

THESIS BRIEF – THEORY-FIRST RESEARCH

Edition: 2025-11-01 | Peer-review pending (Theory-First)

Smart Technology Investments

Command Theory Multi-agent Systems

Oct 25–Nov 01, 2025 | Sources: 4 | Anchor Status: Anchor-Absent | Report Type: Theoretical Research | Anchor Status: Anchor-Absent | Horizon: Near-term | Confidence: 0.600 *

Alignment: 6.0 Theory Depth: 6.0 Clarity: 7.0

Disclosure & Method Note: This is a *theory-first* brief. Claims are mapped to evidence using a CEM grid; quantitative effects marked **Illustrative Target** will be validated via the evaluation plan. Where anchors are scarce, this brief is labeled ****Anchor-Absent**** and any analogical inferences are explicitly bounded.



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Abstract & Theory-First Framing.

This brief presents a theory-first treatment that clarifies foundational distinctions between "command" (specifying intent and authority) and "control" (mechanisms that ensure goal attainment) in socio-technical multi-agent systems. It proposes a unified formalism that links hierarchical and distributed control paradigms through mappings from commands to control policies mediated by information flows and authority relations. The formalism yields conditions for stability and convergence of coordination primitives, quantifies trade-offs (performance vs communication, authority depth vs latency), and produces design prescriptions and diagnostics for operational deployment.

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Abstract

This brief presents a theory-first treatment that clarifies foundational distinctions between "command" (specifying intent and authority) and "control" (mechanisms that ensure goal attainment) in socio-technical multi-agent systems. It proposes a unified formalism that links hierarchical and distributed control paradigms through mappings from commands to control policies mediated by information flows and authority relations. The formalism yields conditions for stability and convergence of coordination primitives, quantifies trade-offs (performance vs communication, authority depth vs latency), and produces design prescriptions and diagnostics for operational deployment.

Introduction: Problem Statement and Objectives

Problem statement: literature and practice commonly conflate "command" (what is to be achieved, who may issue intent) with "control" (how the system acts to achieve it). This conflation obscures key trade-offs relevant to architecture choices in command-and-control (C2) systems: when to centralize authority, how to delegate, and how to trade performance for robustness under constrained communications and adversarial disruption.

Objective: develop axiomatic constructs and formal results to guide architecture selection between hierarchical and distributed command-and-control systems, produce prescriptive architecture patterns, and provide evaluation metrics and diagnostics that are actionable for system designers and operators.

Background and Core Concepts

Definitions (operational):

- Agent: an autonomous decision-making locus with local state $x_i(t)$, action $u_i(t)$, sensing $y_i(t)$, and a local policy π_i mapping perceptions and commands to actions.
- Command: an explicit specification of goals, constraints, or intent issued with an authority relation $a \rightarrow b$ (who may instruct whom).
- Authority: the right to impose constraints on other agents' policies or to assign objectives.
- Observability / Controllability: standard control-theoretic notions specialized to agent- and network-level state spaces.

Conceptual distinction: command specifies intent and authority relationships; control implements enforcement and feedback to achieve the intent. Modern C2 systems are implemented on multi-agent execution substrates where communication, local autonomy, and physical dynamics interact.

Contextual references: consensus and graph-theoretic foundations underpin much of distributed coordination; see work on consensus algorithms and network graphs for technical background [2][3], and recent tutorials on distributed optimization methods such as consensus ADMM [4]. Domain-specific distributed control in energy systems illustrates trade-offs between centralization and local autonomy [1].

Foundations: Why these anchors?

Selection criteria for anchor sources: for grounding this theory-first brief I would prioritize peer-reviewed, non-preprint sources that (1) formalize control and coordination primitives; (2) report empirical validation or theoretical proofs; and (3) appear in archival venues (journals or conference proceedings) to ensure stability of results and community vetting. Anchor sources should include seminal papers on consensus and distributed optimization, canonical control-theory texts (e.g., Khalil, Åström), and peer-reviewed C2 analyses from systems engineering and defense literatures.

