# Reference Sheet for Elementary Category Theory

## Categories

A **category** $\mathcal{C}$ consists of a collection of "objects" `Obj` $\mathcal{C}$`,` a collection of "morphisms" `Mor` $\mathcal{C}$, an operation `Id` associating a morphism $\mathtt{Id}_a$ `: a → a` to each object `a`, a parallel pair of functions `src, tgt : Mor` $\mathcal{C}$ `→ Obj` $\mathcal{C}$, and a "composition" operation `_⨾_ : ∀{A B C : Obj} → (A → B) → (B → C) → (A → C)` where for objects `X` and `Y` we define the *type* `X → Y` as follows

$$f : X \to Y \quad \equiv \quad \mathsf{src}\, f = X \;\wedge\; \mathsf{tgt}\, f = Y \qquad \text{defn-Type}$$

Moreover composition is required to be associative with `Id` as identity.
Instead of `src` and `tgt` we can instead assume primitive a ternary relation `_:_→_` and regain the operations precisely when the relation is functional in its last two arguments:

$$f : A \to B \;\wedge\; f : A' \to B' \implies A = A' \;\wedge\; B = B' \qquad \text{unique-Type}$$

When this condition is dropped, we obtain a *pre-category*; e.g., the familiar *Sets* is a pre-category that is usually treated as a category by making morphisms contain the information about their source and target: `(A, f, B) : A → B` rather than just `f`.

A categorical statement is an expression built from notations for objects, typing, morphisms, composition, and identities by means of the usual logical connectives and quantifications and equality.

Even when morphisms are functions, the objects need not be sets: Sometimes the objects are *operations* –with an appropriate definition of typing for the functions. The categories of F-algebras are an example of this.

Example Categories.

- Each digraph determines a category: The objects are the nodes and the paths are the morphisms typed with their starting and ending node. Composition is catenation of paths and identity is the empty path.
- Each preorder determines a category: The objects are the elements and there is a morphism `a → b` named, say, `(a, b)`, precisely when *ab*.

## Further Reads

- Roland Backhouse
- Grant Malcolm
- Lambert Meertens
- Jaap van der Woude