VECTORS AND SCALARS

Scalar Quantity:

The physical quantity which is completely specified by its magnitude and proper unit is known as scalar quantity.
Example: length, mass, time, volume, density, work etc.

- These quantities are denoted by alphabets.i.e. l, m, t, v, ρ.
- They can be added, subtracted, multiplicated and divided by simple algebraic rules.
- Please give me {Magnitude} → (2) (kg) ← {Unit} sugar.

Vector Quantity:

The physical quantity which is completely specified by its magnitude, proper unit and appropriate direction is known as vector quantity.
Example: acceleration, velocity, displacement, force, torque etc.

- These quantities are denoted by alphabets and an arrow above them. I.e. $\vec{a}$, $\vec{V}$, $\vec{d}$, $\vec{F}$ etc. sometimes vector is represented by $\vec{OA}$, $\vec{AB}$, in this way the first latter shows the tail of a vector and the second latter shows the head of a vector.
- These quantities can be added by geometrical or graphical method (parallelogram method, head to tail rule) and by trigonometric method (rectangular component method), and can be multiplied by dot or cross product rules.
- The magnitude of a vector is denoted as |A| and read as modulus of A or simply A.
- A car is moving with acceleration of {Magnitude} →(5) (m/s^2) ←{Unit} (towards east) → {direction}.

Unit Vector (â):

A unit vector is a vector whose magnitude is 1 which is used to specify the direction of a vector and it is represented by small alphabets with a cap "^" i.e. a.

- Unit vector can be calculated as $\hat{a} = \dfrac{\vec{A}}{|A|}$
- Note that: A vector is composed of its two parts, its magnitude and its direction. $\vec{A} = \hat{a}|A|$

Negative of a vector ($-\vec{A}$):

Negative of a vector is a vector whose magnitude is same but the direction is opposite to the given vector.

Example: $\vec{A} = |A|$

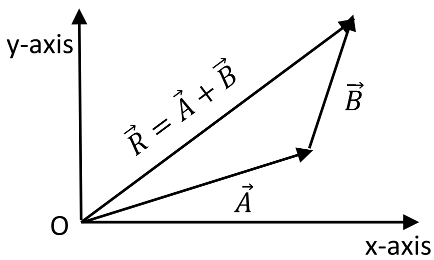
$-\vec{A} = |A|$

Note that: if a vector is represented by $\overrightarrow{OP}$ then negative vector will be represented as $\overrightarrow{PO}$

Resultant vector ($\vec{R}$):

Resultant of two or more vectors is a single vector whose magnitude and direction produces same effect as the combined effect of the given vectors. Which is formed by joining the tail of the first vector to the head of the last vector.

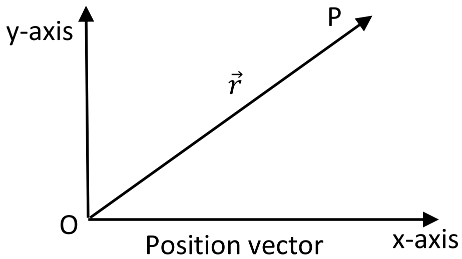
Example:



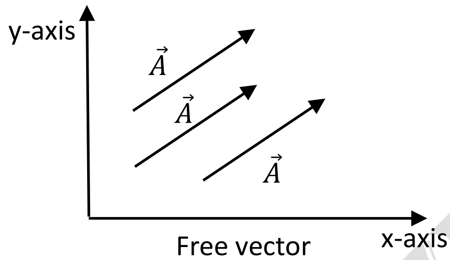
Prepared by: Sir. Hasnain Cell: 0331-2189275 Email Address: Hasnainkhankqs@gmail.com

Position vector ($\vec{r}$):

A vector which is used to specify the position of a point with respect to a fixed point such as origin of a coordinate system in other words it tells us that how far and in what direction a point is it from the origin of a coordinate system. It is denoted by $\vec{r}$. As shown in following left figure:



Position vector



Free vector

Free vector:

A vector which can be displaced parallel to itself and can be applied at any point is called free vector in other words free vector is a vector which is free to move itself. As shown in above right figure.

Null vector:

A vector which has magnitude equal to zero and has no direction is called null vector and it is obtained when two vectors of equal magnitude but opposite directions are added.

Example:

$$\vec{A} + (-\vec{A}) = 0$$

Scalar Product:

When multiplication of two vectors results a scalar quantity then the product is known as scalar product. Or it is the product of magnitude of two vectors and the cosine of angle between them.

- It is denoted as $\vec{A}\,.\,\vec{B} = |A||B|\cos\theta$
- It is represented by placing a dot between two vectors hence it is also called dot product.
- It obeys Commutative law $\vec{A}\,.\,\vec{B} = \vec{B}.\vec{A}$, Associative law $m\vec{A}\,.\,n\,\vec{B} = mn\vec{B}.\vec{A}$ and Distributive law
  $\vec{A}\,.\,(\vec{B} + \vec{C}) = \vec{A}\,.\vec{B} + \vec{A}\,.\vec{C}$

Example:

$W = \vec{F}\,.\vec{d}$ $\quad\quad$ $P = \vec{F}\,.\vec{V}$

$W = |F||d|\cos\theta$ $\quad\quad$ $P = |F||V|\cos\theta$

❖ Properties of unit vectors with respect to dot product.

