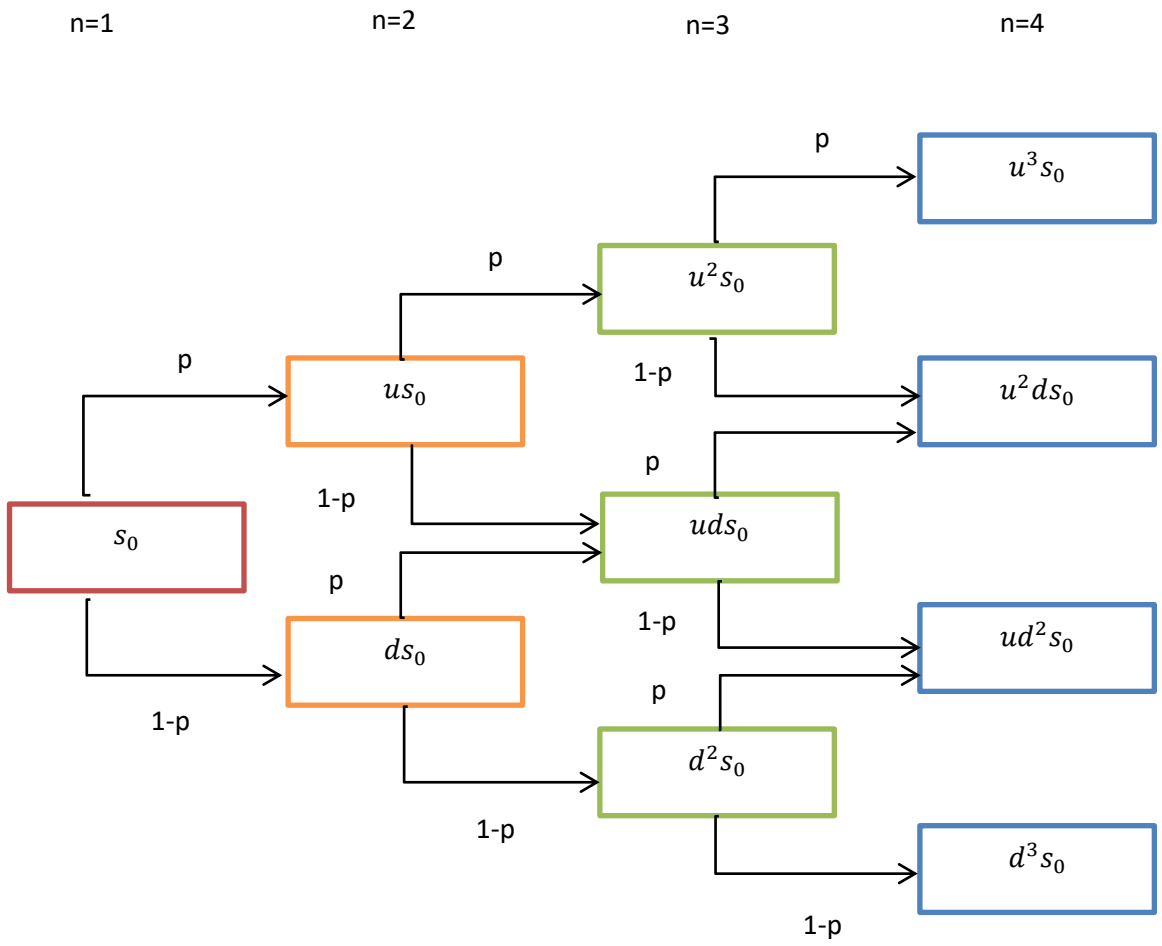
Assignment 1

Binomial pricing paths for stocks

Index: s15537

Method

The binomial pricing tree shows the evolution of the stock's price in discrete-time for a number of time steps between the valuation and expiration dates. Each node in the tree represents a possible price of the stock at a given time.



Here, $p = \frac{e^{rt/n} - d}{u - d}, u = e^{\sigma\sqrt{t/n}}$ and $d = e^{-\sigma\sqrt{t/n}}$

At each step, it is assumed that the underlying instrument will move up or down by a specific factor $(u \; or \; d)$ per step of the tree $(where, by \; definition, u \geq 1 \; and \; 0 < d \leq 1)$. So, if S is the current price, then in the next period the price will either be $S_{up} = SU \; or \; S_{down} = Sd$.

The up and down factors are calculated using the underlying volatility,σ, and the time duration of a step,t, measured in years $(using \; the \; day \; count \; convention \; of \; the \; underlying \; instrument)$. From the condition that the variance of the log of the price is $\sigma^2 t$, we have:

$$u = e^{\sigma\sqrt{\Delta t}}, d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u}$$

Here,

- $r: interest \; rate$
- $d: down \; rate$
- $u: upper \; rate$
- $p: upper \; probability$
- $\sigma: volatitlity$
- $t: expiration \; time$ measured in year
- $n: number \; of \; days$
- $\Delta t: time \; spet$