

EN2550  
Final Exa...Index no.: 

UNIVERSITY OF MORATUWA, SRI LANKA  
Faculty of Engineering  
Department of Electronic and Telecommunication Engineering  
B.Sc. Engineering  
Semester 4 Final Examination

**EN2550—FUNDAMENTALS OF IMAGE PROCESSING AND MACHINE VISION****Time Allowed:** 2 hours

January 2020

**ADDITIONAL MATERIAL**

None.

**INSTRUCTIONS TO CANDIDATES**

- This paper contains 4 questions on 12 pages.
- Answer **all** the questions.
- This examination accounts for 70% of the module assessment.
- This is a closed-book examination.
- The symbols used in this paper have their usual meanings.
- Clearly state any assumptions that you may make.
- Answer the questions in the question paper itself. You may not attach extra sheets. Any extra sheet will be disregarded in assessing.
- Neat and orderly presentation is important.

**Marks**

Q01	
Q02	
Q03	
Q04	
<b>Total</b>	

Q1. (a) State an example each of

- i. Intensity transformations, and  $\rightarrow$  Gamma, Identity, Negative, Intensity windowing  
 ii. Spatial filtering.  $\rightarrow$  Smoothing  $\rightarrow$  Noise reduction, Sharpening edges, Corners, Blobs  
 Average, Gaussian, Median, Unsharp Mask, Gaussian Filter

(b) Consider the intensity transformation given by

$$s = c \ln(r), \quad r \in [0, 255], \quad \text{like a Gamma low value}$$

where  $r$  is the input intensity,  $s$  is the output intensity, and  $c$  is a constant to be estimated.

i. State the effect of this transformation.

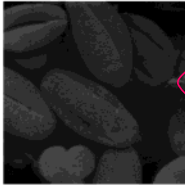
Compresses the Dynamic range  
 $\rightarrow$  Increase brightness in dark regions, maybe reducing brightness of lighter regions

ii. In a program, what is the usual way of applying a transform of this nature?

$$S = \text{cv.LUT}(img, c * np.log(r_{255}) * 255) \quad \text{or} \quad S = t[r] \quad \text{depends on "c"}$$

iii. A student's implementation of this transformation resulted in Fig. Q1(b). What is the reason for the incorrectness?

$\ln(0) \rightarrow -\infty$  undefined and closer values to zero is also negative  $\rightarrow$  Use  $S = c \ln(1+r)$   
 High values  $\rightarrow$  Scaling is important.



i. Original image.



ii. Intensity transformed image.

Figure Q1(b): Intensity transformation Q1b.

(c) Figure Q1(c) shows the probability mass function  $p_I(i)$  and cumulative distribution function of the intensities  $I$  of an 8-bit image. The darkest pixels (10%) and the brightest pixels (10%) are not important in this particular image. Consider the intensity transformation

$$s(i) = \begin{cases} 0, & \text{if } i \leq L \\ \frac{(i-L)255}{U-L}, & \text{if } L < i < U \\ 255, & \text{if } i \geq U \end{cases}$$

where

$$\sum_{i=0}^L p_I(i) = 0.1, \quad \text{and} \quad \sum_{i=U}^{255} p_I(i) = 0.1.$$

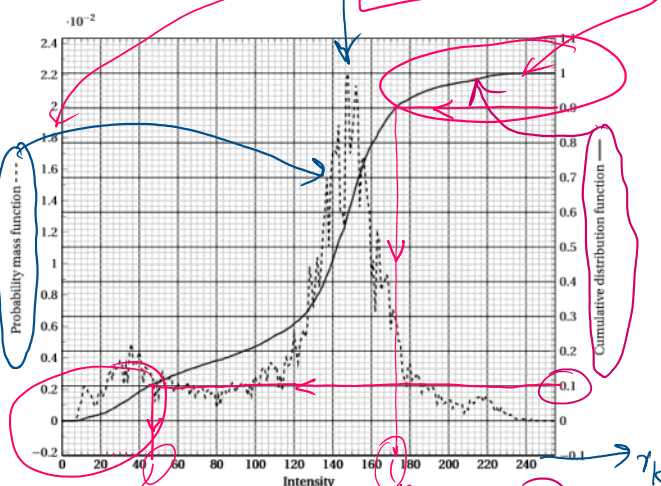
i. Briefly explain the effect of this transformation.

