



# Artificial Intelligence and Machine Learning

## Classification

# Lecture 2: Outline

- Linear Regression (Review)
- Intro to Classification
- Logistic Regression (Linear Classifier)
- Classification Metrics
- Other ML Models
- Optimizers

# Recap

Design your model

- Input scalar linear model (line fitting)
- Fitting polynomials (synthetically designing features from a one-dimensional input)

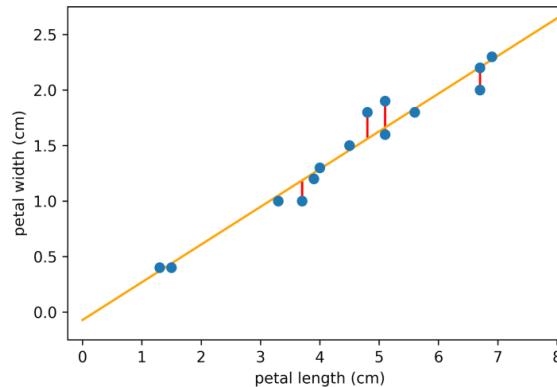
$$\{(\mathbf{x}_i, y_i)\}_{i=1}^N, \mathbf{x}_i \in \mathbb{R}^n, y_i \in \mathbb{R}$$

Design your loss function

- We used mean squared error loss throughout

Finding optimal parameter fitting

- Closed form solution to the linear least squares?
- Why is it linear leastsquares?
- Solution is closed form



# Regression VS classification



## Regression

What is the temperature going to be tomorrow?

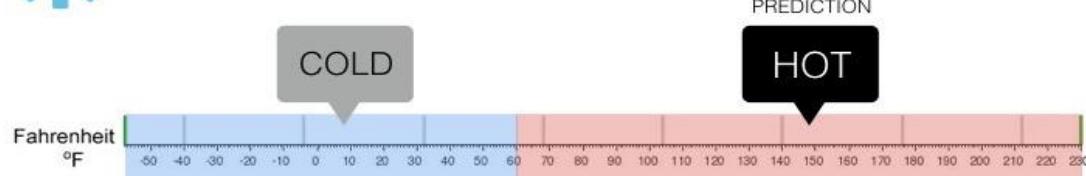
=> Continuous Values



## Classification

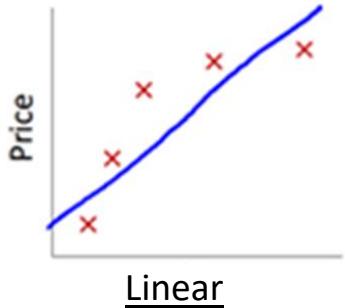
Will it be Cold or Hot tomorrow?

=> Discrete Values

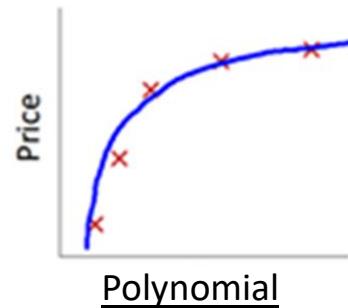


# Regression VS classification

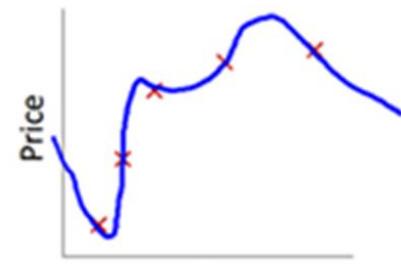
**Regression:**



Linear

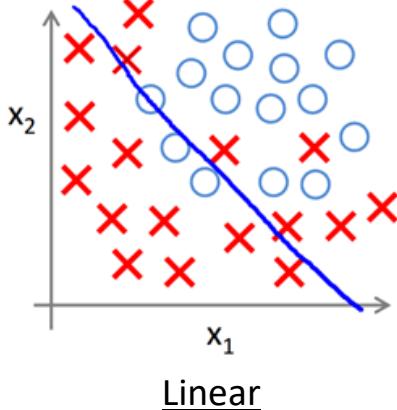


Polynomial

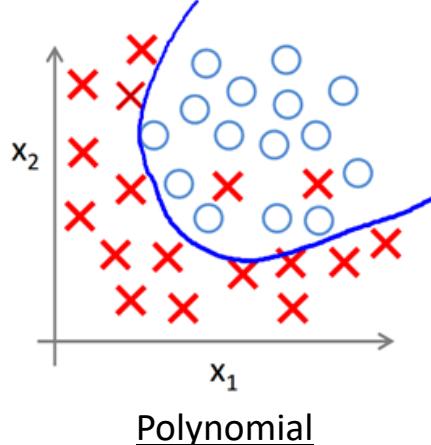


Very complex (overfitting)

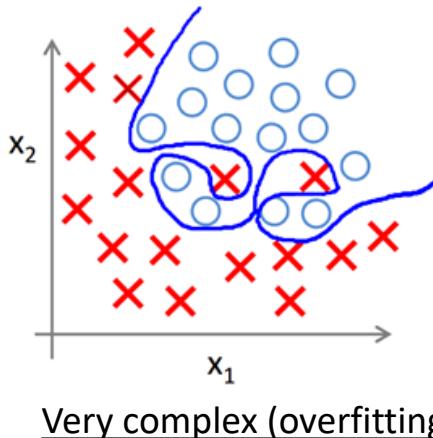
**Classification:**



Linear

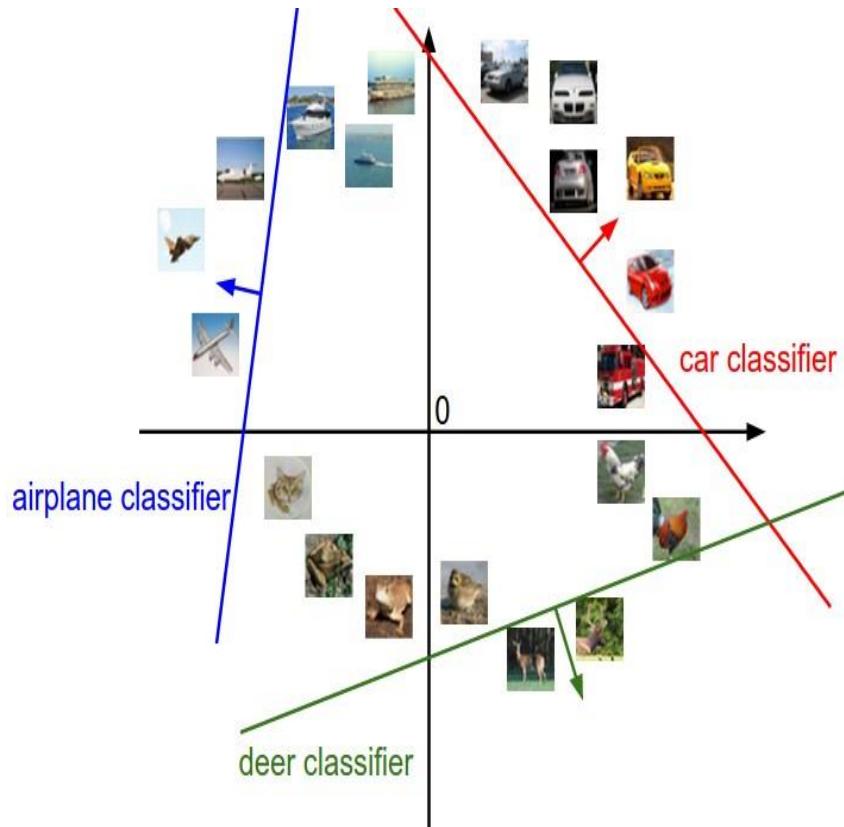


Polynomial



Very complex (overfitting)

# Interpreting a Linear Classifier



$$f(x_i, W, b) = Wx_i + b$$



**[32x32x3]**  
array of numbers 0...1  
(3072 numbers total)

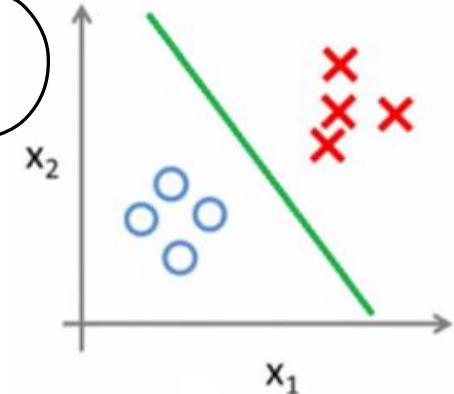
# Classification Types

**Binary Classification:** Two possible outcomes (e.g., Yes/No, Spam/Not Spam).

**How?** Single Classifier.



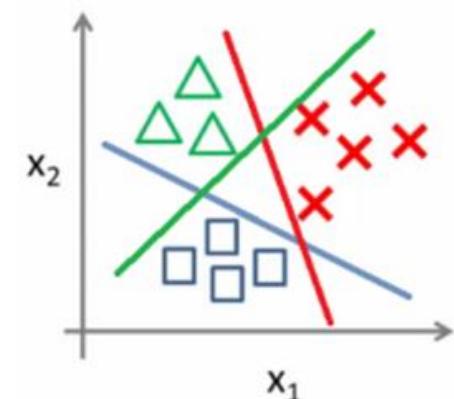
1 / 0



**Multiclass Classification:** More than two outcomes (e.g., Class A, B, C).

**How?** Multiple \*Single Classifiers\*.

1  
2  
3  
4



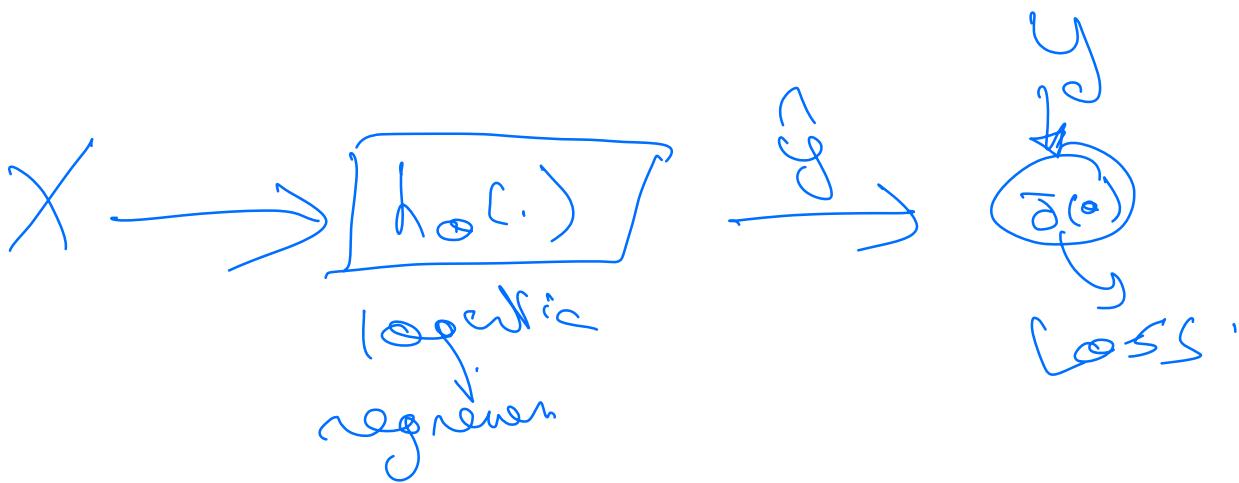
## \* Logistic regression ; (classification)

- Linear model:  $\hat{y} = w^T x$

Probability between 0 and 1  
using sigmoid function.

$$\hat{y} = \sigma(w^T x)$$

$$\sigma(z) = \frac{1}{1 + \exp(-z)}$$



## Binary cross entropy (BCE)

$$J(w) = \frac{1}{N} \sum y_i \log(\sigma(w^T x_i)) + (1-y_i) \log(1-\sigma(w^T x_i))$$

$$y_i = 0 \rightarrow J(w) = \log(1 - \sigma(w^T x_i))$$

$$y_i = 1 \rightarrow J(w) = \log(\sigma(w^T x_i))$$

$\Rightarrow w$  ?? (weight)

$\nabla_w J(w) = 0 \Rightarrow$  no nice closed solution

we will do gradient descent!!

