Type-directed search with dependent types

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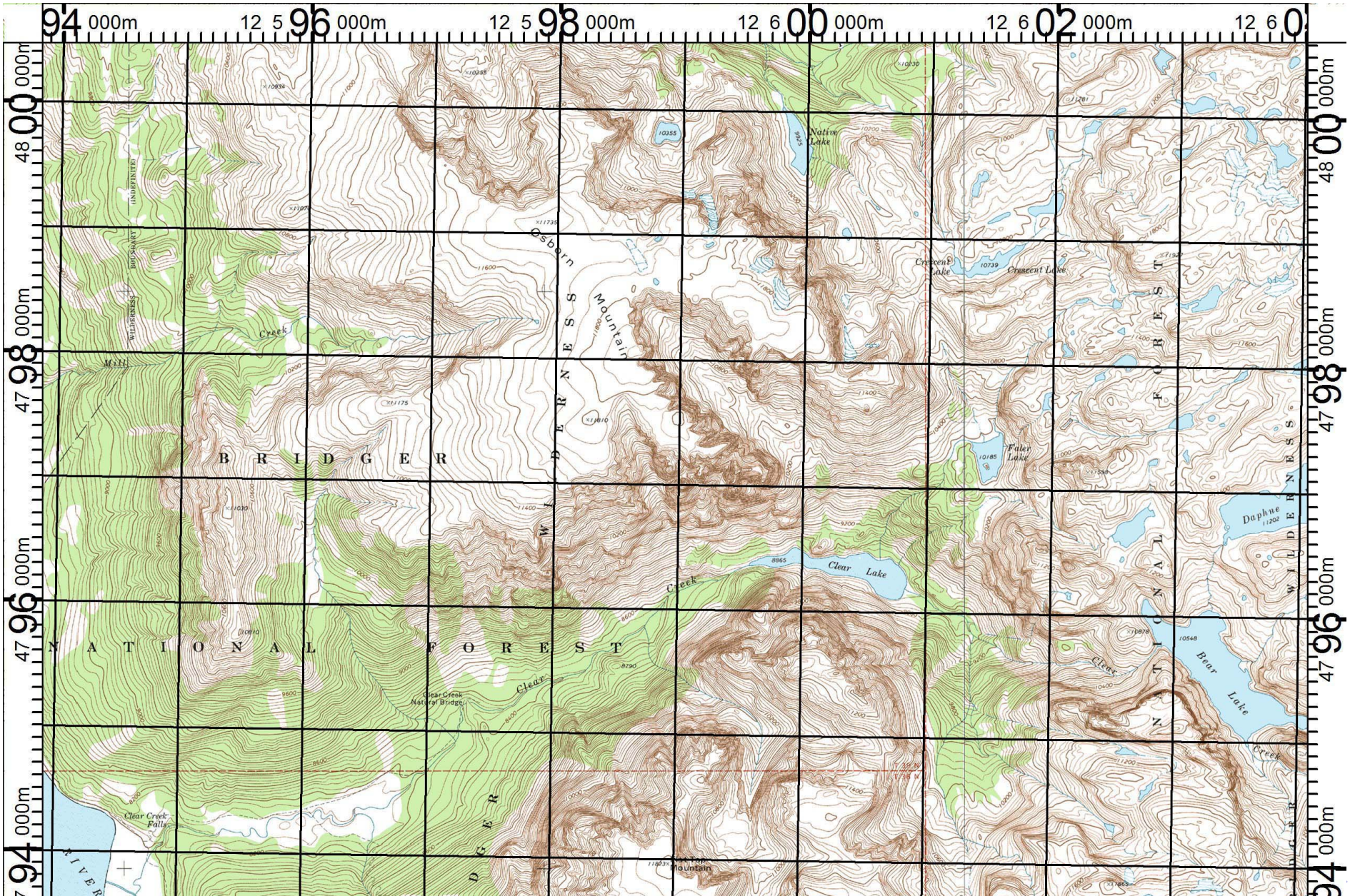
Type systems are like jigsaw puzzles

