FAST National University of Computer and Emerging Sciences, Karachi

FAST School of Computing

Midterm-2 Examination, Fall-2021

November 25, 2021, 10:30 am – 11:30 am

Course Code: CS1005	**Course Name: Discrete Structures**
Instructor Names: Mr. Shoaib Raza, Mr. Musawar Ali, and Ms. Safia	
Student Roll No:	**Section No:**

Instructions:

- Return the question paper together with the answer script. Read each question completely before answering it. There are **3 questions** written on **2 pages.**
- In case of any ambiguity, you may make assumptions. However, your assumptions should not contradict any statement in the question paper.
- Attempt all the questions in given sequence of the question paper. Show all steps properly in order to get full points.

Total Time: 60 Minutes **Maximum Points:** 24

Question # 1: [Sequence & Series + Relations] **[4x2=08 points]**

(a) Find the sum of number between 150 and 500 which are divisible by 11.

(b) Express in sigma notation the sum of the first 100 terms of the series 2 + 4 + 6 + 8 + 10 + 12 +. ...

(c) Let $R = \{(a, c), (c, e), (d, a), (e, b), (b, d)\}$ be the relation on {a, b, c, d, e}. Write $R \circ R^{-1}$ in matrix form.

(d) Prove or disprove that relation given in figure # 1 digraph is partial order or equivalence relation. Discuss all properties.

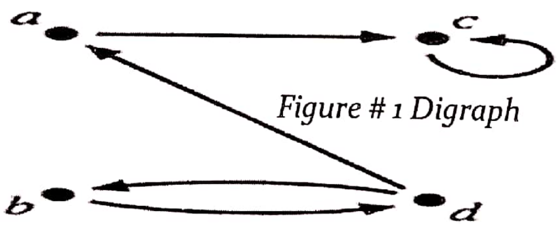

Figure # 1 Digraph

Question # 2: [Proofs + Mathematical Inductions] **[3x2= 06 points]**

(a) Suppose that $w^2 + x^2 + y^2 = z^2$, where w, x, y, and z always denote positive integers. Prove the proposition by using direct proof: "z is even if and only if w, x, and y are even."

(b) Let x be an integer and P is the following statement. P: "If $x^2 - (x - 2)^2$ is not divisible by 8, then x is even." Prove by contraposition.

(c) Prove using mathematical induction that $1^2 + 3^2 + 5^2 + \ldots + (2a - 1)^2 = \frac{a(2a-1)(2a+1)}{3}$, whenever "a" is a nonnegative integer.

<u>Question # 3:</u> [Number Theory + Cryptography] **[5x2=10 points]**

Alice and Bob are cousins. Both are good in Number Theory. They challenged one-another to solve different problems.

(a) Alice gifted Bob a book entitled "Discrete Mathematics and Its Applications". Suppose that first 9 digits of ISBN-10 of the textbook are *125973128*. How can Bob find the check digit to validate the originality of the book?

(b) Bob has asked Alice to find the greatest common divisor, d, of 250 and 29 and determine integers x and y such that d = 250x + 29y.

(c) Now it is Alice's turn to ask a question. He asks Bob to show her that:
(i) 67 is an inverse of 7 modulo 26. (ii) 854 is an inverse of 123 modulo 4567.

(d) Bob ask Alice to use the hashing function h(x) = (x + 9) mod 65 to determine the memory locations at which the values 63, 509, 197, 83, and 652 are stored.

(e) Suddenly their cousin Eve enters the room, Alice tells Bob "*STOP SPEAKING*". Encrypt the message using the RSA system with n = 5 · 7 and e = 11. Translate each letter into integers and write in the form of cipher text equation.

ALL THE BEST