$$w_{\text{new}} = w_{\text{old}} - \gamma \cdot \nabla_w J(w^k)$$

learning rate

$$③ \hat{J} = -\frac{1}{N} \sum y_i \log(\hat{y}_i) + (1-y_i) \log(1-\hat{y}_i)$$

$$④ \hat{g} = \sigma(z) = \frac{1}{1 + e^{-z}}$$

$$① z = w^T x$$

$$\frac{\partial J}{\partial w} = \frac{\partial J}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial z}{\partial w}$$

\* For one sample:

$$L(w) = -[y \log \hat{y} + (1-y) \log(1-\hat{y})]$$

$$\frac{\partial L}{\partial \hat{y}} = -\left[y \cdot \frac{1}{\hat{y}} + (1-y) \cdot \frac{-1}{1-\hat{y}}\right]$$

$$= -\frac{y}{\hat{y}} + \frac{1-y}{1-\hat{y}}$$

$$= \frac{-y(1-\hat{y}) + (1-y)\hat{y}}{\hat{y}(1-\hat{y})}$$

$$= \frac{-y + y\hat{y} + \hat{y} - y\hat{y}}{\hat{y}(1-\hat{y})}$$

$$\frac{\partial L}{\partial \hat{y}} = \frac{\hat{y} - y}{\hat{y}(1-\hat{y})}$$

$$\star \frac{\partial \hat{y}}{\partial z} = ? \quad \hat{y} = \begin{cases} 1 \\ 0 \end{cases} + \exp(-z)$$

$$\left( \begin{array}{c} u \\ v \end{array} \right) \rightarrow \frac{u'v - v'u}{\sqrt{2}}$$

$$\frac{\partial \hat{y}}{\partial z} = 0 \rightarrow \frac{-\exp(-z) \cdot 1}{[1 + \exp(-z)]^2}$$

$$= \frac{\exp(-z)}{[1 + \exp(-z)]^2} = \frac{1 - 1 + \exp(-z)}{[1 + \exp(-z)]^2}$$

$$\frac{1 + \exp(-z)}{[1 + \exp(-z)]^2} =$$

$$\frac{(1 + \exp(-z))}{[1 + \exp(-z)]^2} - \frac{1}{[1 + \exp(-z)]^2} =$$

$$\frac{1}{(1 + \exp(-z))} - \left[ \frac{1}{1 + \exp(-z)} \right]^2 =$$

$$\Gamma(z) - \Gamma(z)^2 =$$

derivative  
of sigmoid

$$\Gamma(z) \cdot [1 - \Gamma(z)]$$

$$\frac{\partial z}{\partial w} = ?? \quad z = w^T x$$

$$\Leftrightarrow \frac{\partial z}{\partial w} = x$$

$$\frac{\partial L}{\partial w} = \frac{\hat{y} - y}{\hat{y}(1-\hat{y})} \cdot g'(l, \hat{y}) \cdot x$$

$$\boxed{\frac{\partial L}{\partial w} = (\hat{y} - y) \cdot x}$$

general one:

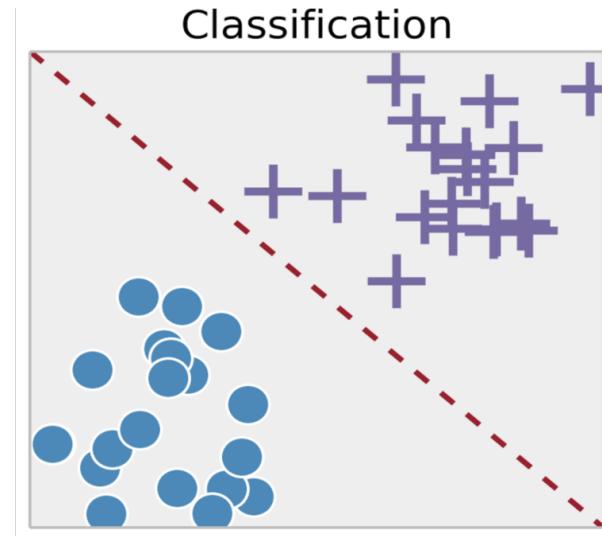
$$\nabla J = \frac{1}{n} (\hat{y} - y) \cdot x$$

# Linear Model in Disguise

$$\hat{y} = \mathbf{w}^T \mathbf{x}$$

$$\mathbf{w} = [w_0, w_1, \dots, w_m]^T$$

$$\mathbf{x} = [1, x^1, \dots, x^m]^T$$



$$\{(\mathbf{x}_i, y_i)\}_{i=1}^N, \mathbf{x}_i \in \mathbb{R}^n, y_i \in \{0, 1\}$$

# Linear Model in Disguise

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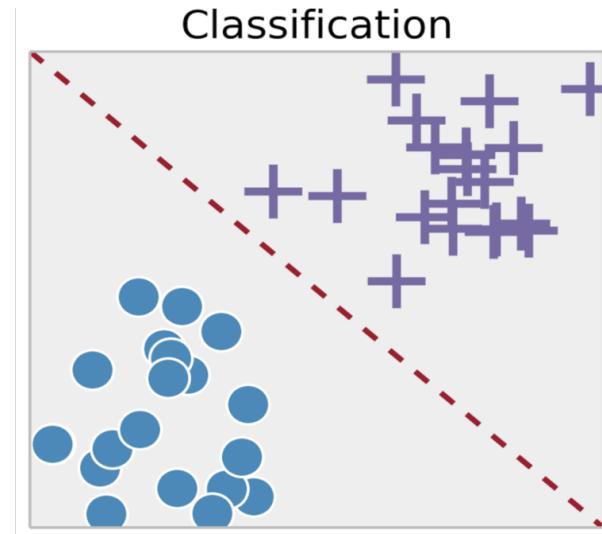
$$\mathbf{x} = [1, x^1, \dots, x^m]^T$$

---

$$\hat{y} \approx y$$

Recall that the output/label  $y$  is binary

How to map the predictions to binary?



$$\{(\mathbf{x}_i, y_i)\}_{i=1}^N, \mathbf{x}_i \in \mathbb{R}^n, y_i \in \{0, 1\}$$

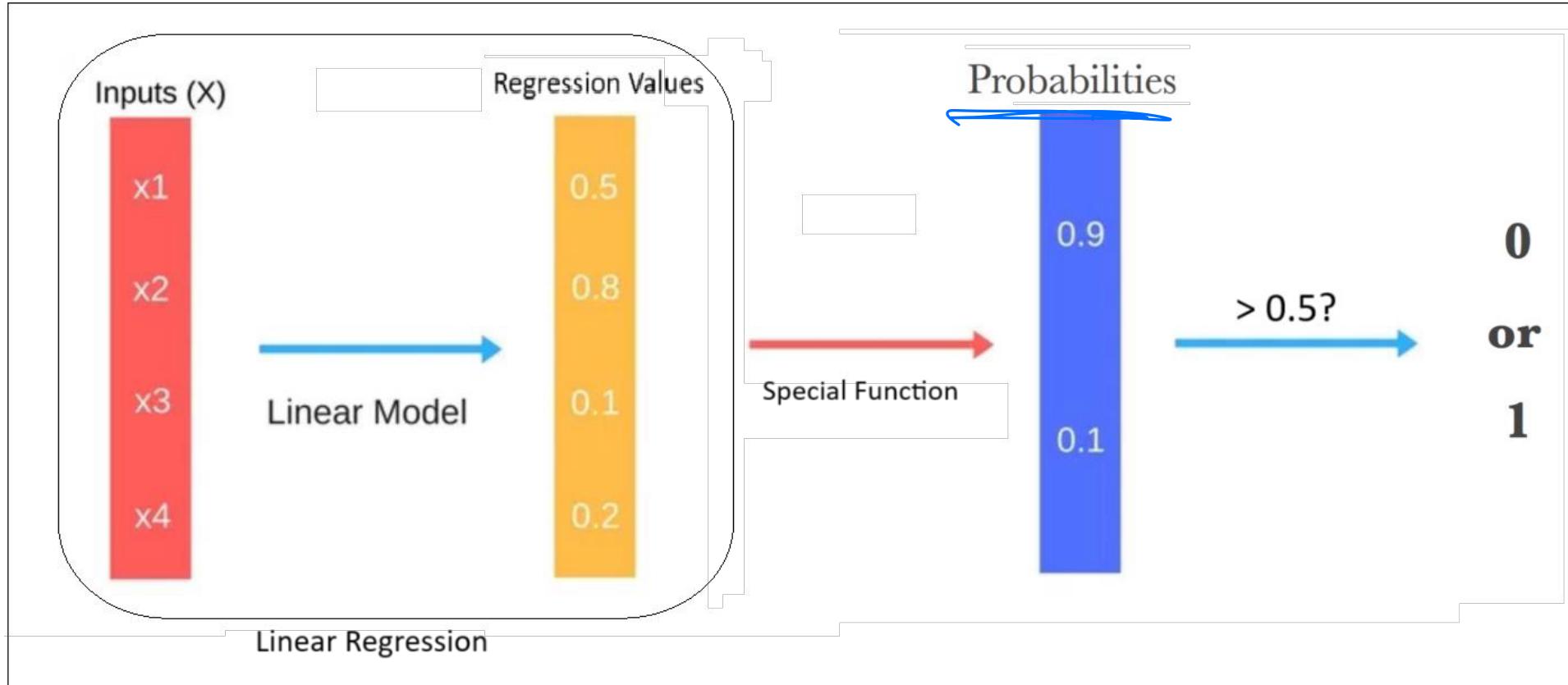
# Classifiers

Classifiers are **regression** models with two key modifications:

- Pass the regression outputs through a **special function** to convert them into probabilities.
- Replace the Mean Squared Error loss with a **new loss function** designed for probabilities.

Applying these two to the Linear Regression should give Linear Classifier (called **Logistic Regression**).

# Logistic Regression



# Special Function



We are searching for a function that have the following characteristics:

1. Could convert any arbitrary input values to **[0,1] range (Probabilities)**.
2. **Smooth and Differentiable**. Because we want to find the minima later, right?

Any ideas?

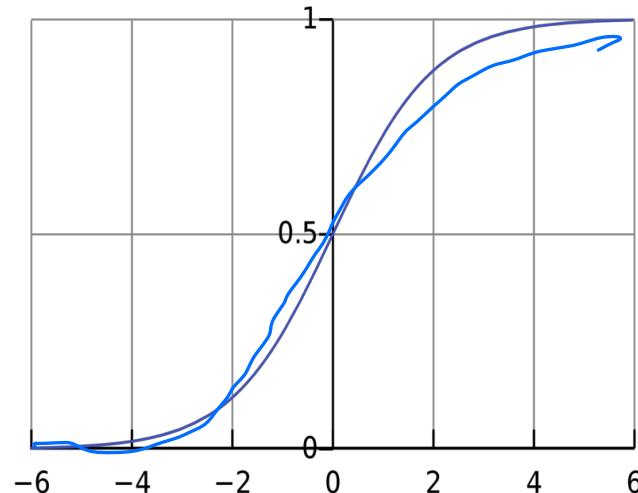
# Sigmoid Function (for Binary Classification)

$$\sigma(z) = \frac{1}{1 + \exp(-z)}$$

$$\sigma'(z) = \sigma(z)(1 - \sigma(z))$$

Assuming you have negative and positive classes,  
its output would be the probability of the  
positive class.

But what if we have multiclass  
problem? 🤔



$$\lim_{z \rightarrow -\infty} \sigma(z) = 0$$

$$\lim_{z \rightarrow \infty} \sigma(z) = 1$$

# Softmax Function (for Multiclass Classification)

$$\text{Softmax}(z_i) = \frac{\exp(z_i)}{\sum_{j=1}^n \exp(z_j)}$$

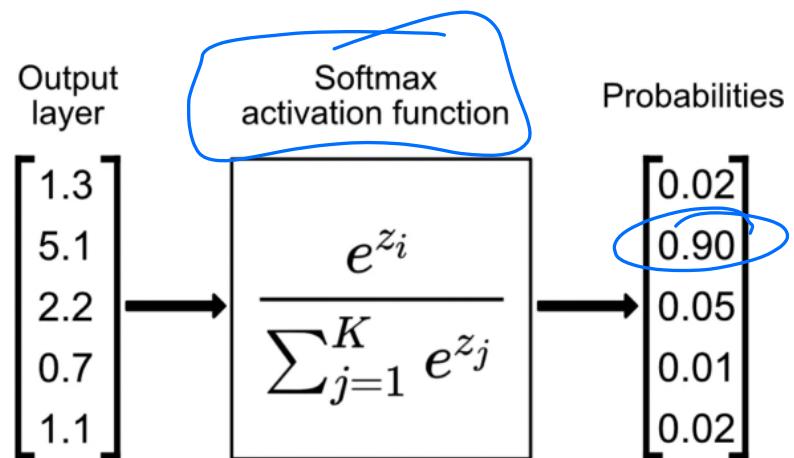
Numerator: The exponential of the  $i$ th class  $\exp(z_i)$ .

