



Introduction

A binary indexed tree is a data structure stored as an array that can efficiently calculate prefix sum and perform update operations. It is also called a Fenwick Tree. Given an array of inputs, the tree will store the sum of some elements in each of its nodes. There is another data structure called a segment tree which can also perform similar tasks. However, in contrast to a segment tree, a binary indexed tree will consume less space, and the implementation of a binary indexed tree is also easier.

Internal understanding of binary indexed tree

In a scenario where we store the sum of given inputs in a binary indexed tree, we can express each integer value as the sum of powers of 2. For example, 21 can be expressed as $16+4+1$ where each term is a power of 2. Therefore, each node in a binary indexed tree will store sum of n elements as a power of 2.

Operations on a binary indexed tree

We can perform query operations and update operations in a standard binary indexed tree.

Advantages of a Binary indexed tree

- A binary indexed tree will always be more space efficient compared to a segment tree.
- A binary indexed tree is easier and faster in terms of implementation in code.

Sample Code

```
1 #include <iostream>
2
3 using namespace std;
4
5 int getSum(int tree[], int index)
6 {
7     int sum = 0;
8     index = index + 1;
9
10    while (index>0)
11    {
```

```

12
13         sum += tree[index];
14         index -= index & (-index);
15     }
16     return sum;
17 }
18
19 void update(int tree[], int n, int index, int val)
20 {
21     index = index + 1;
22     while (index <= n)
23     {
24         tree[index] += val;
25         index += index & (-index);
26     }
27 }
28
29 int *constructTree(int arr[], int n)
30 {
31     int *tree = new int[n+1];
32     for (int i=1; i<=n; i++)
33         tree[i] = 0;
34     for (int i=0; i<n; i++)
35         update(tree, n, i, arr[i]);
36     return tree;
37 }

```

Complexity Analysis

The construction of a binary indexed tree has a time complexity of $O(n \log n)$. Similar to a segment tree, both `query()` and `update()` operation has a time complexity of $O(\log n)$.

Problems involving binary indexed tree

Range sum query

In this problem, we are required to calculate the sum of some elements of a given array within an input range. We also have to update numbers for given indices throughout our calculations.

Solution

We can use a binary indexed tree and sum of multiple ranges will be stored inside that tree so that we can find sum of our desired range within $O(\log n)$ time complexity.