

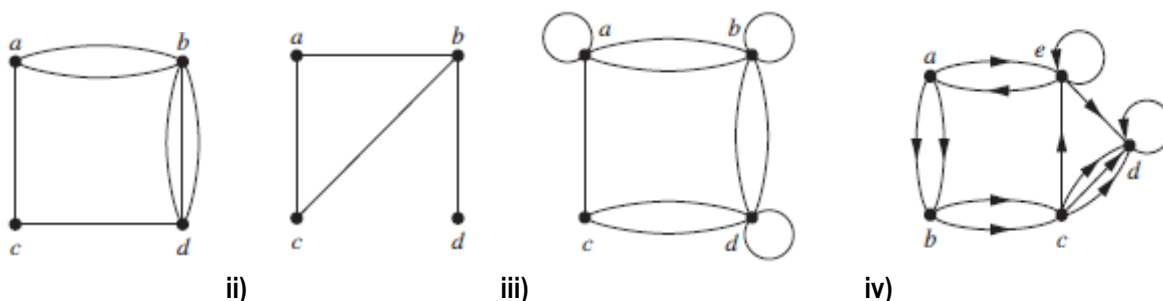
**Instructions:**

Max. Points: Task 1: 78 (26x 3) Task II: 96 (32 x 3)

- 1- This is hand written assignment.
- 2- Just write the question number instead of writing the whole question.
- 3- You can only use A4 size paper for solving the assignment.

**Task I**

1. Determine whether the graph shown in figure i to iv has directed or undirected edges, whether it has multiple edges, and whether it has one or more loops. Use your answers to determine the type of graph.

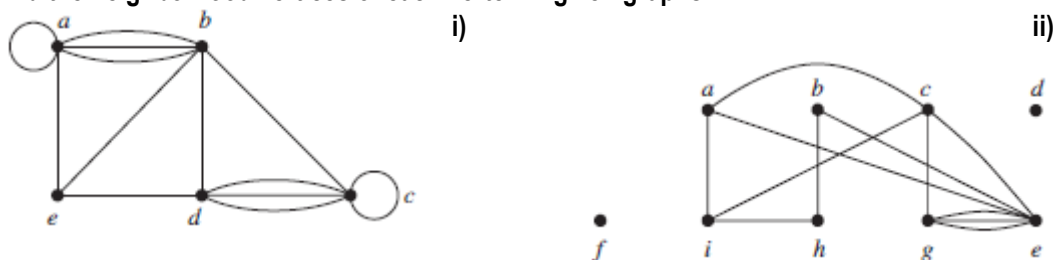


2. The intersection graph of a collection of sets  $A_1, A_2, \dots, A_n$  is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two sets if these sets have a nonempty intersection. Construct the intersection graph of these collections of sets.

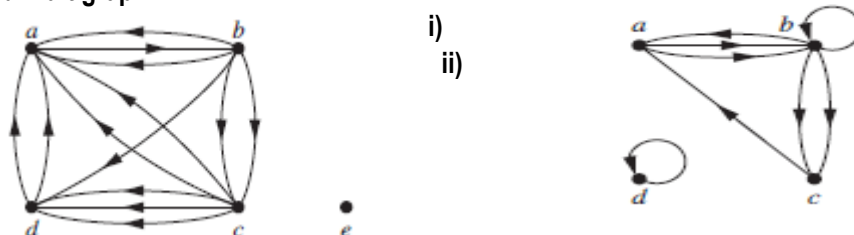
i)  $A_1 = \{0, 2, 4, 6, 8\}$ ,  $A_2 = \{0, 1, 2, 3, 4\}$ ,  $A_3 = \{1, 3, 5, 7, 9\}$ ,  $A_4 = \{5, 6, 7, 8, 9\}$ ,  $A_5 = \{0, 1, 8, 9\}$

ii)  $A_1 = \{\dots, -4, -3, -2, -1, 0\}$ ,  $A_2 = \{\dots, -2, -1, 0, 1, 2, \dots\}$ ,  $A_3 = \{\dots, -6, -4, -2, 0, 2, 4, 6, \dots\}$ ,  $A_4 = \{\dots, -5, -3, -1, 1, 3, 5, \dots\}$ ,  $A_5 = \{\dots, -6, -3, 0, 3, 6, \dots\}$

3. (a) Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Also find the neighborhood vertices of each vertex in given graphs.



- (b) Determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph.