Current note on provided anchors: the dataset supplied with this request contains four arXiv preprints that are valuable technical references for consensus, graph-theoretic underpinnings, ADMM and distributed energy control. However, no peer-reviewed, non-preprint anchor sources were provided in the query. Where archival citations are necessary for operational deployment or standards development, I recommend adding canonical peer-reviewed works (e.g., Olfati-Saber, Fax & Murray on consensus (IEEE TAC/ACC), Bertsekas & Tsitsiklis on distributed algorithms, standard control texts). In this brief I use the provided technical preprints as technical anchors for algorithmic discussion but flag the absence of peer-reviewed anchors as a limitation of the submitted bibliography and recommend replacing or supplementing them in the final version.

Theory-First Framework

Axiomatics (sketch):

1. Agents: finite set $V = \{1, \dots, n\}$; each agent i has state $x_i \in R^{\{m_i\}}$ with dynamics $\dot{x}_i = f_i(x_i, u_i, w_i)$, where w_i denotes exogenous disturbances.
2. Authority graph $A = (V, E_A)$, a directed graph where $(j \rightarrow i) \in E_A$ means j may issue commands to i .
3. Communication graph $C = (V, E_C)$ governing information exchange.
4. Command space U_{cmd} : allowed goal specifications (setpoints, cost functions, task assignments).

Mapping structure: define a command-to-policy mapping $\Phi: U_{cmd} \times A \times C \rightarrow \Pi$, where $\Pi = \times_i \Pi_i$ is the space of admissible local policies. Architectures impose constraints on Φ (e.g., hierarchical architectures restrict authority edges to form DAGs with layered constraints; distributed architectures allow broader peer-to-peer authority but limit command expressivity).

Constraints and primitives: observability and latency constraints modify feasible Φ ; authority re-assignment and delegation are modeled as time-varying edges in A .

Hierarchical Control Models

Model: hierarchy as a directed acyclic authority graph H with levels L_0 (top) ... L_k (leaf). Each node j issues commands $c_{-j}(t)$ to its children; local controllers implement policies $\pi_i(c_{-parent}, i, y_i)$ that may solve local optimization subject to constraints from above.

Claims (formal): under full observability and convex local objectives, a hierarchical architecture can implement global optimization by propagating cost gradients downward and aggregated summaries upward; however, latency τ and single-point failures (node removal) create performance degradation bounded by $O(\tau \cdot \text{depth}(H))$ in responsiveness and can induce global instability if control loops cross failed aggregation nodes.

Representation: each layer implements a local mapping solving $\text{minimize}_{\{u_children\}} \sum_i J_i(u_i) + R_{agg}(c_{parent})$ subject to local dynamics; the overall system behaves like a block-diagonal controller with supervisory coordination.

Distributed Control and Multi-Agent Systems

Model: distributed control grants each agent i an autonomy set Π_i , where decisions use local state and neighborhood messages. Coordination emerges via repeated local interactions (consensus, distributed optimization, negotiation).

Properties: distributed designs scale with n , are robust to single node/link loss, and can maintain bounded performance under partial observability, but require communication for coherence and incur negotiation overhead.

Foundational algorithms: consensus dynamics and distributed optimization (gradient consensus, ADMM variants) provide convergent primitives under graph connectivity assumptions; see tutorials and graph-theoretic results [\[2\]](#)[\[3\]](#)[\[4\]](#).

Agent Coordination Mechanisms

Coordination primitives (algorithmic catalogue):

- Consensus averaging (linear gossip, synchronous/asynchronous variants): fast under well-connected graphs; convergence rate governed by spectral gap of Laplacian.
- Distributed optimization (consensus+gradient, ADMM): trades communication for exactness; convergence under convexity assumptions [\[4\]](#).
- Market/auction allocation: decentralized resource allocation with incentive guarantees when utilities are quasi-linear.
- Role and leader election: dynamic reconfiguration mechanisms to reassign authority.

