

Discrete Structure (CS1005)

Final Exam

Date: 22nd May 2024

Course Instructor

Mr. Muhammad Jamil

Total Time (Hrs): 03

Total Marks: 80

Total Questions: 05

Roll No

Section

Student Signature

Do not write below this line.

Attempt all the questions.

CLO # 2: Construct formal logic, proofs and/ or informal for solving problems in discrete structure.

Q1:

16 Marks

- a. Find an equivalent compound proposition that does not involve conditionals. [2]
 $(p \rightarrow \neg q) \rightarrow (\neg p \rightarrow q)$
- b. Rewrite the following statements in the if-then form. Then write the converse, inverse, and contra positive of your implication. [2]
"Divisibility by 21 is a sufficient condition for divisibility by 7".
- c. Determine the truth value of the following statement if the domain consist of all real numbers.[2]
 - i. $\exists x(x^3 = -1)$
 - ii. $\forall x((-x)^2 = x^2)$
 - iii. $\exists x(x^4 < x^2)$
 - iv. $\forall x(2x > x)$
- d. Let $P(x)$ and $q(x)$ be given by $p(x)$: x is odd , $q(x)$: $x^2 - 1$ is even and the statement " if x is odd , then $x^2 - 1$ is even " [2]
 - i. Translate the statements into logical expression using predicates, quantifiers, and logical connectives
 - ii. Write negation of the above expression in Q1 part d(i).
- e. Prove or disprove : $x^2 - x, \forall x \in \mathbb{Z}$ is an even integer.[2]
- f. Use mathematical induction to show that $n^3 - 7n + 3$ is divisible by 3 for all positive integer n . [2]
- g. Write each of the following arguments in symbolic form. Then check the validity of the argument using truth table. [4]

'If Rochelle gets the supervisor's position and works hard, then she'll get a raise. If she gets the raise, then she'll buy a new car. She has not purchased a new car. Therefore, either Rochelle did not get the supervisor's position or she did not work hard'.

CLO #1. Explaining and solving mathematical logic of sets, relations, counting and permutations in Discrete structure.

Q2:

20 Marks

- a. Find the coefficient of x^3y^2 in the expansion of $(3x - 7y)^5$. [2]
- b. Let $f(x) = \lfloor x^2/3 \rfloor$ find $f(s)$ if $s = \{-2, -1, 0, 1, 2, 3\}$. [2]
- c. Find an explicit formula for the following linear homogeneous recurrence relation: [2]
 $3a_n = 7a_{n-1} - 2a_{n-2}$, if $n > 1$, Where $a_0 = -2$ and $a_1 = 1$.

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- d. In how many ways can 12 different books be distributed among four children of school, so that each child gets three books. [2]
- e. What is the least number of area codes needed to guarantee that the 25 million phones in a state can be assigned distinct 10 -digit telephone numbers? (Assume that telephone numbers are of the form $NXX - NXX - XXXX$, where the first three digits form the area code, N represents a digit from 2 to 9 inclusive, and X represents any digit.) [3]
- f. During freshman orientation at a small liberal arts college, two showing of the latest James Bond movie were presented. Among the 600 freshman ,80 attended the first showing and 125 attended the second showing, while 450 didn't make it to either showing. How many of the 600 freshman attended twice, use Inclusion-Exclusion principal. [3]
- g. List all the ordered pairs in the relation $R = \{(a, b) | a \text{ divides } b\}$ on the set $\{1,2,3,4,5,6\}$ and check whether the relation R is Irreflexive , Antisymmetric or Asymmetric (justify) [4]

CLO # 3: Use discrete structure in solving other computing problems such as formal specification, verification, databases, artificial intelligence and cryptography.

Q3:

6*2+4=16 Marks

- a. What is the greatest common divisor (GCD) and least common multiple(LCM) of pair of integers? [2]
 $3^7.5^3.7^3$ and $2^{11}.3^5.5^9$
- b. Without using Fermat's theorem find the remainder of $12^{43} \text{ mod } 713$. [2]
- c. Let f be the function from the set $X = \{2,3,4,5,6,7\}$ into the set $Y = \{0,1,2,3,4\}$ defined by $f(x) = 2x \text{ (mod } 5)$. Write f as a set of ordered pairs. Is f one-one or onto? Explain. [2]
- d. The first nine digits of ISBN-10 of the 7th edition of elementary number theory and its application is 007288008 . What is the check digit? [2]
- e. Find the sequence of pseudorandom numbers generated by the power generator with $p = 7, d = 3$ and seed $x_0 = 2$ [2]
- f. Solve the linear congruence $17x \equiv 4 \text{ (mod } 36)$ using modular inverse. [4]
- g. Use RSA algorithm ,Encrypt the Message "HELLO" where $p = 7, q = 11$ and $e = 17$. [4]

CLO # 1: Use discrete structure in solving other computing problems such as formal specification, databases, graphs and Trees.