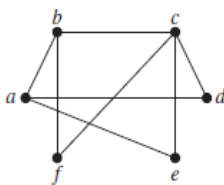


4. (a) Suppose that a new company has five employees: Zamora, Agraharam, Smith, Chou, and Macintyre. Each employee will assume one of six responsibilities: planning, publicity, sales, marketing, development, and industry relations. Each employee is capable of doing one or more of these jobs: Zamora could do planning, sales, marketing, or industry relations; Agraharam could do planning or development; Smith could do publicity, sales, or industry relations; Chou could do planning, sales, or industry relations; and Macintyre could do planning, publicity, sales, or industry relations. Model the capabilities of these employees using appropriate graph.

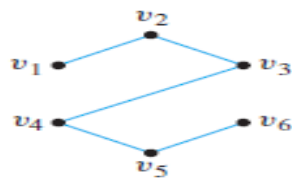
(b) Suppose that there are four employees in the computer support group of the School of Engineering of a large university. Each employee will be assigned to support one of four different areas: hardware, software, networking, and wireless. Suppose that Ping is qualified to support hardware, networking, and wireless; Quiggley is qualified to support software and networking; Ruiz is qualified to support networking and wireless, and Sitea is qualified to support hardware and software. Use appropriate graph to model the four employees and their qualifications.

5. Find which of the following graphs are bipartite. Redraw the bipartite graphs so that their bipartite nature is evident. Also write the disjoint set of vertices.

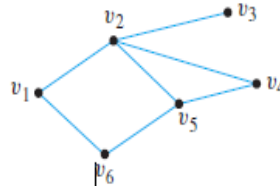
i)



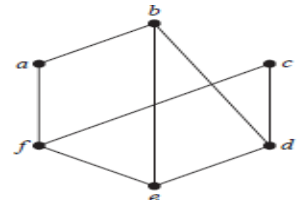
ii)



iii)



iv)



6. Draw a graph with the specified properties or show that no such graph exists.

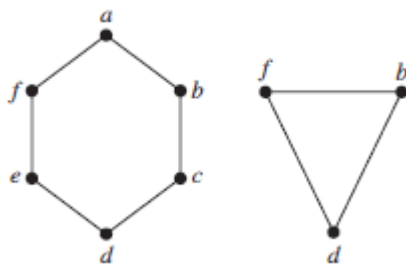
- A graph with four vertices of degrees 1, 1, 2, and 3
- A graph with four vertices of degrees 1, 1, 3, and 3
- A simple graph with four vertices of degrees 1, 1, 3, and 3

7. a) In a group of 15 people, is it possible for each person to have exactly 3 friends? Explain. (Assume that friendship is a symmetric relationship: If  $x$  is a friend of  $y$ , then  $y$  is a friend of  $x$ .)

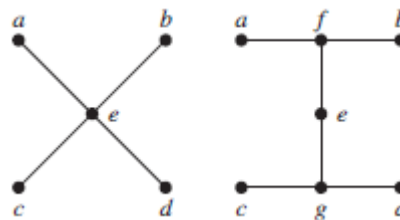
b) In a group of 4 people, is it possible for each person to have exactly 3 friends? Why?

8. (a) Find the union of the given pair of simple graphs. (Assume edges with the same endpoints are the same.)

i)



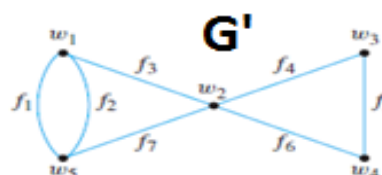
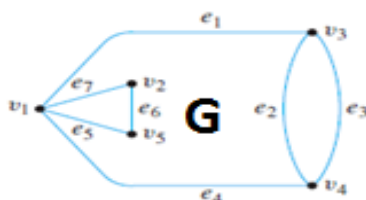
ii)



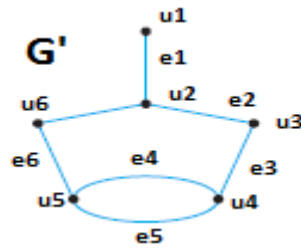
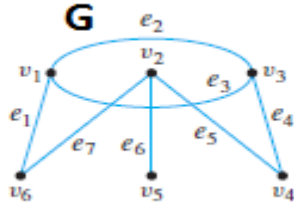
b) How many vertices does a regular graph of degree four with 10 edges have?

9. For given pair  $(G, G')$  of graphs. Determine whether they are isomorphic. If they are, give function  $g: V(G) \rightarrow V(G')$  that define the isomorphism. If they are not, give an invariant for graph isomorphism that they do not share.