Denominator: The sum of the exponentials of all classes  $\sum_{j=1}^n \exp(z_j)$ .

E.g. Divide the cat value by the cat, dog and deer values to get the cat probability.

Assuming you have  $K$  classes, its output would be the probability of the  $i$ th class. So, you will run it  $K$  times to get the probability of each class.

Note: Sigmoid is a special case of Softmax.  
Optional: Can you prove it?



# Classification Loss

We are searching for a loss that have the following characteristics:

1. **Probabilistic behaviour:** Should work with probabilities, not raw numbers.
2. **Sensitivity:** It should heavily penalize incorrect confident predictions (e.g., predicting 0.9 when the true label is 0).
3. **Differentiable:** Must be smooth and differentiable to enable optimization.

Any ideas?

# Classification Loss



We are searching for a loss that have the following characteristics:

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Any ideas?

**Logarithm is all you need :)**

# Binary Cross Entropy (LogLoss)

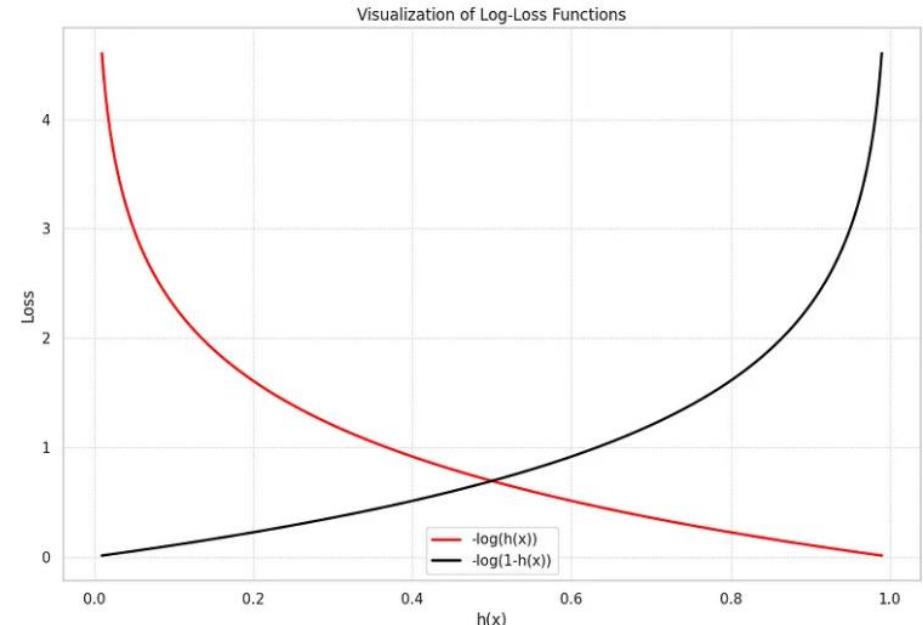
- For one sample, this can be represented mathematically by:

$$\text{loss}(y, p) = \begin{cases} -\log(p), & \text{if } y = 1 \\ -\log(1 - p), & \text{if } y = 0 \end{cases}$$

If  $y = 1$  and prediction is close to 1, loss will be close to **zero**. (Great!)

If  $y = 1$  and prediction is close to 0, loss will be close to **infinity**. (Very bad!)

Same behaviour for  $y = 0$



# Binary Cross Entropy (LogLoss)

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- Let's rewrite it in one line:


$$\text{loss}(y, p) = -(y * \log(p) + (1 - y) * \log(1 - p))$$

But we have many samples, not only one, right?

# Binary Cross Entropy (LogLoss)

- For one sample, this can be represented mathematically by:

$$\text{loss}(y, p) = \begin{cases} -\log(p), & \text{if } y = 1 \\ -\log(1 - p), & \text{if } y = 0 \end{cases}$$

- Let's write it in one line:

$$\text{loss}(y, p) = -(y * \log(p) + (1 - y) * \log(1 - p))$$

- Sum and average:



$$\text{logloss}(Y, P) = -\frac{1}{N} \sum_{i=1}^N (y_i * \log(p_i) + (1 - y_i) * \log(1 - p_i))$$

# How to find optimal Parameters?

$$J(\mathbf{w}) = -\frac{1}{N} \sum_{i=1}^N y_i \log(\sigma(\mathbf{w}^T \mathbf{x}_i)) + (1 - y_i) \log(1 - \sigma(\mathbf{w}^T \mathbf{x}_i))$$

Just like before, simply take

$$\nabla_{\mathbf{w}} J(\mathbf{w}) = 0$$

However, this does not have a nice closed solution  
Just like the MSE case

# How to find optimal Parameters?

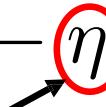
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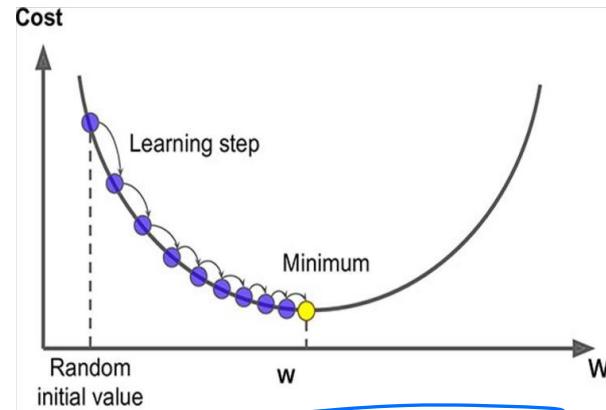
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However, this does not have a nice closed solution  
Just like the MSE case

  $\mathbf{w}^{k+1} = \mathbf{w}^k - \eta \nabla_{\mathbf{w}} J(\mathbf{w}^k)$

 **Learning rate**



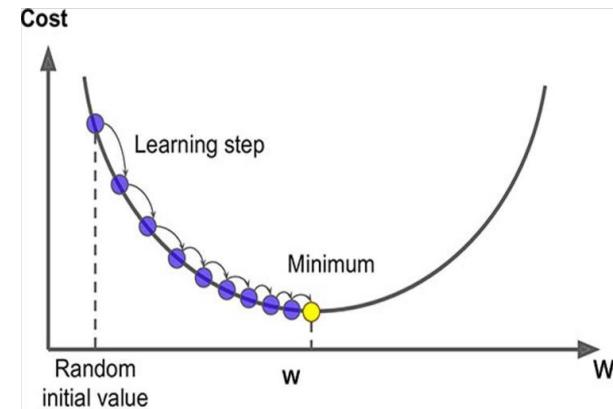
Instead, use gradient descent  
(optimizer)!

# Logistic Regression

$$J(\mathbf{w}) = -\frac{1}{N} \sum_{i=1}^N y_i \log(\sigma(\mathbf{w}^T \mathbf{x}_i)) + (1 - y_i) \log(1 - \sigma(\mathbf{w}^T \mathbf{x}_i))$$

$$\mathbf{w}^{k+1} = \mathbf{w}^k - \eta \nabla_{\mathbf{w}} J(\mathbf{w}^k)$$

Learning rate



- We have a linear model for prediction
- For classification, we want to output a probability
- We map the prediction to probabilities with a sigmoid function
- We have a loss function (BCE) to compare models

# Logistic Regression

Linear Regression	Logistic Regression
For Regression	For Classification
We predict the target value for any input value	We predict the probability that the input value belongs to the specific target
Target: Real Values	Target: Discrete values
Graph: Straight Line	Graph: S-curve

Binary

• Sigmoid

• BCE

Multiclass

• Softmax:  $\frac{\exp(z_i)}{\sum_{i=1}^k \exp(z_i)}$

• Cross entropy

$$L(y, \hat{y}) = -\sum_{j=1}^k y_j \log(\hat{y}_j)$$

where  $y_j$  is zero except  
for the true label  $y_j = 1$ .

# Classification Metrics

- **Accuracy:**

Accuracy measures the proportion of correctly classified samples out of the total number of samples.

$$Accuracy = \frac{Correct\ Samples}{All\ Sample}$$

**Question:** Why not optimizing for the accuracy directly instead of using LogLoss?

# Classification Metrics

- **Accuracy:**

Accuracy measures the proportion of correctly classified samples out of the total number of samples.

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**Question:** Why not optimizing for the accuracy directly instead of using LogLoss?  
Non-differentiable

# Classification Metrics

- **Accuracy:**

Accuracy measures the proportion of correctly classified samples out of the total number of samples.

$$\text{Accuracy} = \frac{\text{Correct Samples}}{\text{All Sample}}$$

Could also be written as:

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$

Where:

TP: True Positives

TN: True Negatives

FP: False Positives

FN: False Negatives

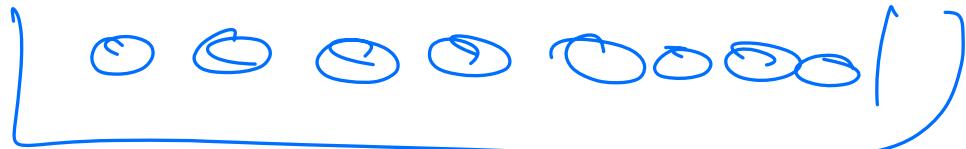
**Question:** Why not optimizing for the accuracy directly instead of using LogLoss?

Non-differentiable

# Classification Metrics

But accuracy isn't always enough!

Imagine an imbalanced dataset with 95% negative labels (e.g., "not spam").  
A model predicting "negative" all the time will be 95% accurate but useless.



$$\frac{90}{100}$$

# Classification Metrics

**But accuracy isn't always enough!**

Imagine an imbalanced dataset with 95% negative labels (e.g., "not spam").  
A model predicting "negative" all the time will be 95% accurate but useless.

**Different goals require different metrics**

- Do you care more about **catching all positives** (e.g., cancer detection)?
- Or avoiding **false alarms** (e.g., fraud detection)?

# Classification Metrics

- **Precision:**

Among all the positive predictions the model made,  
how many are actually correct?

important when **minimizing false alarms** is critical,  
like in fraud detection or spam filtering.

$$\text{Precision} = \frac{\text{True Positives (TP)}}{\text{True Positives (TP)} + \text{False Positives (FP)}}$$

# Classification Metrics

- **Recall:**

Among all the actual positives, how many did the model correctly identify?

important when **catching all positives** is vital, such as in cancer detection.

$$Recall = \frac{True\ Positives\ (TP)}{True\ Positives\ (TP) + False\ Negatives\ (FN)}$$

negative

# Classification Metrics

- **F1 Score:**

What if both precision and recall are important?  
Take their (harmonic) mean.

