National University of Computer & Emerging Sciences, Karachi

Fall-2017 CS-Department

CS211-Discrete Structures

Practice Assignment-III A

Note:

1- This is hand written assignment.
2- Just write the question number instead of writing the whole question.
3- You would submit this assignment to your respective CR.
4- CR's are instructed to submit this assignment at my office by 10 am.

Submission date: Monday, 23th October, 2017 by 9:00 am

1. Let R be the following relation defined on the set {a, b, c, d}:

   R = {(a, a), (a, c), (a, d), (b, a), (b, b), (b, c), (b, d), (c, b), (c, c), (d, b), (d, d)}.

   Determine whether R is:
   (a) Reflexive (b) Symmetric (c) Antisymmetric (d) Transitive

2. Let *R* be the following relation on the set of real numbers:

   $aRb \leftrightarrow \lfloor a \rfloor = \lfloor b \rfloor$, where $\lfloor x \rfloor$ is the floor of x.

   Determine whether R is:
   (a) Reflexive (b) Symmetric (c) Antisymmetric (d) Transitive

3. Let $f(x) = \lfloor x^2/3 \rfloor$. Find $f(S)$ if
   a) $S = \{-2, -1, 0, 1, 2, 3\}$.
   b) $S = \{0, 1, 2, 3, 4, 5\}$.
   c) $S = \{1, 5, 7, 11\}$.
   d) $S = \{2, 6, 10, 14\}$.

4. Why is f not a function from **R** to **R** if
   a) $f(x) = 1/x$?
   b) $f(x) = \sqrt{x}$?
   c) $f(x) = \pm\sqrt{(x^2 + 1)}$?

5. Determine whether f is a function from **Z** to **R** if
   a) $f(n) = \pm n$.
   b) $f(n) = \sqrt{n^2 + 1}$.
   c) $f(n) = 1/(n^2 - 4)$.

6. Find these values.
   a) $\lceil \frac{3}{4} \rceil$
   b) $\lfloor \frac{7}{8} \rfloor$
   c) $\lceil -\frac{3}{4} \rceil$
   d) $\lfloor -\frac{7}{8} \rfloor$
   e) $\lceil 3 \rceil$
   f) $\lfloor -1 \rfloor$
   g) $\lfloor \frac{1}{2} + \lceil \frac{3}{2} \rceil \rfloor$
   h) $\lfloor \frac{1}{2} \cdot \lfloor \frac{5}{2} \rfloor \rfloor$

7.

Determine whether each of these functions from $\{a, b, c, d\}$ to itself is one-to-one.

a) $f(a) = b,\ f(b) = a,\ f(c) = c,\ f(d) = d$
b) $f(a) = b,\ f(b) = b,\ f(c) = d,\ f(d) = c$
c) $f(a) = d,\ f(b) = b,\ f(c) = c,\ f(d) = d$

8.

Which functions in Exercise 10 are onto?

9.

Determine whether each of these functions is a bijection from $\mathbf{R}$ to $\mathbf{R}$.

a) $f(x) = 2x + 1$
b) $f(x) = x^2 + 1$
c) $f(x) = x^3$
d) $f(x) = (x^2 + 1)/(x^2 + 2)$

10.

Let $f: \mathbf{R} \to \mathbf{R}$ and let $f(x) > 0$ for all $x \in \mathbf{R}$. Show that $f(x)$ is strictly decreasing if and only if the function $g(x) = 1/f(x)$ is strictly increasing.

11.

Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from $\mathbf{R}$ to $\mathbf{R}$.

12.

Prove that if x is a real number, then $\lfloor -x \rfloor = -\lceil x \rceil$ and $\lceil -x \rceil = -\lfloor x \rfloor$.

13. List the ordered pairs in the relation R from A = {0, 1, 2, 3, 4} to B = {0, 1, 2, 3}, where (a, b) ∈ R if and only if

a) a = b. **b) a + b = 4.** **c) a > b.**
d) a | b. **e) gcd(a, b) = 1.** **f) lcm (a, b) = 2.**

14. List all the ordered pairs in the relation R = {(a, b) | a divides b} on the set {1, 2, 3, 4, 5, 6}. Display this relation graphically, as well in matrix form.

15. For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

a) {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)} **b) {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}**
c) {(2, 4), (4, 2)} **d) {(1, 2), (2, 3), (3, 4)}**
e) {(1, 1), (2, 2), (3, 3), (4, 4)} **f) {(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)}**

16. Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where (a, b) ∈ R if and only if

a) a is taller than b. **b) a and b were born on the same day.**
c) a has the same first name as b. **d) a and b have a common grandparent.**

17. Give an example of a relation on a set that is

a) both symmetric and antisymmetric.

b) neither symmetric nor antisymmetric.

18. Consider these relations on the set of real numbers:

$R1 = \{(a, b) \in R^2 \mid a > b\}$, the "greater than" relation,

$R2 = \{(a, b) \in R^2 \mid a \geq b\}$, the "greater than or equal to "relation,

$R3 = \{(a, b) \in R^2 \mid a < b\}$, the "less than" relation,

$R4 = \{(a, b) \in R^2 \mid a \leq b\}$, the "less than or equal to "relation,

$R5 = \{(a, b) \in R^2 \mid a = b\}$, the "equal to" relation,

$R6 = \{(a, b) \in R^2 \mid a \neq b\}$, the "unequal to" relation.

a) Find:

a) $R2 \cup R4$.
b) $R3 \cup R6$.
c) $R3 \cap R6$.
d) $R4 \cap R6$.
e) $R3 - R6$.
f) $R6 - R3$.
g) $R2 \oplus R6$.
h) $R3 \oplus R5$.

b) Find:

Find

a) $R2 \circ R1$.
b) $R2 \circ R2$.
c) $R3 \circ R5$.
d) $R4 \circ R1$.
e) $R5 \circ R3$.
f) $R3 \circ R6$.
g) $R4 \circ R6$.
h) $R6 \circ R6$.

19.

Represent each of these relations on $\{1, 2, 3\}$ with a matrix (with the elements of this set listed in increasing order).

a) $\{(1, 1), (1, 2), (1, 3)\}$

b) $\{(1, 2), (2, 1), (2, 2), (3, 3)\}$

c) $\{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$

d) $\{(1, 3), (3, 1)\}$

20.

List the ordered pairs in the relations on $\{1, 2, 3\}$ corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$