

## Paul's Online Notes

Home / Calculus I / Derivatives

---

### Chapter 3 : Derivatives

---

In this chapter we will start looking at the next major topic in a calculus class, derivatives. This chapter is devoted almost exclusively to finding derivatives. We will be looking at one application of them in this chapter. We will be leaving most of the applications of derivatives to the next chapter.

Here is a listing of the topics covered in this chapter.

**The Definition of the Derivative** – In this section we define the derivative, give various notations for the derivative and work a few problems illustrating how to use the definition of the derivative to actually compute the derivative of a function.

**Interpretation of the Derivative** – In this section we give several of the more important interpretations of the derivative. We discuss the rate of change of a function, the velocity of a moving object and the slope of the tangent line to a graph of a function.

**Differentiation Formulas** – In this section we give most of the general derivative formulas and properties used when taking the derivative of a function. Examples in this section concentrate mostly on polynomials, roots and more generally variables raised to powers.

**Product and Quotient Rule** – In this section we will give two of the more important formulas for differentiating functions. We will discuss the Product Rule and the Quotient Rule allowing us to differentiate functions that, up to this point, we were unable to differentiate.

**Derivatives of Trig Functions** – In this section we will discuss differentiating trig functions. Derivatives of all six trig functions are given and we show the derivation of the derivative of  $\sin(x)$  and  $\tan(x)$ .

**Derivatives of Exponential and Logarithm Functions** – In this section we derive the formulas for the derivatives of the exponential and logarithm functions.

**Derivatives of Inverse Trig Functions** – In this section we give the derivatives of all six inverse trig functions. We show the derivation of the formulas for inverse sine, inverse cosine and inverse tangent.

**Derivatives of Hyperbolic Functions** – In this section we define the hyperbolic functions, give the relationships between them and some of the basic facts involving hyperbolic functions. We also give the derivatives of each of the six hyperbolic functions and show the derivation of the formula for hyperbolic sine.

**Chain Rule** – In this section we discuss one of the more useful and important differentiation formulas, The Chain Rule. With the chain rule in hand we will be able to differentiate a much wider variety of functions. As you will see throughout the rest of your Calculus courses a great many of derivatives you take will involve the chain rule!

**Implicit Differentiation** – In this section we will discuss implicit differentiation. Not every function can be explicitly written in terms of the independent variable, e.g.  $y = f(x)$  and yet we will still need to know what  $f'(x)$  is. Implicit differentiation will allow us to find the derivative in these cases. Knowing implicit differentiation will allow us to do one of the more important applications of derivatives, Related Rates (the next section).

**Related Rates** – In this section we will discuss the only application of derivatives in this section, Related Rates. In related rates problems we are give the rate of change of one quantity in a problem and asked to determine the rate of one (or more) quantities in the problem. This is often one of the more difficult sections for students. We work quite a few problems in this section so hopefully by the end of this section you will get a decent understanding on how these problems work.

**Higher Order Derivatives** – In this section we define the concept of higher order derivatives and give a quick application of the second order derivative and show how implicit differentiation works for higher order derivatives.

**Logarithmic Differentiation** – In this section we will discuss logarithmic differentiation. Logarithmic differentiation gives an alternative method for differentiating products and quotients (sometimes easier than using product and quotient rule). More importantly, however, is the fact that logarithm differentiation allows us to differentiate functions that are in the form of one function raised to another function, *i.e.* there are variables in both the base and exponent of the function.