No types


94 000m
96 000m
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02 000m
48 00 000m
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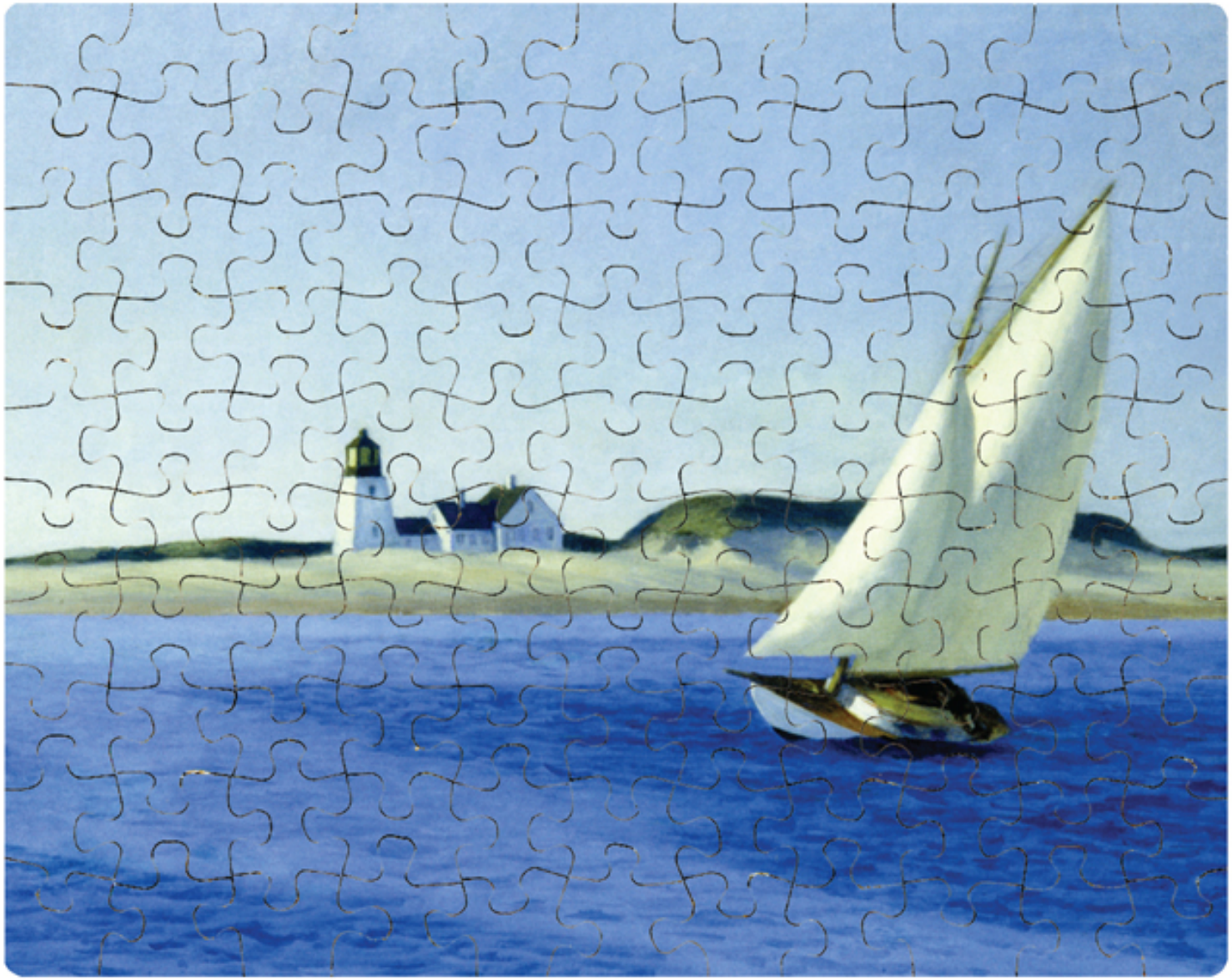
B R I D G E R
N A T I O N A L F O R E S T
W I L D E R N E S S
Native Lake
Crescent Lake
Faler Lake
Daphne
Clear Lake
Bear Lake
Clear Creek Falls
Clear Creek Natural Bridge
Osborn Mountain
Mill Creek

Simple types



Expressive types

Sorting a list

Haskell:

```
1 sort :: Ord a ⇒ [a] → [a]
```

Idris (my example, $>$ 150 LOC):

```
1 quickSort : TotalOrder a lte
2           → (xs : List a)
3           → (ys : List a ** (IsSorted lte ys, Permutation xs ys))
```

Type-driven development

Types

- Prove properties stronger than any test can show
- Are documentation that is never wrong or outdated
- Provide an exact specification

Type-directed search

- Tool of choice for type-driven developers
- Can choose your name; can't choose your type!

Distinction without a difference

- Even though $\mathrm{a} \to \mathrm{b} \to \mathrm{c}$ and $(\mathrm{a},\ \mathrm{b}) \to \mathrm{c}$ are distinct types, they "mean the same thing."

Type isomorphism

Definition

Types A and B are *isomorphic* if there are functions

$f : A \to B$ and $g : B \to A$

such that

$(x : A) \to (g \circ f)(x) = x$ and

$(y : B) \to (f \circ g)(y) = y$.

(Type *equivalence* in HoTT)

(What does $=$ mean?)

Isomorphism is not enough!

Suppose we want to compare two values whose type has instance Ord for equality. We search

```
1 Ord a ⇒ a → a → Bool
```

We'd like to find

```
1 (==) :: Eq a ⇒ a → a → Bool
```

Its type is strictly *more* general than what we asked for.

