Fall 2022-2023

Operations Research
Solution to Assignment #2 - Graphical Method

1)

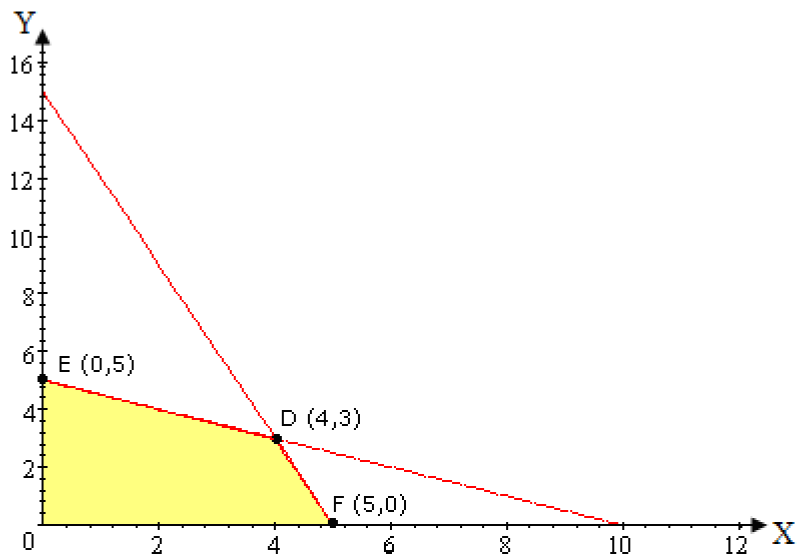
	X	Y	Available hours
A	12	4	60
B	4	8	40
	\$10	\$6	

Maximize: $P = 10X + 6Y$

Subject to: $12X + 4Y \leq 60$

$4X + 8Y \leq 40$

All variables ≥ 0



At E (0, 5) : $P = (10)(0) + (6)(5) = 30$
At F (5, 0) : $P = (10)(5) + (6)(0) = 50$
At D (4, 3) : $P = (10)(4) + (6)(3) = 58$

$\Rightarrow$ The optimal solution is: P = \$58 at point D.
T = 4
C = 3

2)

	X	Y	Available
E1	0.2	0.1	48
E2	0.3	0.1	30
E3M	0.5	0.8	60
	\$9	\$6	

Maximize: $P = 9X + 6Y$

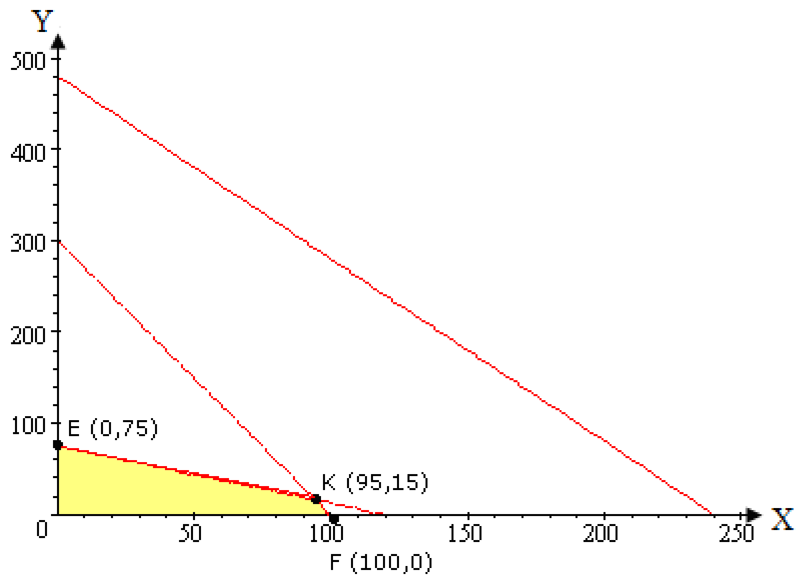
Subject to:

$0.2X + 0.1Y \leq 48$

$0.3X + 0.1Y \leq 30$

$0.5X + 0.8Y \leq 60$

All variables ≥ 0



At E (0, 75) : P = (9) (0) + (6) (75) = 450

At F (100, 0) : P = (9) (100) + (6) (0) = 900

At K (95, 15) : P = (9) (95) + (6) (15) = 945

⇒ The optimal solution is: P = \$945 at point K

X= 95

Y = 15

3)

