Bessel Function Zeros

The Bessel function of order n, for $n = 0, 1, 2, \ldots$, can be defined by the definite integral

$$J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(x \sin\theta - n\theta)\, d\theta.$$

Compute the first five positive roots $j_{n,k}$, ($k = 1, 2, \ldots, 5$), of the first six Bessel functions $J_n(x)$, ($n = 0, 1, \ldots,$

Script ❓

💾 Save   ⟳ Reset   📖 MATLAB Documentation (https://www.mathworks.com/help/)

```
 1  num_roots=5; num_functions=6;
 2  %initial guess for roots (from Wolfram MathWorld)
 3  zeros_guess=[2.4,3.8,5.1,6,7.5,8.7;...
 4      5.5,7,8.4,9.7,11,12;...
 5      8.6 10,11.6,13,14,16;...
 6      11.8,13,15,16,18,19;...
 7      15,16.4,18,19.4,21,22];
 8  %Compute first five roots of first six Bessel functions
 9  %Put in variable bzeros with size(bzeros) = [5, 6]
10
11  integrand = @(theta,x,n) cos(x.*sin(theta)-n*theta);
12  J_n = @(x) integral(@(theta)integrand(theta,x,n),0,pi);
13  for n=0:num_functions-1
14      J_n = @(x) integral(@(theta)integrand(theta,x,n),0,pi);
15      for k=1:num_roots
16          bzeros(k,n+1) = fzero(J_n, zeros_guess(k,n+1));
17      end
18  end
19
20
21
22
23  %print table
24  fprintf('k      J0(x)      J1(x)      J2(x)      J3(x)      J4(x)      J5(x)\n')
25  for k=1:num_roots
26      fprintf('%i',k)
27      for n=0:num_functions-1
28          fprintf('%10.4f',bzeros(k,n+1));
29      end
30      fprintf('\n');
31  end
32
```

▶ Run Script   ❓

Tests: All Tests Passed

✅ **Test Bessel function zeros**

Output

k	J0(x)	J1(x)	J2(x)	J3(x)	J4(x)	J5(x)
1	2.4048	3.8317	5.1356	6.3802	7.5883	8.7715
2	5.5201	7.0156	8.4172	9.7610	11.0647	12.3386
3	8.6537	10.1735	11.6198	13.0152	14.3725	15.7002
4	11.7915	13.3237	14.7960	16.2235	17.6160	18.9801
5	14.9309	16.4706	17.9598	19.4094	20.8269	22.2178