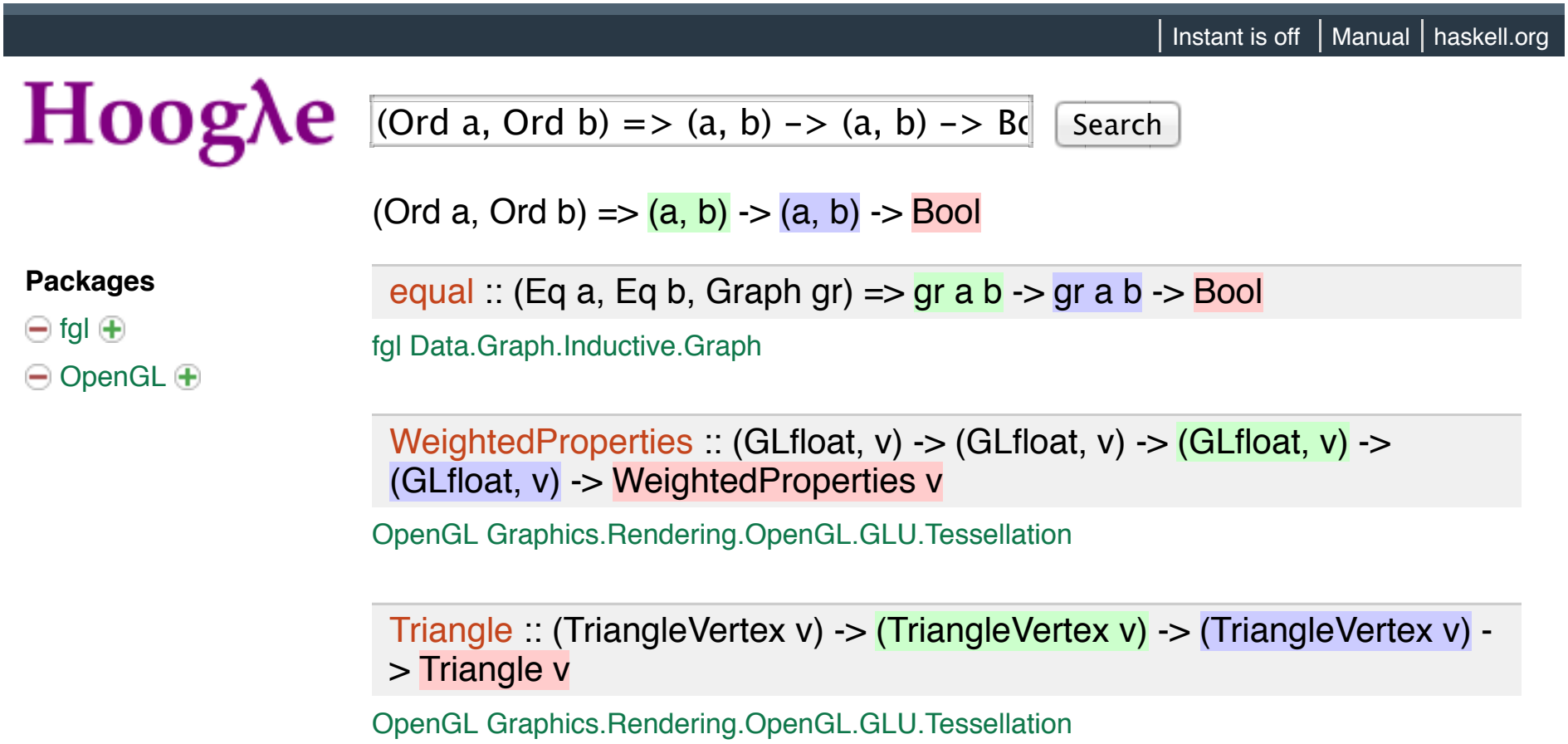
Type containment

We want a partial order $\leq$ that defines isomorphism: that is,

If $A \leq B$ and $B \leq A$, then $A \cong B$.

Demo

Beating Hoogle



"Kind" search for free

Instant is off | Manual | haskell.org

Hoogλe

* -> *

Search

* -> *

Parse error: (line 1, column 2): unexpected " " expecting letter

For information on what queries should look like, see the user manual.

The algorithm

- Try messing with the types in lots of different ways.
- Find the best results first (Dijkstra's algorithm)!

(More details: `https://github.com/idris-lang/Idris-dev/wiki/Type-directed-search-(:search)`)

Next up?

- Produce the corresponding "data" for the search results
- Inlining single-constructor datatypes
- Find isomorphic datatypes
- What do **you** think?

Pi in the sky

- Big database of libraries (with code that feels like programs and code that feels like proofs)
- Type-driven development
- Search the types you must implement; if there's a result, use the library with confidence that it meets the specification