$\rightarrow$  Unwanted Low & High 10% intensities will be saturated.  
 $\rightarrow$  (L-U) scale intensities will go from (0-255)

$$n_k = (MN) \times P_r(r_k)$$

PDF  $P_r(r_k)$

$$\text{CDF} = \int_0^r P_r(w) dw$$



Intensity

Figure Q1(c): Intensity transformation Q1c.

$$L = 46.666$$

$$U = 173.33$$

$$U - L = 173.33 - 46.666 = 126.666$$

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ii. State the values of  $L$  and  $U$ . [2]

$L = 46.666$

$U = 173.33$

iii. Compute the resulting values of pixels with intensity  $I_1 = 100$  and  $I_2 = 200$ ? [2]

$I_1 = \frac{255}{(U-L)} (I - L) = \frac{255}{126.666} (100 - 46.666) = 107.369$

$I_2 = 255$

(d) The Laplacian for a function  $f(x, y)$  is

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Discretization of this can result in a  $3 \times 3$  Laplacian kernel.

i. Use the forward difference formula and backward difference formula together to obtain a symmetric expression for  $\frac{\partial^2 f}{\partial x^2}$  if  $f$  is a digital image. [2]

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) - 2f(x, y) + f(x-1, y)$$

ii. Hence, write an expression for the Laplacian  $\nabla^2 f$  if  $f$  is a digital image. [2]

$$\nabla^2 f = [f(x+1, y) - 2f(x, y) + f(x-1, y)] + [f(x, y+1) - 2f(x, y) + f(x, y-1)]$$

iii. Show this  $3 \times 3$  Laplacian as a kernel. [2]

$$\nabla^2 f = \begin{bmatrix} 0 & +f(x, y+1) & 0 \\ f(x-1, y) & -4f(x, y) & +f(x+1, y) \\ 0 & +f(x, y-1) & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \cdot f(x, y)$$

iv. Figure Q1(d) shows a different version of the  $3 \times 3$  Laplacian kernel and an image. What is the resulting value at  $(1, 1)$  of the Laplacian image? Top-left pixel is  $(0, 0)$ . [2]

$$10 + 50 + 200 + 20 + 190 + 40 + 190 + 180 - 8(50) = 460$$

v. If the image is zero padded, what would be the sum of values of the resulting Laplacian image? [1]

$$(-8+5) + (-8+3) + (-8+8) = -7790$$

i. Kernel.

1	1	1
1	-8.0	1
1	1	1

3x3

5x5

7x7

Figure Q1(d) Intensity transformation Q1d.

5x5

7x7

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$7/2 = 3 \text{ zero lines}$

$$\frac{\partial f}{\partial x} = \frac{f(x+1, y) - f(x, y)}{1} + \frac{f(x, y) - f(x-1, y)}{1}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial x} \right) = \frac{f(x+1, y) - f(x, y)}{1} - \frac{f(x, y) - f(x-1, y)}{1}$$

$$= f(x+1, y) - 2f(x, y) + f(x-1, y)$$

**Q2.** (a) Total least squares is useful in vision problems.

- i. Show how to find the line parameters expressed as  $ax + by = d$  using this method in terms of point coordinates  $(x_i, y_i)$ ,  $i = 1, \dots, N$ , and  $U$ , in regular notation. [3]

- ii. In a particular case,  $\bar{x} = 5.0$ ,  $\bar{y} = 13.1$ . The eigenvalues of  $U^T U$  are 21.2, 4324.1 and corresponding eigenvectors are  $[-0.8974, 0.4411]^T$ , and  $[-0.4411, -0.8974]^T$ . Find the gradient  $m$  and intercept  $c$  of the total-least-squares-fit line. [4]

- (b) Figure Q2(b) shows the image 1 and 4 from the graffiti images, and the homography that maps image 1 to 4. Note the convention of axes.

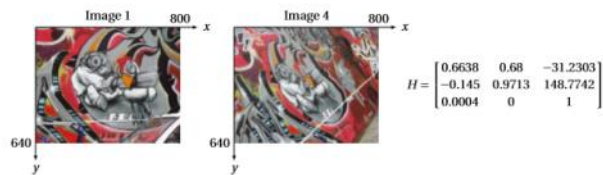


Figure Q2(b): Image 1, image 4 and homography mapping image 1 to 4.

