

The variance of our 20 assets portfolio can be written out as $\sigma_p^2 = w^T \cdot \Omega \cdot w$, where w is a vector containing the weights of our assets, Ω is our variance-covariance matrix of our assets, and w^T is the transpose of our weight vector. Technically, the portfolio with the minimum variance is one where all weights in w would equal 0, however we have the extra conditions that $\sum_{i=1}^{20} w_i = 1 \rightarrow \sum_{i=1}^{20} w_i - 1 = 0$. Therefore, we have a constraint optimization problem, where we must minimize the Lagrange function to find the weights that give us the minimum variance for our portfolio, but also satisfy the given constraint,

Ω : asset Variance – Covariance matrix of daily returns

w : weight vector , w^T : weight vector transpose

λ : Lagrange Multiplier

$$J = w^T \cdot \Omega \cdot w + \lambda * \sum_{i=1}^{20} w_i - 1$$

$$\forall i \in (1, 2, \dots, 20)$$

$$\frac{\partial J}{\partial w_i} = \frac{\partial}{\partial w_i} \left(w^T \cdot \Omega \cdot w + \lambda * \sum_{i=1}^{20} w_i - 1 \right)$$

$$0 = \frac{\partial w_j^T}{\partial x_i} \Omega_{jl} w_l + w_j^T \Omega_{jl} \frac{\partial w_l}{\partial x_i} + \lambda$$

$$0 = \delta_{ij} \Omega_{jl} w_l + w_j^T \Omega_{jl} \delta_{il} + \lambda$$

$$0 = \Omega_{il} w_l + w_j^T \Omega_{ji} + \lambda$$

$$0 = 2 * \sum_{j=1}^{j=20} \sigma_{ij} w_j + \lambda$$

$$0 = \sum_{j=1}^{j=20} \sigma_{ij} w_j + \frac{\lambda}{2}$$

$$\frac{\partial J}{\partial \lambda} = 0 = \frac{\partial}{\partial \lambda} \left(w^T \cdot \Omega \cdot w + \lambda * \sum_{i=1}^{20} w_i - 1 \right)$$

$$0 = \sum_{i=1}^{20} w_i - 1$$

$\lambda = 0.0001$, and weights listed above