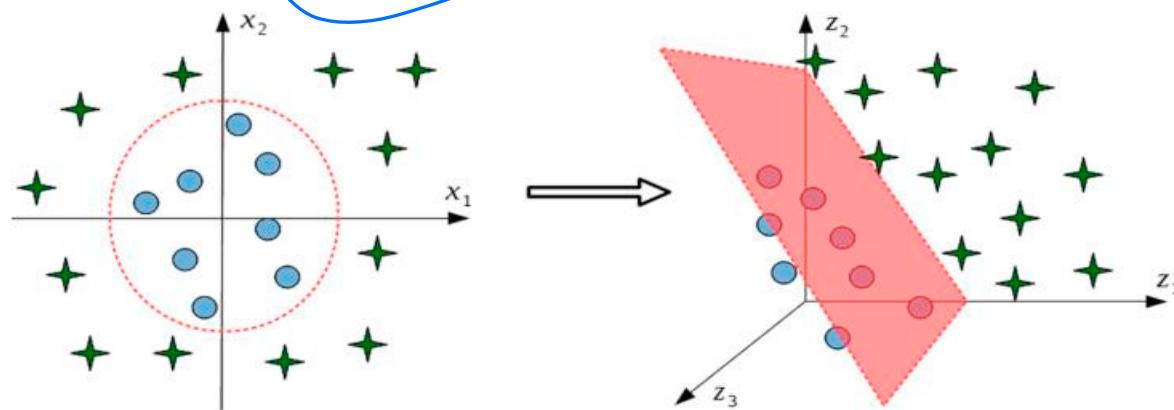
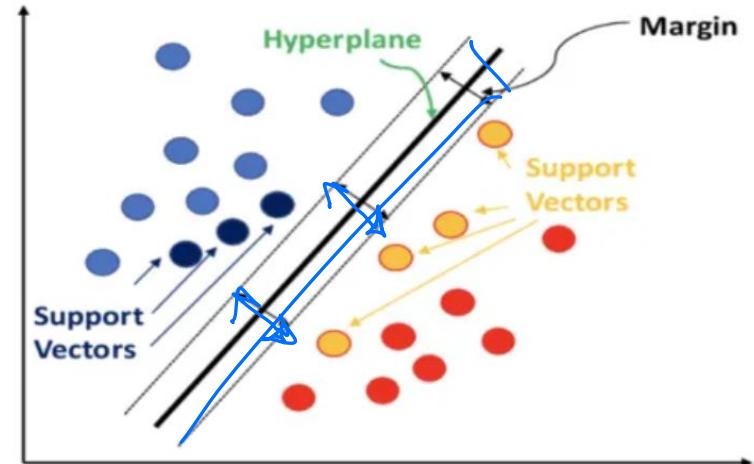
$$F1\ score = \frac{2}{\frac{1}{Precision} + \frac{1}{Recall}}$$

# Other ML Models

- **Support Vector Machine (SVM):**

**Key Idea:**

- SVM finds the best boundary (hyperplane) that separates classes by maximizing the margin between them.
- Works well with high-dimensional data and small datasets.
- Can be computationally expensive for large datasets.

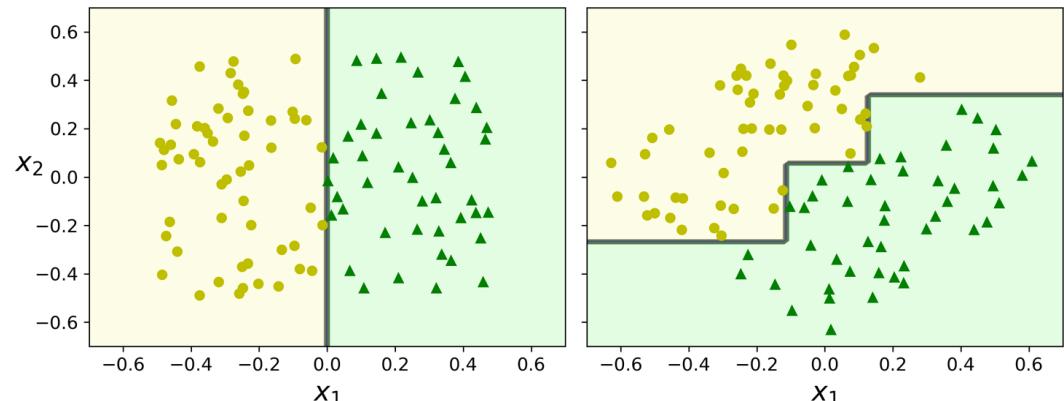
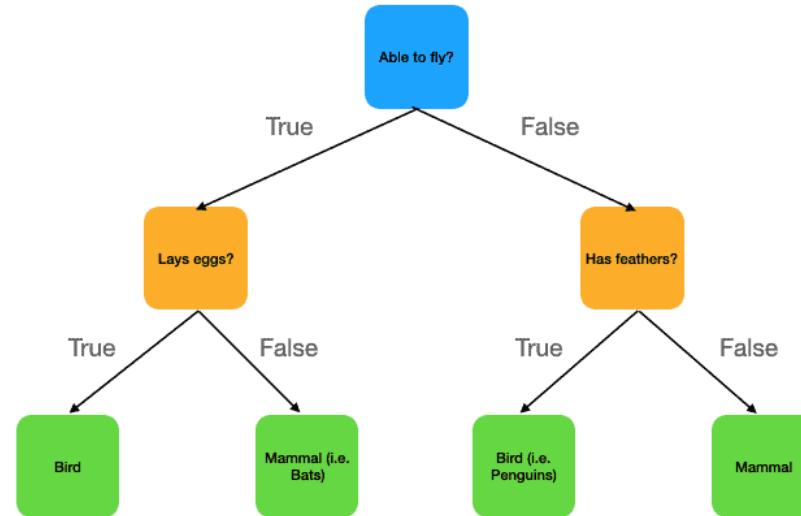


# Other ML Models

- **Decision Trees:**

**Key Idea:**

- A tree-based model splits data into subsets based on feature thresholds, creating a flowchart-like structure for decision-making.
- Overfits very easily.
- could work bad if relation between features and the target is linear. (Check the right figure)



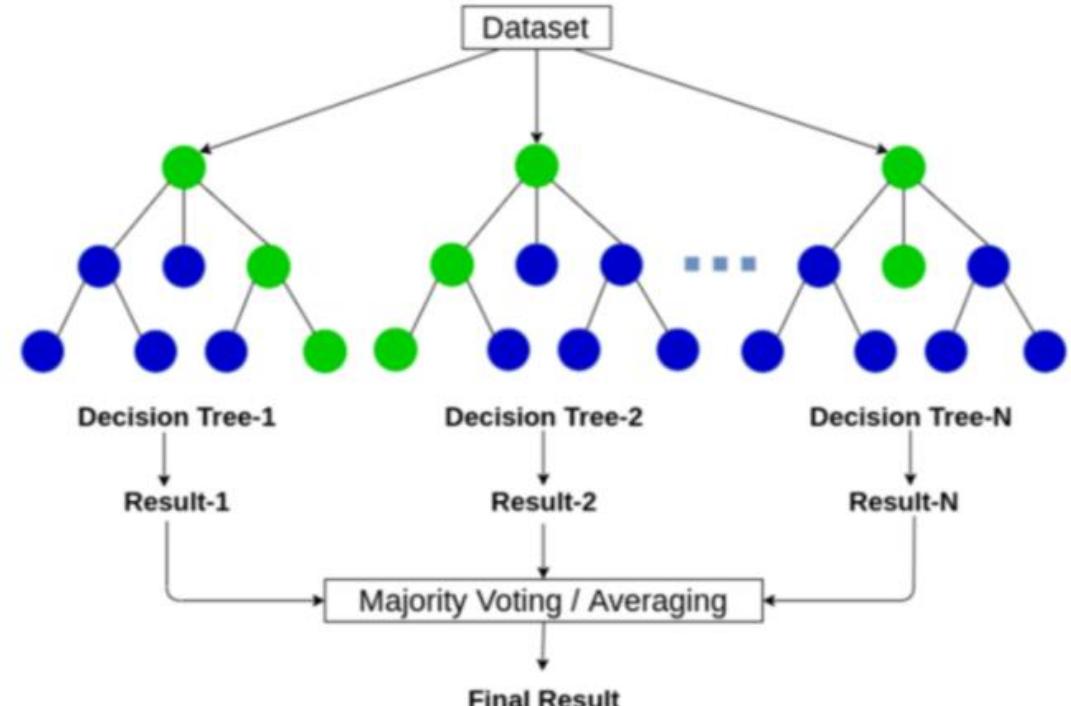
# Other ML Models

- **Random Forest:**

Instead of training one tree, let's train a forest :)

**Key Idea:**

- Training K decision trees independently, where each one is fed with a different subset of samples and features.
- Then average the results (Regression) or Take the mode (Classification).
- Powerful model.

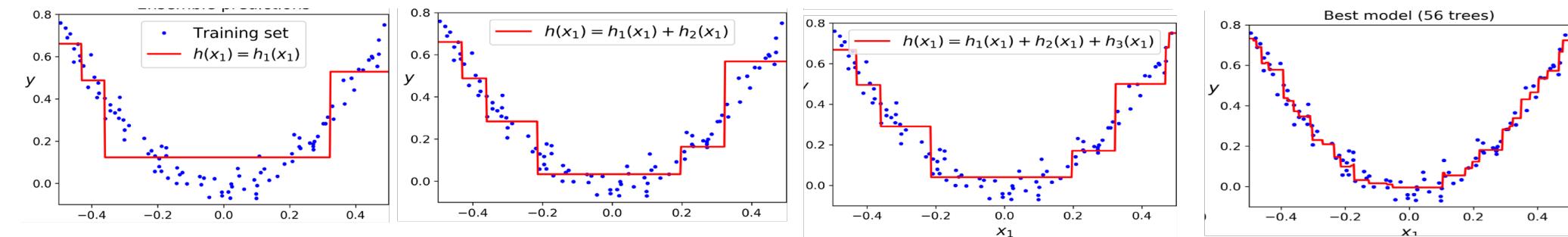
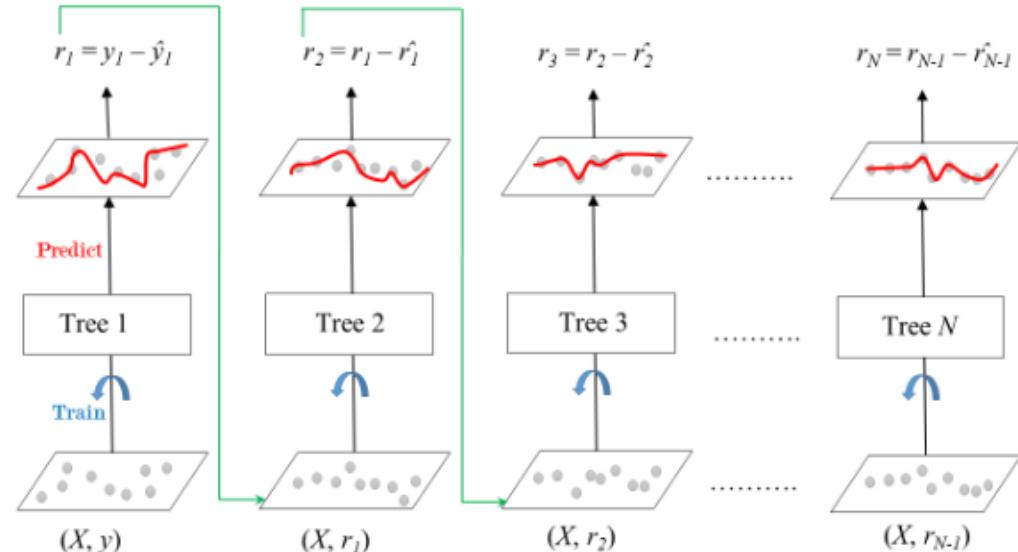


# Other ML Models

- **Gradient Boosting:**

**Key Idea:**

- Training K decision trees independently, where each tree corrects the errors of the previous one.
- Many versions: XGBoost, LightGBM, CatBoost,...etc
- Top ML models.



# A Slight Detour: A Look at Optimization Tools

# Introduction

- **Optimizers** are algorithms that adjust the weights of the model to minimize the loss function during training.
- They are one of the main components of Deep Learning models.
- We will go through several types of optimizers in this section.