➢ When vectors are parallel to each other.

- $\hat{\imath}\,.\,\hat{\imath} = \hat{\jmath}\,.\,\hat{\jmath} = \hat{k}\,.\,\hat{k} = 1$
- $i\,i\cos 0° = j\,j\cos 0° = k\,k\cos 0° = 1$

$\boxed{\because \cos 0° = 1}$

➢ When vectors are perpendicular to each other.

- $\hat{\imath}\,.\,\hat{\jmath} = \hat{\jmath}.\,\hat{k} = \hat{k}.\hat{\imath} = 0$
- $i\,j\cos 90° = j\,k\cos 90° = k\,i\cos 90° = 0$

$\boxed{\because \cos 90° = 0}$

❖ If $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

$\vec{A}\,.\vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}).(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$

$\vec{A}\,.\vec{B} = A_x B_x + A_y B_y + A_z B_z$

Vector Product:

When multiplication of two vectors results a Vector quantity then the product is known as vector product. Or it is the product of magnitude of two vectors and the sine of angle between them.

- It denoted as $\vec{A} \times \vec{B} = |A||B| \sin\theta$
- It is represented by placing a cross between two vectors hence it is also called cross product
- In terms of magnitude and direction it is represented as $|A||B| \sin\theta\, \hat{u}$
- It does not obey commutative law. $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$
- It obeys Associative law $m.(\vec{A} \times \vec{B}) = m\vec{A} \times \vec{B}$ and distributive law $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

Where û is a unit vector pointing in the direction of the product vector .The direction of a vector product is determine by right hand rule. According to this rule the direction of vector product always perpendicular to the plan or perpendicular to both the vectors $\vec{A}$ and $\vec{B}$ as well.

Example:

Torque; $\vec{\tau} = \vec{F} \times \vec{d}$

Force experienced by charge particle moving with a velocity in a magnetic field $\vec{F} = \vec{q}\,(\vec{V} \times \vec{B})$

❖ Properties of unit vectors with respect to cross product.

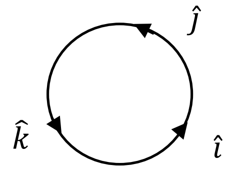
- When vectors are parallel to each other.
  - $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$
  - $i\, i \sin 0° = j\, j \sin 0° = k\, k \sin 0° = 0$

$\because \sin 0° = 0$

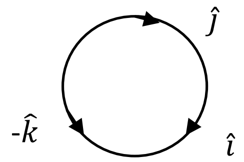
- When vectors are perpendicular to each other.
  - $\hat{\imath} \times \hat{\jmath} = \hat{\jmath} \times \hat{k} = \hat{k} \times \hat{\imath} = 1$
  - $i\, j \sin 90° = j\, k \sin 90° = k\, i \sin 90° = 1$

$\because \sin 90° = 1$

- Moving anticlockwise;
  - $\hat{\imath} \times \hat{\jmath} = \hat{k}$
  - $\hat{\jmath} \times \hat{k} = \hat{\imath}$
  - $\hat{k} \times \hat{\imath} = \hat{\jmath}$



- Moving clockwise;
  - $\hat{\jmath} \times \hat{\imath} = -\hat{k}$
  - $\hat{k} \times \hat{\jmath} = -\hat{\imath}$
  - $\hat{\imath} \times \hat{k} = -\hat{\jmath}$



❖ If $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

$$\vec{A} \times \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

By using unit vector's properties

$$\vec{A} \times \vec{B} = A_x B_y\hat{k} - A_xB_z\hat{\jmath} - A_yB_x\hat{k} + A_yB_z\hat{\imath} + A_z B_x\hat{\jmath} - A_zB_y\hat{\imath}$$

$$\vec{A} \times \vec{B} = A_yB_z\hat{\imath} - A_zB_y\hat{\imath} + A_z B_x\hat{\jmath} - A_xB_z\hat{\jmath} + A_x B_y\hat{k} - A_yB_x\hat{k}$$

$$\vec{A} \times \vec{B} = \hat{\imath}\,(A_yB_z - A_zB_y) + \hat{\jmath}\,(A_z B_x - A_xB_z) + \hat{k}\,(A_x B_y - A_yB_x)$$

$$\vec{A} \times \vec{B} = \hat{\imath}\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} + \hat{\jmath}\begin{vmatrix} A_z & A_x \\ B_z & B_x \end{vmatrix} + \hat{k}\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \implies \vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_y \end{vmatrix}$$

Prepared by: Sir. Hasnain Cell: 0331-2189275 Email Address: Hasnainkhankqs@gmail.com