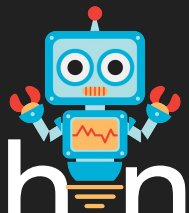
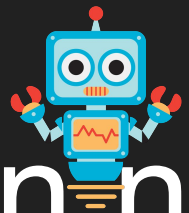


Mach ne Learn ng.



#Clustering

Eng. Hassan Elseoudy

Table of Contents.

Introduction

What is Clustering?

What is Clustering?

Clustering is the task of **dividing the population or data points** into a **number of groups** such that data points in the same groups are more **similar** to other data points in the same group than those in other groups.

What is Clustering?

the aim is to **make groups** with **similar attributes** and assign them into **clusters**.

Clustering Types

Hard Clustering: each data point either belongs to a cluster **completely or not**.
Examples : **K – means clustering**.

Soft Clustering: a probability or likelihood of that data point to be in those clusters is assigned.
Examples : **Fuzzy C-means** .

Supervised vs Unsupervised learning

Supervised learning:
Given (x_i , y_i), $i = 1, \ldots , n$.
$f(x) : X \rightarrow Y$.

- Categorical -> classification.
- Continuous -> regression.

Unsupervised learning: Given only (x_i), $i = 1, \ldots , n$, can we infer the underlying structure of X?

Unsupervised learning

Example
Iris Dataset

	Sepal length X_1	Sepal width X_2	Petal length X_3	Petal width X_4
$\mathbf{x}_1$	5.9	3.0	4.2	1.5
$\mathbf{x}_2$	6.9	3.1	4.9	1.5
$\mathbf{x}_3$	6.6	2.9	4.6	1.3
$\mathbf{x}_4$	4.6	3.2	1.4	0.2
$\mathbf{x}_5$	6.0	2.2	4.0	1.0
$\mathbf{x}_6$	4.7	3.2	1.3	0.2
$\mathbf{x}_7$	6.5	3.0	5.8	2.2
$\mathbf{x}_8$	5.8	2.7	5.1	1.9
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathbf{x}_{149}$	7.7	3.8	6.7	2.2
$\mathbf{x}_{150}$	5.1	3.4	1.5	0.2

Unsupervised learning

Why do unsupervised learning?

- **Raw data** is cheap.
- **Save** memory/computation.
- **Reduce noise** in high-dimensional data.
- Often a **pre-processing step** for supervised learning.

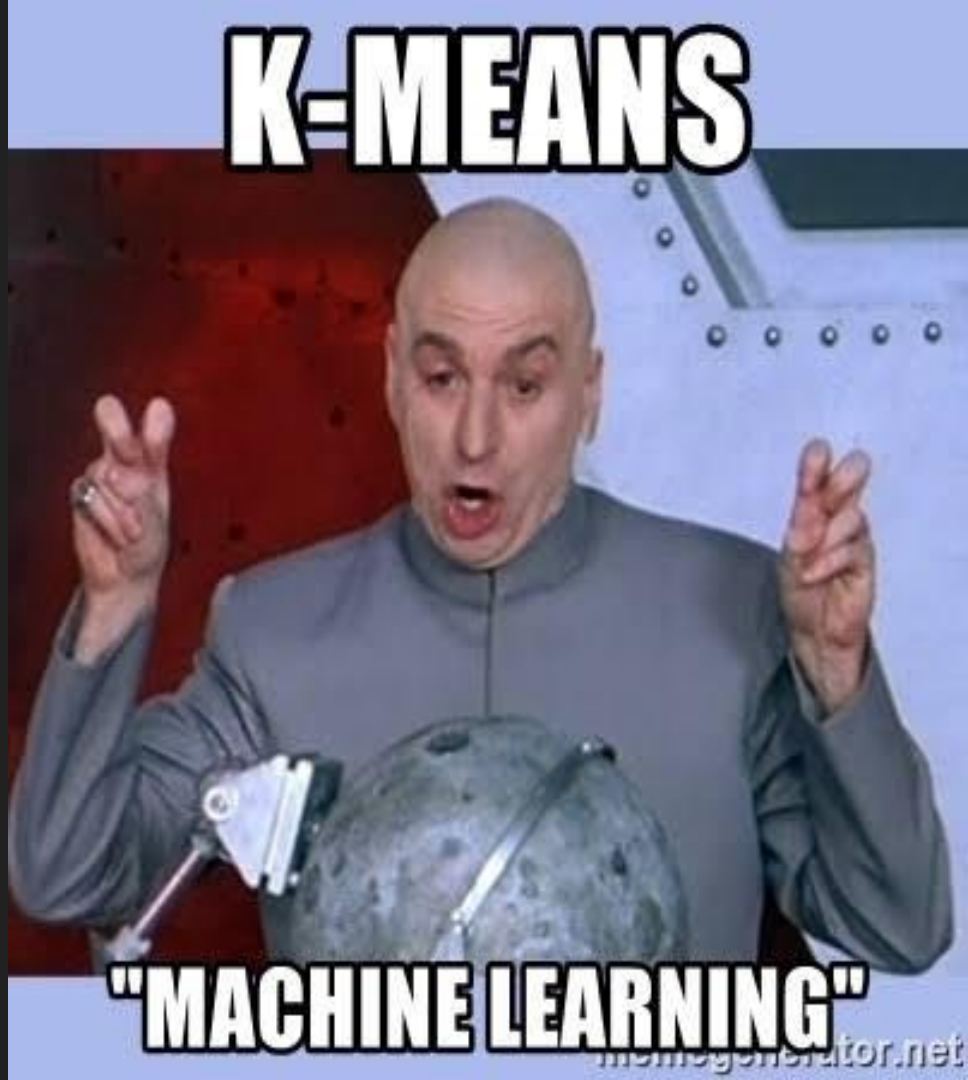
Common Distances

- Euclidean distance
  - *L2- Norm of the difference vector*
  - $\boldsymbol{\partial(a,b) = \|a-b\|_2 = \sqrt{\sum_{i=1}^{d}(a_i-b_i)^2}}$
- Manhattan Distance
  - *L1- Norm of the difference vector*
  - $\boldsymbol{\partial(a,b) = \|a-b\|_1 = \sum_{i=1}^{d} |a_i-b_i|}$
- L^p distance
  - *L^P norm of the difference vector*
  - $\boldsymbol{\partial(a,b) = \|a-b\|_p = \sqrt[p]{\sum_{i=1}^{d}(a_i-b_i)^p}}$

Question?

K-Means Clustering

What is K-Means?



