**Why Zero Padding Doesn’t Increase Frequency Resolution**

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**1. INTRODUCTION**

Frequency resolution measures the ability to discern nearby frequencies. When working with the DFT, bin spacing and frequency resolution are often used interchangeably. But this equivalence can be misleading.

Looking at this equation naively, we might conclude that increasing the number of samples will always increase our resolution, allowing us to resolve frequency components that are nearby. And so, instead of increasing the number of data points measured by a sampling device, we are compelled to zero-pad a signal afterward to increase artificially.

In this case study, we will observe how zero-padding does not represent an increase in true spectral resolution but only offers extra graphical detail through interpolation.

To make the difference clear, we study a simple test case: a sum of two pure sinusoids whose frequencies are only 50 Hz apart.

**2. SET UP**

Consider a signal composed of two similar frequencies , and .

*Time domain plot of .*

A blue line graph with white text

Description automatically generated

The frequency transform of is as follows.

*Plot of the frequency spectrum of .*

A graph of an electrical waveform

Description automatically generated with medium confidence

In the plot above, we see two impulses corresponding to the frequencies present in

An implicit assumption of the spectrum is that is infinite, but in real applications, we can only sample a finite length of a signal. This is mathematically represented by multiplying the signal with the rectangular window function, which takes a snapshot of the signal in time of duration *T*.

*The definition of the rectangular window*.

Performing with produces the following plot.

*Time domain plot of ;*

A graph of a windowed length

Description automatically generated

We see that only the information between the time interval of -5 ms and +5 ms is saved—reflecting the capture of only a section (10 ms) of an incoming signal. We can now look at in the frequency domain.

Given the convolution theorem, we see that the effect of performing in the time domain results in a spectrum convolved with a sinc function (the frequency dual of the rect function).

Compared to the spectrum of the spectrum of replaces each of the impulses with a sinc function—an oscillating waveform with side lobes— centered at the frequencies and .

*Spectrum ;*

A graph with a blue line

Description automatically generated

Because the two frequencies are similar, it is difficult to distinguish the two sincs; so, the plot below decomposes the spectrum into the individual sincs components.

*Decomposed spectrum ;*

A graph of a function

Description automatically generated

Again, in real applications, we observe the frequency spectrum in a discrete form (DFT). This can be mathematically represented as the spectrum multiplied by an impulse train, or Dirac comb function.

*The definition of the Dirac comb function.*

Where defines the spacing between each of the discrete points.

In the plot below, .

*Sampling of spectrum ;* .

A graph of a number of waves

Description automatically generated with medium confidence

We see the smooth continuous function replaced by a series of impulses with an envelope that closely resembles the original spectrum. This is how a DFT would present the frequency information.

To observe how this affects the signal in the time domain, we can apply the convolution theorem again. The result is a time domain signal that repeats at an interval of (aliasing).

*Aliased plot of ;*

A graph of blue lines

Description automatically generated

Interestingly, we also observe empty space (zero-padding) in each cycle. The duration of this space is the time difference between and .

The amount of zero-padding is dependent on our chosen, and so we can now evaluate whether increasing the amount of zero-padding (by decreasing will resolve the two frequencies in the signal.

**3. OBSERVATIONS**

Here we decrease the bin spacing to

*Aliased plot of ;*

A graph with blue lines

Description automatically generated

We see that we have increased the zero-padding, but when we observe the frequency dual of the signal, we still cannot resolve the two frequencies.

*Sampling of spectrum ;* .

A graph with a line and numbers

Description automatically generated

In fact, we will never be able to resolve the two frequencies. As we converge on the continuous spectrum we started with, where the sinc lobes of the two components still overlap and are difficult to distinguish. It increases the smoothness of the plot by sampling more of the ideal continuous spectrum, but it cannot improve the underlying frequency resolution.

This is the essence of what zero-padding does. It can only interpolate points between the frequencies. It cannot increase the resolution of the signal because no new information is created. What was already present is just revealed by a denser sampling of the spectrum—it will only make the plot prettier and approach a more continuous form.

So, in order to increase the frequency resolution, we must acquire more information about the signal. This is done by increasing the snapshot length *T,* thus capturing more data points.

We can observe the effect of increasing (increasing our observation window and thereby recording more samples).

Performing with produces the following plot.

*Time domain plot of with*

A graph showing a number of blue lines

Description automatically generated

Then we observe the spectrum with .

*spectrum ;*

A graph of a frequency

Description automatically generated

Immediately we are able to able to resolve the two frequencies. We can still decompose it into its individual sincs to observe the lobe spacing.

*Decomposed spectrum ;*

A graph of a sine wave

Description automatically generated

From the plot above the sincs have a lobe spacing of about .

Turns out, the lobe spacing is inversely proportional to the length . Minimizing the lobe spacing, thus maximizing is what prevents the two lobes from interfering and producing a spectrum that obscures the peaks.

**4. CONCLUSION**

Zero-padding reduces the DFT bin spacing, but it does not shrink the main-lobe width set by the observation window length T, which is what governs the ability to resolve nearby frequencies. Extra zeros only interpolate between existing spectral samples; only a longer record of actual data points provides gains in frequency resolution.