Paul's Online Notes

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Chapter 2 : Partial Derivatives

In Calculus I and in most of Calculus II we concentrated on functions of one variable. In Calculus III we will extend our knowledge of calculus into functions of two or more variables. Despite the fact that this chapter is about derivatives we will start out the chapter with a section on limits of functions of more than one variable. In the remainder of this chapter we will be looking at differentiating functions of more than one variable. As we will see, while there are differences with derivatives of functions of one variable, if you can do derivatives of functions of one variable you shouldn't have any problems differentiating functions of more than one variable. You'll just need to keep one subtlety in mind as we do the work.

Here is a list of topics in this chapter.

Limits – In the section we'll take a quick look at evaluating limits of functions of several variables. We will also see a fairly quick method that can be used, on occasion, for showing that some limits do not exist.

Partial Derivatives – In this section we will look at the idea of partial derivatives. We will give the formal definition of the partial derivative as well as the standard notations and how to compute them in practice (i.e. without the use of the definition). As you will see if you can do derivatives of functions of one variable you won't have much of an issue with partial derivatives. There is only one (very important) subtlety that you need to always keep in mind while computing partial derivatives.

Interpretations of Partial Derivatives – In the section we will take a look at a couple of important interpretations of partial derivatives. First, the always important, rate of change of the function. Although we now have multiple 'directions' in which the function can change (unlike in Calculus I). We will also see that partial derivatives give the slope of tangent lines to the traces of the function.

Higher Order Partial Derivatives – In the section we will take a look at higher order partial derivatives. Unlike Calculus I however, we will have multiple second order derivatives, multiple third order derivatives, etc. because we are now working with functions of multiple variables. We will also discuss Clairaut's Theorem to help with some of the work in finding higher order derivatives.

Differentials – In this section we extend the idea of differentials we first saw in Calculus I to functions of several variables.

Chain Rule – In the section we extend the idea of the chain rule to functions of several variables. In particular, we will see that there are multiple variants to the chain rule here all depending on how many variables our function is dependent on and how each of those variables can, in turn, be written in terms of different variables. We will also give a nice method for writing down the chain rule for pretty much any situation you might run into when dealing with functions of multiple variables. In addition, we will derive a very quick way of doing implicit differentiation so we no longer need to go through the process we first did back in Calculus I.

Directional Derivatives – In the section we introduce the concept of directional derivatives. With directional derivatives we can now ask how a function is changing if we allow all the independent variables to change rather than holding all but one constant as we had to do with partial derivatives. In addition, we will define the gradient vector to help with some of the notation and work here. The gradient vector will be very useful in some later sections as well. We will also give a nice fact that will allow us to determine the direction in which a given function is changing the fastest.

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