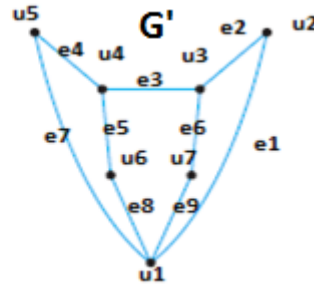
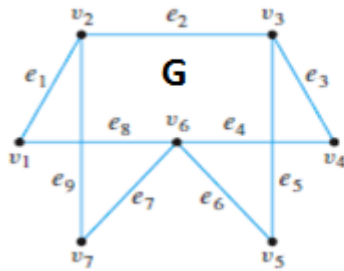
i)



ii)

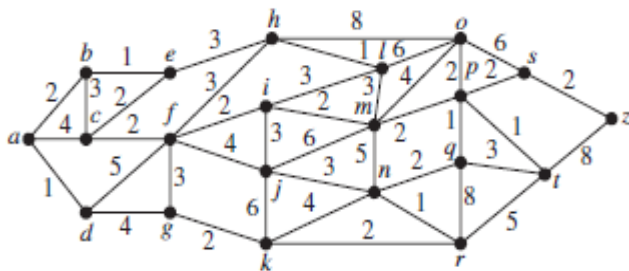


iii)

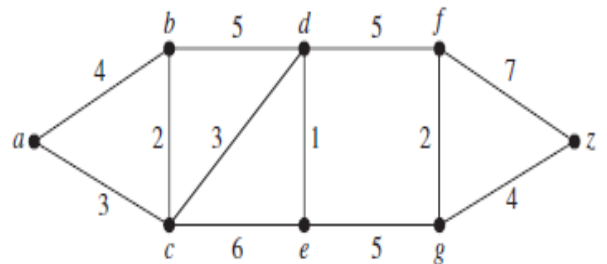


10. Find the length of a shortest path between  $a$  and  $z$  in the given weighted graph by using Dijkstra's algorithm.

i)

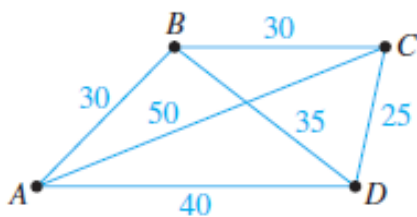


ii)

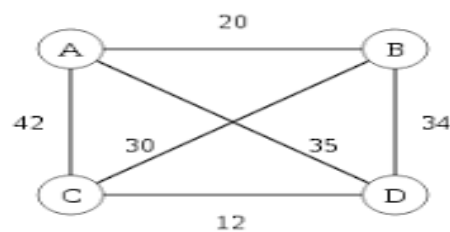


11. Imagine that the drawing below is a map showing four cities and the distances in kilometers between them. Suppose that a salesman must travel to each city exactly once, starting and ending in city A. Which route from city to city will minimize the total distance that must be traveled?

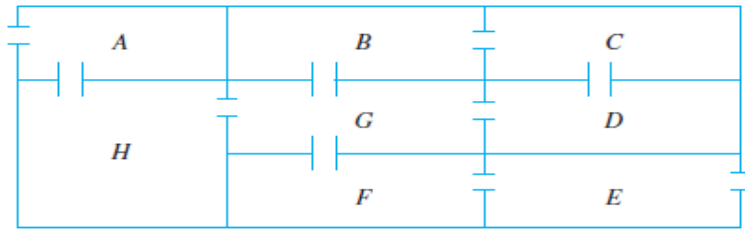
i)



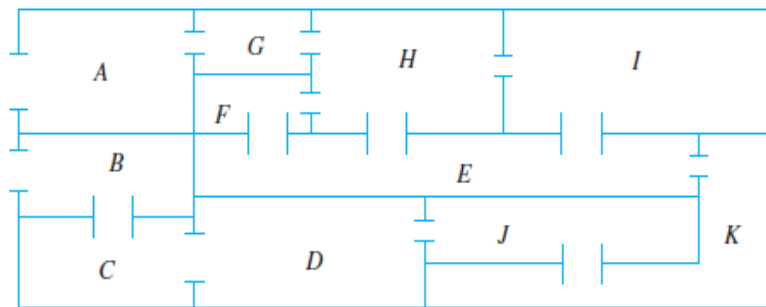
ii)



12. (a) The following is a floor plan of a house. Is it possible to enter the house in room A, travel through every interior doorway of the house exactly once, and exit out of room E? If so, how can this be done?

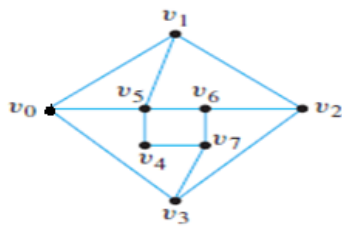


(b) The floor plan shown below is for a house that is open for public viewing. Is it possible to find a trail that starts in room A, ends in room B, and passes through every interior doorway of the house exactly once? If so, find such a trail.

