

## Paul's Online Notes

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### Chapter 4 : Series And Sequences

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In this chapter we'll be taking a look at sequences and (infinite) series. In fact, this chapter will deal almost exclusively with series. However, we also need to understand some of the basics of sequences in order to properly deal with series. We will therefore, spend a little time on sequences as well.

Series is one of those topics that many students don't find all that useful. To be honest, many students will never see series outside of their calculus class. However, series do play an important role in the field of ordinary differential equations and without series large portions of the field of partial differential equations would not be possible.

In other words, series is an important topic even if you won't ever see any of the applications. Most of the applications are beyond the scope of most Calculus courses and tend to occur in classes that many students don't take. So, as you go through this material keep in mind that these do have applications even if we won't really be covering many of them in this class.

Here is a list of topics in this chapter.

**Sequences** – In this section we define just what we mean by sequence in a math class and give the basic notation we will use with them. We will focus on the basic terminology, limits of sequences and convergence of sequences in this section. We will also give many of the basic facts and properties we'll need as we work with sequences.

**More on Sequences** – In this section we will continue examining sequences. We will determine if a sequence is an increasing sequence or a decreasing sequence and hence if it is a monotonic sequence. We will also determine a sequence is bounded below, bounded above and/or bounded.

**Series – The Basics** – In this section we will formally define an infinite series. We will also give many of the basic facts, properties and ways we can use to manipulate a series. We will also briefly discuss how to determine if an infinite series will converge or diverge (a more in depth discussion of this topic will occur in the next section).

**Convergence/Divergence of Series** – In this section we will discuss in greater detail the convergence and divergence of infinite series. We will illustrate how partial sums are used to determine if an infinite series converges or diverges. We will also give the Divergence Test for series in this section.

**Special Series** – In this section we will look at three series that either show up regularly or have some nice properties that we wish to discuss. We will examine Geometric Series, Telescoping Series, and Harmonic Series.

**Integral Test** – In this section we will discuss using the Integral Test to determine if an infinite series converges or diverges. The Integral Test can be used on a infinite series provided the terms of the series are positive and decreasing. A proof of the Integral Test is also given.

**Comparison Test/Limit Comparison Test** – In this section we will discuss using the Comparison Test and Limit Comparison Tests to determine if an infinite series converges or diverges. In order to use either test the terms of the infinite series must be positive. Proofs for both tests are also given.

**Alternating Series Test** – In this section we will discuss using the Alternating Series Test to determine if an infinite series converges or diverges. The Alternating Series Test can be used only if the terms of the series alternate in sign. A proof of the Alternating Series Test is also given.

**Absolute Convergence** – In this section we will have a brief discussion on absolute convergence and conditionally convergent and how they relate to convergence of infinite series.

**Ratio Test** – In this section we will discuss using the Ratio Test to determine if an infinite series converges absolutely or diverges. The Ratio Test can be used on any series, but unfortunately will not always yield a conclusive answer as to whether a series will converge absolutely or diverge. A proof of the Ratio Test is also given.

**Root Test** – In this section we will discuss using the Root Test to determine if an infinite series converges absolutely or diverges. The Root Test can be used on any series, but unfortunately will not always yield a conclusive answer as to whether a series will converge absolutely or diverge. A proof of the Root Test is also given.

**Strategy for Series** – In this section we give a general set of guidelines for determining which test to use in determining if an infinite series will converge or diverge. Note as well that there really isn't one set of guidelines that will always work and so you always need to be flexible in following this set of guidelines. A summary of all the various tests, as well as conditions that must be met to use them, we discussed in this chapter are also given in this section.

**Estimating the Value of a Series** – In this section we will discuss how the Integral Test, Comparison Test, Alternating Series Test and the Ratio Test can, on occasion, be used to estimating the value of an infinite series.

**Power Series** – In this section we will give the definition of the power series as well as the definition of the radius of convergence and interval of convergence for a power series. We will also illustrate how the Ratio Test and Root Test can be used to determine the radius and interval of convergence for a power series.

**Power Series and Functions** – In this section we discuss how the formula for a convergent Geometric Series can be used to represent some functions as power series. To use the Geometric Series formula, the function must be able to be put into a specific form, which is often impossible. However, use of this formula does quickly illustrate how functions can be represented as a power series. We also discuss differentiation and integration of power series.

**Taylor Series** – In this section we will discuss how to find the Taylor/Maclaurin Series for a function. This will work for a much wider variety of function than the method discussed in the previous section at the expense of some often unpleasant work. We also derive some well known formulas for Taylor series of  $e^x$ ,  $\cos(x)$  and  $\sin(x)$  around  $x = 0$ .

**Applications of Series** – In this section we will take a quick look at a couple of applications of series. We will illustrate how we can find a series representation for indefinite integrals that cannot be evaluated by any other method. We will also see how we can use the first few terms of a power series to approximate a function.

**Binomial Series** – In this section we will give the Binomial Theorem and illustrate how it can be used to quickly expand terms in the form  $(a + b)^n$  when  $n$  is an integer. In addition, when  $n$  is not an integer an extension to the Binomial Theorem

can be used to give a power series representation of the term.