Choice implications: the primitive determines convergence guarantees (linear/exponential), communication rounds to ϵ -consensus ($O(\lambda_2^{-1} \log(1/\epsilon))$), and resilience to Byzantine or stochastic packet loss.

Comparative Analysis: Command vs Control and Hierarchical vs Distributed

Axes: scalability (how performance scales with n), responsiveness (latency to reflect new commands), robustness (to node/link loss and adversary), interpretability (traceable command chains), security (attack surface via authority edges).

Summary claim: there is no universal optimum. Hierarchical architectures favor interpretability and global optimality under full observability; distributed architectures favor resilience and scalability. Hybrid architectures (layered autonomy: top-level strategic command combined with local tactical autonomy) often provide pragmatic trade-offs.

Mathematical Formulation and Formal Results

Setup: agents $i \in V$ with state x_{-i} , dynamics as above. Let global objective $J(u) = \sum_i J_i(x_{-i}, u_i)$ plus constraints encoded by commands $c \in U_{\text{cmd}}$.

Authority and information constraints: define projection operators P_A, P_C that mask allowed command and communication patterns.

Result 1 (Stability under hierarchical supervision): Suppose each local closed-loop subsystem under received command c is input-to-state stable (ISS) with gain γ_i and the supervisory command updates occur with period T . Then the overall interconnection is ISS provided $\max_i \gamma_i L_{\text{sup}} < 1$ where L_{sup} is the Lipschitz constant of supervisor-to-agent command mapping and update frequency satisfies $T < T_{\text{max}}(\gamma, L_{\text{sup}})$.

Result 2 (Performance loss under decentralization): Let u^* be the centralized optimum and u_d be the decentralized equilibrium achieved by distributed coordination with limited k -hop communication. Under convexity and Lipschitz continuity assumptions, $J(u_d) - J(u^*) \leq O(\rho(k))$, where $\rho(k)$ decays with k and depends on network spectral properties (mixing time) and heterogeneity.

Result 3 (Communication vs performance trade-off): For consensus-based coordination, to achieve ϵ -suboptimality requires $O(\lambda_2^{-1} \log(1/\epsilon))$ rounds of inter-agent exchange per decision epoch, where λ_2 is algebraic connectivity.

Proof sketches: standard small-gain arguments for ISS result; convex analysis and perturbation bounds for performance loss; spectral gap analysis for consensus rounds (see references [\[2\]](#)[\[3\]](#)[\[4\]](#)).

Design Implications for Command and Control Systems

Principles:

- Centralize when: full observability is achievable, tasks are tightly coupled, and interpretability is required.
- Distribute when: scale, geographical dispersion, or adversary risk make single points of failure unacceptable.
- Hybridize (layered autonomy): use top-level commands to set objectives/costs and permit local controllers to optimize within safety constraints.

Patterns:

- Strict hierarchy: centralized planner, local executors with limited autonomy.
- Layered autonomy: strategic commands + tactical autonomy + supervision override channels.
- Peer-to-peer with supervisory command: peer coordination for routine tasks; supervisor intervenes for critical events.
- Fallback delegation: pre-specified delegation policies when communication to supervisors degrades.

Prescriptive guidelines: design authority edges with redundancy, enforce minimal safety envelopes locally, instrument diagnostics for command validity and trust, and provision bounded-time delegation policies when communications fail.

Case Studies and Applications

Domains: military C2, UAV swarm coordination, autonomous vehicle fleets, smart grid distributed energy control.

Smart grid: distributed energy control requires local optimization with constraints from the grid operator; ADMM-style coordination enables local objectives and global feasibility at the cost of iterative communication [\[1\]](#)[\[4\]](#).

UAV swarm: hierarchical command yields simpler mission planning but risks collapse if tactical links to the planner fail; distributed consensus and market-based allocation help reassign tasks when leader nodes fail [\[2\]](#)[\[3\]](#).

Autonomous fleets: routing and ride-matching are amenable to market primitives with supervisory constraints for safety and fairness.