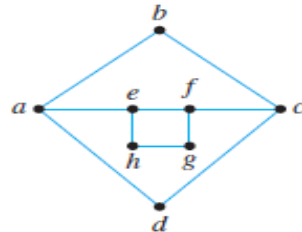


13. Find Hamiltonian circuits AND Path for those graphs that have them. Explain why the other graphs do not.

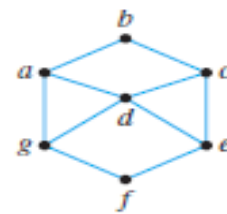
i)



ii)

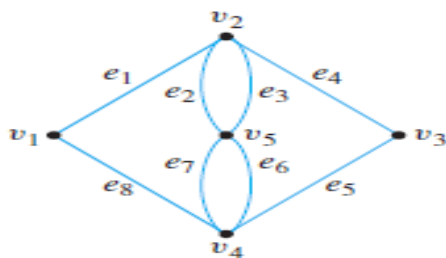


iii)

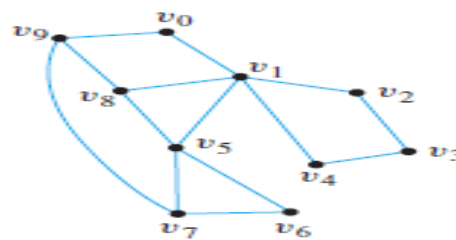


14. a) Determine which of the graphs have Euler circuits. If the graph does not have an Euler circuit, explain why not. If it does have an Euler circuit, describe one.

i)

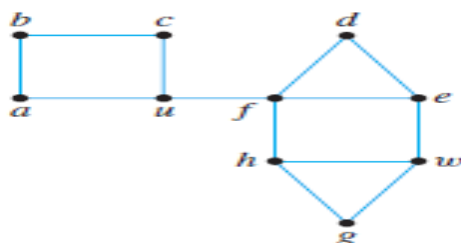


ii)



b) Determine whether there is an Euler path from u to w. If the graph does not have an Euler path, explain why not. If it does have an Euler path, describe one.

i)

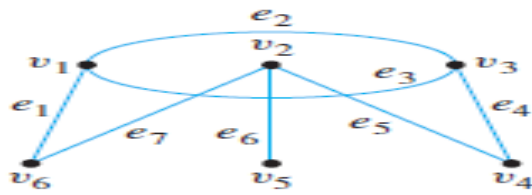


ii)

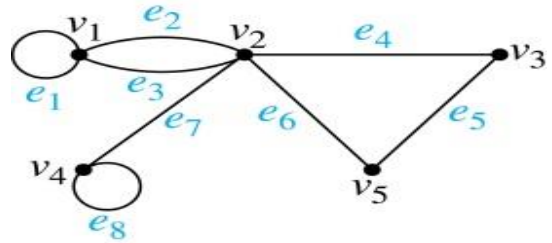


15. (a) Use an incidence matrix to represent the graph shown below.

i)



ii)



(b) Draw a graph using below given incidence matrix.

i)

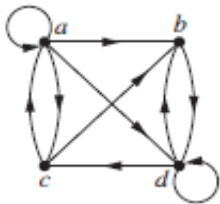
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

ii)

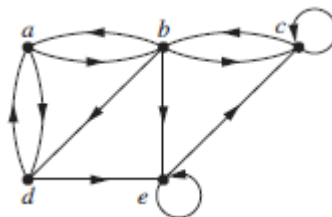
$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

16. Use an adjacency list and adjacency matrix to represent the given graph.

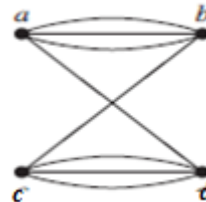
i)



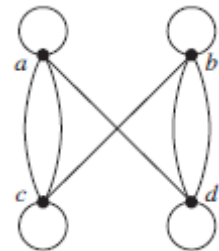
ii)



iii)



iv)



17. Consider the tree shown at right with root a.

i) What is the level of n?

iii) What is the height of this rooted tree?

v) What is the parent of g?

vii) What are the descendants of f?

ix) What are the ancestors of z?

