PART 3

Human Learning and Human–Machine Interaction

10

Adaptive Methods in Machine Learning

The signals used by non-invasive BCIs (EEG, MEG, etc.) fluctuate strongly over time, between different sessions, but also within each session itself, based on the subject's levels of fatigue, motivation or physical changes such as sensor positioning and impedance. The goal of adaptive methods is to tackle the problem of the *non-stationarity* of the signal, or more precisely, the variability of irrelevant information as opposed to the variability of relevant information which allows to control the various degrees of freedom of the interface. These adaptive methods act by modifying the response function during BCI use, in order to maintain (or even improve) the performance of the interface.

10.1. The primary sources of variability

Human biomedical research distinguishes between two principal types of variability, particularly in the domain of cognitive neurosciences and neuroimagery: intrasubject variability (within single subjects) and intersubject variability (between subjects) [FRI 02]. BCIs are affected by both types of variability. Indeed, the goal of a BCI is to optimize the interaction afforded to a given individual. Especially in medical contexts, the success of the BCI is largely dependent on its capacity to adapt to each patient, depending on the medical circumstances of the individual. In order to be

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applicable to as many patients as possible, BCIs must be capable of adapting to intersubject differences. Furthermore, to guarantee high levels of stability and performance, BCIs must be robust against the signal variability that occurs at individual scales. This last point is particularly crucial for BCIs that, by definition, operate online; these BCIs must rapidly establish a link between small numbers of observations, corrupted by noise, and machine decisions.

This section offers a brief review of the primary factors that govern and influence these two types of variability.

10.1.1. *Intrasubject variability*

In essence, there are two major sources of intrasubject variability: technical factors and human factors. Both can affect the robustness of a BCI, during usage, or from one session to the next.

Technical factors include anything that does not depend on the subject but that might potentially alter the reproducibility of the relevant features of the recorded signals. It is difficult to guarantee identical recording conditions over multiple sessions (positioning and impedance of the electrodes for EEG; development of fibrosis during invasive recordings). Within the same session, the signal from one of the EEG sensors can suddenly begin to deteriorate, either due to poor contact (insufficient gel, electrode slipping or becoming detached) or due to the sensor developing a spontaneous defect. In particular, if the impedance or the signal-to-noise ratio of the reference electrode changes, it will affect all of the signals (see [CLE 16], sections 8.2 and 9.3). If a BCI uses non-adaptive algorithms whose parameters are trained during a calibration phase and then remain unchanged for subsequent usage, these kinds of events can have a dramatic impact on the performance. Imagine, for example, a spatial filter with a high weight on one particular sensor (see Chapter 7), which then suddenly loses signal: the BCI is immediately rendered ineffective. If we wish to prevent degraded performance, given that most BCIs are intended for repeated use over multiple sessions for extended periods of time, the use of adaptive methods appears unavoidable.

The *human factors* are the changes in mental state that might alter the physiological markers on which the interaction is based. The markers of interest such as event-related responses and spontaneous rhythms are affected by many of these factors, including for example fluctuations in vigilance, decreased motivation, moments of inattention or even habituation [KOH 07], which also equally affect learning. The goal is to eliminate the negative effects of these fluctuations, or alternatively to be able to measure and interpret them for the opportunity of exploiting them [BER 07]. However, the neurophysiological correlates of attention levels, fatigue and even learning are still poorly understood; they are currently the object of their own lines of research in cognitive neuroscience. This research specifically aims to improve our understanding of the origins of the intrasubject variability in behavioral performance [WEI 06, DRE 11, BOM 15]. In future, the results of this research may greatly enhance the development of more robust, adaptive BCIs. Finally, intentions rather than objects some differences in cerebral activity are rooted in the context, which defines the objects and the expectations of the user. The user context (at hospital, at home, alone or accompanied) plays an important role in this, but so does the experimental context, such as the fact of being online with a working BCI, or offline during a calibration phase. This phenomenon can limit the capacity of generalization of the calibration data, and provides another reason for choosing to implement adaptive methods.

10.1.2. *Intersubject variability*

More difficult to tackle than intrasubject variability, intersubject variability is another one of the major challenges of BCIs. Ideally, while remaining robust against variability at individual scales, BCIs should build upon working principles and neurophysiological markers that are applicable to the majority of subjects or patients.

Intersubject variability can be anatomical in origin, for example occurring due to differences in the gyrification of the cortex. Since EEG is very sensitive to the orientation of the cortical generators, sources of cerebral activity that are close to each other but oriented in different ways can produce very different measurements at the electrodes. Intersubject variability has been identified in a number of cortical structures, for example the sensorimotor cortex [GRA 91] and even the medial cingulate cortex, which is composed of either one or two sulci depending on the subject [AMI 13]. The medial cingulate cortex is the primary location of markers known as error potentials (see section 5.5). The intersubject variability of these markers is particularly significant in the context of BCIs.

