National University of Computer & Emerging Sciences, Karachi
Fall/Spring/Summer-2011 CS-Department
MidTerm 1
23rd February 2017, 10:30 am – 11:30 am

Course Code: CS211	Course Name: Discrete Structures
Instructor Name : Jalaluddin Qureshi	
Student Roll No:	Group:

Instructions:

- Return the question paper.Read each question completely before answering it. There are **5questions and 1 page.**
- In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.
- Invigilators/ instructor can not assist you in understanding the question.
- All the answers must be solved according to the sequence given in the question paper.
- Marks will be awarded iff justifications has been provided.

Time: 60 minutes. **Max Marks**: 10mark/question x 5questions = 50 marks

Part A (Set Theory)

Question 1:

Using the following relationship: |AUB|=|A|+|B|- |A∩B| (1),

Show that the following can be derived:

|AUBUC|=|A|+|B|+|C|- |A∩B|- |A∩C|- |C∩B|+|A∩B∩C| (2).

Based on this exercise, and using Equation (1) and/or (2) derive the formula for |A U B U C U D|.

Solution:

Use substitution method, let, T=BUC, so we have,

|AUT|=|A|+|T|- |A∩T|

|AUBUC|=|A|+|BUC|- |A∩(BUC)|

Since |BUC|=|B|+|C|- |B∩C|,

|AUBUC|=|A|+|B|+|C|- |B∩C|- |A∩(BUC)|

|AUBUC|=|A|+|B|+|C|- |B∩C|- |(A∩B) U (A∩C)|

|AUBUC|=|A|+|B|+|C|- |B∩C|- (|(A∩B)|+|(A∩C)| - |A∩B∩C |)

|AUBUC|=|A|+|B|+|C|- |A∩B|- |A∩C|- |C∩B|+|A∩B∩C|

The same approach of substitution can be used to show that,

|A U B U C U D|= |A|+|B|+|C| +|D| - |A∩B|- |A∩C|- |C∩B| - |A∩D| - |B∩D| -
|C∩D|+|A∩B∩C|+|A∩B∩D|+|D∩B∩C|+|A∩C∩D| - |A∩B∩C∩D |

Question 2:

If A={2,3,4,g,FAST,star}, B={4,f,yoyo, 2, golf, 2}, C={yoyo, 10, 7, f, 4, FAST},
$\mathcal{U}$= A U B U C U {3, golf, 7, tent, 0, 11}, (Hint: $\mathcal{U}$ is the universal set). Find the following, and plot these on a Venn Diagram (by shading appropriate region).

