National University of Computer & Emerging Sciences, Karachi

Fall-2020 - Department of Computer Science

Bachelor of Science (Computer Science)

Final Examination

January 05, 2021, 03:00 pm – 06:00 pm

Course Code: CS211	Course Name: Discrete Structures
Instructor Names: Dr. Fahad Samad, Mr. Shoaib Raza and Ms. Bakhtawar Abbasi	
Student Roll No:	Section No:

Instructions:

- Return the question paper together with the answer script. Read each question completely before answering it. There are **6 questions and 4 pages**. Each question consists of **6 parts.**
- In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.
- For the problems below, we can award partial credit only if you show your work.
- Attempt all the questions (parts) in the given sequence of the question paper to get bonus point.

Total Time: 3 Hours **Maximum Points**: 72 Points

Question # 1: Propositional Logic, Rules of Inference, Predicate Logic and Quantifiers **[2x6 =12 points]**

(i) Consider the following system specifications and translate each of them into notations of propositional logic. Propositions are given as follows:

M = in Multiuser state, **N = Operating normally,** **K = Kernel is functioning,** **I = in Interrupt mode**

(a) The system to be in multiuser state is necessary and sufficient for system to operate normally.

(b) The kernel is functioning if the system is operating normally.

(c) The kernel is not functioning or the system is in interrupt mode.

(d) If the system is not in multiuser state, then it is in interrupt mode.

(ii) Determine using laws of logic if the given expression is a tautology, contradiction or a contingency.

$$((p \vee q) \wedge (p \rightarrow r)) \rightarrow (q \vee r)$$

(iii) Using Rules of inference, show that the following argument is valid.

$$((\neg r \rightarrow (s \rightarrow \neg t)) \wedge (\neg r \vee w) \wedge (\neg p \rightarrow s) \wedge (\neg w)) \rightarrow (t \rightarrow p)$$

Suppose F (p, q) is the predicate "**p understands q**", the universe of discourse for p is "the set of students in your class", and the universe of discourse for q is "the set of examples in the lecture notes."(For parts (iv) & (v) only)

(iv) Write the following predicate expressions in good English without using variables in your answers:

(a) $\exists p \, \forall q \, F(p, q)$

(b) $\forall q \, \exists p \, F(p, q)$

(v) Write the predicate expressions of the following statements using variables and any needed quantifiers:

(a) Every student in this class understands at least one example in the notes."

(b) There is an example in the notes that every student in this class understands."

(vi) Determine the truth value of each of these statements if the domain for all variables consists of all real numbers.

(a) $\forall x \, \exists y \, (x^2 = y)$

(b) $\forall x \, \exists y \, (x = y^2)$

QUESTION # 2: Set Theory, Relations and Functions

[2x6 =12 points]

(i) Let X and Y be two sets. Prove or disprove using the set builder notation that $X - (X \cap Y) = (X - Y)$.

(ii) Let R be the relation on {a, b, c, d, e} represented by the digraph shown in figure 1. What is $R \circ R$? (draw digraph).

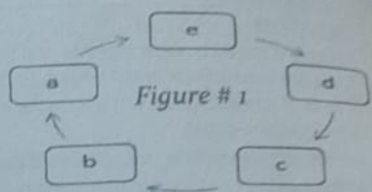


Figure # 1

(iii) A tournament graph G = (V, E) is a directed graph such that there is either an edge from u to v or an edge from v to u for every distinct pair of nodes u and v. (The nodes represent players and an edge u → v indicates that player u beats player v.) Consider the "beats" relation implied by a tournament graph. Indicate whether Partial order or Equivalence relation hold for all tournament graphs and briefly explain your reasoning. You may assume that a player never plays herself.

(iv) Consider the function g: Z→Z defined by $g(n)=n^2+1$. Find g (1) and g ({1}). Is g (1) = g ({1}) are equal if not why?

(v) Consider the function f: N→N given by f(0)=0 and f(n+1)=f(n)+2n+1. Find f (6).

(vi) How many functions are there from a set with four elements to a set with three elements?

Question # 3: Graph Theory and Trees

[2x6=12 points]

(i) Construct the Minimum spanning tree (MST) for the given graph in figure # 2 using PRIM'S and KRUSKAL'S algorithms.

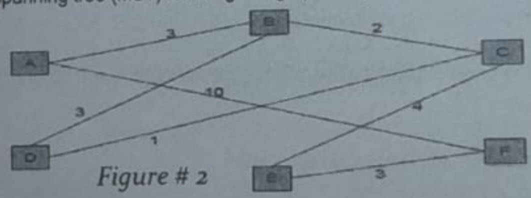
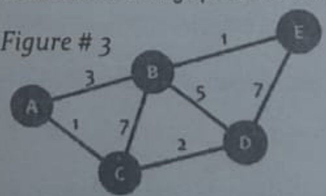


Figure # 2

(ii) Find the Shortest path from Node C to all other nodes in graph as shown in figure # 3 using Dijkstra's algorithm.

Figure # 3



(iii) Below is a graph as shown in figure # 4 representing friendships between a group of students (each vertex is a student and each edge is a friendship). Is it possible for the students to sit around a round table in such a way that every student sits between two friends? What does this question have to do with paths?