Intersubject variability may also have a functional origin; the responses to a stimulus differ from subject to subject, all the more so that their latency is long: the variability of P300 waves is much higher than that of the spikes of early auditory responses [DAL 97]. Similarly, there is intersubject variability in the frequency of spontaneous rhythms. For instance in MEG, the posterior alpha activity (occipital and parietal) was measured at 10.3 Hz with an intersubject standard deviation of 2.8 Hz [HAE 14]. These differences are the reason behind the standard practice of starting with a calibration session for each new user, for BCIs exploiting event-related responses as well as oscillatory activities. The calibration session optimizes physiological markers for each individual by characterizing their temporal, spectral and spatial features (see section 7.2). Transfer learning approaches aim to eliminate the necessity of calibration session by initializing the BCI with a database of other subjects or patients (see section 9.2.2). This process is naturally limited by intersubject variability, but its implementation alongside adaptive methods (explained below, section 10.3.2) is a promising prospect for optimizing both usage, comfort and BCI performance.

Intersubject differences can also arise due to psychological factors, personality traits or simply age differences. These aspects will need to be accounted for when designing BCIs for widespread usage.

Finally, although currently BCIs are mostly being tested and evaluated with healthy subjects, we should not forget that there can be important differences in cerebral activity between healthy subjects and the patients who are the target population of this technology. Moreover, interpatient variability is possibly even stronger than intersubject variability within the general population, due to differences in the etiology and the evolution of the patients' pathologies.

For example, the event-related auditory responses of patients in a coma or waking from a coma can be very atypical, very variable, weaker and delayed or even absent, depending on neurological impairments and the cause of the coma (trauma or anoxia), and also depending on the evolution of the state of the patient over time (see [MOR 14] and Chapter 1 in Volume 2 [CLE 16]).

These parameters must be taken into consideration when developing and validating BCIs for clinical purposes.

10.2. Adaptation framework for BCIs

Figure 10.1 provides a general framework for studying variability in the context of BCIs. The BCI attempts to interpret the measurements, denoted $\mathbf{x}$, originating from the cerebral activity of a subject with the objective of making a decision. The decision is based on the analysis of certain indicators y of the cerebral state of the subject when he or she receives the stimulus or feedback u.

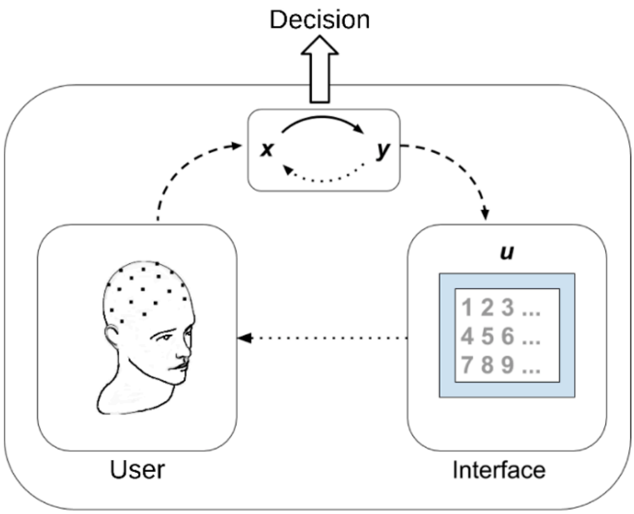

Figure 10.1. *General framework for studying variability in BCIs. The relationship between the cerebral state y and the measurements $\mathbf{x}$ is subject to variability, and this must be taken into account when estimating the cerebral states y*

The relationship between the cerebral state y and the measurements $\mathbf{x}$ can be expressed in two different ways:

– as a function explicitly linking $\mathbf{x}$ and y according to the logic of the statistical decoding of the cerebral activity (classification of this activity). One can then think of y as a label;

– as a generative model describing as realistically as possible the causality between y and $\mathbf{x}$. y is then thought of as a hidden variable representing the user's intention. Decoding is performed by inverting the model (Bayesian approach).

In each of these two approaches, the same objects are sometimes called different names. For the sake of clarity, Table 10.1 lists the links between the different instances of terminology for this chapter. The rest of the chapter is organized in two parts, which presents the two approaches for describing variability: statistical decoding and generative models.

	Statistical decoding	Generative model
$\mathbf{x}$	Data, observation, feature, covariable	Observation, feature
y	Label, variable	State, hidden variable
u	Stimulations	Stimulations, experimental design
f	Prediction function, classifier	
$M(\theta)$		Generative model, potentially dependent on parameters θ

Table 10.1. *Commonly used terminology from two different perspectives: statistical decoding and generative modeling*

10.3. Adaptive statistical decoding

This section adopts the perspective of a decoding process applied to the recorded activity; the goal is to estimate a prediction function f that, given any observation $\mathbf{x}$, returns the predicted label $y = f(\mathbf{x})$.

10.3.1. *Covariate shift*

As explained in section 9.1, in supervised learning, the prediction function is estimated using a dataset whose labels are known. This dataset, known as the training dataset, is usually obtained during a dedicated calibration session. The prediction function may then be used to label new data, known as the test data. Applications of BCIs rely on being able to label the test data online.

Over the course of a single session, the statistical distribution of the data might be subject to change (and *a fortiori* between different sessions). If these changes occur between the training and testing phases, the labels of the new test data risk being incorrect, consequently leading to incorrect decoding (upon which the BCI relies).