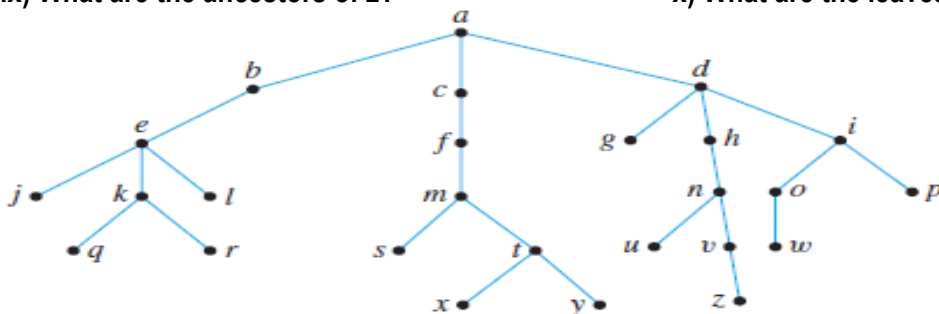
ii) What is the level of a?

iv) What are the children of n?

vi) What are the siblings of j?

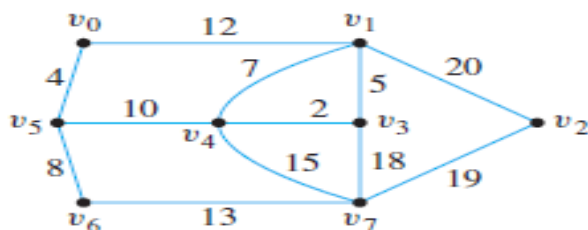
viii) What are the internal nodes?

x) What are the leaves?

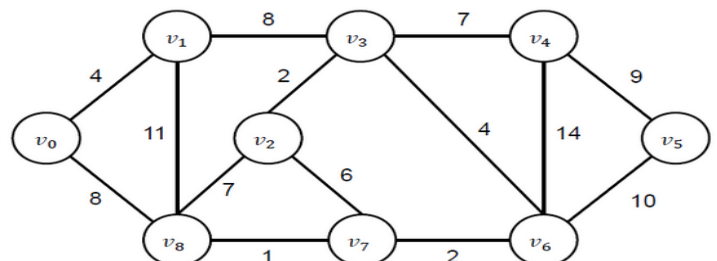


18. Use Prim's algorithm to find a minimum spanning tree starting from  $V_0$  for given graphs. Indicate the order in which edges are added to form each tree.

i)

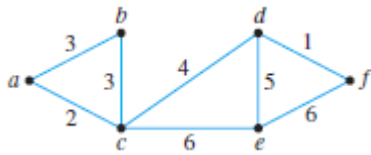


ii)

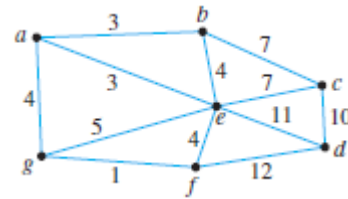


19. Use Kruskal's algorithm to find a minimum spanning tree for given graphs. Indicate the order in which edges are added to form each tree.

i)



ii)



20. (a) i) Build a binary search tree for the word's banana, peach, apple, pear, coconut, mango, and papaya using alphabetical order.

ii) Build a binary search tree for the word's oenology, phrenology, campanology, ornithology, ichthyology, limnology, alchemy, and astrology using alphabetical order.

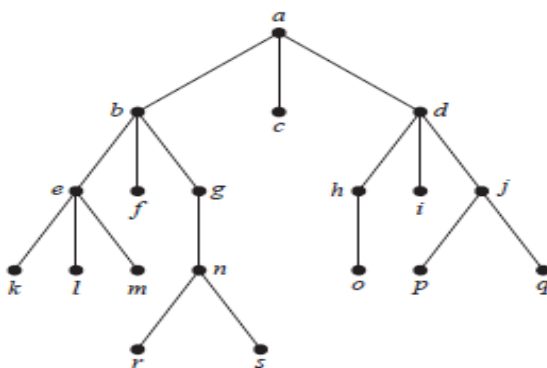
(b) Represent these expressions using binary trees.

i)  $(x + xy) + (x / y)$

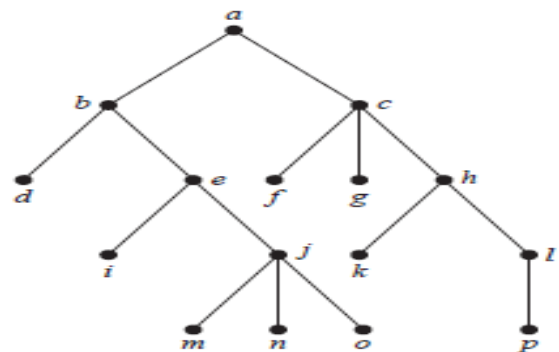
ii)  $x + ((xy + x) / y)$

21. Determine the order in which preorder, Inorder and Postorder traversal visits the vertices of the given ordered rooted tree.

i)



ii)



22. (a) How many edges does a tree with 10000 vertices have?