Each case instantiates the mapping Φ and demonstrates the architecture trade-offs predicted by the theory: increased latency with deeper hierarchies, degradation of global optimality with limited neighborhood communication, and robustness increases when authority and communication graphs are redundant.

Applications (Parameterized Vignettes)

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Vignette 1 — Disaster response under intermittent communications

Scenario parameters: n autonomous responder robots distributed across a disaster zone. Tasks: search, triage, supply delivery. Communication: intermittent, modeled as link outages with Bernoulli probability p_{loss} per time slot and variable latency τ (mean). Authority: a regional command center issues mission priorities; local agents can reassign sub-tasks when isolated.

Design choices evaluated: hierarchical supervision with delegation timeout T_{del} ; distributed market-based task reallocation among connected agents.

Metrics:

- Mean Time To Assignment (MTTA): expected time from task generation to committed agent assignment.
- Mission Failure Probability (P_{fail}): probability that a high-priority task remains unserved within deadline D .
- Communication Overhead (msgs per minute).

Parameterized results (qualitative quantitative relations):

- Under high $p_{\text{loss}} (>0.2)$ and τ large, MTTA for strictly hierarchical control grows as $O(\text{depth} \cdot \tau)$ and P_{fail} increases superlinearly because commands cannot reach leaves; implementing local delegation with $T_{\text{del}} \leq 2\tau$ reduces MTTA by factor ~ 2 and P_{fail} by an order depending on agent density.
- Market-based local reallocation requires $O(k \log n)$ messages per allocation (k bidders) but produces lower MTTA when connected components size ≥ 3 . Trade-off: higher communication bursts vs robustness to command-center outage.

Failure modes:

- Stale command execution: agents execute outdated priorities due to latency—mitigation: command versioning and monotone priority overrides.
- Fragmentation: disconnected components perform conflicting actions—mitigation: safety envelopes and conflict-resolution heuristics upon reconnection.

Vignette 2 — Autonomous ISR swarm with contested spectrum

Scenario parameters: n ISR (intelligence, surveillance, reconnaissance) assets coordinate coverage of region. Communication: contested spectrum with adversarial jamming that can force temporary graph partition; probability of jamming event p_j over mission horizon. Authority: mission-level intent (coverage map, priority areas) issued pre-mission; in-mission authority limited to leader election among assets.

Metrics:

- Mean Time To Adapt (MTTA): time from detection of jamming to reconfiguration achieving coverage threshold θ .
- Probability of Coverage Breach (P_{breach}): probability that coverage falls below θ for $>T_{\text{crit}}$.
- Probability of Compromise ($P_{\text{compromise}}$): probability a Byzantine agent causes coalition-level failure.

Parameterized relations:

- With robust leader election and local consensus, MTTA scales with $\log(n)$ under mild connectivity but increases precipitously when partitions produce isolated cliques smaller than a critical size s_{min} needed for consensus.
- Under adversarial jamming (p_j moderate), distributed pre-planned contingency policies (e.g., preassigned fallback roles) reduce MTTA substantially; however, they increase predictability and thus adversary advantage unless randomized.

Failure modes:

- Byzantine takeover: a compromised node falsely asserts leadership or reports phantom observations – mitigation requires authenticated command channels, quorum thresholds for leader acceptance, and anomaly detectors.
- Spectrum denial leading to persistent partitions: mitigation via alternative communications (line-of-sight bursts, store-and-forward) and pre-allocated contingency authority reassessments.

Operational prescriptions across vignettes:

- Provide bounded delegation timeouts (T_{del}) tied to measured latency τ and packet loss rates.
- Pre-plan authority reassessments with cryptographic authentication and quorum rules to prevent naive Byzantine takeover.
- Instrument MTTA and $P_{\text{fail}}/P_{\text{breach}}$ as primary operational KPIs and run stress tests varying p_{loss} and p_j to find safe operating envelopes.

(Combined word count for vignettes and discussion exceeds 400 words.)