This situation may be modeled using the concept of *covariate shift*: the distribution of the observations (or covariates) $\mathbf{x}$ is non-stationary, whereas the conditional distribution $P(y|\mathbf{x})$ on the other hand is stationary (Figure 10.2). Note that the condition that $P(y|\mathbf{x})$ is stationary implies that, regardless of its shift, a covariate $\mathbf{x}$ that returns to a previous state at a later point in time still corresponds to the same label y. Covariate shift models provide reasonable formalisms for BCIs when $\mathbf{x} \in \mathbb{R}^d$ with relatively high d, and when the values of the labels y are discrete (classification).

Figure 10.2. *Illustration of covariate shift, in two dimensions ($\mathbf{x} \in \mathbb{R}^2$). Each circle (square) indicates an observation $\mathbf{x}_i$ for task 1 (2). The covariate shift may be seen in the differences between the probability distributions of the colored points (training data) and the white points (test data)*

Estimating the prediction function is traditionally achieved by minimizing an empirical risk term (equation [9.2], Chapter 9) recalled below:

$$R_{emp}(f) = \frac{1}{n}\sum_{i=1}^{n} L(y_i, f(\mathbf{x}_i))$$

In order to compensate for covariate shift, the empirical risk should be weighted by an "importance" function: the ratio (assumed finite) of the probabilities of the test and training data:

$$\frac{p_{test}(\mathbf{x})}{p_{training}(\mathbf{x})}\ . \qquad [10.1]$$

The importance-weighted empirical risk then becomes:

$$R_{\mathcal{I}emp}(f) = \frac{1}{n}\sum_{i=1}^{n} \frac{p_{test}(\mathbf{x}_i)}{p_{training}(\mathbf{x}_i)} L(y_i, f(\mathbf{x}_i)) \qquad [10.2]$$

If the importance function [10.1] is not known, it may be estimated, for example using parametric models with Gaussian kernels [SUG 07, LI 10].

10.3.2. *Classifier adaptation*

10.3.2.1. *Sliding window retraining*

In mathematical terms, adaptive learning is an optimization problem in which the current performance must be optimized while preserving the performances acquired during previous training phases. A mathematical analysis in [KIV 04], that, in order to follow the variations of the environment without overloading the storage and computational capacity, it is often necessary to delete the oldest datapoints to make space for more recent information. This approach is also known as sliding window retraining.

Several methods for BCIs have been suggested to allow online data to be leveraged at regular intervals. The most common method is to begin with an initial, generic classifier and then *retrain* this classifier in real time. The generic classifier is calculated from a multisubject database that is supplied beforehand (transfer learning, see section 9.2.2). In principle, this classifier can be used immediately with any new subject, without calibration. However, it can be expected to perform fairly poorly. Adaptive approaches attempt to improve this initial classifier by regularly updating it with information gathered during usage. The method suggested by [VID 11] updates the classifier in three stages. The first two stages use a supervised adaptive method, for which the subject must copy a series of words, and the last stage uses an unsupervised adaptive method, during which the subject uses the interface freely. In contrast, the approaches suggested by [LI 06, KIN 12a], inspired by the EM algorithm [DEM 77], are entirely unsupervised. A probabilistic model is constructed from an initial, multisubject dataset. The data acquired during usage are gradually added to the initial dataset to refine the model, *without any labeling information*. The simulations in [KIN 12b] show that in most cases the algorithm is capable of constructing useful classifiers, even without any initial data.

10.3.2.2. *Gradient descent*

Another family of adaptive methods is the family of "stochastic gradient methods". These methods were suggested at a very early stage for neural networks. The approach assumes that it is possible to estimate the *gradient of* *the error* at each new trial. Training may therefore be achieved by modifying the classifier f after each trial in the direction *opposite* to the error [WID 62, RUM 88, WIL 92]. If $\tilde{g}(\mathbf{x}, f)$ estimates the error at point $(\mathbf{x}, f)$, the change in f may be written:

$$f \leftarrow f - \eta\tilde{g}(\mathbf{x}, f)$$

where η is a "small" scalar. For example, in the linear case, where $f(\mathbf{x}) = \langle\mathbf{x}, \mathbf{w}\rangle$, the estimator of the gradient of the least squares error is given by $\tilde{g}(\mathbf{x}, \mathbf{w}) = (\langle\mathbf{x}, \mathbf{w}\rangle - y)\,\mathbf{x}$ and the update operation is given by:

$$\mathbf{w} \leftarrow \mathbf{w} - \eta\left(\langle\mathbf{x}, \mathbf{w}\rangle - y\right)\mathbf{x}.$$

where y is the expected response and $(\langle\mathbf{x}, \mathbf{w}\rangle - y)$ is the prediction error.