# Direction of maximum increase and decrease for a function

- Gradient direction is the direction of maximum increase for a function
- Negative gradient is the direction of maximum decrease for a function

# Gradient Descent

```
# Vanilla Gradient Descent
```

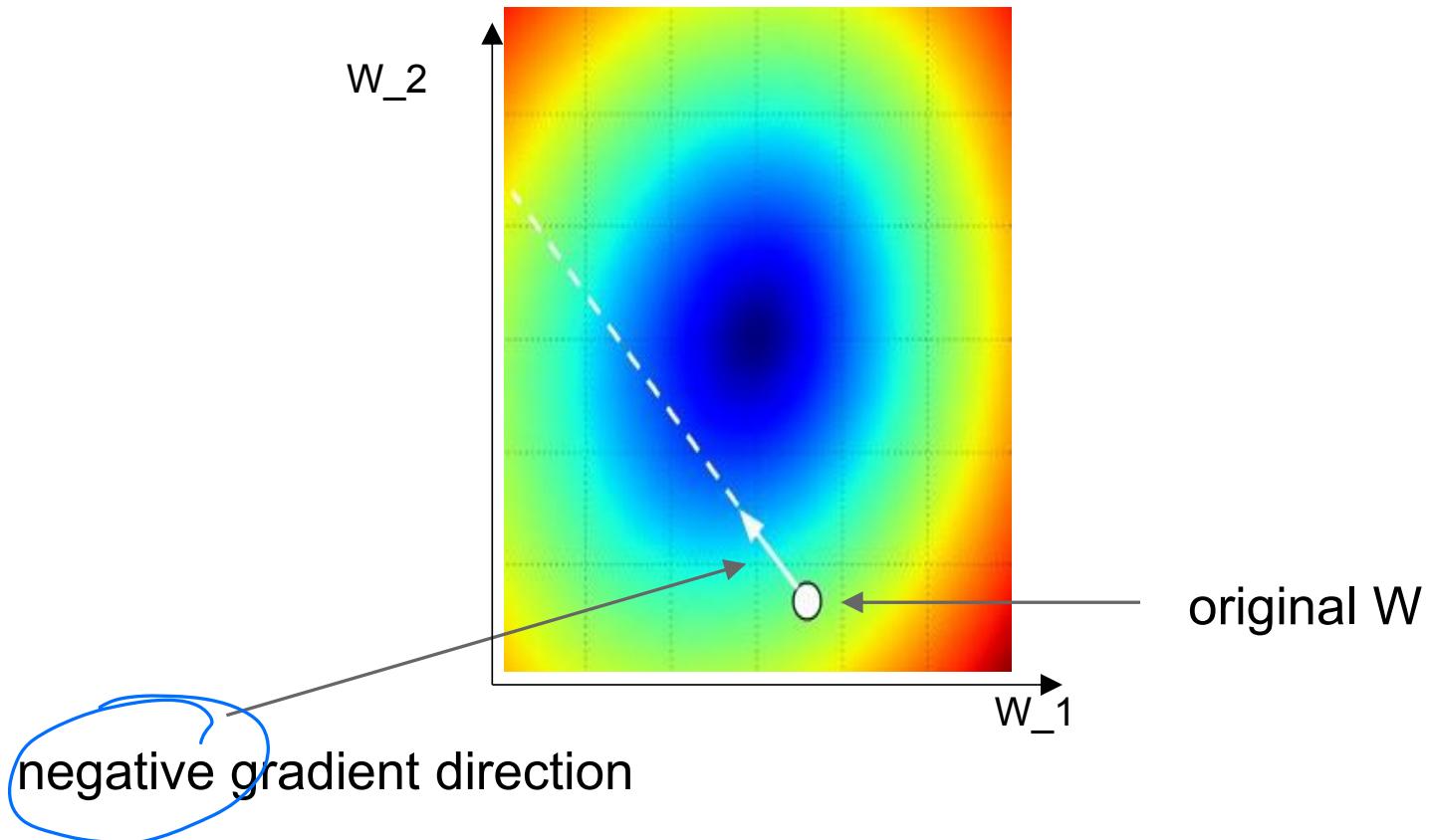
```
while True:
```

```
    weights_grad = evaluate_gradient(loss_fun, data, weights)
```

```
    weights += - step_size * weights_grad # perform parameter update
```

derivative

$$w = w - \gamma \cdot \text{gradient}.$$



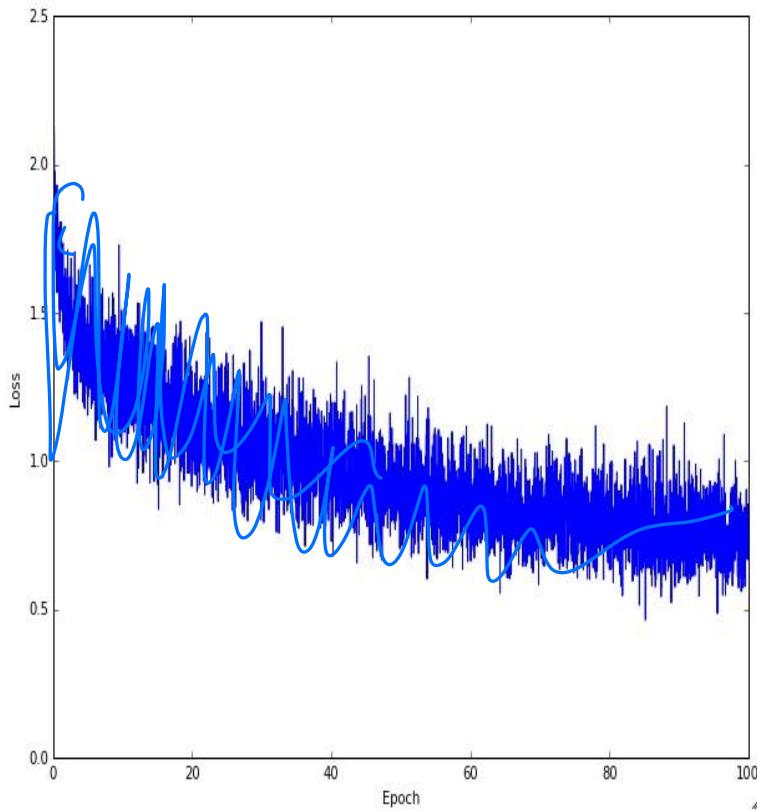
# Mini-batch Gradient Descent

- only use a small portion of the training set to compute the gradient.

```
# Vanilla Minibatch Gradient Descent

while True:
    data_batch = sample_training_data(data, 256) # sample 256 examples
    weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
    weights += - step_size * weights_grad # perform parameter update
```

Common mini-batch sizes are 32/64/128 examples  
e.g. Krizhevsky ILSVRC ConvNet used 256 examples



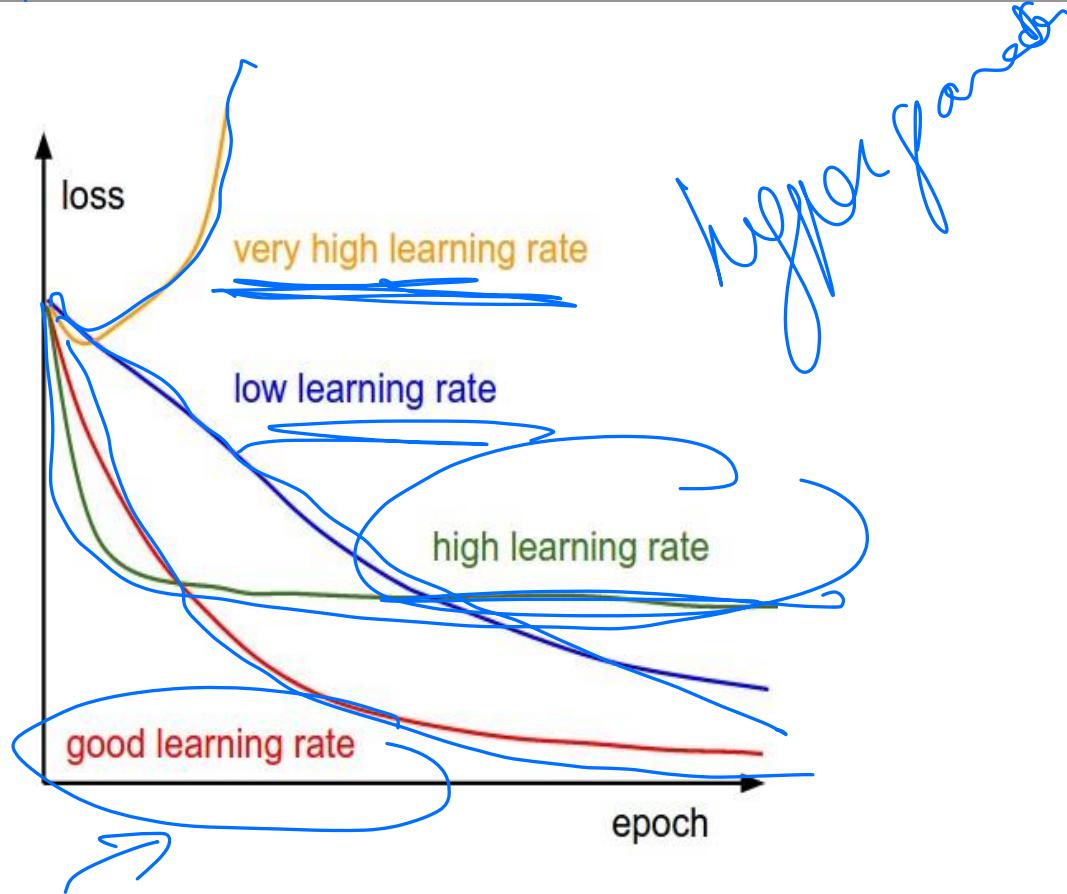
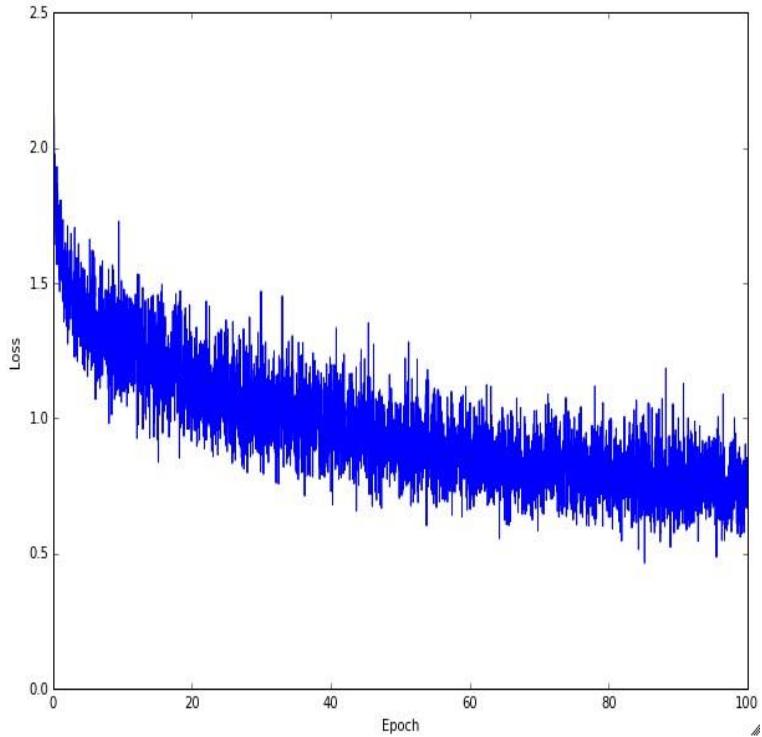
Example of optimization progress while training a neural network.

(Loss over mini-batches goes down over time.)

Since we can not see all the data, the update will not be exactly decreasing, we will have some noise (oscillation) until we reach the local minimum.



# The effects of step size (or “learning rate”)



# Mini-batch Gradient Descent

- only use a small portion of the training set to compute the gradient.