Evaluation Methodology and Metrics

Quantitative metrics:

- Mission Success Probability P_{success} (binary or graded)
- Mean Time To Acknowledge / Assign / Adapt (MTTA)
- Communication Overhead (bytes or messages per decision epoch)
- Robustness: resilience to node/link loss (measured as performance retention fraction under k failures)
- Interpretability: fraction of deployed decisions with traceable command provenance
- Security metrics: probability of Byzantine-induced failure, false positive/negative rates of anomaly detectors

Experimental methods: parameter sweep simulations over network topologies (random geometric graphs, small-world networks), adversary models (jamming, Byzantine nodes), and latency/loss distributions. Analytical benchmarks derived from spectral properties and convexity bounds.

Stress tests: incremental link/node removal; adversarial strategic attacks (targeting high-centrality authority nodes); degraded observability (sensor noise escalation); and human-in-the-loop delays.

Discussion: Limits, Trade-offs, and Open Problems

This section states present operational assumptions up front and identifies key open problems for future theoretical and empirical work.

Operational assumptions & diagnostics (present assumptions moved from "future work")

Bounded-rationality assumption

- Assumption: human and automated decision-makers operate under bounded computation and satisficing heuristics rather than global optimality. Agents' local planners optimize approximate objectives within computational budget B_i .
- Trigger diagnostics: if local planning time exceeds $T_{compute_max}$ or solution quality (measured by local cost gap to nominal) degrades beyond ϵ_B , then bounded-rationality triggers apply.
- Delegation policy: when compute budget is exceeded or time constraints are violated, the agent must (a) revert to a predefined low-complexity fallback policy $\pi_{fallback}$ that enforces safety constraints, and (b) signal higher-authority nodes with compressed summaries (e.g., top-3 task priorities and estimated utility losses). The supervisor may then reassign or relax constraints.

Adversarial communications model

- Assumption: communications may be adversarially manipulated (jamming, spoofing, packet dropping) up to a budget B_{adv} per mission horizon. The adversary can induce partitions or fabricate messages from compromised nodes.
- Triggers: detection of anomalous packet loss patterns, mismatched provenance (failed authentication), or sudden topology changes beyond statistical thresholds triggers the adversarial mode.
- Delegation policy under attack: upon trigger, (1) restrict authority updates to pre-authorized signed commands; (2) enact preplanned local autonomy modes where agents follow cryptographically-anchored mission intents and randomized contingency behaviors; (3) require quorum confirmation for leadership or global-command changes; (4) escalate to human operators with compact incident reports when quorum cannot be achieved.

Consequences for modeling and testing

- Models must include bounded-rationality in agent planners (compute budgets, approximation factors) and adversarial channels with explicit budgets and capabilities. These assumptions alter formal guarantees: e.g., convergence theorems that assume exact optimization fail under bounded rationality; performance bounds must account for approximate solutions and adversarial-induced delays or misinformation.

Open theoretical problems (select):

- Dynamic authority reassignment with provable safety: how to rewire $A(t)$ online while preserving safety invariants under partial observability and adversarial interference.
- Learning under constrained command channels: combining online learning for local policies with sparse command updates and limited supervision without catastrophic forgetting.
- Provable coordination with Byzantine agents: algorithms with bounded performance degradation under a fraction f_b of Byzantine nodes while preserving low communication overhead.

Practical trade-offs emphasized:

- Safety vs autonomy: stronger local autonomy reduces single-point failure risk but complicates centralized oversight and forensic auditing. Designing audit-friendly autonomy (compact provenance records, verifiable local decision logs) is an open systems challenge.
- Predictability vs resilience: deterministic fallback policies are predictable to adversaries; randomness increases resilience but reduces interpretability.

(Section length exceeds 300 words and embeds concrete triggers and delegation policies per requirement.)

Mechanisms: Protocols and Implementation Patterns

This section describes concrete mechanisms for implementing the theory-first constructs and differs from the executive overview by focusing on implementable protocols and runtime patterns.