During usage, BCIs, by definition, do not know the response expected by the subject. The value of the prediction error therefore remains unknown, and classical gradient descent methods are not applicable as such. But we can use a weaker hypothesis stating that we have access to a simple *indication* of the "quality" of the response. For example, in the case of a P300 virtual keyboard, we will be able to tell whether the suggested letter is correct or incorrect, but if the label is wrong, we will not know the correct label. This indication is denoted r; $r = 1$ means that the response is correct, and $r = -1$ means that it is incorrect. Gradient descent methods that are applicable with this setup are based on a probabilistic approach. The output $\{f(\mathbf{x}, i, \mathbf{w})\}_{i\in 1,\ldots,K}$ of the classifier is a probability for each label $1, ..., K$, and the response $\tilde{y}$ is the result of a random event with these probability values. The "policy gradient" algorithm [WIL 92] updates the classifier with:

$$\mathbf{w} \leftarrow \mathbf{w} + \eta r \nabla_{\mathbf{w}} \log f(\mathbf{x}, \tilde{y}, \mathbf{w})$$

A numerical study published in [DAU 15] shows that the performance of the policy gradient applied to P300 classifiers is close to that of the logistic gradient with complete information. In BCI applications, the problem at hand is to measure in real time the "value" of the response delivered by the interface. There are various ways of achieving this, such as measuring an "error potential" [FAL 91] that indicates a mismatch between the subject's expectation and the response produced by the algorithm [BUT 06], or using a “backspace” character indicating that the *previous* letter was incorrect [DAU 15], or even language-based statistics indicating that the chosen letter is unlikely given the previous letters.

10.3.3. *Subject-adapted calibration*

Before starting to use a BCI, an initial calibration stage is often necessary to determine certain parameters, or choose between the available options. For example, for SSVEPs, the frequency of the flashes needs to be selected to ensure that it appears in the EEG spectrum with a good signal-to-noise ratio [BRY 13]. For motor imagery, tasks must be found that the user can perform in a reproducible manner, in view of promoting their correct classification.

One traditional approach to the initial calibration stage is to ask the subject to repeat a certain set of tasks a large number of times, corresponding to the different options under consideration. The EEG data thus obtained is analyzed *offline* to determine the best choices for the *online* phase. However, this approach has multiple disadvantages as follows:

– data are acquired uniformly across the different options (the same amount of data for each option), but certain options will be unusable; this time would be better spent gathering data from tasks that can be effectively exploited, which will make it possible to establish better classifiers;

– users receive zero feedback about the interpretation of their cerebral activity; the signals gathered in the phase could potentially be significantly different from the signals in the *online* exploitation phase, since user expectations and tasks can vary strongly between the two phases [SHE 06].

Calibration is expensive in terms of time and attention required from the subject: it is therefore important for calibration to be performed rapidly and efficiently [DOB 09].

10.3.3.1. *Reinforcement learning*

Reinforcement learning is a sequential method that selects the best-performing actions for a chosen objective, given the observed state of the system. For BCIs, this method provides a promising way of exploring the available options, while simultaneously exploiting them online. Reinforcement learning searches for a compromise between exploration and exploitation. Thus, the two disadvantages of traditional precalibration listed above can be corrected: data acquisition is no longer necessarily uniform across the available options (exploration), and data processing is performed online, providing feedback to the user (exploitation). The most simple algorithm of reinforcement learning is the *multiarmed bandit algorithm* [AUE 02]. The algorithm was named after a series of slot machines ("one-armed bandits"), in front of which a player would naturally attempt to maximize profit.

10.3.4. *Optimal tasks*

The multiarmed bandit algorithm has been used to rapidly select from a set of motor imagery tasks the most discriminating task relative to the idle state [FRU 13]. There is a random factor coming from the low signal-to-noise ratio of cerebral signals, which explains why it is necessary to measure multiple trials for each task in order to classify them. Our knowledge of the target function (here, the classification rate that we are attempting to maximize) is imprecise, but we can find bounds that confine it almost surely within a confidence interval. The bounds of the confidence interval are calculated from past observations, and are updated with each new observation. The bandit algorithm performs sequentially: at each stage, it selects from the K possible tasks the task that maximizes an upper bound of the target function. This is illustrated for $K = 2$ in three successive steps in Figure 10.3(top). Over the course of a single experiment, the algorithm chooses a sequence of tasks for the subject to perform, as shown at the bottom of Figure 10.3. Table 10.2 shows the results of five successive optimal task selection experiments with the same subject, each time selecting the best task, with the number of trials (observations) of each class and their final classification rate. We can see that in each of the experiments, feet motion achieved the best classification rate. Tongue motion, which had the lowest classification rate, was performed fewer times than the other tasks.