K-Means

Introduction

- Uses Distance Function
- Uses Mean as representative. Called **centroid**
- It has a parameter **K** that you need to guess before clustering.
- Iterative **two-step** approach
  - Cluster Assignment.
  - Centroid Update

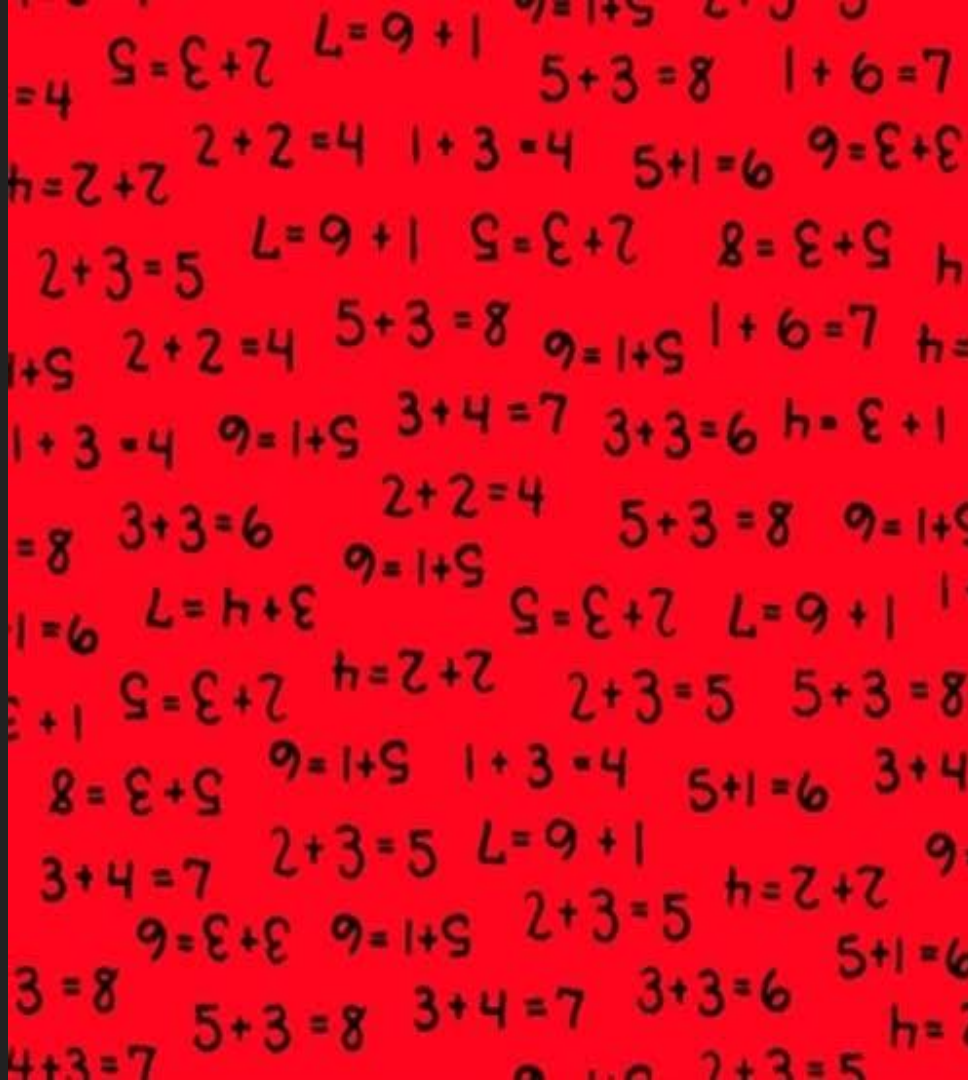
K-Means

Objective Function

Minimize the sum of the errors between samples in a cluster and their representative (**centroid**).

MATH ALERT!

Algorithm



ALGORITHM 13.1. K-means Algorithm

K-MEANS $(\mathbf{D}, k, \epsilon)$:

$t = 0$

Randomly initialize k centroids: $\boldsymbol{\mu}_1^t, \boldsymbol{\mu}_2^t, \ldots, \boldsymbol{\mu}_k^t \in \mathbb{R}^d$

repeat

  $t \leftarrow t + 1$

  $C_j \leftarrow \emptyset$ for all $j = 1, \cdots, k$

  `// Cluster Assignment Step`

  foreach $\mathbf{x}_j \in \mathbf{D}$ **do**

    $j^* \leftarrow \arg\min_i \left\{ \left\| \mathbf{x}_j - \boldsymbol{\mu}_i^{t-1} \right\|^2 \right\}$ `// Assign` $\mathbf{x}_j$ `to closest centroid`

    $C_{j^*} \leftarrow C_{j^*} \cup \{\mathbf{x}_j\}$

  `// Centroid Update Step`

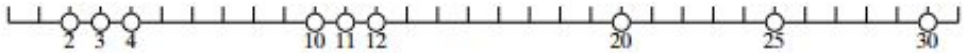
  foreach $i = 1$ *to* k **do**

    $\boldsymbol{\mu}_i^t \leftarrow \frac{1}{|C_i|} \sum_{\mathbf{x}_j \in C_i} \mathbf{x}_j$

until $\sum_{i=1}^k \left\| \boldsymbol{\mu}_i^t - \boldsymbol{\mu}_i^{t-1} \right\|^2 \leq \epsilon$

