Combinatorial Auction Mechanism(VCG) for Computation Distribution

for

Advanced Topics in Intelligent Systems - CSE6369

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Problem Statement:

Given a range of available resources on a computational cluster (divided into 3 types: processing, memory, disk space, each with a specific amount), implement an efficient (VCG) auction mechanism that can distribute these to N bidding agents that each want to schedule a randomly constructed job (i.e. with random resource requirements) with a true valuation that is additive in terms of the values of the required resources.

Vickrey Clarke Groves Auction:

In auction theory, a Vickrey–Clarke–Groves (VCG) auction is a type of sealed-bid auction of multiple items. Bidders submit bids that report their valuations for the items, without knowing the bids of the other people in the auction. The auction system assigns the items in a *socially optimal manner*.

VCG Auction:

For any set of auctioned items $M = \{t_1, \ldots, t_m\}$ and any set of bidders $N = \{b_1, \ldots, b_n\}$, let V_N^M be the social value of the VCG auction for a given bid-combination. That is, how much each person values the items they've just won, added up across everyone. The value of the item is zero if they do not win. For a bidder b_i and item t_j, let the bidder's bid for the item be $v_i(t_j)$. The notation $A \setminus B$ means the set of elements of A which are not elements of B.

Assignment

A bidder b_i whose bid for an item t_j is an "overbid", namely $v_i(t_j)$, wins the item, but pays $V_{N\setminus\{b_i\}}^M - V_{N\setminus\{b_i\}}^{M\setminus\{t_j\}}$, which is the social cost of their winning that is incurred by the rest of the agents.

Explanation

Indeed, the set of bidders other than b_i is $N \setminus \{b_i\}$. When item t_j is available, they could attain welfare $V_{N\setminus\{b_i\}}^M$. The winning of the item by b_i reduces the set of available items to $M \setminus \{t_j\}$, however, so that the attainable welfare is now $V_{N\setminus\{b_i\}}^{M\setminus\{t_j\}}$. The difference between the two levels of welfare is therefore the loss in attainable welfare suffered by the rest of the bidders, as predicted, **given** the winner b_i got the item t_j. This quantity depends on the offers of the rest of the agents and is unknown to agent b_i.

Winner's utility:

The winning bidder whose bid is the true value A for the item t_j, $v_j\{t_j\}$=A derives maximum $A - \left(V_{N\setminus\{b_i\}}^M - V_{N\setminus\{b_i\}}^{M\setminus\{t_j\}}\right)$

Project: *vcg-auction.ipnyb*

```
pip3 install jupyter

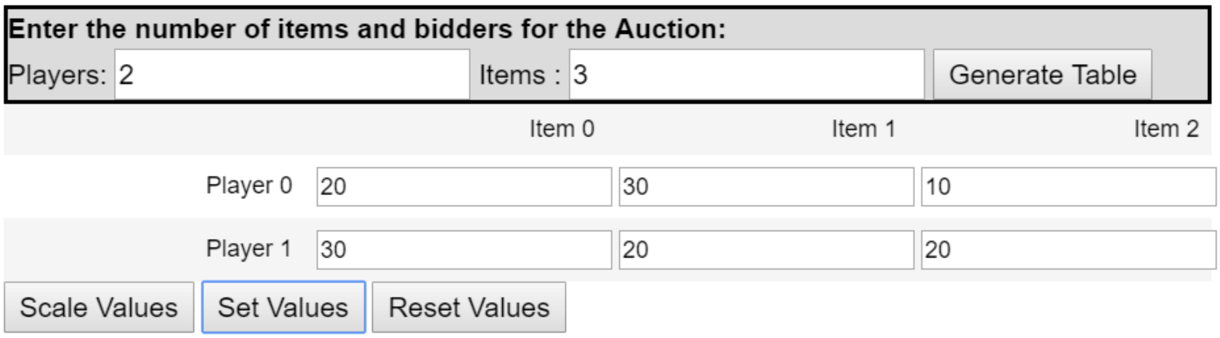
jupyter notebook
```

Code Structure:

Enter the number of *players* and number of *items* and click on *generate table*. Here players are the agents and items are the resources.

After giving the inputs click on *Set Values*.

For Example,



- First compute all different one-to-one allocations of items to bidders.
- valuations are: [array([20, 30, 10]), array([30, 20, 20])]
- Identify all possible allocations of items to bidders
- The value of auction matrix is :

```
[[ 0  0  0 60  0 60]
 [ 0  0  1 50 20 70]
 [ 0  1  0 30 20 50]
 [ 0  1  1 20 40 60]
 [ 1  0  0 40 30 70]
 [ 1  0  1 30 50 80]
 [ 1  1  0 10 50 60]
 [ 1  1  1  0 70 70]]
```

In the following auction matrix i0,i1 and i2 are the items and the v0, v1 are the valuations and v_tot is the total valuation. There are total of 8(2^3) combinations for which the valuations of the players if they get the resources are displayed.

- The auction matrix is

```
   i0  i1  i2  v0  v1  v_tot
0   0   0   0  60   0     60
1   0   0   1  50  20     70
2   0   1   0  30  30     60
3   0   1   1  20  50     70
4   1   0   0  40  30     70
5   1   0   1  30  50     80
6   1   1   0  10  60     70
7   1   1   1   0  80     80
```

Here we are using the **Clarke-Pivot Rule**

$$h_i(v_{-i}) = -\max_{x\in X} \sum_{j\neq i} v_j(x)$$

(value of others if agent/player were absent) - (value of others when agent/player were present).

- By using the Clarke Pivot Rule, we only take the cases where we get the max evaluation

```
    i0  i1  i2  v0  v1  v_tot
 0   1   0   1  30  50     80
 1   1   1   1   0  80     80
```

- ```
  Items:  ['i0', 'i1', 'i2']
  Valuation of Player to their allocation:  ['v0', 'v1']
  Payment of Player for their allocation:  ['p0', 'p1']
  ```

	i0	i1	i2	v0	v1	v_tot	p0	p1
0	1	0	1	30	50	80	20	30

VCG Payments:

The VCG Payments are given by
Player 1: v0-p0
Player 2: v1-p1

```
VCG Cost for Player  0 :  10
30 - 20
VCG Cost for Player  1 :  20
50 - 30
```

References:

https://pathtogeek.com/vickrey-clarke-groves-vcg-auction

https://en.wikipedia.org/wiki/Vickrey%E2%80%93Clarke%E2%80%93Groves_auction

https://www.youtube.com/watch?v=8-t_BmeNWgs
```