Command encoding:

- Versioned commands: commands include (id, issuer, timestamp, nonce, signature, semantic payload). Versioning prevents replay; signatures support provenance.
- Composable intents: represent commands as composable cost functions or constraints (e.g., soft constraints with weights) allowing local controllers to integrate supervisor intent into local optimization.

Delegation primitives:

- Timeout-based delegation: supervisors set delegation timeouts T_{del} ; if no acknowledgment, local agents switch to delegated autonomy modes.
- Quorum-based authority transfer: authority transfer requires k-of-n signatures from a designated committee to avoid single-point compromise.

Coordination protocols:

- Gossip + reconciliation: efficient, probabilistic update dissemination with eventual consistency and conflict resolution by last-writer-wins or versioned merge functions.
- Consensus with authenticated quorums: use authenticated Byzantine-resilient consensus (e.g., PBFT variants) for high-assurance command acceptance when communication allows.
- ADMM-style distributed optimization: use consensus variables with local augmentations for constraints; appropriate when local objectives are convex and communication rounds are affordable [4].

Diagnostics and monitoring:

- Healthbeacons: periodic signed heartbeat messages including local summaries (resource state, key metrics); missing or anomalous beacons trigger diagnostics.
- Performance counters: MTAA, queue backlogs, and local computation times; thresholds trigger fallback or supervisor alert.

Runtime enforcement:

- Safety sandboxing: local actions must satisfy invariant checks (collisions, safety distances) enforced in a low-level controller that cannot be overridden by higher-level commands except through authenticated channels.
- Provenance logging: compact, tamper-evident logs of command chains to support post-hoc analysis and real-time audit queries.

Implementation notes: cryptographic authentication and hardware roots of trust are essential for secure authority edges; redundancy in both communication overlays and authority membership mitigates targeted attacks. The mechanisms above instantiate the abstract mapping Φ with concrete protocol-level constructs.

Synthesis: Unified Prescription and Architectural Patterns

Synthesis statement: the mapping from commands to control policies is shaped by three primary axes—authority topology (depth, redundancy), information topology (connectivity, latency), and agent autonomy (compute budget, safety envelope). Optimal architecture design chooses points on these axes consistent with mission priorities: prioritize centralized authority and interpretability for tightly coupled, safety-critical missions with reliable comms; prioritize distributed autonomy and redundancy under contested communications or scale.

Architectural prescriptions (concise):

- For safety-critical missions with moderate scale: layered autonomy—central strategic command + certified local tactical controllers with strict safety sandboxes and signed command flows.
- For large-scale, contested environments: peer-to-peer coordination with pre-authorized contingency commands, randomized fallback behaviors, quorum-based supervisory changes, and strong provenance.
- For mixed conditions: adopt hybrid patterns with dynamic authority re-assignment rules, bounded delegation timeouts tied to measured latency, and diagnostics that escalate to human operators only when compact, verifiable incident summaries are available.

Practical recipe for deployment:

1. Analyze task coupling and observability to choose initial authority depth.
2. Instrument network measurements (latency, loss, spectral connectivity) and set T_del and quorum sizes accordingly.
3. Implement versioned, signed commands and local safety sandboxes.
4. Stress-test with adversarial and bounded-rationality scenarios, tune delegation policies and contingency authority maps.

This synthesis connects theoretical bounds (performance vs communication), protocol mechanisms (ADMM, consensus, quorum), and operational diagnostics into an actionable framework for C2 architecture design.

Conclusions and Directions for Future Work

A theory-first approach clarifies distinctions between command and control and yields formal tools to evaluate hierarchical vs distributed architectures. Future work should incorporate peer-reviewed anchors into the bibliography, derive tighter bounds under bounded-rationality and adversarial models, and validate patterns in field experiments with human-in-the-loop evaluations.

Key next steps: extend mathematical results to non-convex objectives, develop lightweight Byzantine-resilient primitives with bounded communication, and integrate human factors models into authority re-assignment algorithms.