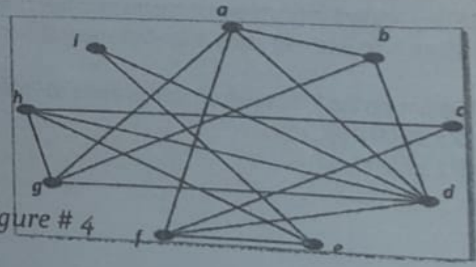


Figure # 4

Determine if the following two graphs G and H are isomorphic as shown in figure # 5. If they are, give function F: V(G) → V (H) that define the isomorphism. If they are not, give the reason why?

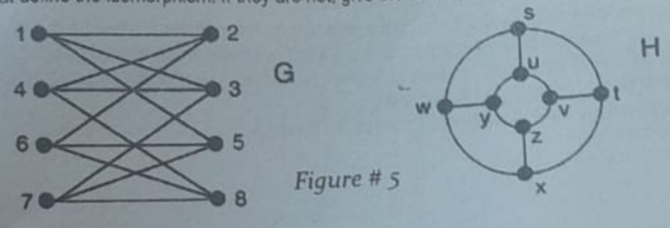
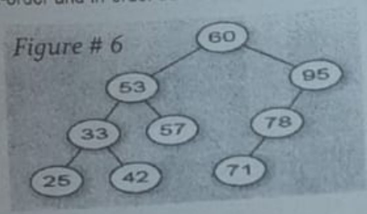


Figure # 5

(v) Construct Pre-order, Post-order and In-order traversals of the given tree as shown in figure # 6.



(vi) Convert the given expression into postfix and prefix notations. (A + B) * C - (D - E) * (F + G).

[2x6=12 points]

Question # 4: Combinatorics and Discrete Probability

Suppose that you roll five 6-sided dice that are fair and mutually independent. (For parts (i) & (ii) only)

(i) What is the probability that all five dice show different values?
Example: (1, 2, 3, 4, 5) is a roll of this type, but (1, 1, 2, 3, 4) is not.

(ii) What is the probability that two dice show the same value and the remaining three dice all show different values?
Example: (6, 1, 6, 2, 3) is a roll of this type, but (1, 1, 2, 2, 3) and (4, 4, 4, 5, 6) are not.

(iii) Nine chairs are numbered 1 to 9. Three women and four men wish to occupy one chair each. First the women chose the chairs from amongst the chair marked 1 to 5; and then the men select the chairs from amongst the remaining. The number of possible arrangements is?

(iv) How many ways are there of choosing k things from {1 . . . n} if 1 and 2 can't both be chosen? (Suppose n, k ≥ 2.)

(v) How many ways are there to distribute four distinct balls evenly between two distinct boxes (two balls go in each box)?

(vi) Suppose that all license plates have three uppercase letters followed by three digits.
(a) How many license plates begin with B and end in 1?
(b) How many license plates are possible in which all the letters and digits are distinct?

Question # 5: Number theory, Binomial theorem and Pigeon hole Principle [2x6=12 points]

(i) A shipwrecked sailor passes the time of day by counting the coconuts he has gathered. When he counts by threes, there are 2 coconuts left over. When he counts by fives, there are 4 left over, and when he counts by sevens, there are 5 left over, and when he counts by eleven, there is only one coconut left. How many coconuts has the sailor gathered if he is positive that he had fewer than 150 coconuts.
In this problem, you are supposed to state and use of the following Theorems:
a. Chinese Remainder Theorem
b. The Euclidean Algorithm Lemma
c. Bézout's Theorem
d. Linear congruences

(ii) Use Fermat's little theorem to calculate the remainder of 9^{2579} mod 79.

(iii) What is the co-efficient of x^7y^2 in the expansion of $(x + 3y)^9$.

Suppose that every student in a discrete mathematics class of 25 students is a freshman, a sophomore, or a junior.

(iv) Show that there are at least nine freshmen, at least nine sophomores or at least nine juniors in the class.

(v) Show that there are either at least three freshmen, at least 19 sophomores, or at least five juniors in the class.

(vi) Find the check digit of the following Universal Product Code (UPC): 69277198116.

Question # 6: Proofs, Mathematical Induction and Cryptography [2x6=12 points]

(i) Using Direct proof method, prove that If a is an integer such that a – 2 is divisible by 3, then a^2 - 1 is divisible by 3.

(ii) Using Contradiction method, prove that there are no integer x and y such that $x^2 = 4y + 2$.

(iii) Prove by Contraposition that for all integers a and b, if a + b is odd then a is odd or b is odd.

(iv) Prove using mathematical induction that $1^3 + 2^3 + 3^3 + ... + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ whenever n is a nonnegative integer.

(v) Prove or disprove by counterexample: The sum of squares of two numbers is an odd number.

(vi) Jack is sending Tommy a message with RSA. The public key is 3, while n = p*q is 55. What is the value of d that Tommy must use to decrypt the message?
Hint: Public key <e, n> and Private Key <d, n>

BEST OF LUCK ☺