(b) How many edges does a full binary tree with 1000 internal vertices have?

(c) How many vertices does a full 5-ary tree with 100 internal vertices have?

23. (a) Write these expressions in Prefix and Postfix notation:

i)  $(x + xy) + (x / y)$

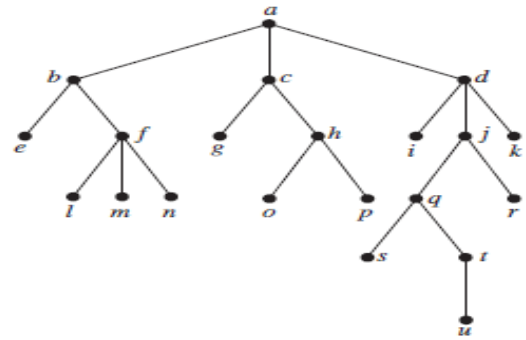
ii)  $x + ((xy + x) / y)$

(b) i) What is the value of this prefix expression  $+ - \uparrow 3 2 \uparrow 2 3 / 6 - 4 2$

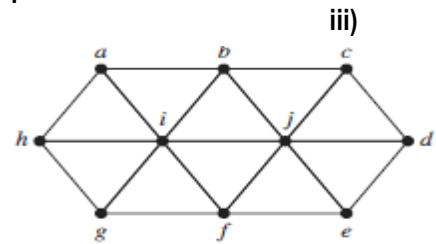
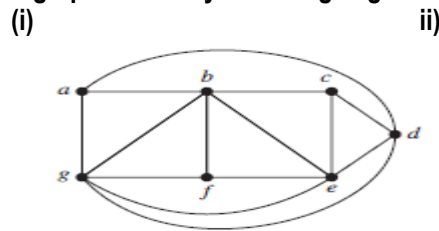
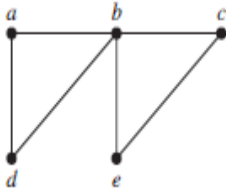
ii) What is the value of this postfix expression  $4 8 + 6 5 - * 3 2 - 2 2 + * /$

**24. Answer these questions about the rooted tree illustrated.**

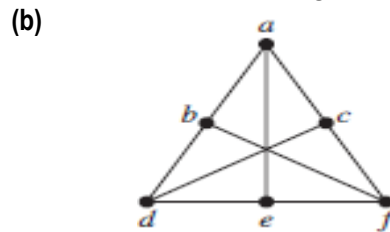
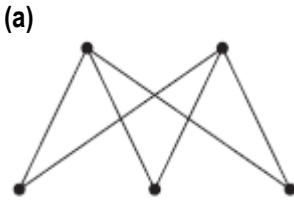
- i) Is the rooted tree a full m-ary tree?  
 ii) Is the rooted tree a Balanced m-ary tree?  
 iii) Draw the subtree of the tree that is rooted at  
 a) c.                      b) f.                      c) q.



25. Find a spanning tree for the graph shown by removing edges in simple circuits. Write down the removed edges.



**26. Determine whether the given graph is planar. If so, draw it so that no edges cross.**



## Task II

27. What are the quotient and remainder when:

- |                           |                           |                          |
|---------------------------|---------------------------|--------------------------|
| a) 19 is divided by 7?    | b) -111 is divided by 11? | c) 789 is divided by 23? |
| d) 1001 is divided by 13? | e) 10 is divided by 19?   | f) 3 is divided by 5?    |
| g) -1 is divided by 3?    | h) 4 is divided by 1?     |                          |

28. (a) Find  $a \div m$  and  $a \bmod m$  when

- |                            |                             |
|----------------------------|-----------------------------|
| i) $a = -111, m = 99.$     | ii) $a = -9999, m = 101.$   |
| iii) $a = 10299, m = 999.$ | iv) $a = 123456, m = 1001.$ |

(b) Decide whether each of these integers is congruent to 5 modulo 17.

- |       |         |          |          |
|-------|---------|----------|----------|
| i) 80 | ii) 103 | iii) -29 | iv) -122 |
|-------|---------|----------|----------|

29. (a) Determine whether the integers in each of these sets are pairwise relatively prime.

- |               |                |                     |                 |
|---------------|----------------|---------------------|-----------------|
| i) 11, 15, 19 | ii) 14, 15, 21 | iii) 12, 17, 31, 37 | iv) 7, 8, 9, 11 |
|---------------|----------------|---------------------|-----------------|

(b) Find the prime factorization of each of these integers.

