Operations Research

Regular Simplex Method

Prepared By: Doaa Ezzat

Steps of Regular Simplex Method

1. Transform the linear program to the standard form:

$$\text{Optimize: } \boldsymbol{z} = \boldsymbol{c}^T \boldsymbol{x}$$

$$\text{Sub. To: } \mathbf{A}\,\mathbf{X} = \mathbf{B}$$

$$\mathbf{X} \geq 0$$

Where $\mathbf{B} \geq 0$, BS $\boldsymbol{X_0}$ is known

Steps of Regular Simplex Method

2. Get the Initial Basic Feasible Solution IBFS.

3. Draw the tableau.

4. Test the optimality.

5. Improve the optimality.

How to transform to the standard form

1. <u>All variables must be ≥ 0 : -</u>
   a) If there is a variable X_1 in the problem must be ≤ 0 :
      put X_1 = - X_3 where $X_3 \geq 0$
   b) If there is a variable X_1 in the problem may be positive, negative or 0:
      put $X_1 = X_3 - X_4$ where $X_3, X_4 \geq 0$
2. <u>All the right hand side values in the constraints must be ≥ 0 : -</u>

   If one constraint has a negative right hand side value then multiply the both sides by -1 and don't forget to invert the inequality operator.

How to transform to the standard form

3. <u>All constraints must be equations : -</u>
   a) If a constraint has a less than operator:
      We need a slack variable, as follows:
      (ex: X_1 - X_2 ≤ 5 ⟹ X_1 - X_2 + S_1= 5)
   b) If a constraint has a greater than operator:
      We need a surplus variable, as follows:
      (ex: X_1 - X_2 ≥ 5 ⟹ X_1 - X_2 - S_1= 5)
   c) Sometimes we also need artificial variables but we will discuss them later.

<u>Note:</u> Both slack and surplus variables must be added to the objective function with zero coefficients.

Let's solve an example

Ex (1) :-

Min: z = $4x_1 - x_2$

Sub. To:

$2x_1 + x_2 \leq 8$

$x_2 \leq 5$

$x_1 - x_2 \leq 4$

$x_1, x_2 \geq 0$

Solution:-

1. Transformation:-

Min: z = $4x_1 - x_2 + 0s_1 + 0s_2 + 0s_3$

Sub. To:

$2x_1 + x_2 + s_1 = 8$

$x_2 + s_2 = 5$

$x_1 - x_2 + s_3 = 4$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

Let’s solve an example

Solution:-

1. Transformation:-

Min: z = 4x_1- x_2 + $0s_1$+ $0s_2$+ $0s_3$

Sub. To:

$2x_1 + x_2 + s_1 = 8$

$x_2 + s_2 = 5$

$x_1 - x_2 + s_3 = 4$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

Basic Variables

2. IBFS:-

Non-Basic Variables

Let $x_1 = x_2 = 0$

So → $s_1 = 8$

$s_2 = 5$

$s_3 = 4$

$$\boldsymbol{x}_0 = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \\ 4 \end{pmatrix}$$

Let's solve an example

3. The Tableau:-

Min: z = $4x_1 - x_2 + 0s_1 + 0s_2 + 0s_3$

Sub. To:

$2x_1 + x_2 + s_1 = 8$

$x_2 + s_2 = 5$

$x_1 - x_2 + s_3 = 4$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

4- Optimality test:-

This Solution is not optimal because there is a negative number in the last row

The smallest value

		x_1 4	WC x_2 -1	s_1 0	s_2 0	s_3 0	RHS	Ratio
s_1	0	2	1	1	0	0	8	8
s_2	0	0	1*	0	1	0	5 WR	5
s_3	0	1	-1	0	0	1	4	
		4	-1	0	0	0		

The most negative

Let's solve an example

5- Improving the optimality:-

Multiply by -1

		x_1	x_2	s_1	s_2	s_3	RHS	Ratio
		4	-1	0	0	0		
s_1	0	2	1	1	0	0	8	8
s_2	0	0	1*	0	1	0	5	5
s_3	0	1	-1	0	0	1	4	
		4	-1	0	0	0		

		x_1	x_2	s_1	s_2	s_3	RHS
		4	-1	0	0	0	
s_1	0	2	0	1	-1	0	3
x_2	-1	0	1	0	1	0	5
s_3	0	1	0	0	1	1	9
		4	0	0	1	0	

There is no negative numbers in this row, so this solution is optimal

Let’s solve an example

- The optimal solution is at :

x_1 = 0, x_2 = 5, s_1 = 3, s_2 = 0, s_3 = 9

Z* = 4 (0)- 5+0+0+0 = $\boxed{-5}$

Another Example

Ex (2) :-

Max: $z = 5x_1 + 3x_2 + x_3$

Sub. To:

$$x_1 + x_2 + 3x_3 \leq 6$$

$$5x_1 + 3x_2 + 6x_3 \leq 15$$

$$x_1, x_2 \geq 0$$

We put: $x_3 = x_4 - x_5$

Solution:-

1. Transformation:-

Max: $z = 5x_1 + 3x_2 + x_4 - x_5 + 0s_1 + 0s_2$

Sub. To:

$$x_1 + x_2 + 3x_4 - 3x_5 + s_1 = 6$$

$$5x_1 + 3x_2 + 6x_4 - 6x_5 + s_2 = 15$$

$$x_1, x_2, x_4, x_5, s_1, s_2 \geq 0$$

Another Example

Solution:-

1. Transformation:-

Max: z = $5x_1 + 3x_2 + x_4 - x_5 + 0s_1 + 0s_2$

Sub. To:

$$x_1 + x_2 + 3x_4 - 3x_5 + s_1 = 6$$
$$5x_1 + 3x_2 + 6x_4 - 6x_5 + s_2 = 15$$
$$x_1, x_2, x_4, x_5, s_1, s_2 \geq 0$$

2. IBFS:-

Let $x_1 = x_2 = x_4 = x_5 = 0$

So → $s_1 = 6$

$s_2 = 15$

$$\boldsymbol{x}_0 = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} 6 \\ 15 \end{pmatrix}$$

Another Example

3. The Tableau:-

Max: $z = 5x_1 + 3x_2 + x_4 - x_5 + 0s_1 + 0s_2$

Sub. To:

$$x_1 + x_2 + 3x_4 - 3x_5 + s_1 = 6$$

$$5x_1 + 3x_2 + 6x_4 - 6x_5 + s_2 = 15$$

$$x_1, x_2, x_4, x_5, s_1, s_2 \geq 0$$

4- Optimality test:-

This Solution is not optimal because there are negative numbers in the last row

The smallest value

The most negative

		WC							
		x_1 5	x_2 3	x_4 1	x_5 -1	s_1 0	s_2 0	RHS	Ratio
s_1	0	1	1	3	-3	1	0	6	6
s_2	0	5*	3	6	-6	0	1	15	3
		-5	-3	-1	1	0	0	WR	

Let's solve an example

5- Improving the optimality:-

		x_1	x_2	x_4	x_5	s_1	s_2	RHS	Ratio
		5	3	1	-1	0	0		
s_1	0	1	1	3	-3	1	0	6	6
s_2	0	5*	3	6	-6	0	1	15	3
		-5	-3	-1	1	0	0		

+

Multiply by 1/5

WC

		x_1	x_2	x_4	x_5	s_1	s_2	RHS
		5	3	1	-1	0	0	
s_1	0	0	2/5	9/5	-9/5	1	-1/5	3
x_1	5	1	3/5	6/5	-6/5	0	1/5	3
		0	0	5	-5	0	1	

Multiply by -1

The most negative

Unbounded Solution