = 44.5 - 39.5 = 5$

$c = 16$

$$m = 40 + \frac{5}{11}\left(\frac{48}{2} - 16\right)$$

$m = 43.6363$

Quartiles:

Q_1:

For first quartile: $\therefore Q_1 = \left(\frac{n}{4}\right)^{th}$

Note: We are using $\therefore Q_1 = \left(\frac{n}{4}\right)^{th}$ because $\left(\frac{n}{2}\right)$ is an integer.

$Q_1 = \left(\frac{48}{4}\right) = 12$, as 10 is an integer so,

$$\therefore Q_1 = \frac{1}{2}\left[\left(\frac{n}{4}\right)^{th} + \left(\frac{n}{4} + 1\right)^{th}\right] observation$$

$$Q_1 = \frac{1}{2}\left[\left(\frac{48}{4}\right)^{th} + \left(\frac{48}{4} + 1\right)^{th}\right] observation$$

$$Q_1 = \frac{1}{2}\left[\left(\frac{48}{4}\right)^{th} + \left(\frac{48+4}{4}\right)^{th}\right] observation$$

$$Q_1 = \frac{1}{2}\left[(12)^{th} + (13)^{th}\right] observation$$

, Both 12^{th} and 13^{th} lies in C.F = 16 whose value of x is 37. so,

$$Q_1 = \frac{1}{2}\left[(37) + (37)\right]$$

$$Q_1 = \frac{74}{2}$$

$Q_1 = 37$

Q_3:

Third quartile = $Q_3 = \left(\frac{3n}{4}\right)$

$Q_3 = \left(\frac{3(48)}{4}\right) = \left(\frac{144}{4}\right) = 36$, where 36 is an integer so,

$$\therefore Q_3 = \frac{1}{2}\left[\left(\frac{3n}{4}\right)^{th} + \left(\frac{3n}{4}+1\right)^{th}\right] observation$$

$$Q_3 = \frac{1}{2}\left[\left(\frac{3(48)}{4}\right)^{th} + \left(\frac{3(48)}{4}+1\right)^{th}\right] observation$$

$$Q_3 = \frac{1}{2}\left[(36)^{th} + (37)^{th}\right] observation$$

, Both 36 and 37 lies in C.F = 39 whose value of x is 47. so,

$$Q_3 = \frac{1}{2}\left[(47)+(47)\right] = \frac{94}{2}$$

$$Q_3 = 47$$

Mode:

Class limit	Class Boundary	Frequency
25 – 29	24.5 – 29.5	5
30 – 34	29.5 – 34.5	4
35 – 39	34.5 – 39.5	7
40 – 44	39.5 – 44.5	11
45 – 49	44.5 – 49.5	12
50 – 54	49.5 – 54.5	8
55 - 59	54.5 – 59.5	1
		$\sum f = 48$

$$\therefore Mode = L + \left(\frac{(f_M - f_{M-1})\times h}{2f_M - f_{M-1} - f_{M+1}}\right)$$

NOTE: By using this formula and the given data. Find the value of mode. (Reference: See the approach of finding mode in Q8 and Q9 of Ex: 22.3).

Aims Public School