- |       |         |          |          |         |         |
|-------|---------|----------|----------|---------|---------|
| i) 88 | ii) 126 | iii) 729 | iv) 1001 | v) 1111 | vi) 909 |
|-------|---------|----------|----------|---------|---------|

30. Use the extended Euclidean algorithm to express  $\gcd(144, 89)$  and  $\gcd(1001, 100001)$  as a linear combination.

31. Solve each of these congruences using the modular inverses.

- |                              |                              |
|------------------------------|------------------------------|
| a) $55x \equiv 34 \pmod{89}$ | b) $89x \equiv 2 \pmod{232}$ |
|------------------------------|------------------------------|

32. (a) Use the construction in the proof of the Chinese remainder theorem to find all solutions to the system of congruences.

- |   |
|---|
| i) $x \equiv 1 \pmod{5}, x \equiv 2 \pmod{6}, \text{ and } x \equiv 3 \pmod{7}.$                        |
| ii) $x \equiv 1 \pmod{2}, x \equiv 2 \pmod{3}, x \equiv 3 \pmod{5}, \text{ and } x \equiv 4 \pmod{11}.$ |

(b) An old man goes to market and a camel step on his basket and crushes the oranges. The camel rider offers to pay for the damages and asks him how many oranges he had brought. He does not remember the exact number, but when he had taken them out five at a time, there were 3 oranges left. When he took them six at a time, there were also three oranges left, when he had taken them out seven at a time, there was only one orange was left and when he had taken them out eleven at a time, there was no orange left. What is the number of oranges he could have had?

33. Find an inverse of  $a \bmod m$  for each of these pairs of relatively prime integers.

- |                       |                        |
|-----------------------|------------------------|
| a) $a = 2, m = 17$    | b) $a = 34, m = 89$    |
| c) $a = 144, m = 233$ | d) $a = 200, m = 1001$ |

34. (a) Encrypt the message STOP POLLUTION by translating the letters into numbers, applying the given encryption function, and then translating the numbers back into letters.

- |                              |                                |
|------------------------------|--------------------------------|
| i) $f(p) = (p + 4) \bmod 26$ | ii) $f(p) = (p + 21) \bmod 26$ |
|------------------------------|--------------------------------|

(b) Decrypt these messages encrypted using the Shift cipher.  $f(p) = (p + 10) \bmod 26.$

- |                  |                  |
|------------------|------------------|
| i) CEBBOXNOB XYG | ii) LO WI PBSOXN |
|------------------|------------------|

35. Use Fermat's little theorem to compute  $5^{2003} \bmod 7$ ,  $5^{2003} \bmod 11$ , and  $5^{2003} \bmod 13$ .



36. (a) Encrypt the message I LOVE DISCRETE MATHEMATICS by translating the letters into numbers, applying the Caesar Cipher Encryption function and then translating the numbers back into letters.
- (b) Decrypt these messages encrypted using the Caesar Cipher.  
 i) PLG WZR DVVLJQPHQW      ii) IDVW QXFHV XQLYHUVLWB
37. (a) Which memory locations are assigned by the hashing function  $h(k) = k \bmod 97$  to the records of insurance company customers with these Social Security numbers?  
 i) 034567981      ii) 183211232      iii) 220195744      iv) 987255335
- (b) Which memory locations are assigned by the hashing function  $h(k) = k \bmod 101$  to the records of insurance company customers with these Social Security numbers?  
 i) 104578690      ii) 432222187      iii) 372201919      iv) 501338753
38. What sequence of pseudorandom numbers is generated using the linear congruential generator?  
 $x_{n+1} = (4x_n + 1) \bmod 7$  with seed  $x_0 = 3$ ? Do at least 20 iterations.
39. (a) Determine the check digit for the UPCs that have these initial 11 digits.  
 i) 73232184434      ii) 63623991346
- (b) Determine whether each of the strings of 12 digits is a valid UPC code.  
 i) 036000291452      ii) 012345678903
40. (a) The first nine digits of the ISBN-10 of the European version of the fifth edition of this book are 0-07-119881. What is the check digit for that book?
- (b) The ISBN-10 of the sixth edition of Elementary Number Theory and Its Applications is 0-321-500Q1-8, where Q is a digit. Find the value of Q.
41. Encrypt the message ATTACK using the RSA system with  $n = 43 \cdot 59$  and  $e = 13$ , translating each letter into integers and grouping together pairs of integers.
42. (a) An office building contains 27 floors and has 37 offices on each floor. How many offices are in the building?
- (b) A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in three sizes for each sex. How many different types of this shirt are made?
43. (a) How many different three-letter initials can people have?
- (b) How many different three-letter initials with none of the letters repeated can people have?
44. (a) A wired equivalent privacy (WEP) key for a wireless fidelity (WiFi) network is a string of either 10, 26, or 58 hexadecimal digits. How many different WEP keys are there?
- (b) How many strings are there of four lowercase letters that have the letter x in them?
45. (a) How many functions are there from the set  $\{1, 2, \dots, m\}$ , where  $m$  is a positive integer, to the set  $\{0, 1\}$ ?
- (b) How many one-to-one functions are there from a set with five elements to sets with five elements?
46. (a) Use a tree diagram to determine the number of subsets of  $\{3, 7, 9, 11, 24\}$  with the property that the sum of the elements in the subset is less than 28.