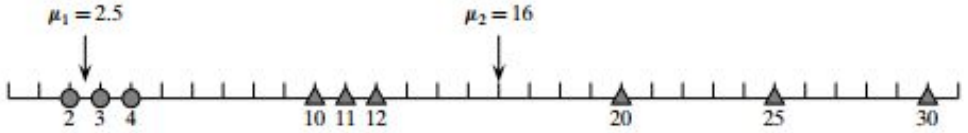
K-Means

Example



(a) Initial dataset

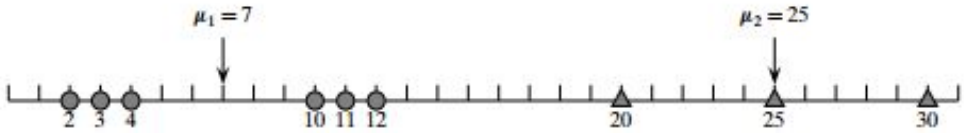
(b) Iteration: $t = 1$



(c) Iteration: $t = 2$

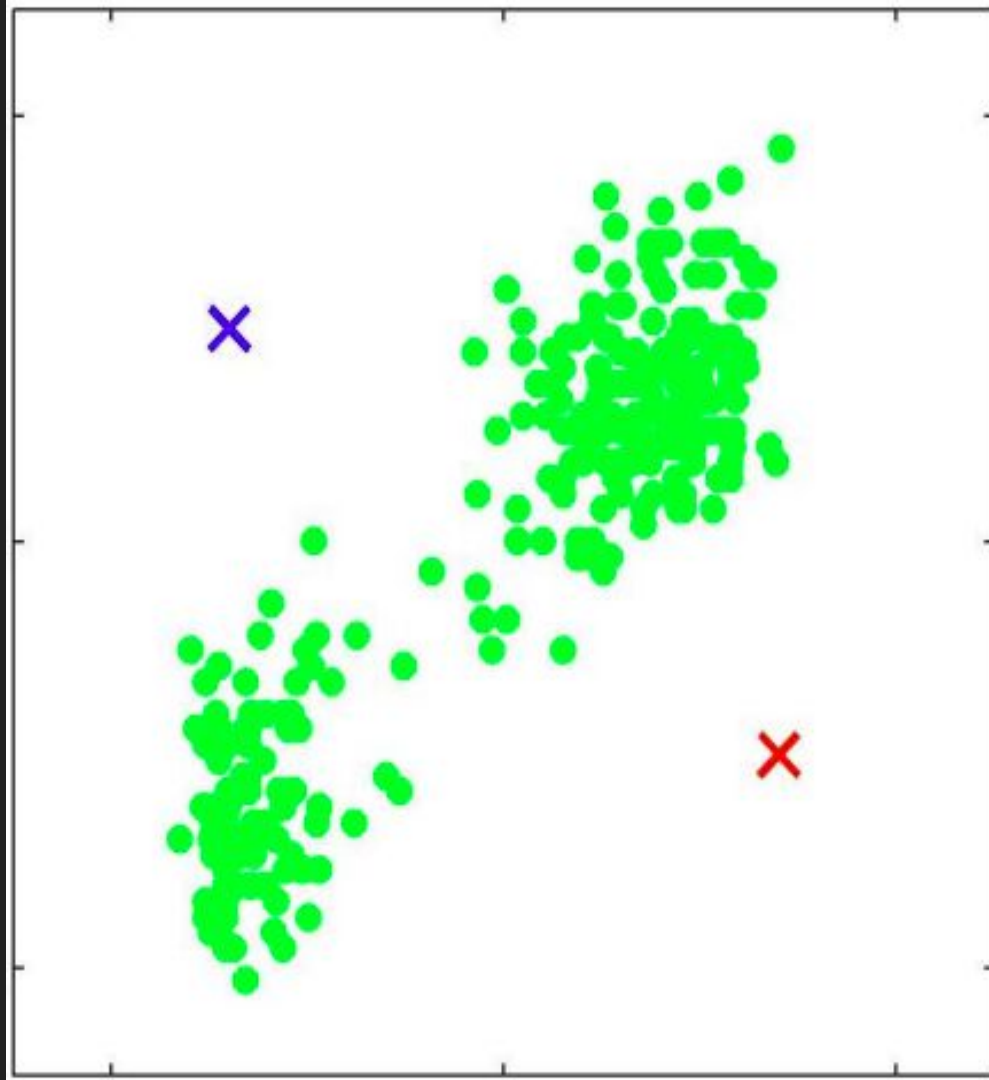
(d) Iteration: $t = 3$

(e) Iteration: $t = 4$



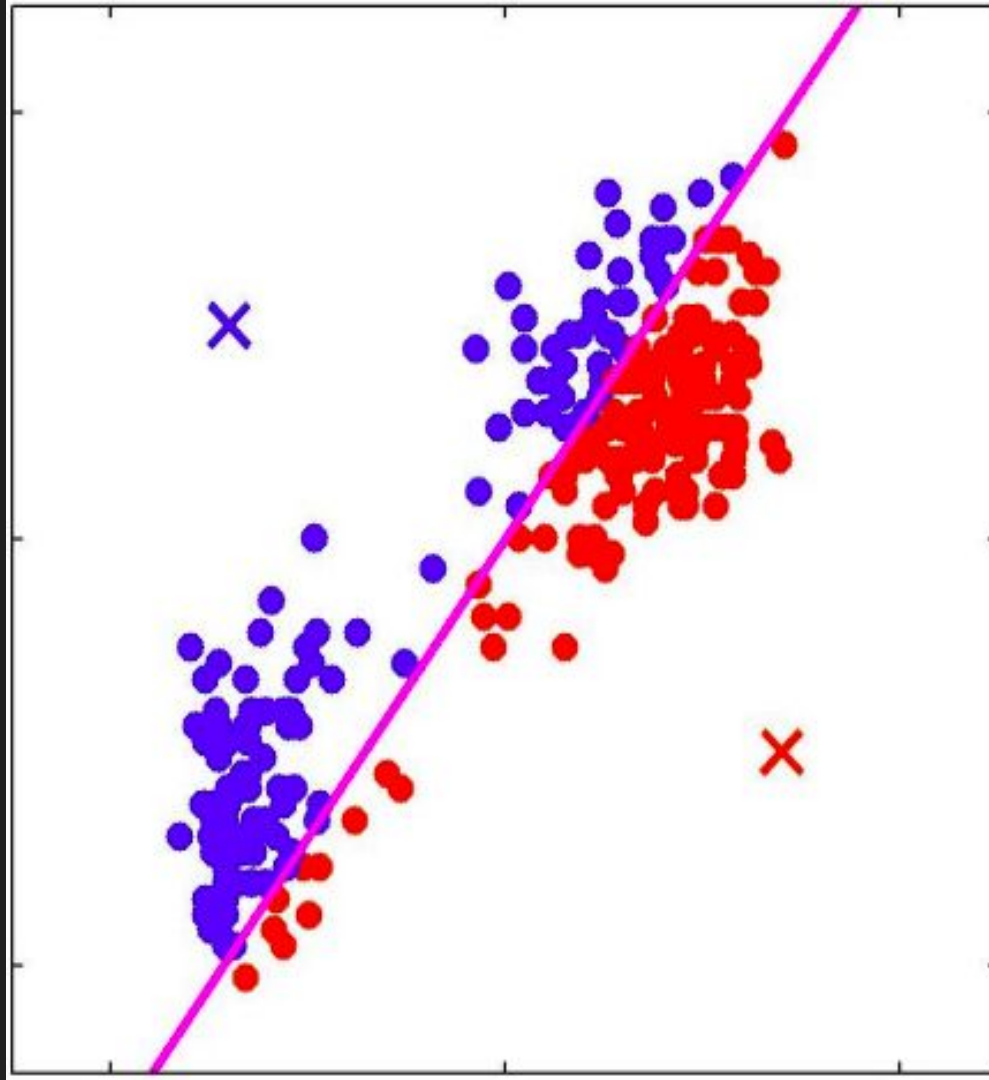
K-Means