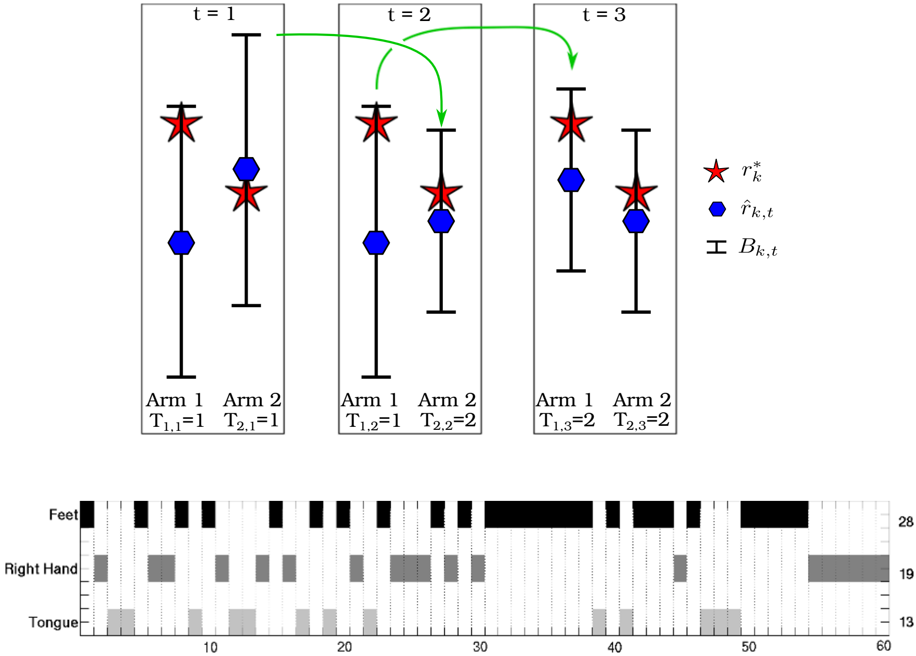

Figure 10.3. *Top: Three stages of the multiarmed bandit algorithm $t \in \{1, 2, 3\}$. Each task $k \in \{1, 2\}$ has a theoretical classification rate r_k^*. At stage t, this rate is estimated by $\hat{r}_{k,t}$ with a confidence interval $B_{k,t}$ whose size decreases as the number of realizations of task k (given by $T_{k,t}$) increases. Bottom: Sequence of selected motor imagery tasks as the bandit method is iterated during an online experiment (last row of Table 10.2); motor imagery of the feet is the most commonly performed task, and the most discriminating. For a color version of this figure, see www.iste.co.uk/clerc/interfaces1.zip*

Right hand		Feet		Tongue		
Number of trials	Classification rate (%)	Number of trials	Classification rate (%)	Number of trials	Classification rate (%)	Selected task
16	80	28	97	16	78	Feet
19	72	30	87	11	40	Feet
16	70	34	86	10	57	Feet
15	64	34	88	11	55	Feet
19	73	28	86	13	76	Feet

Table 10.2. *Results of five successive instances of an experiment selecting the best task using the multiarmed bandit algorithm, performed with the same subject*

10.3.5. *Correspondence between task and command*

The characteristic features of cerebral responses are assigned classes by the classifier, but classes and the commands do not necessarily have a one-to-one correspondence. Although most BCIs fix this correspondence *a priori*, certain studies have suggested allowing the system to learn the relationship in real time. The first study that used reinforcement learning in this way was performed on rats, whose unitary activity in the motor cortex was recorded [DOB 09]. By online optimization of a multilayer perceptron, the rat could access a reward by operating a 3D robot arm toward a predefined target. A more recent study uses partially observable Markov decision processes with SSVEPs [BRY 13]. Users were able to modify the correspondence between three flashing zones and three selectable targets. Unlike the motor study in the rat, this last process was supervised because the sequence of targets to select was known *a priori*.

10.4. Generative model and adaptation

10.4.1. *Bayesian approach*

The Bayesian framework is as simple as it is generic, because it can be applied to any probabilistically formulated inference problem and is based on a single formula, Bayes' law:

$$p(y|\mathbf{x}, M) = \frac{p(\mathbf{x}|y, M)p(y|M)}{p(\mathbf{x}|M)} \ , \qquad [10.3]$$

where we recall that $\mathbf{x}$ is the vector of observations or relevant features of the signal and y is the hidden variable that we wish to estimate (the command or mental state that defines the interaction). Equation [10.3] explicitly exhibits the dependency on the chosen model M composed of all the hypotheses about the causal process, linking the intent or state y to the observations $\mathbf{x}$.

Defining M is equivalent to defining the distributions $p(\mathbf{x}|y, M)$ (the likelihood of the observations, given the variable y describing the state) and $p(y|M)$ (our *a priori* knowledge about y). Given these distributions, the Bayesian inference step simply applies equation [10.3] to obtain the *a posteriori* distribution $p(y|\mathbf{x}, M)$ and the evidence $p(\mathbf{x}|M)$ in favor of the model M, which is just a (scalar) normalization factor.

Although Bayes' law itself is simple, its application can prove difficult if the model M is nonlinear, or if the distributions that define it cannot be solved analytically. In these cases, it is necessary to resort to approximation techniques; these techniques can provide numerical solutions that are precise but expensive to calculate (this is the case for sampling methods [ROB 04]), or analytical solutions that are very efficient computationally but potentially less precise (this is the case for variational methods [FRI 07]).

But, more generally, the states y are not the only hidden variables. The model M usually includes parameters θ that must be estimated, specific to each subject or patient. The equation to be solved then becomes:

$$p(y, \theta|\mathbf{x}, M) = \frac{p(\mathbf{x}|y, \theta, M)p(y, \theta|M)}{p(\mathbf{x}|M)} \qquad [10.4]$$

The purpose of the θ variable is to make certain aspects of the model M explicit, such as the number and the nature of the unknown parameters. But M contains other information as well, such as the nature of the relationship between the hidden parameters and the observed data. For example, when reconstructing the sources for EEG or MEG, models known as distributed models describe cortical activity using a large number of current dipoles with fixed position and orientation. Thus, in addition to the θ parameters describing the amplitude of these dipoles, the model M specifies other hypotheses such as their positions and orientations, which are defined using a reference brain, or from the MR images of the subject [MAT 07].