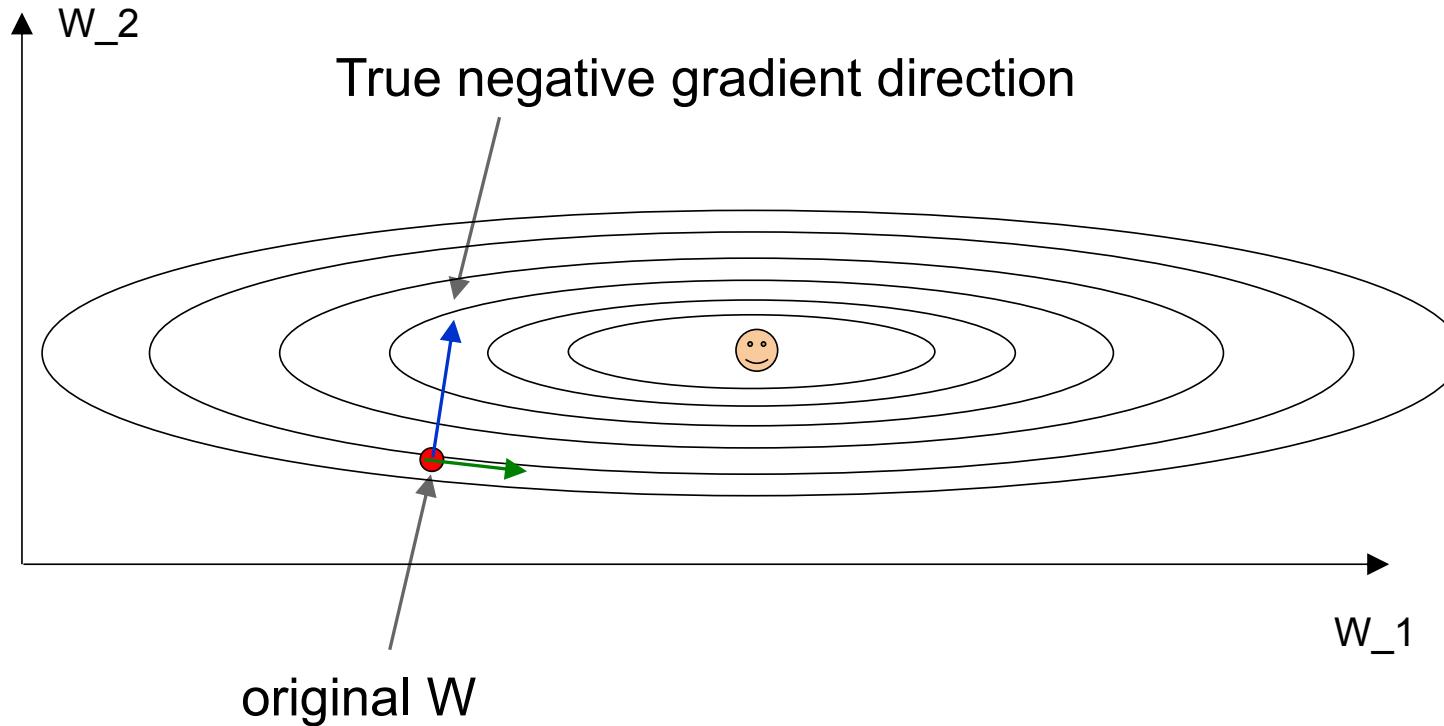
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    weights += - step_size * weights_grad # perform parameter update
```

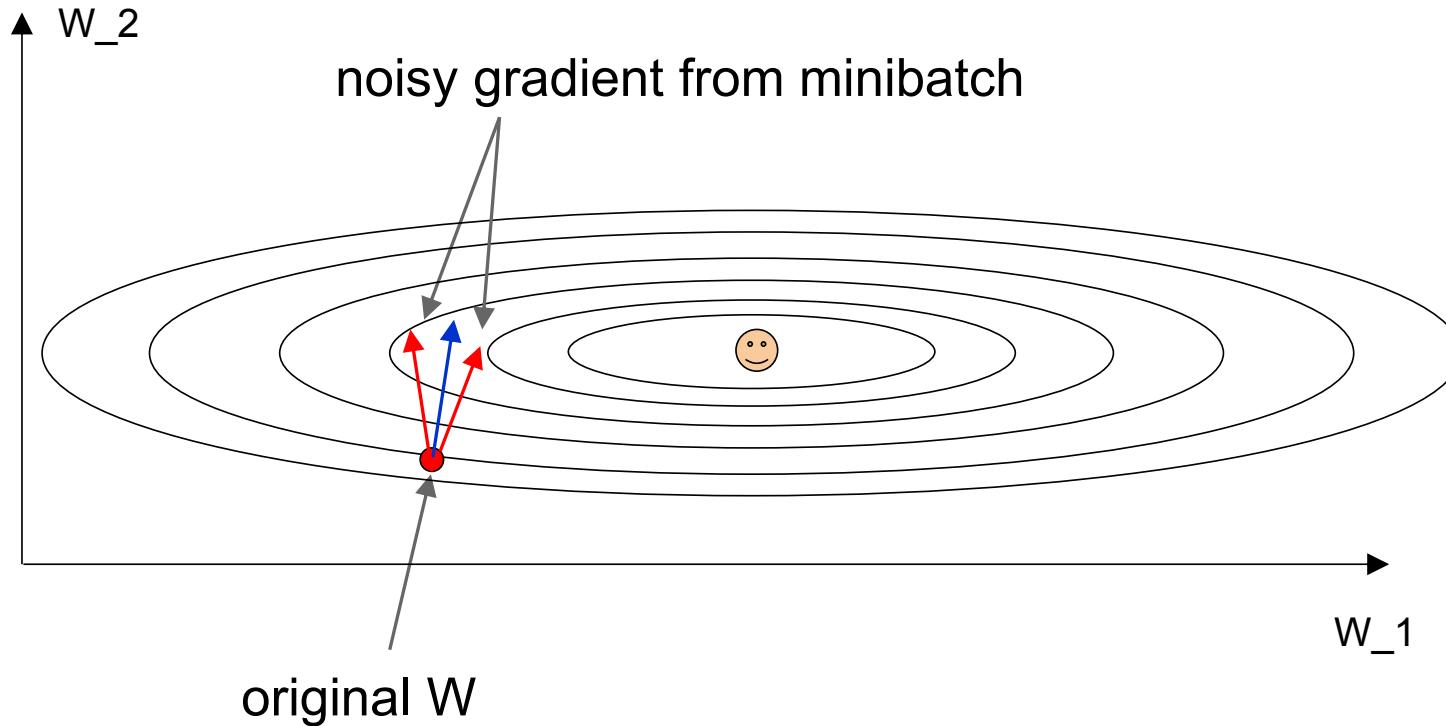
Common mini-batch sizes are 32/64/128 examples  
e.g. Krizhevsky ILSVRC ConvNet used 256 examples

we will look at more fancy update formulas (momentum, Adagrad, RMSProp, Adam, ...)

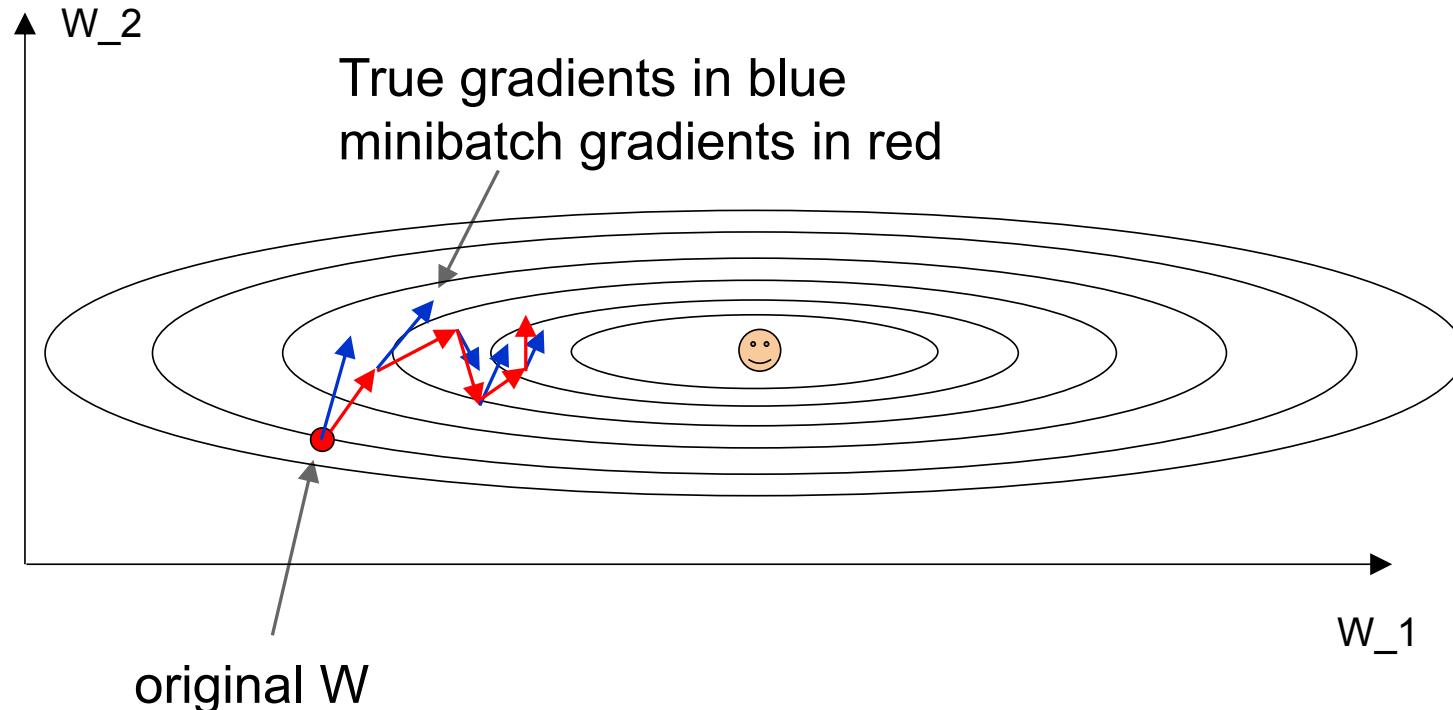
# Minibatch updates



# Stochastic Gradient

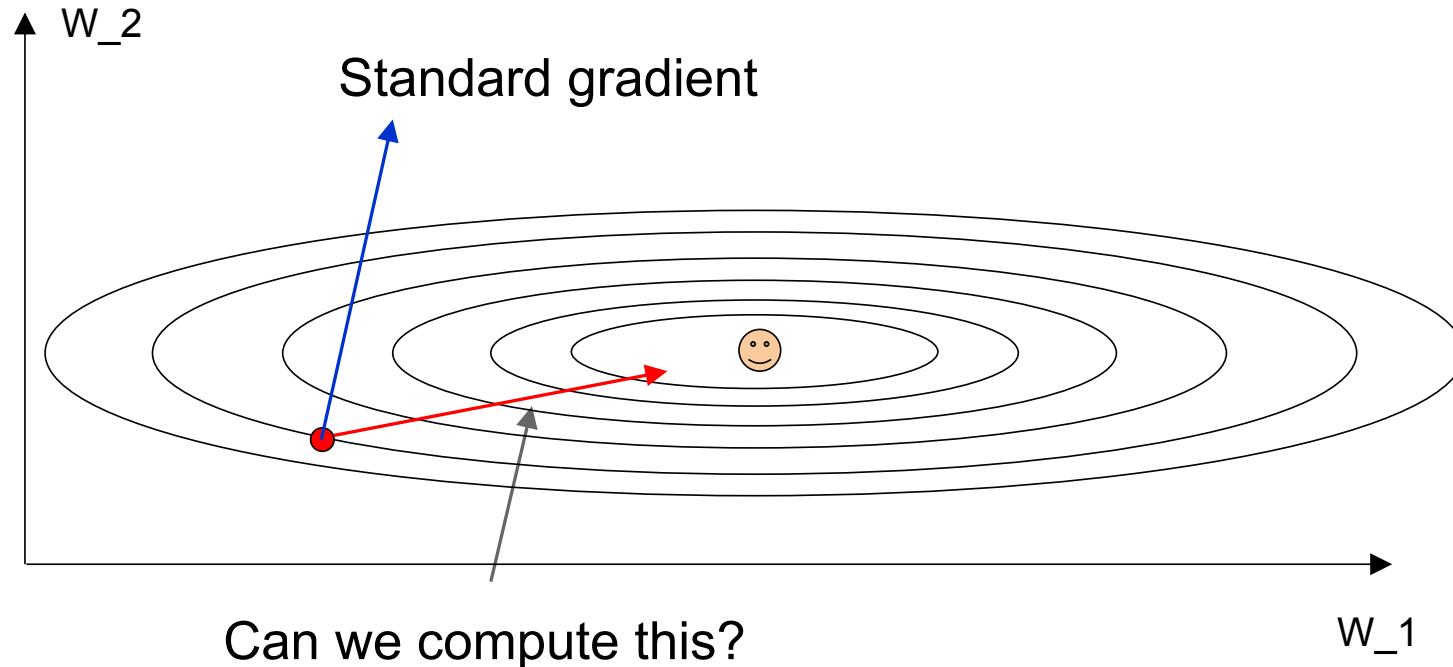


# Stochastic Gradient Descent

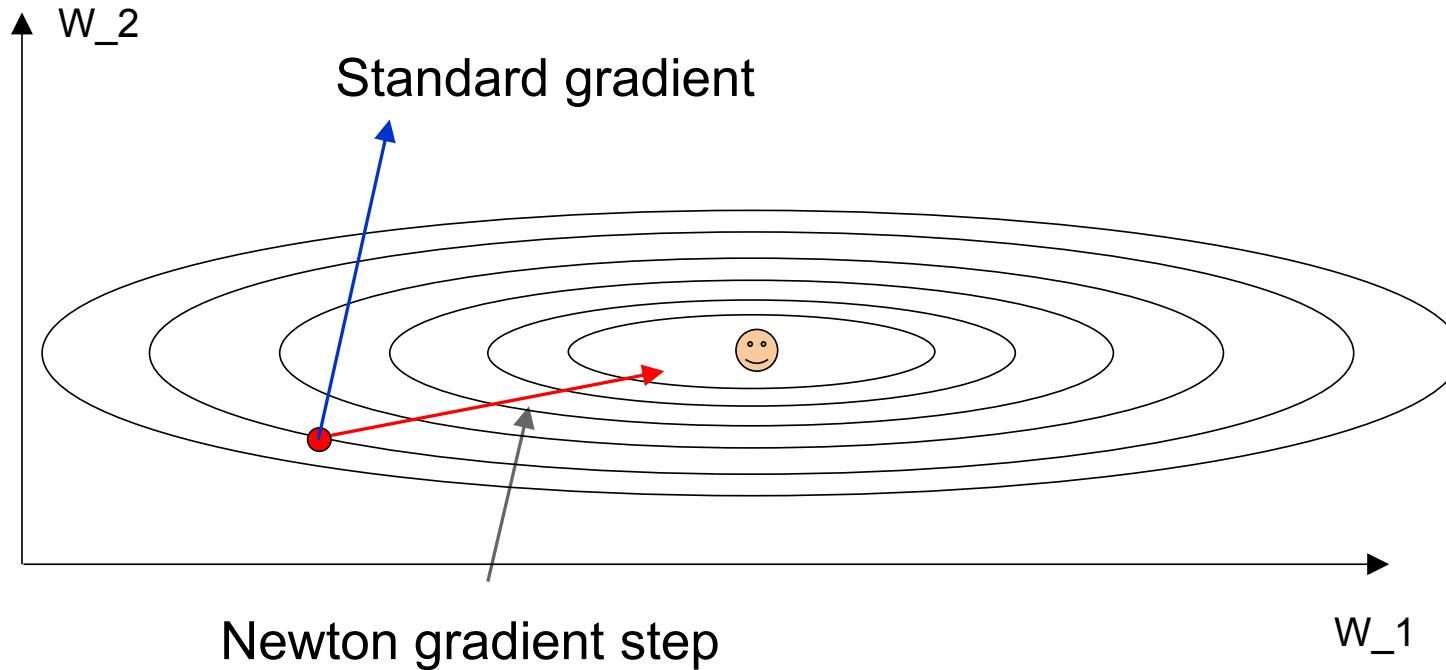


Gradients are noisy but still make good progress on average

# You might be wondering...



# Newton step



# Newton's method for gradients:

The Newton update is:

$$x_{n+1} = x_n - H_f(x_n)^{-1} \nabla f(x_n)$$

Where  $H_f$  is the Hessian matrix of second derivatives of  $f$ .

Converges very fast, but rarely used in DL. Why?

Newton's method has quadratic convergence

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$$x_{n+1} = x_n - H_f(x_n)^{-1} \nabla f(x_n)$$

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Too expensive: if  $x_n$  has dimension M, the Hessian  $H_f(x_n)$  has dimension  $M^2$  and takes  $O(M^3)$  time to invert.

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**Too expensive:** if  $x_n$  has dimension M, the Hessian  $H_f(x_n)$  has dimension  $M^2$  and takes  $O(M^3)$  time to invert.

**Too unstable:** it involves a high-dimensional matrix inverse, which has poor numerical stability. The Hessian may even be singular.

# Momentum update