- i. State the transformation that maps image 4 to image 1. [2]

- ii. Compute the locations of the corners of image 4 when mapped onto image 1. [2]

- iii. What is the size of the arrays needed to accommodate the stitched image after this mapping? [2]

- iv. What are two additional steps to make the stitched result visually pleasing? [2]

- (c) The scale-normalized Laplacian of Gaussian (LoG) for circularly symmetric blob detection is

$$\nabla^2_{\text{norm}} g = \sigma^2 \left( \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \right),$$

where  $g$  is the bi-variate Gaussian.

- i. Obtain an expression for the radius of a black circle in a white background that would give the maximum response to this operator. [2]

- ii. What is the suitable value for  $\sigma$ , if black circles of radius 10 pixels are to be detected. [2]

- iii. Assume that two images of the scene—taken with two level of zoom—have features detected. How can the features be matched? [2]

(d) A conveyor belt carries *several types* wrapped chocolate bars. Images of reference wrapped chocolate bars are available. The factory has the following requirements:

- Counting the number of wrapped chocolate bars that pass along the belt.
- Determining if there are printing mistakes in comparison with references.

i. Propose a system for counting. [2]

ii. Propose a method for locating printing mistakes. [2]

Q9. (a) A camera calibration software gave the following parameters for a particular camera:

$$\alpha_x = 650, \quad \alpha_y = 650, \quad \beta_x = 303, \quad \beta_y = 242.$$

The camera rotation matrix and the translation matrix with respect to the world coordinate system are

$$R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \hat{C} = \begin{bmatrix} 0 & 0 & 2 \end{bmatrix}^T.$$

i. Compute the camera matrix  $P$ . [2]

ii. Compute the Cartesian coordinates of the image of the point  $X = \begin{bmatrix} 2 & 3 & 10 & 1 \end{bmatrix}^T$ . [1]

- iii. What will happen to the size of the image of an object when  $\alpha = \alpha_x = \alpha_y$  increase? [1]

- (b) Consider the epipolar geometry of two cameras. Assume that the first camera  $C$  is at the world origin with axes aligned with the world coordinate frame. The second camera  $C'$  has a rotation  $R$  and a translation  $t$  with respect to the first camera. The intrinsic camera calibration matrices are  $K$  and  $K'$ , respectively.  $x$  and  $x'$  are the images of the world point  $X$  in the first and the second camera, respectively.

- i. Sketch the aforementioned epipolar geometry. [2]

- ii. Obtain an expression for the essential matrix  $E$ . [1]

- iii. Express epipolar lines associated with  $x$  and  $x'$  in terms of  $E$ . [1]

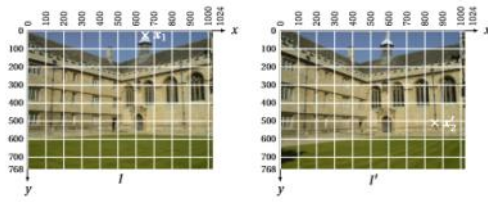
- iv. Derive an expression for the Fundamental matrix  $F$  in terms of  $E$ . [1]

- (c) Figure Q3(c) shows image  $I$  and  $I'$ , and point  $x_1$  and  $x'_2$ . The fundamental matrix relating the two cameras is

$$F = \begin{bmatrix} 0 & -0.1935 & 171.0789 \\ 0.1935 & 0 & 6850.4876 \\ -171.0789 & -6850.4876 & 0 \end{bmatrix}$$

- i. Compute the epipolar line  $l'_1$  and express it in the  $y = mx + c$  form. [2]

- ii. Compute the epipolar line  $l_2$  and express it in the  $y = mx + c$  form. [2]

Figure Q3(c): Image  $I$  and  $I'$ , and point  $x_1$  and  $x_2$ .

	0	1	2	3
0	120	200	240	50
1	50	60	100	120
2	10	40	20	30

i. Thresholding.

	0	1	2	3
0	120	200	240	50
1	50	60	100	120
2	10	40	20	30

ii.  $k$ -means

	0	1	2	3
0	120	200	240	50
1	50	60	100	120
2	10	40	20	30

iii.  $k$ -means with spatial

Figure Q3(d): Image and the grids for segmentation Q3d.



iii. Accurately plot these epipolar lines on the given images.