Besides the fact that the state variable is the primary variable of interest, as it determines the conditions of the interaction, its main difference with the other unknown parameters θ is that it varies over time. Usually, the θ parameters are considered known, once estimated. They can be estimated in advance, e.g. during calibration. If we consider the example of the P300 speller, θ is the vector of parameters corresponding to each class (target and non-target) for a given user, and y is the label assigned to the observation $\mathbf{x}$. But equation [10.4] shows that the θ parameters may also be estimated,jointly with y. This highlights the possibility that the Bayesian approach could be used to develop approaches that would be capable of adapting the model parameters to each user.

Once Bayesian inference has been performed, the *a posteriori* probability distribution $p(y, \theta|\mathbf{x}, M)$ allows the hidden states and the parameters of the model to be inferred. This distribution provides an estimate of the correct command for the BCI. Note that this *a posteriori* estimate, expressed as a probability distribution, gives not just an estimate of the (average) value of these variables, but also an estimate of their variance, which is a measure of the confidence in the estimate. The variance can be used, for example by setting up an adaptive decision-making strategy for optimal stopping [MAT 15] (see section 10.4.2).

The evidence $p(\mathbf{x}|M)$ also allows us to make inferences about the models, and potentially to compare different, alternative models [MAT 07] (see section 10.4.3).

BCIs are essentially interested in estimating states and sometimes parameters, so they focus primarily on the first type of inference. With this objective in mind, equations [10.3] and [10.4] show that Bayes' law is just a rule for updating the state of knowledge in the presence of new observations. Since BCIs work in real time and gather observations sequentially, Bayesian reasoning, applied to each new observation, establishes a set of rules for learning and endows the machine with the capacity to adapt. After explicitly introducing a notion of time to index the sequential events or trials t, Bayesian learning may be written as follows:

$$p(y|\mathbf{x}_t, M) \propto p(\mathbf{x}_t|y, M)p(y|M) \tag{10.5}$$

where the *a priori* distribution of the state y is given by the *a posteriori* distribution learned from earlier observations

$$p(y|M) = p(y|\mathbf{x}_1 ... \mathbf{x}_{t-1}, M) \tag{10.6}$$

Many of the existing methods of feature extraction or classification can be reformulated within the Bayesian framework. Conversely, the Bayesian framework, and in particular the explicit formulation of a generative model for the data, allows for *a priori* information to be integrated into a specific context of experimental interaction, thus making it possible to optimize and explore the possibility of identifying parameters directly online. For example, with the P300 speller, the inference and dynamic decision process may be constrained using a language model that provides *a priori* information about letters and the most probable letter sequences [MAI 15].

10.4.2. *Sequential decision*

It is desirable for the BCI to only make a decision when the confidence in its correctness is sufficiently high. To achieve this, strategies for evidence accumulation have been suggested [VER 12, DAU 14], which classify the user's cerebral activity over a variable time duration, and output a decision when the evidence has exceeded a chosen threshold.

This problem may also be expressed in probabilistic terms. Consider the problem of finding the number of measurements necessary to find the (unique) label $y \in 1, ..., K$ shared by a series of observations $\mathbf{x}_1, ..., \mathbf{x}_t, ...$. If $P(y = k|\mathbf{x}_1, ..., \mathbf{x}_{t-1})$ is known, the probability assigned to the new measurement $\boldsymbol{x}_t$ is given by:

$$P(y = k|\mathbf{x}_1, ..., \mathbf{x}_t) = \frac{P(\mathbf{x}_t|y = k)P(y = k|\mathbf{x}_1, ..., \mathbf{x}_{t-1})}{\sum_{i=1}^{K} P(\mathbf{x}_t|y = i)P(y = i|\mathbf{x}_1, ..., \mathbf{x}_{t-1})} \quad [10.7]$$

Starting from a known initial distribution $P(y = k), k = 1, ..., K$, we can sequentially refine the assigned probability of each label with each new measurement. The sequential decision problem [WAL 47] consists of choosing the optimal moment to terminate the measurement process, given a fixed decision threshold s. The "optimal stopping" algorithm 10.1 allows us to decide after each measurement whether to continue or terminate the measurement process.

Algorithm 10.1. Optimal stopping (s)

1: $P_1 \leftarrow P(y = 1), \ldots, P_K \leftarrow P(y = K)$
2: **while** $\nexists i : P_i \geq s$ **do**
3: Perform a measurement $\boldsymbol{x}$
4: Update $P_1, \ldots, P_K$
5: **end while**

In the example of the virtual keyboard "P300", we denote $\mathbf{x}_1, ..., \mathbf{x}_T$ the sequence of observations obtained from T flashes. The label $y \in 1..K$ corresponds to the target letter (where K is the total number of listed of proposed symbols). We denote $P^+(\mathbf{x}_t)$ the probability that observation $\mathbf{x}_t$ is a P300. If we know the list S_t of letters flashed on the screen at each instant t, we can construct the *a posteriori* instantaneous probability of each symbol [DAU 14], which is:

$$P(y = k|\mathbf{x}_t) = \frac{1}{n}\mathbb{I}_{k \in S_t} P^+(\mathbf{x}_t) + \frac{1}{K-n}\mathbb{I}_{k \notin S_t}(1 - P^+(\mathbf{x}_t)) \qquad [10.8]$$

where n is the number of highlighted symbols for each flash.