Example



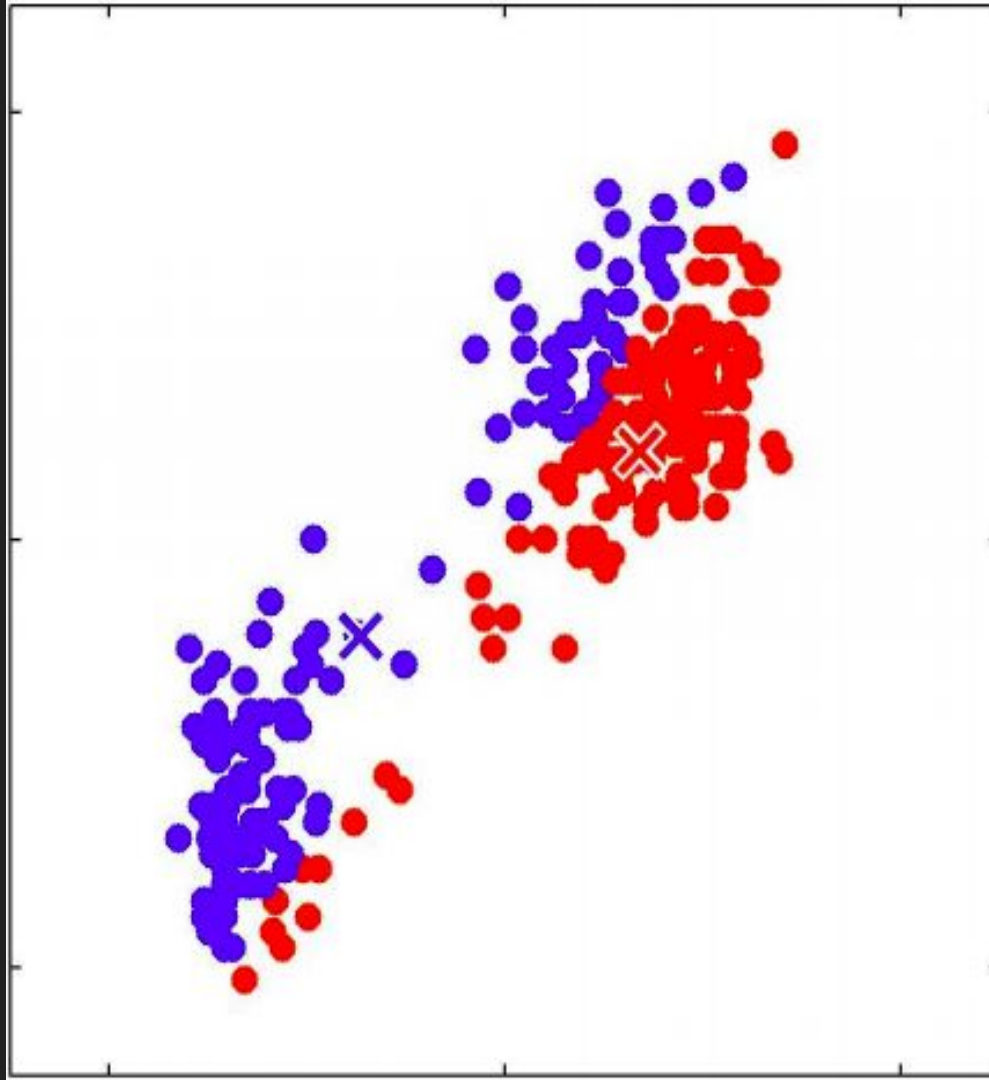
K-Means

Example



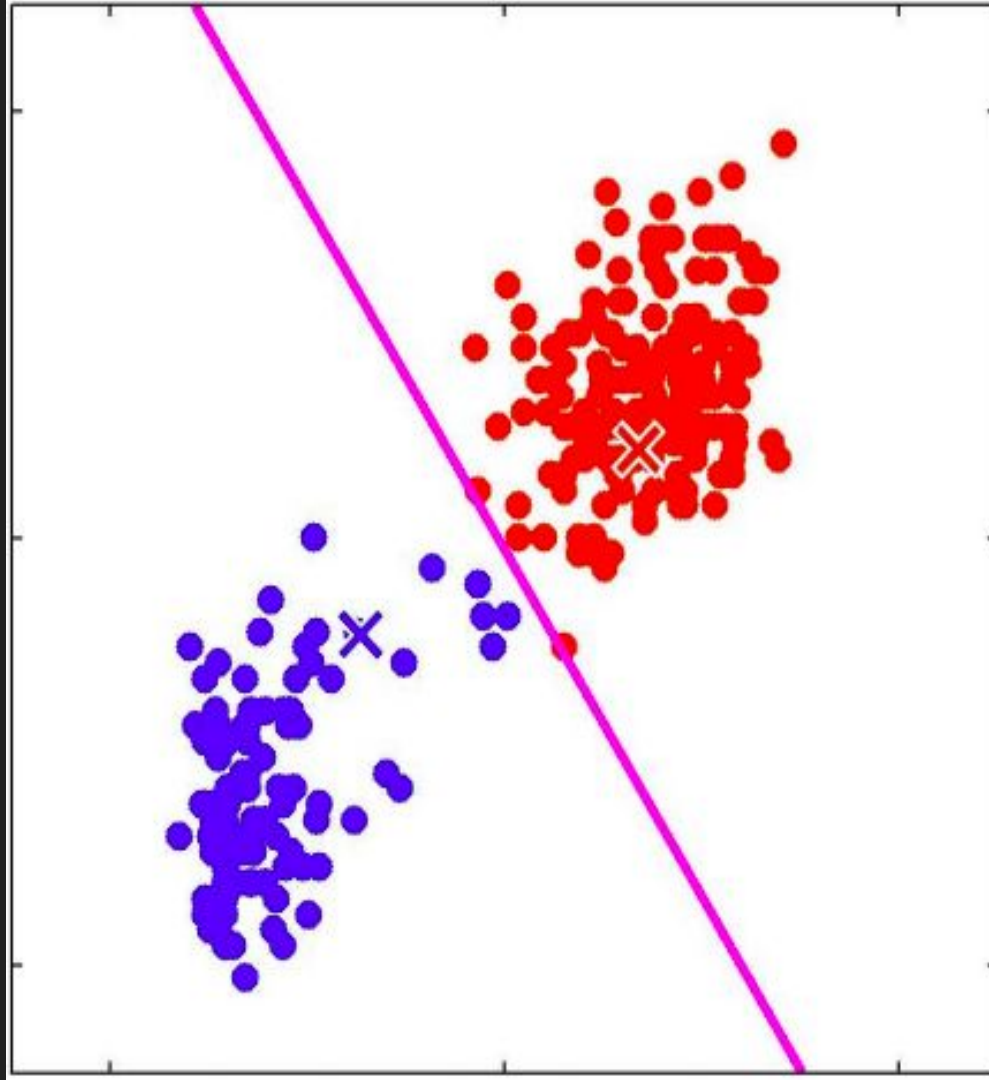
K-Means

Example



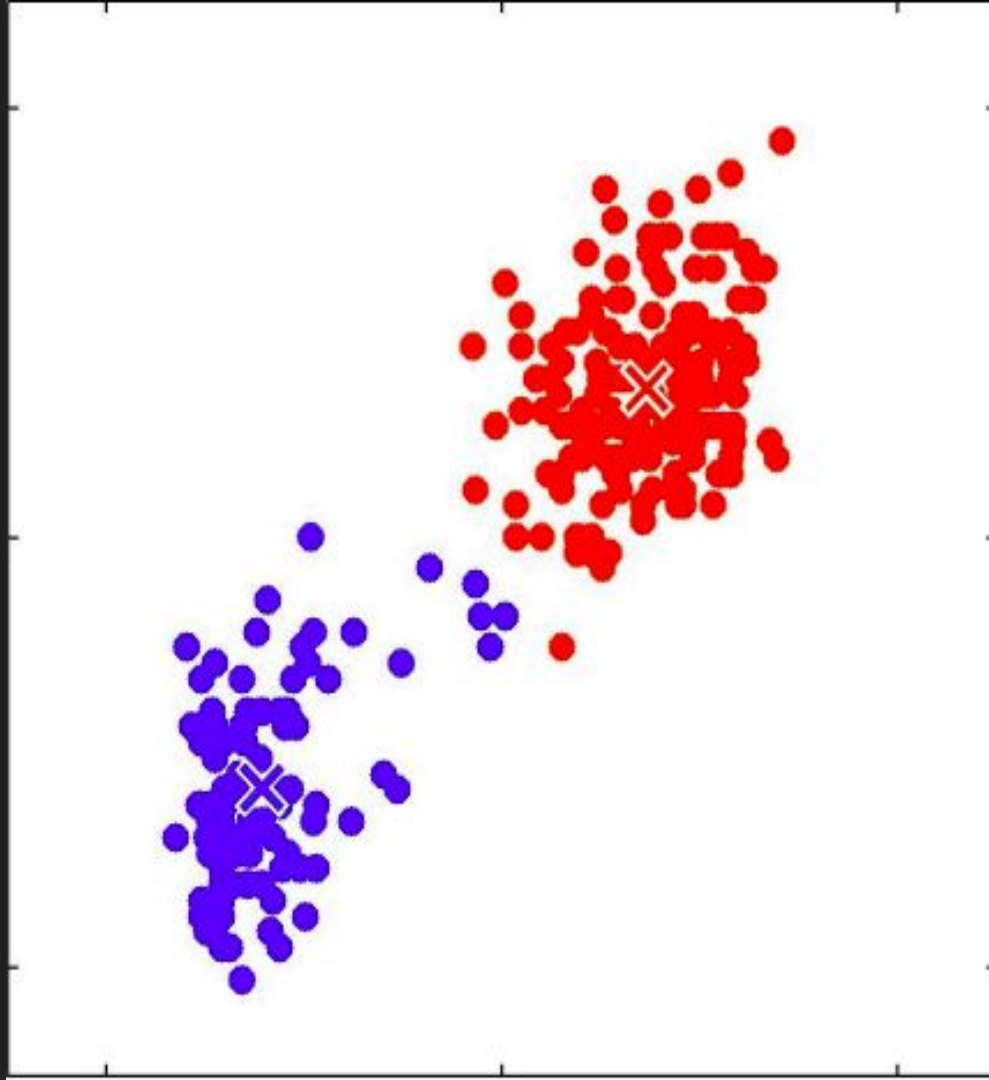
K-Means

Example



K-Means

Example



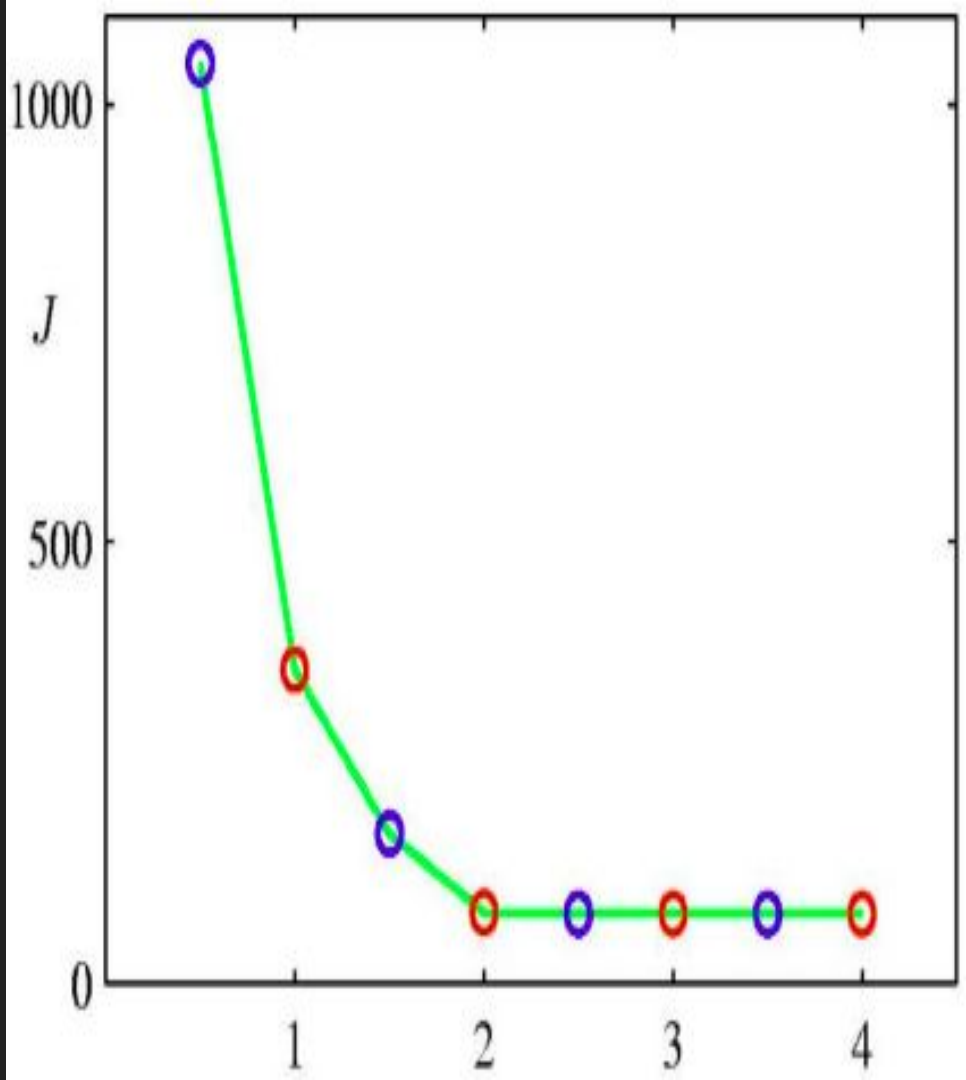
K-Means

Choosing K?

One way to select **K** for the **K-means** algorithm is to try different values of **K**, plot the K-means objective versus K, and look at the "**elbow-point**" in the plot.

K-Means

Choosing K?



K-Means

Limitations

- **Hard assignments** of data points to clusters.
- **Sensitive** to outliers.
- Works poorly on **non-convex** clusters.