[4]

(d) Figure Q3(d) shows a  $3 \times 4$  image.

i. Segment this image by thresholding with the threshold of 80.

[2]

ii. Carry out one iteration of  $k$ -means clustering in the intensity space. Use Manhattan distance and  $K = 2$ . Select  $(0, 3)$  and  $(2, 0)$  as initial cluster centers.

[3]

iii. Carry out one iteration of  $k$ -means clustering in the intensity space and spatial space. Scale the spatial coordinated by 100. Use the parameters mentioned in Q3(d)ii.

[3]

Show the results in the grids in Figure Q3(d)

Q4. (a) State the output of the following vision tasks:

[5]

No.	Task	Output
1.	Image classification	
2.	Object detection	
3.	Semantic segmentation	
4.	Image captioning	
5.	Panoptic segmentation	



- (b) Consider a dataset of three classes of images, say, dogs, cats, and bunnies. An image is represented by a 5-dimensional vector. A liner classifier (after the learning process) is represented by

$$W = \begin{bmatrix} -0.43 & 0.53 & -0.29 & -1.03 & 0.53 \\ -0.55 & -0.69 & 0.95 & -0.06 & 1.16 \\ 0.54 & -0.28 & -1.44 & -0.21 & -0.93 \end{bmatrix}, \text{ and}$$

$$\mathbf{b} = \begin{bmatrix} 2.64 & -0.64 & -3.27 \end{bmatrix}^T.$$

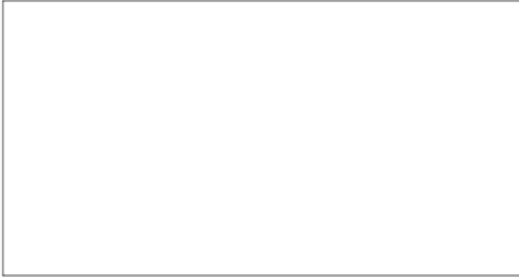
The first row of  $W$  and  $\mathbf{b}$  is the one-vs-all classifier for dogs, the second for cats, and the third for bunnies.

- i. State the steps for representing an image with a 5-dimensional feature vector. [2]

- ii. Determine the class of a image with feature vector  $\mathbf{x} = \begin{bmatrix} 5.31 & 1.51 & 1.61 & 8.1 & -8.45 \end{bmatrix}^T$ . [3]

- (c) Consider a neural network implementation for MNIST digit recognition. Assume that each digit is  $28 \times 28$  in grayscale. There are 10 digits, from 0 to 9. The network flattens the input first. There is a dense layer of 128 nodes, a dropout layer of 20% dropout and a 10-node dense softmax output layer.

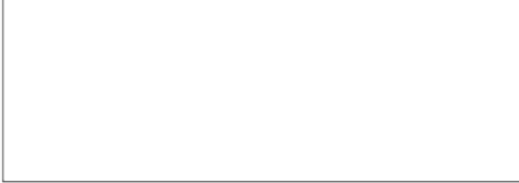
- i. Sketch the network. [3]



ii. Compute the number of learnable parameters in this network. [2]



iii. If the network is altered to have, first, a convolutional layer of  $(3 \times 3)$  convolutions with 64 filters, followed by a max-pooling layer of  $(2 \times 2)$  pooling window, and then the aforementioned dense network, compute the number of learnable parameters. [3]



(d) Consider an in-vehicle vision system that recognizes speed-limit traffic signs along an expressway. There are several such speed limits, e.g., 50, and 100. A speed limit sign is circular of diameter 450 mm with a red border and black numbers in white background (e.g., Figure Q4(d)). Assume that the system has to detect these signs irrespective of the distance within a range of 20 m to a maximum dictated by the camera. The resolution of the camera is  $2,048 \times 1,280$ . Horizontal field of view is  $\pm 30^\circ$ .

i. If the sign must occupy at least a width (or height) of 20 pixels for detection, compute the maximum distance for speed sign detection. [3]



Figure Q4(d): A example of a speed sign.

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ii. Suggest a conventional method for detection.

[3]

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iii. How can the same camera be used for obstacle detection?

[1]

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