	D	C	Available amount
K	8	6	24
P	10	4	20
M	6	12	24
	\$0.80	\$1	

Minimize: $T = 0.8\,D + 1\,C$

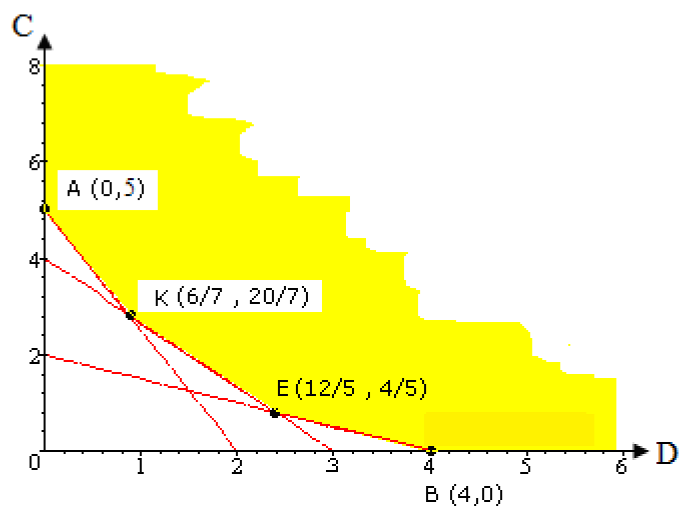
Subject to:

$8\,D + 6\,C \geq 24$

$10\,D + 4\,C \geq 20$

$6\,D + 12\,C \geq 24$

All variables ≥ 0



At A (0, 5) : $T = (0.8)(0) + (1)(5) = 5$

At B (4, 0) : $T = (0.8)(4) + (1)(0) = 3.2$

At E (2.4, 0.8) : $T = (0.8)(2.4) + (1)(0.8) = 2.72$

At K (6/7, 20/7) : $T = (0.8)(6/7) + (1)(20/7) = 3.54$

$\Rightarrow$ The optimal solution is: T = \$2.72 at point E.

D = 2.4

C = 0.8

4)

	X	Y	Available resource
A	-1	2	4
B	3	2	14
C	1	-1	3
	\$3	\$2	

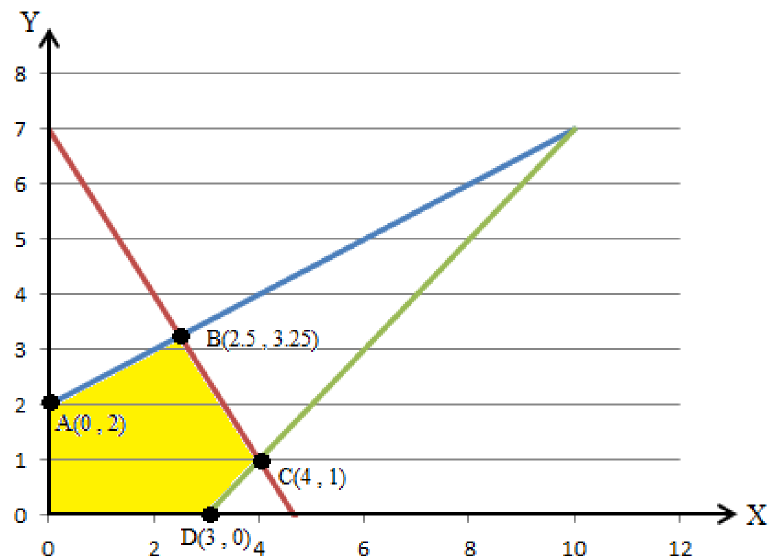
Maximize: $P = 3X + 2Y$

Subject to: $-X + 2Y \leq 4$

$3X + 2Y \leq 14$

$X - Y \leq 3$

All variables ≥ 0



At A (0, 2) : $P = (3)(0) + (2)(2) = 4$

At B (2.5, 3.25) : $P = (3)(2.5) + (2)(3.25) = 14$

At C (4, 1) : $P = (3)(4) + (2)(1) = 14$

At D (3, 0) : $P = (3)(3) + (2)(0) = 9$

$\Rightarrow$ The optimal solution is: P = \$14 at points B & C (Multiple solution).

Hence, the optimal solution is at any point on the line segment BC.

5. **Maximize**: $z = x + y$

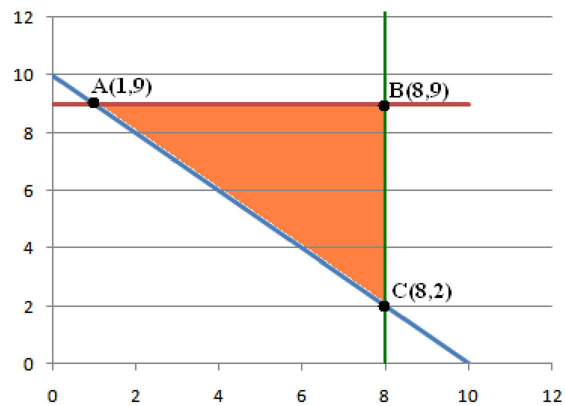
Subject to:

$$\begin{aligned} x + y &\geq 10 \\ x \quad &\leq 8 \\ y &\leq 9 \\ x , y &\geq 0 \end{aligned}$$