The value of $P(y = k|\mathbf{x}_1, ..., \mathbf{x}_t)$ may then be deduced from equation [10.7] using a uniformity hypothesis, namely $\exists C : \forall k, P(y = k|\mathbf{x}_t) = CP(\mathbf{x}_t|y = k)$. An optimal stopping strategy, as illustrated in Figure 10.4, may then be implemented.

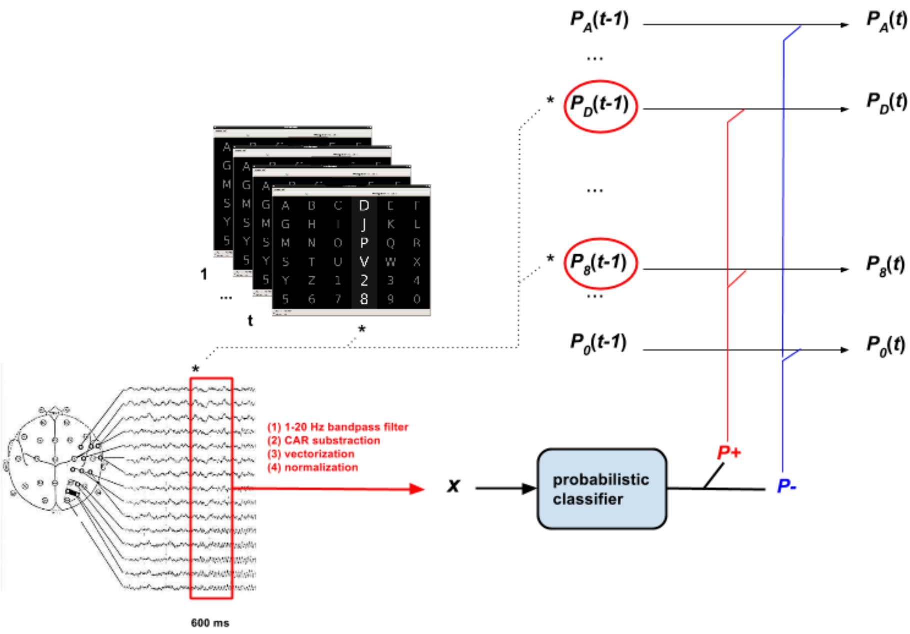

Figure 10.4. *Evidence accumulation with the P300 speller. Each flash is assigned a probability P^+ of eliciting a P300, which allows the vector of probabilities associated with the various different symbols of the virtual keyboard to be updated (see text). For a color version of this figure, see www.iste.co.uk/clerc/interfaces1.zip*

10.4.3. *Online optimization of stimulations*

Sequential hypothesis testing does not just involve determining the optimal moment to make a decision and stop gathering information. The nature of the test must also be optimized, that is to say the nature of the question being asked, or, in the case of reactive BCIs such as the P300 speller, the identity of the flashing items. To see this, we can simply make explicit the role of the stimulation u_t at an instant t when the observed data are generated. With the Bayesian formalism, this leads us to rewrite equation [10.5] in the following form:

$$p(y|\mathbf{x}_t, M_k, u_t) = \frac{p(\mathbf{x}_t|y, M_k, u_t)p(y|M_k)}{p(\mathbf{x}_t|M_k, u_t)} \quad [10.9]$$

where M_k is the kth alternative hypothesis.

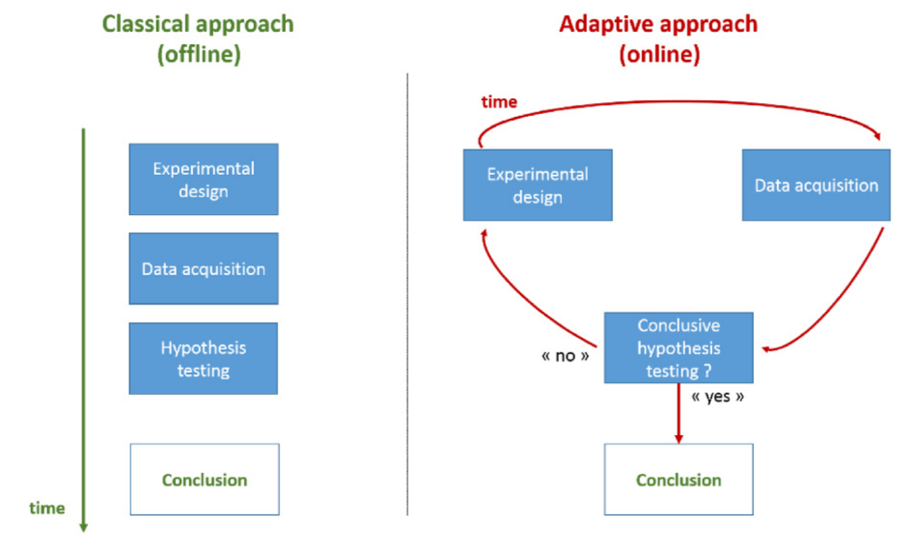

Figure 10.5. *General principles of the traditional experimental design (left panel) and the adaptive experimental design (right panel)*