K-Means

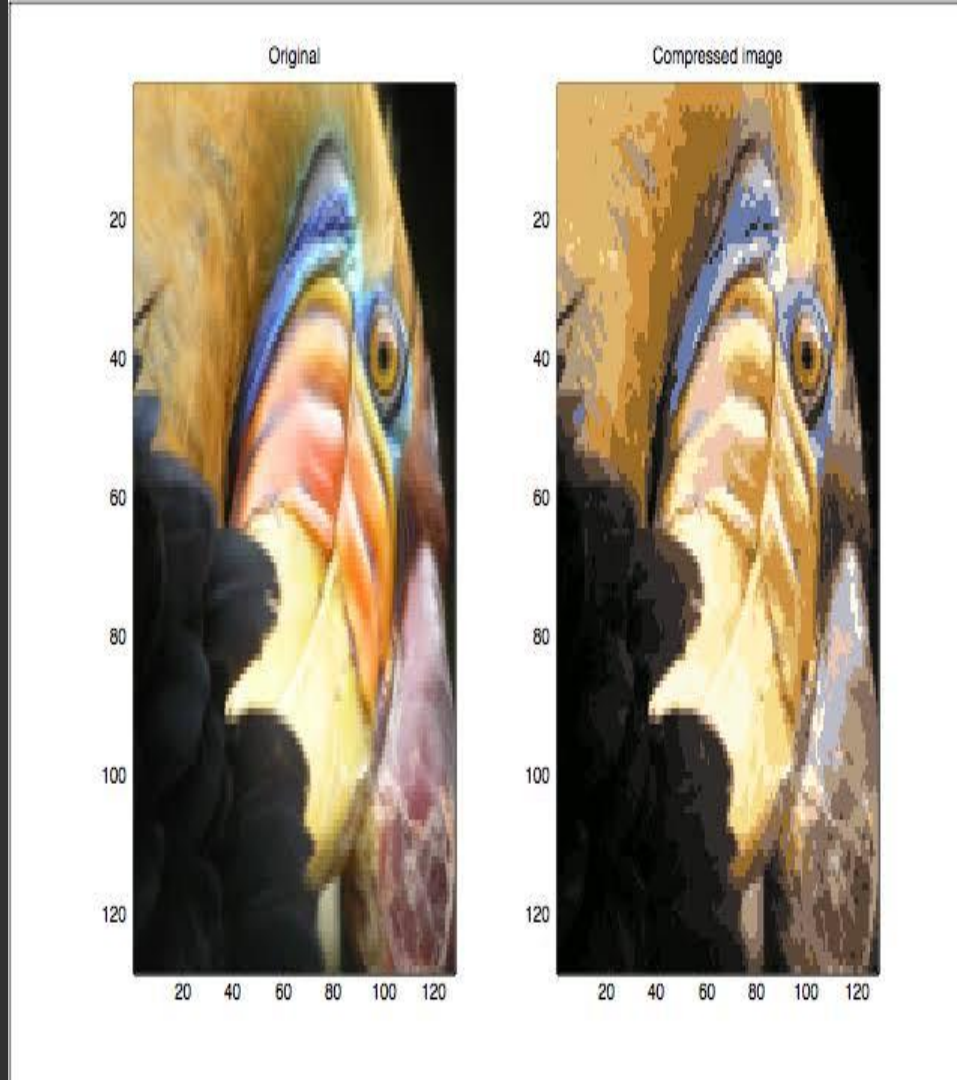
Document Classification

K-Means

Image Segmentation

K-Means

Image Compression



Question?

P U Z Z L E - T I M E

You have 15 L.E with you. You go to a shop and shopkeeper tells you price as 1 L.E per chocolate. He also tells you that you can get a chocolate in return of 3 wrappers. How many maximum chocolates you can eat? 22

Spectral Clustering

What is Spectral Clustering?

Spectral Clustering

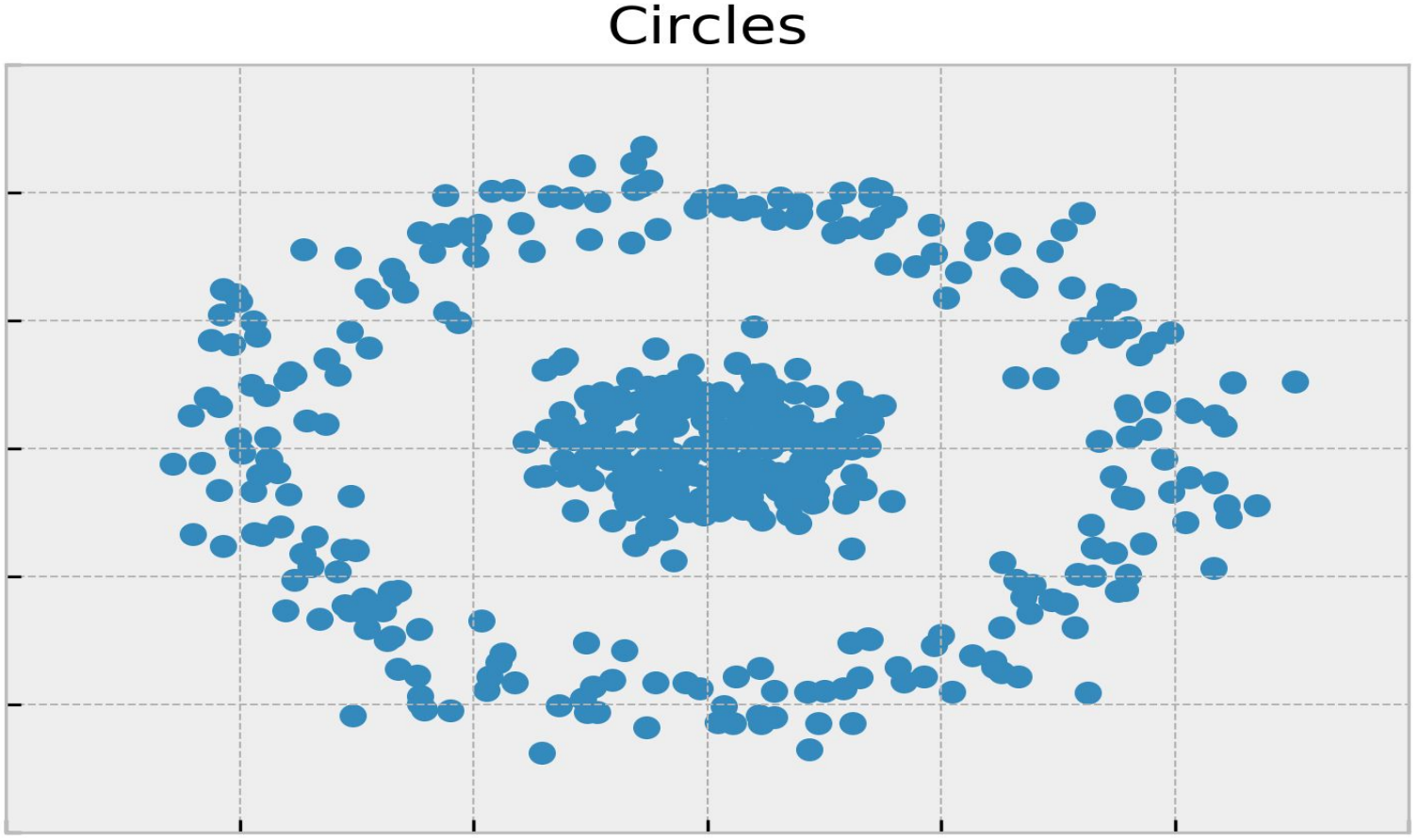
Spectral clustering is a technique with roots in **graph theory**, where the approach is used to identify **communities of nodes** in a graph based on the edges connecting them. The method is **flexible** and allows us to **cluster non graph data** as well.

