Exercise # 6.2

Q1. $u = (-1, 5, 2)$, $v = (2, 4, -9)$

$$\cos\theta = \left(\frac{\langle u,v\rangle}{\|u\|\|v\|}\right)$$

$$\langle u,v\rangle = (-1)(2) + (5)(4) + (2)(-9)$$
$$= -2 + 20 - 18$$
$$= -20 + 20 = 0$$

$$\cos\theta = 0$$

$u = (1, 0, 1, 0)$ $v = (-3, -3, -3, -3)$

$$\cos\theta = \left(\frac{\langle u,v\rangle}{\|u\|\|v\|}\right)$$

$$\langle u,v\rangle = (1)(-3) + 0(-3) + (1)(-3) + 0(-3)$$
$$= -3 + 0 - 3 + 0$$
$$= -6$$

$$\|u\| = \sqrt{1^2 + 1^2} = \sqrt{2}$$
$$\|v\| = \sqrt{9+9+9+9} = \sqrt{36} = 6$$

$$\cos\theta = \left(\frac{-\cancel{6}}{\cancel{6}\sqrt{2}}\right)$$

$$\boxed{\cos\theta = \frac{-1}{\sqrt{2}}}$$

Date

Q3-4: Find the cosine of the angle

$p = -1 + 5x + 2x^2$, $q = 2 + 4x - 9x^2$

$$\cos\theta = \left(\frac{\langle p, q\rangle}{\|p\| \|q\|}\right)$$

$\langle p, q\rangle = (-1)(2) + (5)(4) + (2)(-9)$
$= -2 + 20 - 18$
$= 0$

$\cos\theta = 0$

$p = x - x^2$, $q = 7 + 3x + 3x^2$

$\langle p, q\rangle = (0)(7) + (1)(3) + (-1)(3)$
$= 0 + \cancel{3} - \cancel{3} = 0$

$\cos\theta = 0$

Q5-6: Find the cosine of the angle between A and B

$A = \begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$

$$\cos\theta = \left(\frac{tr(u^T v)}{\sqrt{tr(u^T u)}\sqrt{tr(v^T v)}}\right)$$

$\sqrt{tr(u^T u)} = 2^2 + 6^2 + 1^2 + 3^2$
$\sqrt{tr(v^T v)} = 3^2 + 2^2 + 1^2 + 0^2$

$tr(u^T v) = (2)(3) + (6)(2) + (1)(1) + (-3)(0)$
$= 6 + 12 + 1 + 0$
$= 19$

$$\cos\theta = \frac{19}{10\sqrt{7}}$$

Signature No

Date

Q7-8: Determine whether the vectors are orthogonal w.r.t the euclidean inner product.

$u = (-2, -2, -2) \quad v = (1, 1, 1)$

$\langle u, v\rangle = (-2)(1) + (-2)(1) + (-2)(1)$
$= -2 - 2 - 2 = -6 \neq 0$ not orthogonal.

$u = (-4, 6, -10, 1), \quad v = (2, 1, -2, 9)$

$\langle u, v\rangle = (-4)(2) + (6)(1) + (-10)(-2) + (1)(9)$
$= -8 + 6 + 20 + 9$
$= -2 + 20 + 9$
$= 18 + 9 = 27 \neq 0$ not orthogonal

$u = (u_1, u_2, u_3) \quad v = (0, 0, 0)$

$\langle u, v\rangle = (u_1)(0) + (u_2)(0) + (u_3)(0)$
$= 0$ orthogonal

$p = -1 + x + 2x^2, \quad q = 2x + x^2$

$\langle p, q\rangle = (-1)(0) + (1)(2) + (2)(1)$
$= 0 + 2 + 2 = 4 \neq 0$ not orthogonal

$p = 2 - 3x + x^2, \quad q = 4 + 2x - 2x^2$

$\langle p, q\rangle = (2)(4) + (-3)(2) + (1)(-2)$
$= 8 - 6 - 2$
$= 8 - 8 = 0$ orthogonal.