```
# Gradient descent update  
x += - learning_rate * dx
```



```
# Momentum update  
v = mu * v - learning_rate * dx # integrate velocity  
x += v # integrate position
```

momentum ;

\* Build up velocity along shallow direction (slope is gradual)  
as gradient is small

- Physical interpretation as ball rolling down the loss function + friction (mu coefficient).
- mu = usually ~0.5, 0.9, or 0.99 (Sometimes annealed over time, e.g. from 0.5 -> 0.99)

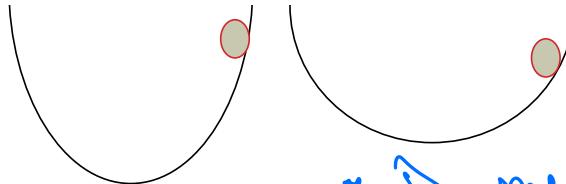
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# Gradient descent update  
x += - learning_rate * dx
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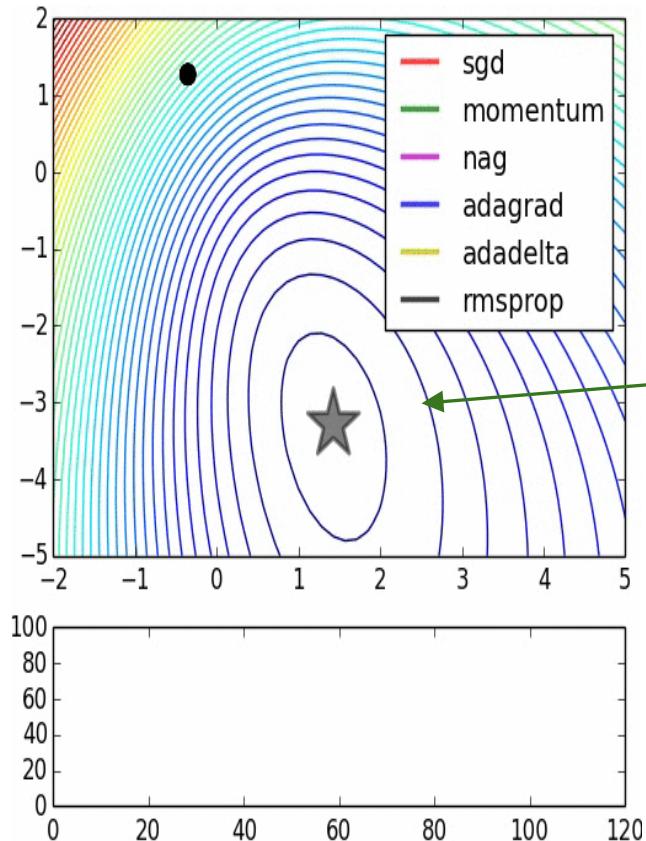
```
# Momentum update  
v = mu * v - learning_rate * dx # integrate velocity  
x += v # integrate position
```

- Allows a velocity to “build up” along shallow directions
- Velocity becomes damped in steep direction due to quickly changing sign



\* Damping velocity  
Steep direction  
(slope change rapidly)  
↳ gradient is large

# SGD VS Momentum



notice momentum  
overshooting the target,  
but overall getting to the  
minimum much faster  
than vanilla SGD.

• Nesterov momentum update (Nag)

⇒ reduce the overshooting

look ahead: reduce the velocity when  
we are near minimum (not  
overshoot)



# AdaGrad update

[Duchi et al., 2011]



```
# Adagrad update  
cache += dx**2  
x += - learning_rate * dx / (np.sqrt(cache) + 1e-7)
```

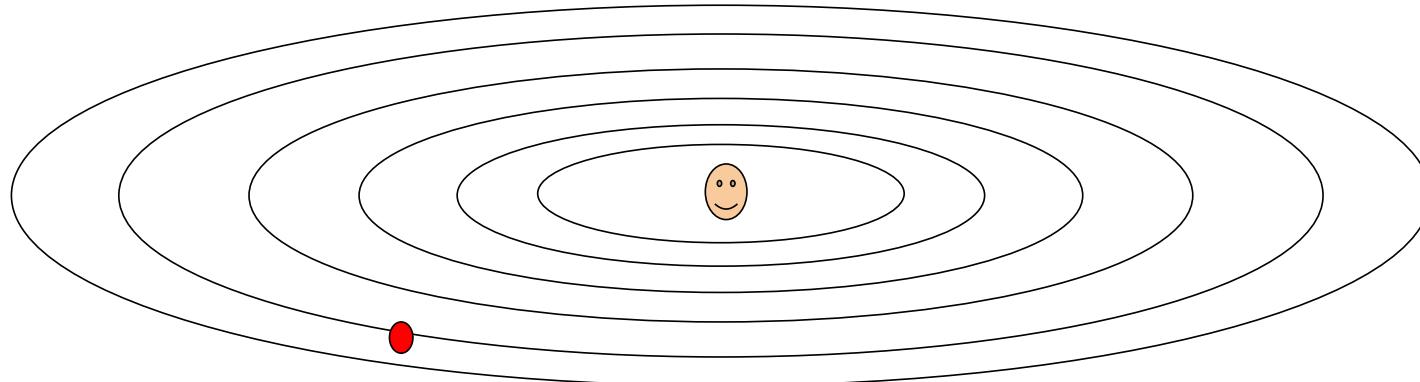
Added element-wise scaling of the gradient based on the historical sum of squares in each dimension

adaptive learning rate  $\rightarrow$  automatic learning adjustment.

- giving a large update to parameters associated with infrequent features which enable a faster adaptation to the rare occurrence of these features.
  - (rarely occur in dataset)
- for these cases we need a large learning rate for faster adaptation.
- \* for frequent features: A large update. a smaller update. this is to prevent their features from dominating the learning process due to their high occurrence.
- \* However, we have potential for raising learning rate.

# AdaGrad update

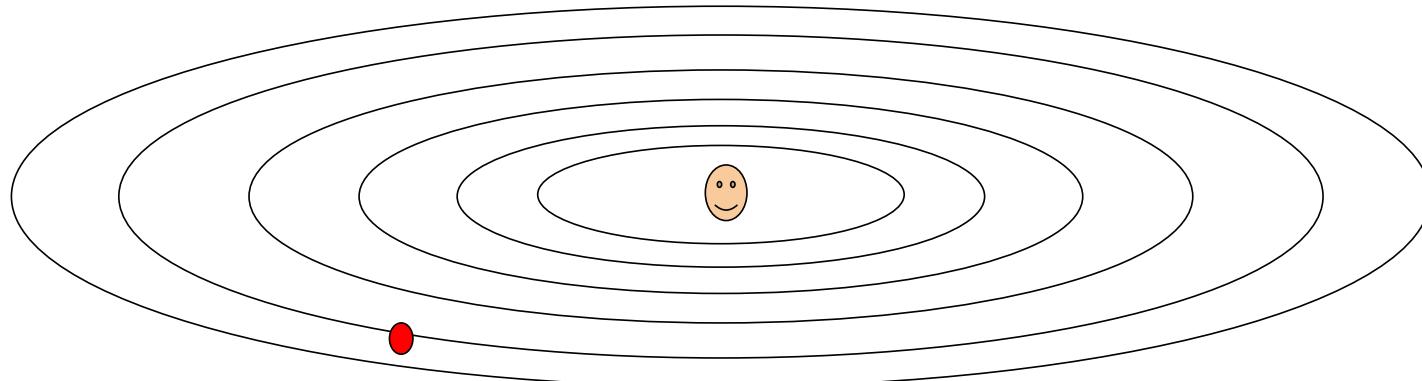
```
# Adagrad update
cache += dx**2
x += - learning_rate * dx / (np.sqrt(cache) + 1e-7)
```



Q: What happens with AdaGrad?

# AdaGrad update

```
# Adagrad update
cache += dx**2
x += - learning_rate * dx / (np.sqrt(cache) + 1e-7)
```



Q2: What happens to the step size over long time?



# RMSProp update

[Tieleman and Hinton, 2012]



```
# Adagrad update
cache += dx**2
x += - learning_rate * dx / (np.sqrt(cache) + 1e-7)
```



we use it to  
avoid having  
the vanishing  
problem..