Spectral Clustering

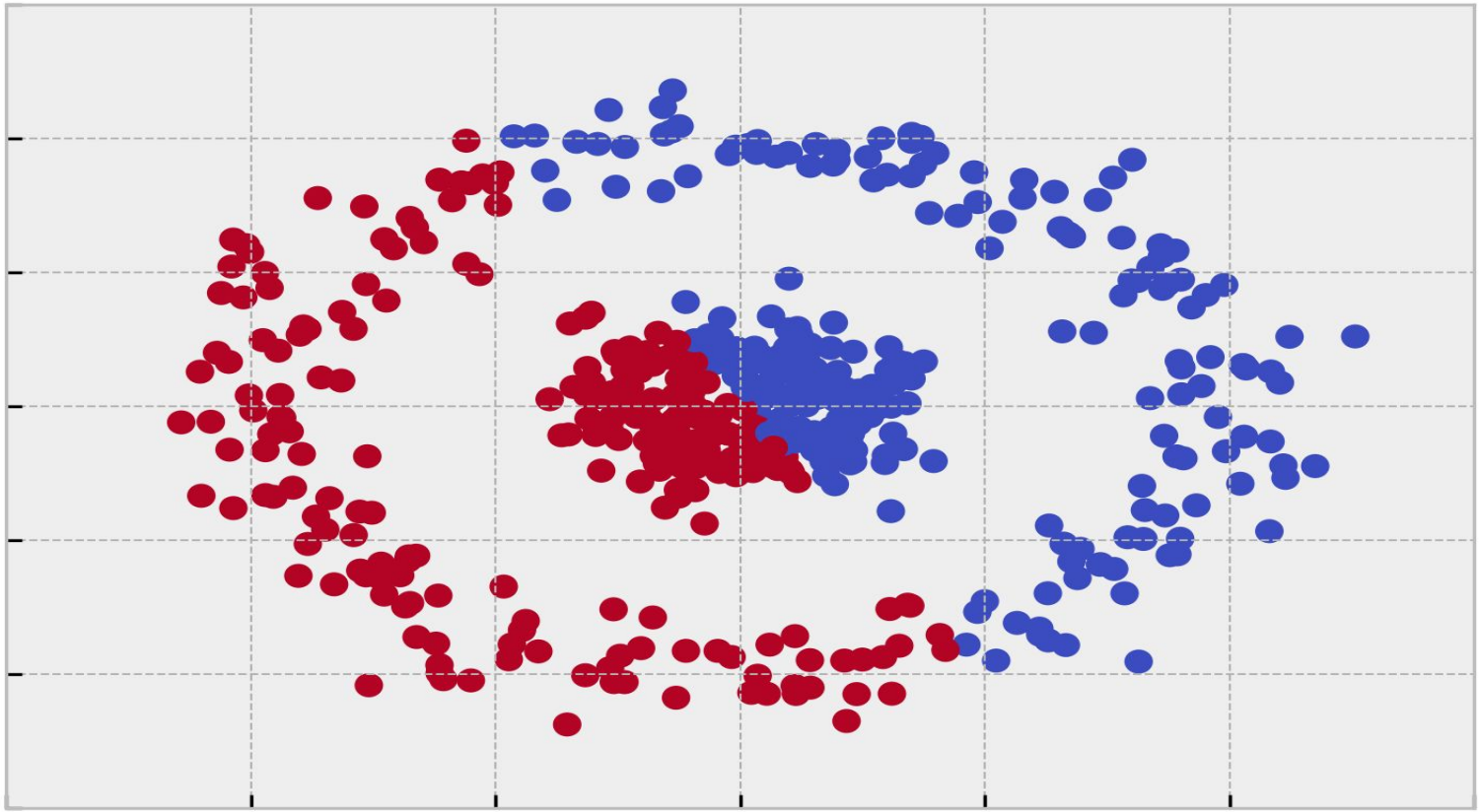
Spectral clustering uses information from the **eigenvalues** (spectrum) of special matrices built from the graph or the data set.

Spectral Clustering

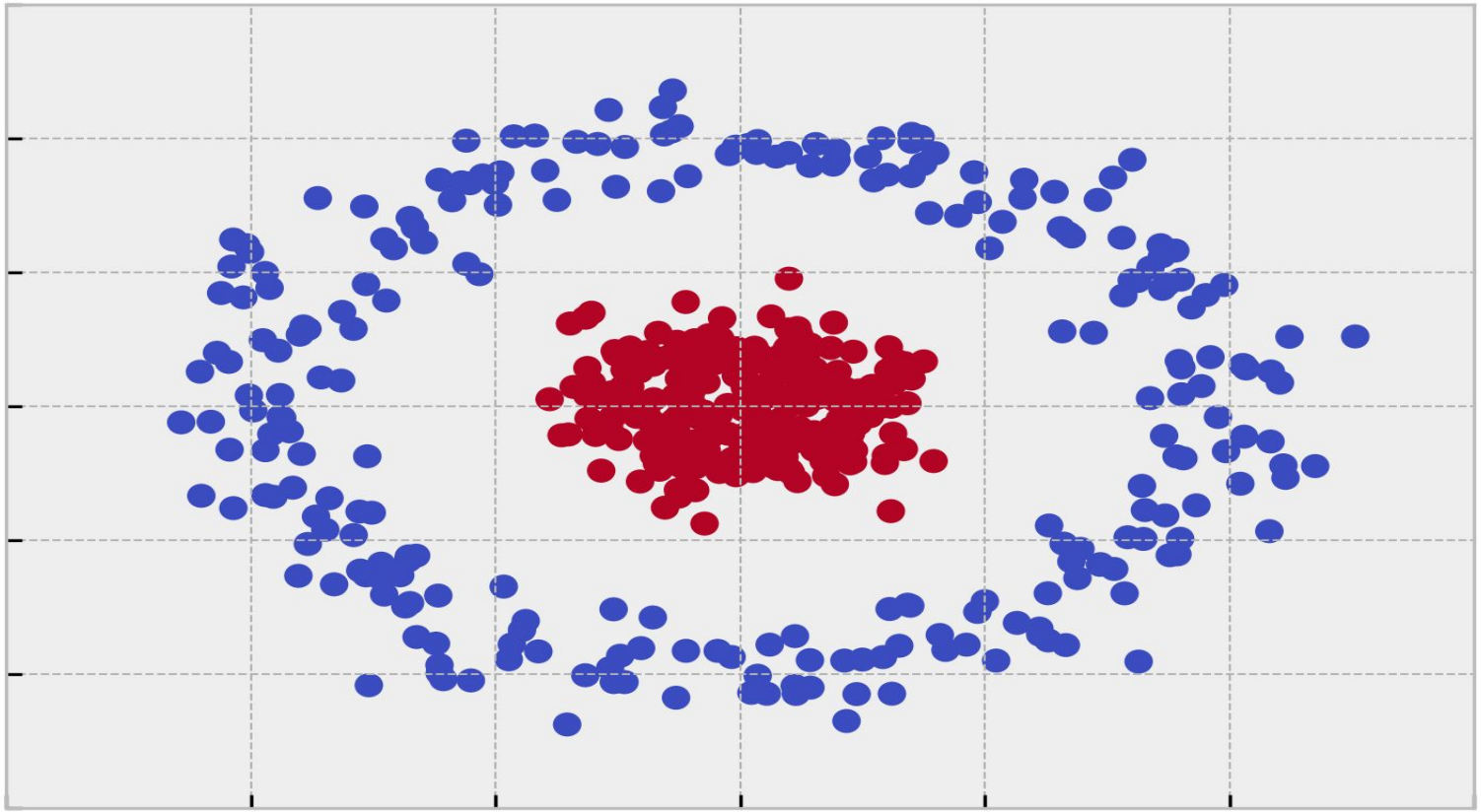
There are a many ways to treat our data as a **graph**. The easiest way is to construct a **k-nearest neighbors graph**. A k-nearest neighbors graph treats every data point as a node in a graph. An edge is then drawn from each node to its k nearest neighbors in the original space. Generally, the algorithm isn’t too sensitive of the choice of k. Smaller numbers like 5 or 10 usually work pretty well.