At A (1, 9) : z = 1 + 9 = 10
At B (8, 9) : Z = 8 + 9 = 17
At C (8, 2) : Z = 8 + 2 = 10

$\Rightarrow$ **The optimal solution is:**

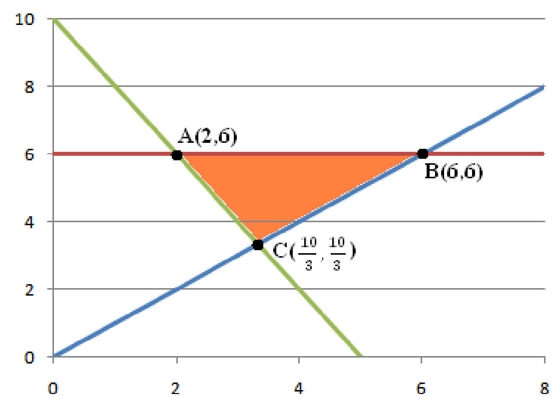
z = 17 at point B (8, 9).



6. **Maximize**: $z = 3x_1 + 4x_2$

Subject to:

$$\begin{aligned} x_1 - x_2 &\leq 0 \\ - x_2 &\geq - 6 \\ 2x_1 + x_2 &\geq 10 \\ x_1 , x_2 &\geq 0 \end{aligned}$$



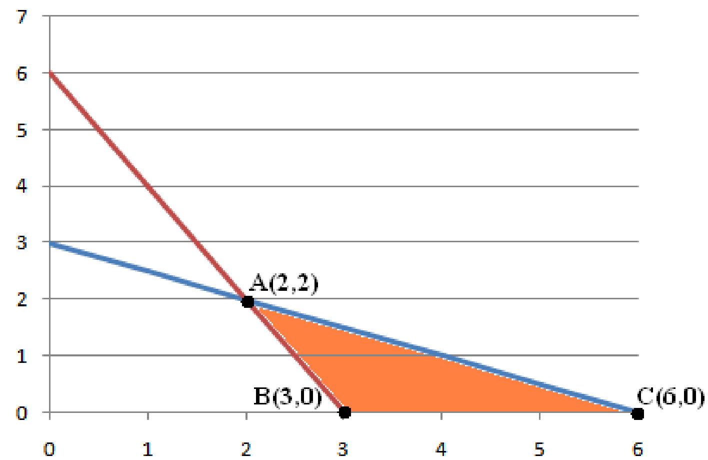
At A (2, 6) : z = (3) (2) + (4) (6) = 30
At B (6, 6) : z = (3) (6) + (4) (6) = 42
At C (10/3, 10/3): z = (3) (10/3) + (2) (10/3) = 50/3

$\Rightarrow$ **The optimal solution is:**

z = 42 at point B (6, 6).

7. **Maximize**: $z = 3x_1 + 2x_2$

Subject to:
$$x_1 + 2x_2 \leq 6$$
$$2x_1 + x_2 \geq 6$$
$$x_1,\ x_2 \geq 0$$



At A (2, 2) : z = (3) (2) + (2) (2) = 10

At B (3, 0) : z = (3) (3) + (2) (0) = 9

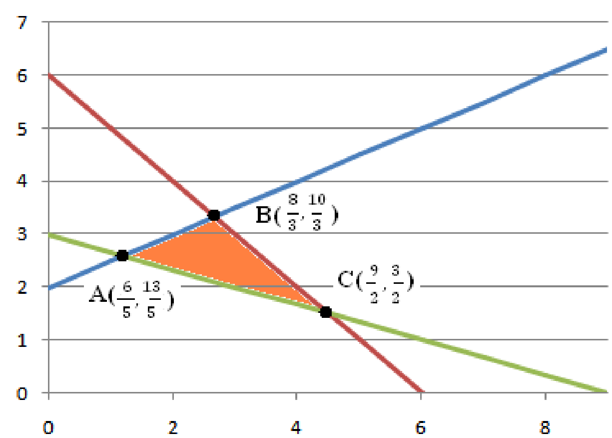
At C (6, 0) : z = (3) (6) + (2) (0) = 18

⇒ **The optimal solution is:**

z = 18 at point C (6, 0).

8. **Minimize** : $z = 2x_1 + 3x_2$

subject to:
$$-x_1 + 2x_2 \leq 4$$
$$x_1 + x_2 \leq 6$$
$$x_1 + 3x_2 \geq 9$$
$$x_1,\ x_2 \geq 0$$



At A (6/5, 13/5) : P = (2) (6/5) + (3) (13/5) = 51/5

At B (8/5, 10/5) : P = (2) (8/3) + (3) (10/3) = 46/3

At C (9/2, 3/2) : P = (2) (9/2) + (3) (3/2) = 27/2

⇒ **The optimal solution is:**

z = 51/5 at point A (6/5, 13/5).

$increases\ (decreases\ )\ the\ profit\ \ by$ \$1.47.