As explained when it was introduced in section 10.4.1, the Bayesian approach allows us to make inferences about the models, by formally comparing several alternative hypotheses. *In fine*, the chosen model will be the model that has an *a posteriori* probability above some chosen threshold (typically 0.95). At time t, for the model M_k, this probability is given by:

$$p(M_k|\mathbf{x}_1...\mathbf{x}_t, u_1...u_t) = \frac{1}{1 + \sum_{M_i \neq M_k} BF_{ik}} \qquad [10.10]$$

where BF_{ik} is the Bayes factor between models M_i and M_k:

$$BF_{ik} = \frac{p(\mathbf{x}_1...\mathbf{x}_t|M_i, u_1...u_t)}{p(\mathbf{x}_1...\mathbf{x}_t|M_k, u_1...u_t)} \qquad [10.11]$$

Given this criterion for comparing hypotheses, which can also be used as an optimal stopping criterion, the choice of the next optimal stimulation u_{t+1} must satisfy the condition that the risk of error in the model selection is minimized [DAU 11]. Figure 10.6 shows the performance increase in realistic Monte Carlo simulations achieved by online adaptation of the stimulations, compared to the classical approach in which the objects are flashed in a random order.

10.4.3.1. *Adaptive experiments in cognitive neuroscience*

Optimizing stimulations, or in other words optimizing the experimental design, is a central challenge in cognitive neuroscience, where the objective of an experiment is usually to compare alternative hypotheses on brain function, whether healthy or pathological. These hypotheses effectively model the relationship between mental processes (learning, decision making, etc.) and neurophysiological or behavioral responses [STE 15]. In computational neuroscience, these models have become more realistic but also more complex; it has become particularly difficult to preoptimize the experimental design so that they may be compared, for a given subject [DAU 11].

Thanks to BCI developments, it is now possible to analyze cerebral activity in real time. Optimization is achieved by adapting directly online the successive stimulations used during experiments in cognitive neuroscience and neuroimaging [SAN 14].

In practice, implementing this adaptive approach involves fitting the data with each of the considered models for each new observation, evaluating the evidence in favor of each model and calculating the discriminative power for deciding between the models of each of the possible values of the upcoming stimulation u. This approach represents a departure from traditional protocols, which fix the experimental design in advance. It provides an alternative strategy, optimized for hypothesis testing, and produces an optimal and specific paradigm for each new subject or patient. This is illustrated in Figure 10.5.

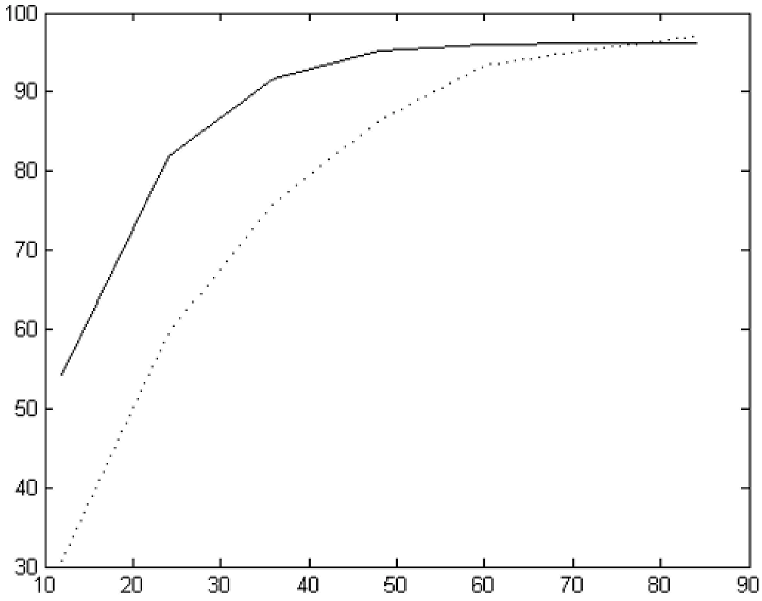

Figure 10.6. *Percentage of correctly spelled letters, for simulated P300 speller data (N = 2,000) as a function of the number of stimulations in the case where the stimulations are ordered pseudo-randomly such that each letter appears an equal number of times (dotted line), and in the case where the choice of stimulation is made online to optimize the detection of the target letter (solid line)*

However, the validity of such an approach remains to be empirically demonstrated. One major obstacle shared by all BCIs is the problem of removing possible artifacts in real time, which traditionally would be performed in fine detail offline. Another example of a possible application for this adaptive approach is the field of diagnostics and prognostics for patients with perturbed states of consciousness, as mentioned in Chapter 1 of Volume 2 [CLE 16].

10.5. Conclusions

We have presented a wide range of methods of adaptive learning, grouped into two families: methods that perform statistical decoding and methods based on a generative model. These methods have not yet found widespread application with BCIs, and the future promises development and progress; the necessity of being able to adapt to the variability of the recorded signals regardless of the origin of this variability can be clearly recognized today. Note that these methods are not necessarily mutually exclusive. Indeed, future developments will most likely draw from the strengths of each of them. Whatever their final form may be, these methods will likely be built with ever-improving mathematical tools suitable for online deployment and will rely on future improvements in our understanding of neurophysiology and cognitive neuroscience, allowing us to create more efficient classification functions or generative models with appropriate adaptive capabilities.

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