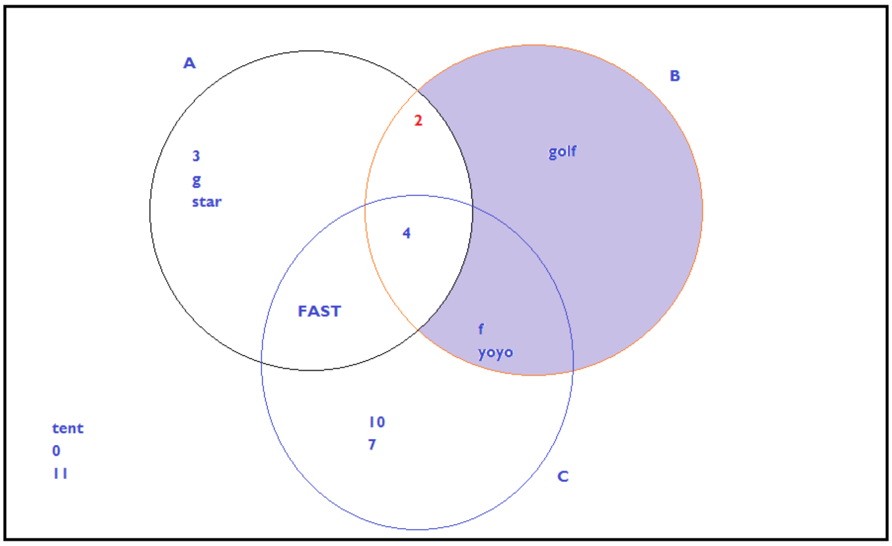
(a) A^c∩B (b) ($\mathcal{U}$ \B)∩ C^c

(c) (A \\$\mathcal{U}$) U (C\B) (d) $(A∩C)^c$ \ B

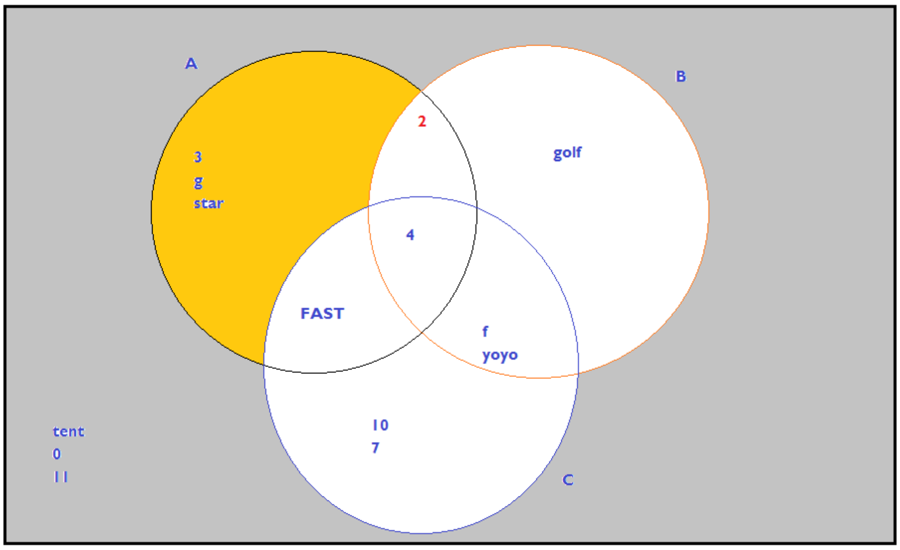
Solution:

The solution is given by the elements in the common region.

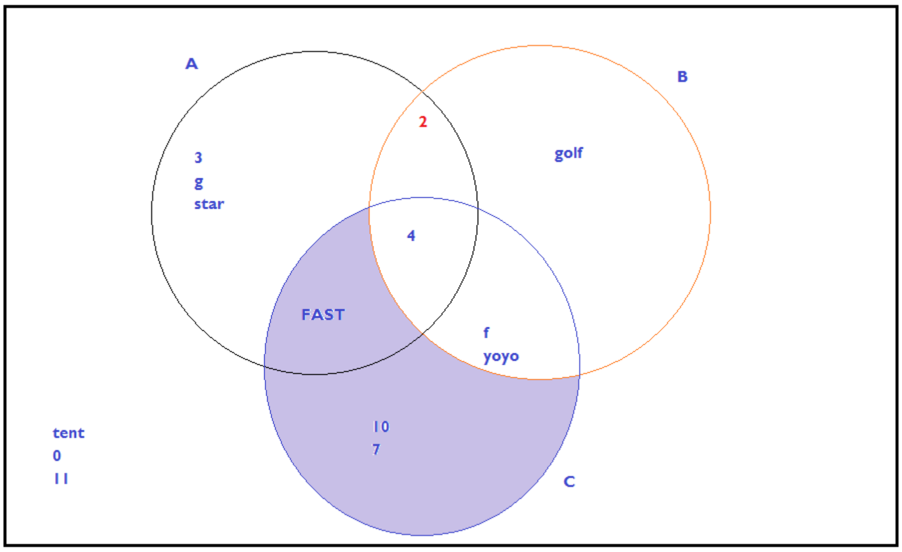
(a)


A
B
2
golf
3
g
star
4
FAST
f
yoyo
tent
0
11
10
7
C

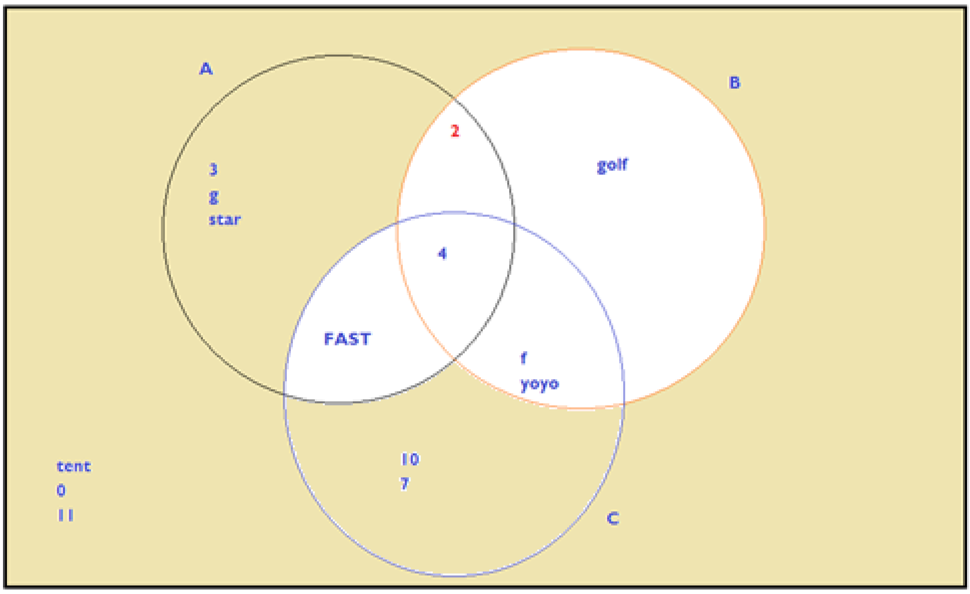
(b)


A
B
2
golf
3
g
star
4
FAST
f
yoyo
tent
0
11
10
7
C

(c)


A
B
2
golf
3
g
star
4
FAST
f
yoyo
tent
0
11
10
7
C

(d)



<u>Part B (Logic Theory)</u>

<u>Question 3:</u>

Determine (using appropriate technique) whether the following relationship is correct/ incorrect:

$P \to (Q \vee R) \equiv (P \to Q) \wedge (\neg P \to R)$.