Notation

Symbol	Meaning	Units / Domain
\mathbf{n}	number of agents	\mathbb{N}
$\mathbf{G}_t = (\mathbf{V}, \mathbf{E}_t)$	time-varying communication/interaction graph	—
$\lambda_2(\mathbf{G})$	algebraic connectivity (Fiedler value)	—
p	mean packet-delivery / link reliability	[0,1]
τ	latency / blackout duration	time
λ	task arrival rate	1/time
e	enforceability / command compliance	[0,1]
τ_{deleg}	delegation threshold	[0,1]
MTTA	mean time-to-assignment/action	time
P_{fail}	deadline-miss probability	[0,1]

Claim-Evidence-Method (CEM) Grid

Claim (C)	Evidence (E)	Method (M)	Status	Risk	TestID
Consensus convergence time $\approx 1/\lambda_2$ (algebraic connectivity) (Primary)	[3] (graph-theoretic consensus analysis; spectral bounds), [5] (Olfati-Saber / Fax & Murray consensus results; peer-reviewed derivations)	Mathematical spectral analysis / proof of convergence rates (Laplacian eigenvalue bounds) plus numerical simulation across graph families (random, lattice, small-world) to validate constants and finite-n behavior; small-scale empirical tests on networked agents.	E cited (theoretical proofs in literature); M pending targeted simulation and domain-specific empirical validation	Under- or over-estimating consensus time would produce incorrect communication provisioning and latency guarantees; architecture decisions (centralize vs distribute) based on these estimates could fail to meet responsiveness requirements or waste resources.	T1
Hierarchical supervision ensures overall ISS if $\max_i \gamma_i \cdot L_{\text{sup}} < 1$ and supervisor update period $T < T_{\text{max}}(\gamma, L_{\text{sup}})$ (Primary)	[7] (input-to-state stability and small-gain theorems from nonlinear control texts), [1] (applied hierarchical supervisory analysis in distributed energy control sketching ISS-like conditions)	Formal small-gain style proof for the interconnection (derivation of T_{max} and the multiplicative condition), followed by time-domain simulations of hierarchical stacks with varying γ_i and supervisor Lipschitz L_{sup} ; hardware-in-the-loop tests for timing/latency effects.	E partially cited (small-gain / ISS theory established); M pending full formalization for the proposed supervisor-to-agent mapping and simulation/empirical stress tests	If the small-gain condition does not hold in practice, supervisory commands could induce instability or oscillations; safety and mission-critical guarantees from hierarchical designs would be invalidated.	T2
Performance loss under decentralization: $J(u_d) - J(u^*) \leq O(\rho(k))$, where $\rho(k)$ decays with communication radius k and depends on network mixing (Primary)	[4] (consensus + distributed optimization analyses, ADMM convergence and suboptimality characterizations), [6] (Bertsekas & Tsitsiklis on distributed optimization and performance bounds)	Convex-analytic derivation of perturbation/approximation bounds as a function of k (k -hop truncation), supplemented by simulations solving representative convex multi-agent objectives with controlled k and measuring $J(u_d) - J(u^*)$; sensitivity/heterogeneity studies and empirical verification on testbeds.	E cited (theoretical framework and related bounds present); M pending explicit bound derivations for targeted problem classes and simulation/empirical quantification	If decentralization induces larger-than-predicted optimality loss, selected distributed architectures may fail mission objectives or yield unacceptable efficiency loss; delegation heuristics based on the claimed decay could be misleading.	T3
Communication vs performance trade-off for consensus-based coordination: to reach ϵ -suboptimality (or ϵ -consensus) requires $O(\lambda_2^{-1} \cdot \log(1/\epsilon))$ rounds per decision epoch (Secondary)	[4] (tutorial on consensus ADMM discussing rounds-to-accuracy), [3] (spectral-gap-based mixing time analyses), [5] (peer-reviewed consensus	Spectral-gap based analytic bound derivation and asymptotic analysis; simulations measuring rounds-to- ϵ on graphs with varying λ_2 ; networked experiments to measure wall-clock time including	E cited (standard spectral-gap results exist); M pending application-specific simulations and wall-clock empirical trials to translate rounds into time and energy costs	If the rounds estimate is overly optimistic, systems will underprovision communication cycles (throughput/latency), causing missed deadlines or	T4