- (b) Teams A and B play in a tournament. The team that wins first two games wins the tournament. Use a tree diagram to find the number of possible ways in which the tournament can occur.
47. (a) Eight members of a school marching band are auditioning for 3 drum major positions. In how many ways can students be chosen to be drum majors?  
 (b) You must take 6 CS elective courses to meet your graduation requirements at FAST-NUCES. There are 12 CS courses you are interested in. In how many ways can you select your elective Courses?  
 (c) Nine people in our class want to be on a 5-person basketball team to represent the class. How many different teams can be chosen?
48. (a) A committee of five people is to be chosen from a group of 20 people. How many different ways can a chairperson, assistant chairperson, treasurer, community advisor, and record keeper be chosen?  
 (b) A relay race has 4 runners who run different legs of the race. There are 16 students on your track team. In how many ways can your coach select students to compete in the race? Assume that the order in which the students run matters.  
 (c) Your school yearbook has an editor in chief and an assistant editor in chief. The staff of the yearbook has 15 students. In how many ways can a student be chosen for these 2 positions?
49. (a) A deli offers 5 different types of meat, 3 types of breads, 4 types of cheeses and 6 condiments. How many different types of sandwiches can be made of 1 meat, 2 bread, 1 cheese, and 3 condiment?  
 (b) Police use photographs of various facial features to help eyewitnesses identify suspects. One basic identification kit contains 15 hairlines, 48 eyes and eyebrows, 24 noses, 34 mouths, and 28 chins and 28 cheeks. Find the total number of different faces.
50. (a) How many bit strings of length 10 either begin with three 0s or end with two 0s?  
 (b) How many bit strings of length 5 either begin with 0 or end with two 1s?
51. (a) Show that if there are 30 students in a class, then at least two have last names that begin with the same letter.  
 (b) Assuming that no one has more than 1,000,000 hairs on the head of any person and that the population of New York City was 8,008,278 in 2010, show there had to be at least nine people in New York City in 2010 with the same number of hairs on their heads.  
 (c) There are 38 different time periods during which classes at a university can be scheduled. If there are 677 different classes, how many different rooms will be needed?
52. (a) What is the coefficient of  $x^5$  in  $(1 + x)^{11}$ ?  
 (b) What is the coefficient of  $a^7b^{17}$  in  $(2a - b)^{24}$ ?
53. A class has 20 women and 16 men. In how many ways can you:  
 (a) put all the students in a row?  
 (b) put 7 of the students in a row?  
 (c) put all the students in a row if all the women are on the left and all the men are on the right?
54. (a) Prove the statement: There is an integer  $n > 5$  such that  $2^n - 1$  is prime.  
 (b) Prove that for any integer  $a$  and any prime number  $p$ , if  $p \mid a$ ,  $P \mid \sqrt[p]{a} + 1$ .
55. (a) Prove the statement: There are real numbers  $a$  and  $b$  such that  $\sqrt{(a + b)} = \sqrt{a} + \sqrt{b}$ .

- (b) Prove that if  $|x| > 1$  then  $x > 1$  or  $x < -1$  for all  $x \in \mathbb{R}$ .
56. (a) Find a counter example to the proposition: For every prime number  $n$ ,  $n + 2$  is prime.  
 (b) Show that the set of prime numbers is infinite.
57. (a) Prove by contradiction method, the statement: If  $n$  and  $m$  are odd integers, then  $n + m$  is an even integer.  
 (b) Prove the statement by contraposition: For all integers  $m$  and  $n$ , if  $m + n$  is even then  $m$  and  $n$  are both even or  $m$  and  $n$  are both odd.
58. (a) Prove by contradiction that  $6 - 7\sqrt{2}$  is irrational.  
 (b) Prove by contradiction that  $\sqrt{2} + \sqrt{3}$  is irrational.
59. By mathematical induction, prove that following is true for all positive integral values of  $n$ .
- (a)  $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$   
 (b)  $1 + 2 + 2^2 + \dots + 2^n = 2^{n+1} - 1$  for all integers  $n \geq 0$   
 (c)  $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{1}{4} n^2(n+1)^2$