Name: Hassaan_Ul_Haq
Byte Wise Limited_Task 9

· **Introduction to Statistics and Probability**

Statistics and Probability from the backbone of data analysis, allowing us to make inferences and Predictions based on data. Here, we will cover some fundamental concepts including measures of central tendency (mean, median, mode), common Probability distributions (normal, binomial, Poisson, and uniform distributions), and basic Probability concepts.

· **Measures of Central Tendency**

**Mean:** The average of a set of numbers, calculated by summing all the numbers and dividing by the count of numbers.

$$\text{Mean } (\mu) = \frac{\sum_{i=1}^{n} x_i}{n}$$

**Median:** The middle value in a dataset when the numbers are arranged in ascending or descending order. If the count of numbers is even, the median is the average of the two middle numbers.

**Mode:** The value that appears most frequently in a dataset.

**Common Probability Distributions**

**Normal Distribution:** A continous Probability distribution characterized by a symmetric, bell-shaped curve. It is defined by its mean ($\mu$) and standard deviation ($\sigma$).

- Probability Density Function:

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

**Binomial Distribution**: A discrete probability distribution representing the number of successes in a fixed number of independent Bernoulli trials (each with success probability $p$).

- Probability Mass Function

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where $\binom{n}{k}$ is the binomial coefficient.

**Poisson Distribution**: A discrete probability distribution expressing the probability of a given number of events occuring in a fixed interval of time or space.

- Probability Mass Function

$$P(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

where $\lambda$ is the average rate

of occurrence.

**Uniform Distribution:** A Probability distribution where all outcomes are equally likely.

- For a discrete uniform distribution over $n$ outcomes:

$$P(X=x)=\frac{1}{n} \text{ for } x\in\{1,2,\ldots,n\}$$

- For a continous uniform distribution over an interval $[a,b]$:

$$f(x)=\frac{1}{b-a} \text{ for } a\leq x\leq b$$

## Basic Probability Concepts

**Probability:** A measure of the likelihood of an event occuring, ranging from 0 to 1.

$$P(A)=\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

**Independent Events:** Two events A and B are independent if the occurence of one does not affect the occurence of the other.

$$P(A \cap B) = P(A) \cdot P(B)$$

**Conditional Probability:** The Probability of event A occuring given that event B has occured.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

**Bayes Theoram:** A formula to find the Probability of an event based on Prior knowledge of condition related to the event.

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

## Solving Problems

**Problem:**

Given the data set: [3, 1, 4, 1, 2, 5, 9]

Solution:

Mean:

$$\text{Mean} = \frac{3+1+4+1+2+5+9}{7} = \frac{25}{7} \approx 3.57$$

Median:

- Arrange the data in ascending order: [1, 1, 2, 3, 4, 5, 9]
- The number of data points (n) is 7, which is odd.
- Median = Middle value = 3

Mode:

- Mode is the most frequent value in the dataset.
- Mode = 1.

Binomial Distribution Problem

Problem:

Find the probability of getting exactly 3 heads in 5 flips of a fair coin.

Solution:

Formula:

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Substitute values:

$$P(x=3) = \binom{5}{3} (0.5)^3 (0.5)^2$$

Calculate the binomial coefficient:

$$\binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{5\times4\times3!}{3!\times2\times1} = 10$$

Calculate the Probability:

$$P(x=3) = 10 \times (0.5)^3 \times (0.5)^2$$

$$= 10 \times 0.125 \times 0.25$$

$$= 10 \times 0.031 = 0.3125$$

**Poisson Distribution Problem**

**Problem:** If the average number of emails received per hour is 4, what is the probability of receiving exactly 2 emails in an hour?

Solution:

Formula:

$$P(X=K) = \frac{\lambda^{K} e^{-\lambda}}{K!}$$

Substitute values:

$$P(X=2) = \frac{4^{2} e^{-4}}{2!}$$

Calculate:

$$4^2 = 16$$

$$e^{-4} \approx 0.0183$$

$$2! = 2 \times 1 = 2$$

$$P(X=2) = \frac{16 \times 0.0183}{2} = \frac{0.2928}{2} = 0.1464$$

$\approx 0.1465.$