```
# RMSProp
cache = decay_rate * cache + (1 - decay_rate) * dx**2
x += - learning_rate * dx / (np.sqrt(cache) + 1e-7)
```

## rmsprop: A mini-batch version of rprop

- rprop is equivalent to using the gradient but also dividing by the size of the gradient.

– The problem with mini-batch rprop is that we divide by a different number for each mini-batch. So why not force the number we divide by to be very similar for adjacent mini-batches?

- rmsprop: Keep a moving average of the squared gradient for each weight

$$\text{MeanSquare}(w, t) = 0.9 \text{ MeanSquare}(w, t-1) + 0.1 \left( \frac{\partial E}{\partial w}(t) \right)^2$$

- Dividing the gradient by  $\sqrt{\text{MeanSquare}(w, t)}$  makes the learning work much better (Tijmen Tieleman, unpublished).

Introduced in a slide in  
Geoff Hinton's Coursera  
class, lecture 6

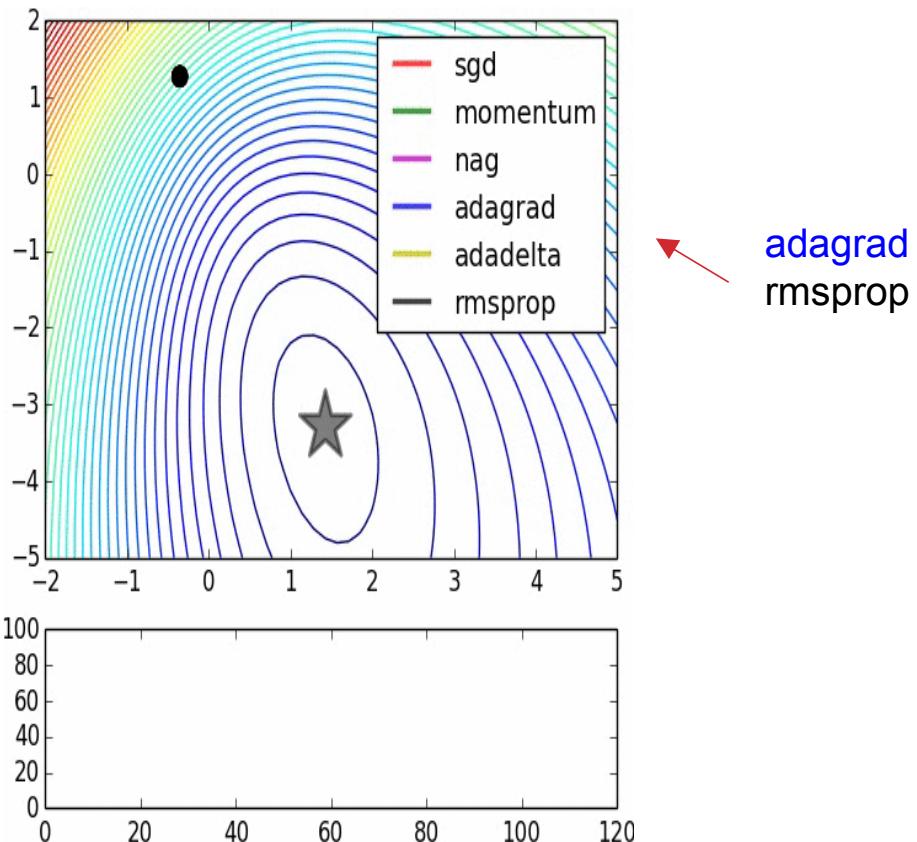
## rmsprop: A mini-batch version of rprop

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Introduced in a slide in  
Geoff Hinton's Coursera  
class, lecture 6

Cited by several  
papers as:

[52] T. Tieleman and G. E. Hinton. Lecture 6.5-rmsprop: Divide the gradient by a running average of its recent magnitude., 2012.





## Adam update

[Kingma and Ba, 2014]



(incomplete, but close)

Like a combination between  
momentum and RMSprop

```
# Adam
m = beta1*m + (1-beta1)*dx # update first moment
v = beta2*v + (1-beta2)*(dx**2) # update second moment
x += - learning_rate * m / (np.sqrt(v) + 1e-7)
```

# Adam update

[Kingma and Ba, 2014]



(incomplete, but close)

```
# Adam
m = beta1*m + (1-beta1)*dx # update first moment
v = beta2*v + (1-beta2)*(dx**2) # update second moment
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```

momentum

RMSProp-like

Looks a bit like RMSProp with momentum

# Adam update

[Kingma and Ba, 2014]



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m = beta1*m + (1-beta1)*dx # update first moment
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x += - learning_rate * m / (np.sqrt(v) + 1e-7)
```

momentum

RMSProp-like

Looks a bit like RMSProp with momentum

```
# RMSProp
cache = decay_rate * cache + (1 - decay_rate) * dx**2
x += - learning_rate * dx / (np.sqrt(cache) + 1e-7)
```

# Adam update

[Kingma and Ba, 2014]



```
# Adam
m,v = #... initialize caches to zeros
for t in xrange(1, big_number):
    dx = # ... evaluate gradient
    m = beta1*m + (1-beta1)*dx # update first moment
    v = beta2*v + (1-beta2)*(dx**2) # update second moment
    mb = m/(1-beta1**t) # correct bias
    vb = v/(1-beta2**t) # correct bias
    x += - learning_rate * mb / (np.sqrt(vb) + 1e-7)
```

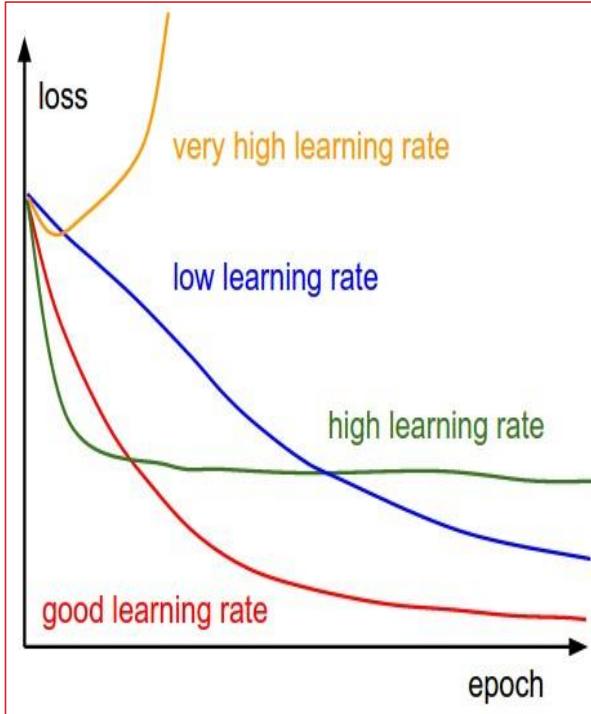
momentum

bias correction  
(only relevant in first few iterations when t is small)

RMSProp-like

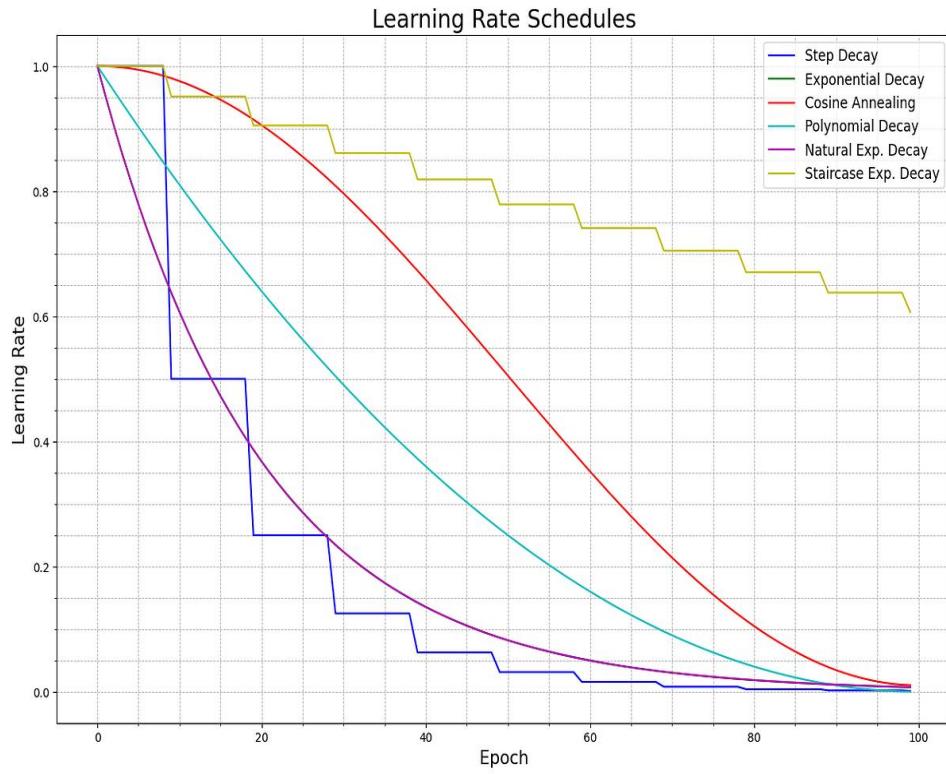
The bias correction compensates for the fact that  $m, v$  are initialized at zero and need some time to “warm up”.

SGD, SGD+Momentum, Adagrad, RMSProp, Adam all have learning rate as a hyperparameter.



Q: Which one of these learning rates is best to use?

SGD, SGD+Momentum, Adagrad, RMSProp, Adam all have learning rate as a hyperparameter.



⇒ Learning rate decay over time (LR Schedulers)!

**Step decay:**

e.g. decay learning rate by half every few epochs.

**Exponential decay:**

$$\alpha = \alpha_0 e^{-kt}$$

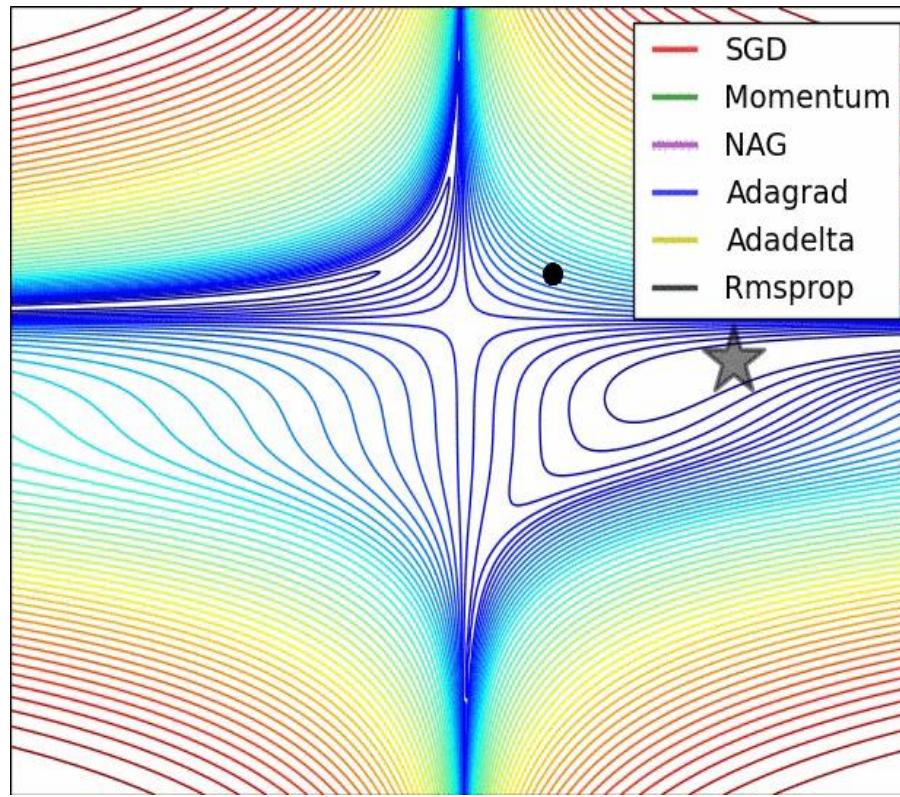
**Cosine decay (Best):**

$$\eta_t = \eta_{min} + \frac{1}{2}(\eta_{max} - \eta_{min}) \left( 1 + \cos \left( \frac{T_{cur}}{T_{max}} \pi \right) \right)$$

# Summary



- **Simple Gradient Methods** like SGD can make adequate progress to an optimum when used on minibatches of data.
- **Second-order** methods make much better progress toward the goal, but are more expensive and unstable.
- **Momentum:** is another method to produce better effective gradients.
- **ADAGRAD, RMSprop** diagonally scale the gradient.
- **ADAM** scales and applies momentum.



(image credits to Alec Radford)

# Questions?