Circles

K-Means Circles



Spectral Circles



Question?

Clustering Validation

Clustering Validation

Goals

- **Evaluation**
  - *Quality*
- **Stability**
  - *Sensitivity to parameters used*
- **Tendency**
  - *Ability to find groups in data if exists*

Clustering Validation

Types

- **External**
  - *Expert specified knowledge.*
- **Internal**
  - *Measures derived from the data.*
- **Relative**
  - *Compare different clustering output , to set the best parameters.*

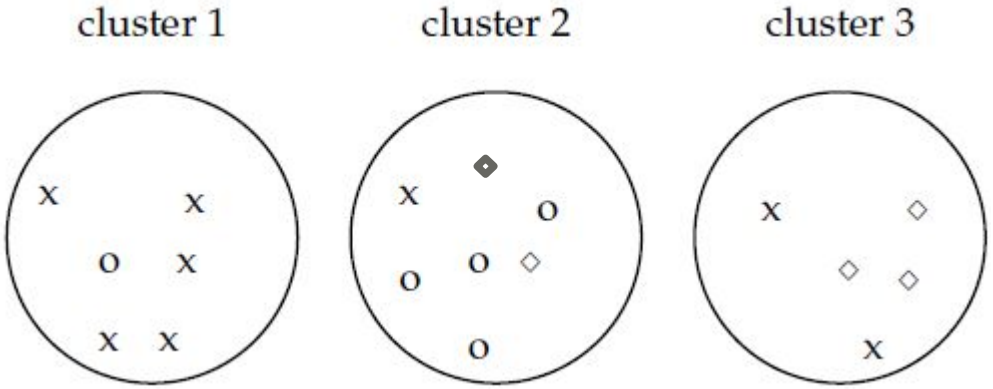
Clustering Validation

Purity

- Matching Based
  - *Purity*
    - $purity_i = \frac{1}{n_i} \max_j^k n_{ij}$, $\text{Purity} = \sum_{i=1}^{r} \frac{n_i}{n} purity_i$

      $purity_1 = \frac{1}{6}(5), purity_2 = \frac{1}{7}(4), purity_3 = \frac{1}{5}(3),$

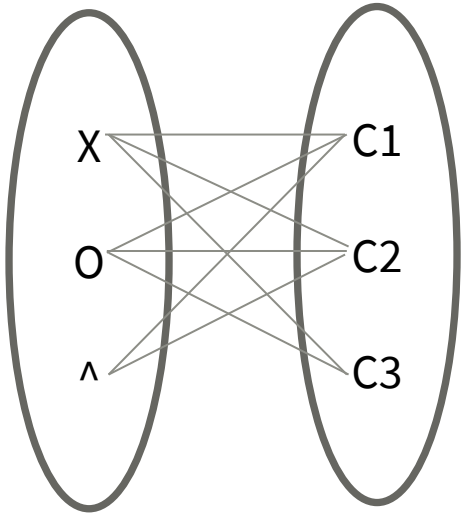
      $Purity = \frac{6}{18}\left(\frac{5}{6}\right) + \frac{7}{18}\left(\frac{4}{7}\right) + \frac{5}{18}\left(\frac{3}{5}\right) = \frac{12}{18}$

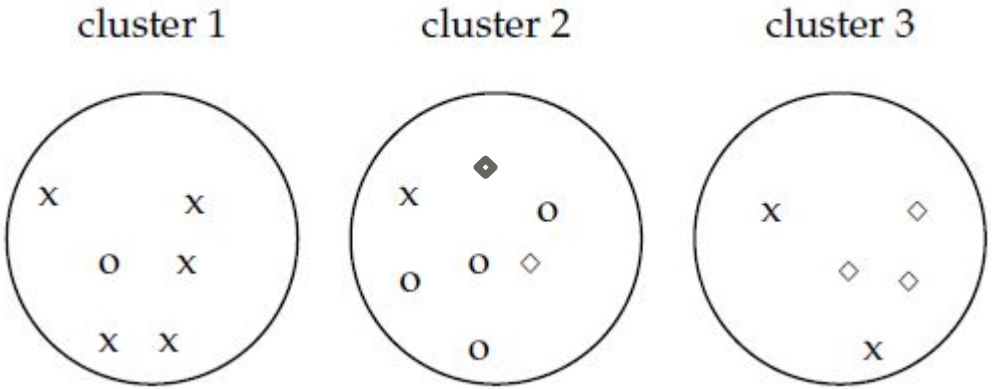


Clustering Validation

Maximum Matching

	x	O	^
C1	5	1	0
C2	1	4	2
C3	2	0	3



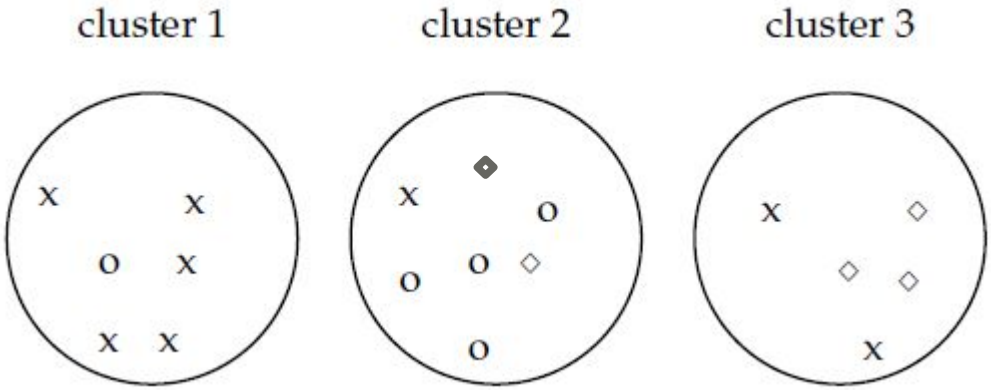


Clustering Validation

F-Measure

- Matching Based
  - *F-measure*
    - For every cluster compute
      - $prec_i = purity_i$
      - $rec_i = \frac{n_{ij_i}}{|Tj_i|}$
      - $F_i = \frac{2*prec_i*rec_i}{prec_i+rec_i}$
      - $F = \frac{1}{r}\sum_{i=1}^{r} F_i$

C1	5/6	5/8	0.714
C2	4/7	4/5	0.666
C3	3/5	3/5	0.666
	F		0.684



P U Z Z L E - T I M E

Know Average Salary without Disclosing Individual Salaries?

Question?

Python (K-Means)

Thank You <3