Introduction to Optimization Modeling

Operations Research can be defined as the use of quantitative methods to assist analysts and decision-makers in:

- **Designing**
- **Analyzing**
- **Improving the performance or operation of systems**

The systems being studied may be any kind of:

- **Financial systems**
- **Scientific or engineering systems**
- **or Industrial systems**

but regardless of the context, practically all such systems lend themselves to scrutiny within the systematic framework of the scientific method.

Definition

- Operations research, management science, or decision science

- Operations Research involves a variety of techniques that aim to
  - create mathematical models that describe real or theoretical systems
  - solve the models for optimal solutions to improve systems efficiency and support decision-making

- It is a scientific approach to making best decisions

- Usually under conditions requiring the allocation of limited resources

The Origin of OR

- British Scientists during WWII

- Optimal allocation of various war supplies
  - deployment of radars
  - management of convoy
  - bombing missions
  - anti-submarine operations

- Military Operations Research, later Operations Research

OR Modeling

- OR requires the use of models, which are mathematical representations of the actual systems

- Modeling: Describing a system at a high level of abstraction (or simplification), which ignores irrelevant details and only represent the relevant details

Success Stories

Organization	Problem	Annual Saving
Continental Airlines	Reassign crews to flights when schedule disruptions occur	$40 million

Organization	Problem	Annual Saving
Samsung	Reduce manufacturing times and inventory levels	$200 million

Organization	Problem	Annual Saving
P&G	Redesign the production and distribution system	$200 million

Organization	Problem	Annual Saving
HP	Product portfolio management	$180 million

Success Stories

Organization	Problem	Annual Saving
Taco Bell	Plan employee work schedules at restaurants	$13 million

Organization	Problem	Annual Saving
Sears	Vehicle routing and scheduling for home services and deliveries	$42 million

Organization	Problem	Annual Saving
Time Inc.	Management of magazine distribution channels	$3.5 million

Organization	Problem	Annual Saving
Merrill Lynch	Pricing analysis for providing financial services	$50 million

Definitions

- **Operations research** is a scientific approach to decision making that seeks to best design and operate a system.
  - Usually requires the allocation of scarce resources.

Definitions

- A **system** is an organization of interdependent components that work together to accomplish a goal.

Definitions

- A **mathematical model** is a simplified mathematical representation of a system or problem.

Definitions

- "A **mathematical programming model** is a mathematical decision model for planning (programming) decisions that optimize an objective function and satisfy limitations imposed by mathematical constraints."

Definitions

- **Optimization** is the process of finding the best decisions to make in solving a problem or arranging a system.

Mathematical Programming

- **Definition**
  - "A **mathematical programming model** is a mathematical decision model for planning (programming) decisions that optimize an objective function and satisfy limitations imposed by mathematical constraints."[1]
- **General symbolic model:**

Maximize (or minimize): $f(x_1, x_2 \ldots x_n)$ } **Objective**

Subject to:

$$\begin{aligned} g_1(x_1, x_2 \ldots x_n) &\ \{\leq, \geq, =\}\ b_1 \\ g_2(x_1, x_2 \ldots x_n) &\ \{\leq, \geq, =\}\ b_2 \\ &\vdots \\ g_m(x_1, x_2 \ldots x_n) &\ \{\leq, \geq, =\}\ b_m \end{aligned}$$

} **Constraints**

… where $x_1, x_2 \ldots x_n$ are the **decision variables**

Mathematical Programming (2)

- **Terminology:**
  - **Decision variables** are quantities you can control, which completely describe the set of decisions to be made.
  - **Constraints** are limitations on the values of the decision variables.
  - The **objective function** is a measure that can be used to rank alternative solutions (e.g., cost, production rate, travel time).
    - The goal is to either maximize or minimize its value.

Mathematical Programming (3)

- **Terminology:**
  - A **solution** is any combination of values for all decision variables.
  - A **feasible solution** is a solution that satisfies all of the constraints.
    - An **infeasible solution** doesn't satisfy some constraint(s).
  - An **optimal solution** is the best feasible solution.

Types of Mathematical Programming

- **Linear Programs (LP):** The objective and constraint functions are linear and the decision variables are continuous.

Linear Programming

- General symbolic form

Maximize: $z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$ } Objective

Subject to:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &\ \{\leq, \geq, =\}\ b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &\ \{\leq, \geq, =\}\ b_2 \\ \vdots & \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &\ \{\leq, \geq, =\}\ b_m \end{aligned}$$

} Constraints

$0 \leq x_j, \quad j = 1, \ldots, n$ } Bounds

…where a_{ij}, b_i, and c_i are the model **parameters**.

Solving Mathematical Programming Problems

- **Graphical method**
  - Only useful for 2 decision variables (maybe 3 if you can handle 3-D chart)
- **Simplex method**
  - Efficient algorithm to solve LP problems by performing matrix operations on the LP Tableau
  - Developed by George Dantzig (1947)
  - Can be used to solve small LP problems by hand
- **Software packages**
  - MS Excel
  - Lindo, LPSolve
  - AMPL/CPLEX: modeling language and "solver" for large and complex LP/IP problems

Classes of Linear Programming Problems

- **Allocation problems**
  - Maximize output with constraints on capacity and resources
  - Divides resources and assigns to competing activities
- **Covering problems**
  - Minimize costs with constraints on required coverage
  - Combines resources and coordinates activities
- **Blending problems**
  - Involves mixing materials with different individual properties, resulting in weighted average properties for the resulting blend
- **Scheduling problems**
  - A special type of covering problem

Classes of Linear Programming Problems (2)

- **Assignment problems**
  - Matching of jobs to machines/workers
- **Equipment replacement problems**
  - How long do we utilize equipment, how do we schedule replacements?
- **Knapsack problems**
  - Fitting different-sized items into finite capacity containers, constraints on item properties, etc.
- **Inventory problems**
  - When and how many do we reorder, constraints on capacity, demands, ordering costs, etc.

Classes of Linear Programming Problems (3)

- **Traveling salesman problems**
  - What order do we visit cities to minimize travel times/distances?
- **Network flow (transhipment) problems**
  - How do we route commodities from supply to demand?

Till next lecture,

take care.