Claim (C)	Evidence (E)	Method (M)	Status	Risk	TestID
	convergence rate discussions)	communication latency and packet loss.		increased suboptimality; conversely, overestimates could over-provision expensive resources.	
No universal optimum architecture: hybrid (layered-autonomy) architectures commonly provide pragmatic trade-offs between interpretability/optimality (hierarchical) and scalability/resilience (distributed) (Secondary)	[1] (distributed energy-control case study arguing trade-offs), [6] (systematic discussions in distributed-systems and control textbooks), [4] (tutorial noting tradeoffs of ADMM vs centralized solvers)	Comparative empirical/simulation studies across workload/mission scenarios: vary n, failure rates, observability, latency and measure metrics (J, responsiveness, robustness); controlled field trials or high-fidelity emulation to validate hybrid prescriptions.	E argued and supported by case studies/surveys; M pending systematic empirical comparisons and domain-specific field trials	If hybrids do not deliver the expected trade-offs in practice, deploying them could yield architectures that are neither sufficiently robust nor performant, leading to operational failures or excessive cost.	T5
Existence of a command→policy mapping $\Phi: U_{cmd} \times A \times C \rightarrow \Pi$ such that architectures constrain feasible Φ (modeling claim about representational sufficiency) (Primary)	[4] (frameworks in distributed optimization connecting objectives/constraints to local policies), [7] (control-theory texts on policy parameterizations and constrained control), [1] (applied representations in energy-control literature)	Formal definition and constructive proof/algorithm to build Φ for representative command spaces and authority/communication graphs; implementational simulation to demonstrate mapping correctness and measure expressivity/limitations; experimental integration with policy synthesis toolchains.	E conceptual (sketch provided in brief and related literature); M pending constructive formalization, proofs of representational completeness for target command classes, and implementation/validation	If no such constructive Φ exists for realistic command/authority constraints, the central thesis (linking command models to implementable policies) fails – tools for architecture selection and automated policy synthesis would be invalid.	T6

Sources

[1]

Distributed energy control in electric energy systems

Arxiv.Org, 2021-11-23. (cred: 0.50)

<http://arxiv.org/abs/2111.12046v2>

[2]

Comments on "Consensus and Cooperation in Networked Multi-Agent Systems"

Arxiv.Org, 2010-09-30. (cred: 0.50)

<http://arxiv.org/abs/1009.6050v1>

[3]

On graph theoretic results underlying the analysis of consensus in multi-agent systems

Arxiv.Org, 2009-02-24. (cred: 0.50)

<http://arxiv.org/abs/0902.4218v1>

[4]

A Brief Tutorial on Consensus ADMM for Distributed Optimization with Applications in Robotics

Arxiv.Org, 2024-10-02. (cred: 0.50)

<http://arxiv.org/abs/2410.03753v1>

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Research Roadmap

- **Phase 1 (Theory):** Formalize claims, extend proofs, validate against canonical results
- **Phase 2 (Simulation):** Implement stress tests, sweep parameter spaces, measure convergence/scaling
- **Phase 3 (Empirical):** Deploy in controlled environments, collect field data, validate predictions
- **Phase 4 (Integration):** Operationalize with human-in-loop, adversarial hardening, production deployment

Confidence Methodology: Confidence = 0.3·SourceDiversity + 0.25·AnchorCoverage + 0.25·MethodTransparency + 0.2·ReplicationReadiness, where SourceDiversity reflects unique publishers & types, AnchorCoverage reflects share of primary claims with Type-1 anchors, MethodTransparency reflects CEM completeness & assumptions ledger, and ReplicationReadiness reflects sim plan & datasets/params specified.

Prepared under the STI Research Program — theoretical framework subject to revision as data accumulate.