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Date

$$U = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}, \quad V = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{aligned} tr(U^T V) &= (2)(-3) + (1)(0) + (-1)(0) + (3)(2) \\ &= -6 + 0 + 0 + 6 \\ &= 0 \quad \text{orthogonal} \end{aligned}$$

Q19 Let P_2 have the evaluation inner product at the points
$x_0 = -2, \quad x_1 = 0, \quad x_2 = 2$
Show that the vectors $p = x$ and $q = x^2$ are orthogonal with respect to this inner product.

$$\begin{aligned} \langle p, q \rangle &= p(x_0)q(x_0) + p(x_1)q(x_1) + p(x_2)q(x_2) \\ &= p(-2)q(-2) + p(0)q(0) + p(2)q(2) \\ &= (-2)(4) + (0)(0) + (2)(4) \\ &= -8 + 0 + 8 = 0 \longrightarrow \text{orthogonal} \end{aligned}$$

Q18. Show that the vectors $u = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ and $v = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$ are orthogonal w.r.t to the inner product on R^2 that is generated by:

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{aligned} &= Au \cdot Av \\ &= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ -8 \end{bmatrix} = \begin{bmatrix} 6+3 \\ 3+3 \end{bmatrix} \cdot \begin{bmatrix} 10-8 \\ 5-8 \end{bmatrix} \\ &= \begin{bmatrix} 9 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \end{bmatrix} = 18 + (-18) = 0 \end{aligned}$$

orthogonal.

Exercise # 6.3 Date:

Q27. let R^2 have the euclidean inner product and use the GSP to transform the basis $\{u_1, u_2\}$ into an orthonormal basis $\rightarrow q_1, q_2$

$u_1 = (1,-3) \quad u_2 = (2,2)$

$v_1 = u_1 = (1,-3)$

$v_2 = u_2 - \frac{\langle u_2, v_1 \rangle}{\|v_1\|^2} v_1$

$= (2,2) - \frac{(2-6)}{\sqrt{10}}(1,-3)$

$= (2,2) - \frac{(-4)^2}{\cancel{16}\,\cancel{10}\,5}(1,-3)$

$= (2,2) - \left(\frac{-2}{5}\right)(1,-3) \Rightarrow (2,2) - \left(\frac{-2}{5}, \frac{6}{5}\right)$

$= (2,2) - \left(\frac{-2}{5}, \frac{6}{5}\right) = \left(\frac{12}{5}, \frac{4}{5}\right)$

$2 + \frac{2}{5} = \frac{10+2}{5}$

$\frac{12}{5}$

$2 - \frac{6}{5} = \frac{10-6}{5}$

$\frac{4}{5}$

$q_1 = \frac{v_1}{\|v_1\|} = \frac{(1,-3)}{\sqrt{10}} = \left(\frac{1}{\sqrt{10}}, \frac{-3}{\sqrt{10}}\right)$

$q_2 = \frac{v_2}{\|v_2\|} = \frac{\left(\frac{12}{5}, \frac{4}{5}\right)}{\sqrt{\frac{160}{25}}} = \left(\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right)$

Q.R DECOMPOSITION

MATRIX $\leftarrow A = QR$

$\downarrow$ ORTHONORMAL

$$Q = \left[\, q_1 \;\middle|\; q_2 \;\middle|\; q_3 \,\right]$$

$$R = \begin{bmatrix} \langle u_1, q_1 \rangle & \langle u_2, q_1 \rangle & \langle u_3, q_1 \rangle \\ 0 & \langle u_2, q_2 \rangle & \langle u_3, q_2 \rangle \\ 0 & 0 & \langle u_3, q_3 \rangle \end{bmatrix}$$

Q. Find Q.R decomposition of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

Solution. $u_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \; u_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \; u_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

Find v_1, v_2 and v_3 by GSP then find q_1, q_2 & q_3

$q_1 = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) \quad q_2 = \left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

$q_3 = \left(\frac{2}{\sqrt{24}}, \frac{4}{\sqrt{24}}, \frac{-2}{\sqrt{24}}\right)$

$$Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{3}} & 2/\sqrt{24} \\ 0 & \frac{1}{\sqrt{3}} & 4/\sqrt{24} \\ \frac{1}{\sqrt{2}} & 1/\sqrt{3} & -2/\sqrt{24} \end{bmatrix}$$

$$R = \begin{bmatrix} \langle u_1, q_1 \rangle & \langle u_2, q_1 \rangle & \langle u_3, q_1 \rangle \\ 0 & \langle u_2, q_2 \rangle & \langle u_3, q_2 \rangle \\ 0 & 0 & \langle u_3, q_3 \rangle \end{bmatrix}$$