Algorithms Project

Team Member

Name	ID
Bishoy Yousry Abdelmalak	**1900733**

Course Name: Design and Analysis of Algorithms

Course Code: CSE332s

Task 1

Assumptions

1- number of barrels is constant equals to 1000

2- numbering all the barrels as binary string is not considered a part of the algorithm, but it is considered as a precondition for the algorithm, and thus the time taken to fill the vector of strings "barrelNumber" is not considered

3- storing the barrels from which each slave drinks is also not considered a part from the algorithm, it is just for visualization

4- All slaves drink from their barrels at the same time

Problem Description

An evil king is informed that one of his 1000 wine barrels has been poisoned. The poison is so potent that a miniscule amount of it, no matter how diluted, kills a person in exactly 30 days. The king is prepared to sacrifice 10 of his slaves to determine the poisoned barrel.

(a) Can this be done before a feast scheduled in 5 weeks?

(b) Can the king achieve his goal with just eight slaves?

Design a Divide and Conquer (NOT decrease and conquer) algorithm to solve this problem.

Solution Description

number all the 1000 barrels from 0 -> 999 in binary format, suppose that we have 10 slaves, then slave#1 will drink from all the barrels that have the first digit equals to 1 for ex. (0000000001, 0000000011, 0000000101, 0000000111, ...etc), and slave#2 will drink from barrels that have the second digit equals to 1 for ex. (0000000010, 0000000011, 0000000110, ...etc) and so on. Then we can determine the poisoned barrel as follows: if only slave#1 dies, then the poisoned barrel index in binary format is 0000000001 which is barrel that has 1 in decimal.

a) Yes, this can be done and we can determine the poisoned barrel within 30 days only.

b) No, we can't. because if the total number of barrels was 1000 (i.e. they are numbered from 0 → 999 then the minimum number of slaves = ceil(log2(999)) = 10 slaves, or we can solve this using only 9 slaves if the poisoned barrel index was = 512 because no one from the nine slaves will die as the representation of the number 512 in binary is 1000000000

Pseudo Code & Solution Steps

Pseudo Code

```
Algorithm
barrelNumber["0000000000", "0000000001", ....., "11 1110 0111"]
divide_and_Conquer(l, r, slavesLives[], slavesBarrels[])
{
```

```
//Input: left and right indices of barrelNumber, binary array of slavesLives, integer array of slavesBarraels.

//Output: slavesLives and slavesBarrels.

m = (l + r) / 2


If(l >= r)

{

for (i = NUMBER_OF_DIGITS – 1 → l = 0)

{
        /*Fill slavesBarrels*/

        if(barrelNumber[l][i] = '1')
           slavesBarrels[NUMBER_OF_DIGITS - i - 1] = l

}

        /*Fill slavesLives*/

    if(l == poisonedIndex)

     {

        for (i = 0 → NUMBER_OF_DIGITS)

        {

            if(barrelNumber[l][NUMBER_OF_DIGITS - i - 1] = '1')
               slavesLives[i] = 1;

        }


     }

return

}

  divide_and_Conquer(l, m, slavesLives, slavesBarrels)

   divide_and_Conquer(m + 1, r, slavesLives, slavesBarrels)

}
```

Steps

1- ckecks the possibility to solve the problem based on the number of slaves entered, and the index of the poisoned barrel

2- divide the vector of "barrelNumber" that has 1000 string numbered from 0 -> 999 into to halves (0 -> 499 and 500 -> 999) then divide each one again into two halves ...etc. until reaching the leaves which has one number

3- check if this barrel is poisoned, then the slaves who drink from this barrel will be marked to die (i.e. set the value one in "slavesLives" vector).

4- convert the binary in slavesLives to decimal which is the decimal index of the poisoned barrel

Complexity Analysis

Here we have the critical operation is the comparison inside the base condition, thus recurrence relation is **T(n) = 2T(n/2) + 10**.

We can solve this recurrence relation using the Master Theorem. The Master Theorem states that if a recurrence relation is of the form T(n) = aT(n/b) + f(n), where a >= 1 and b > 1 are constants and f(n) is an asymptotically positive function, then:

If f(n) = O(n^(log_b(a - ε))) for some constant ε > 0, then T(n) = Θ(n^(log_b(a))).

If f(n) = Θ(n^(log_b(a))), then T(n) = Θ(n^(log_b(a)) log n).

If f(n) = Ω(n^(log_b(a + ε))) for some constant ε > 0, and if af(n/b) <= cf(n) for some constant c < 1 and all sufficiently large n, then T(n) = Θ(f(n)).

In this case, we have a = 2, b = 2, and f(n) = 10. We can see that f(n) is a constant function, so it falls under the first case of the Master Theorem. Therefore, T(n) = Θ(n^(log_2(2))) = Θ(n).

So the solution to the recurrence relation T(n) = 2T(n/2) + 10 is T(n) = Θ(n).

Comparison with another technique

Brute force technique:

- **Explanation:** Iterate though the 1000 binary string vector “barrelNumber”, if the current barrel is the poisoned one, set the lives of the slaves that should drink from it to one, after that convert the binary of the “slavesLives” to decimal to get the poisoned barrel index.
- **Assumptions:**
  1- number of barrels is constant equals to 1000
  2- numbering all the barrels as binary string is not considered a part of the algorithm, but it is considered as a precondition for the algorithm, and thus the time taken to fill the vector of strings "barrelNumber" is not considered
  3- storing the barrels from which each slave drinks is also not considered a part from the algorithm, it is just for visualization
  4- All slaves drink from their barrels at the same time
- **Time complexity: T(n) =** $\sum_{0}^{n-1}\sum_{0}^{9}\mathbf{1} = \sum_{0}^{n-1}\mathbf{10} = \mathbf{10}\sum_{0}^{n-1}\mathbf{1} = \mathbf{10}\boldsymbol{n} \rightarrow \boldsymbol{\Theta}(\boldsymbol{n})$

Output

Two arrays the 1st one consists of binary values **“slaveLives”** which has the slaves lives (the true or 1 value means that this slave died, and false or 0 value means that the slave is alive), and the 2nd one is **“slavesBarrels”** which has the barrels from which each slave drank.

Conclusion

The two algorithms (brute force, and divide and conquer) can solve this problem with the same time complexity = O(n)

References

1- A King, 1000 Bottles of Wine, 10 Prisoners and a Drop of Poison | by Brett Berry | Math Hacks | Medium

2- Personal Finance, Investments, and other things: The King and the Poisoned Wine: An interesting problem, with solution explained (dineshgopalan.com)