<u>Solution:</u>

	P	¬P	Q	R	QVR	P→(QVR)	M (P→Q)	N (¬P→R)	M∧N
1	T	F	T	T	T	T	T	T	T
2	T	F	T	F	T	T	T	T	T
3	T	F	F	F	F	F	F	T	F
4	T	F	F	T	T	T	F	T	F
5	F	T	T	F	T	T	T	F	F
6	F	T	F	T	T	T	T	T	T
7	F	T	F	F	F	T	T	F	F
8	F	T	T	T	T	T	T	T	T

As the two truth tables are not same, the equivalence relationship is false.

As there are two tables (with total 16 entries), at the discretion of the examiner, you will lose 1 marks for a wrong entry.

<u>Question 4:</u>

Determine whether the assignment c⟵c+1 (read as, c is assigned the value given by c plus one) will be executed by the if-statement, where x⟵5, y⟵3, z⟵7.

(a) If ¬{(x<y) ∧ (y≤z)} then c⟵c+1 (b) If ¬{(x=z) ∧ ((x≤z)} then c⟵c+1

(c) If ¬{(x≥y) ∨ (x<z)} then c⟵c+1 (d) If ¬{(x≤z) ∨ (y=z)} then c⟵c+1

<u>Solution:</u>

(a) $\neg\{(x<y) \wedge (y\leq z)\}$ then $c \leftarrow c+1$ TRUE
(x<y) is false, (y≤z) is true, their conjunction is false, and negation of this is true.

(b) $\neg\{(x=z) \wedge ((x\leq z)\}$ then $c \leftarrow c+1$ TRUE
(x=z) is false, (x≤z) is true, their conjunction is false, and negation of this is true.

(c) $\neg\{(x\geq y) \vee (x<z)\}$ then $c \leftarrow c+1$ FALSE
(x≥y) is true, (x<z) is also true, their disjunction is true, and its negation is false.

(d) $\neg\{(x\leq z) \vee (y=z)\}$ then $c \leftarrow c+1$ FALSE
(x≤z) is true, (y=z) is false, their disjunction is true, and its negation is false.

Question 5:

Using the algebra of Propositions, simplify the following:

$(x \vee \neg y \vee z) \wedge (\neg x \vee \neg y \vee z) \vee (w \wedge \neg x)$

After the simplication, draw its truth table.

Solution

$(x \vee \bar{y} \vee z) \wedge (\bar{x} \vee \bar{y} \vee z) \vee (w \wedge \bar{x})$

Conjunction has the highest precedence so,

$= \{[x \wedge (\bar{x} \vee \bar{y} \vee z)] \vee [\bar{y} \wedge (\bar{x} \vee \bar{y} \vee z)]$
$\vee [z \wedge (\bar{x} \vee \bar{y} \vee z)]\} \vee (w \wedge \bar{x})$

$= \{[\cancel{(x \wedge \bar{x})}^{F} \vee (x \wedge \bar{y}) \vee (x \wedge z)] \vee [(\bar{y} \wedge \bar{x}) \vee$
$\cancel{(\bar{y} \wedge \bar{y})}^{\bar{y}} \vee \underline{(\bar{y} \wedge z)}] \vee [(z \wedge \bar{x}) \vee \underline{(z \wedge \bar{y})} \vee$
$\cancel{(z \wedge \bar{z})}^{F}]\} \vee (w \wedge \bar{x})$

$= \cancel{(x \wedge \bar{y})}^{\bar{y}} \vee (x \wedge z) \vee \cancel{(\bar{y} \wedge \bar{x})}^{\bar{y}} \vee \bar{y} \vee \cancel{(\bar{y} \wedge z)}^{\bar{y}}$
$\vee (z \wedge \bar{x}) \vee (w \wedge \bar{x})$

$= \cancel{(x \wedge z)}^{z} \vee \bar{y} \vee \cancel{(z \wedge \bar{x})}^{z} \vee (w \wedge \bar{x}) = \bar{y} \vee z \vee (w \wedge \bar{x})$

It is possible to simplify it further, but the above solution is sufficient, given time limitation in exam.

Based on the above reduction, a truth table for the above can be easily drawn.