Q4:

16 Marks

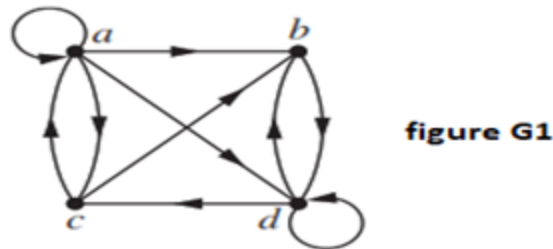
- a. Complete the following table-01 with Yes or No.[2]

S.No.	Type	Edges	Multiple edges allowed ?	Loops allowed?
1	Simple graph	Undirected		
2	Multigraph	Undirected		
3	Pseudo graph	Undirected		
4	Simple directed graph	Directed		
5	Directed multigraph	Directed		

Table-01

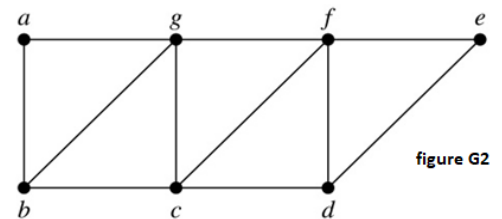
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- b. Write adjacency list, adjacency matrix, in-degree and out-degree of each vertex of figure G1. Is figure G1 connected? [3]



- c. Draw the graph of $K_{3,4}$ and determine graph is planar or not using Euler theorem. [2]
- d. Check graph obtained in Q4 part(c) for Bipartite if yes explain? and what is the chromatic number of $K_{3,4}$ [2]

- e. Determine whether in figure G2 has an Euler circuit or Hamiltonian. Construct such a circuit when one exists. If no Euler or Hamilton circuit exists, provide a clear and concise argument. [2]



- f. Determine whether figure G3 and G4 are isomorphic. If they are, give function $F: V(G3) \rightarrow V(G4)$ that defines the isomorphism. If they are not, give an invariant for graph isomorphism that they do not share. [2]

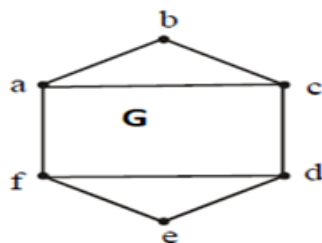


figure G3

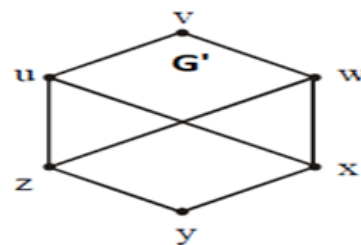
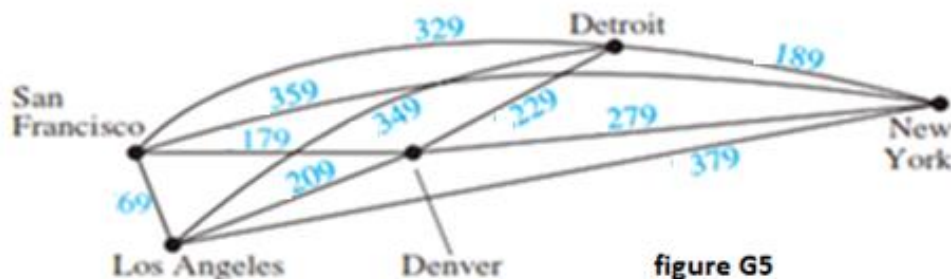


figure G4

- g. Find a route with the least total airfare that visit each of the cities in figure 5. where the weight on an edge is least price available for flight between two cities [3]



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CLO # 4: Distinguish various discrete structure and their relevance within the context of computer science, in the areas of data structure and algorithms, in particular.

Q5

12 Marks

a. Answer the following for the tree shown in figure G6 with root "a". [2]

- what is level of b?
- what are internal nodes?
- what are descendants of j?
- what are sibling of g?

b. Consider the figure G6 .[2]

- Determine the order in which preorder, Inorder and Postorder traversal visits the vertices of the ordered rooted tree in figure G6.
- Is the rooted tree a full m-ary tree or balanced tree? explain

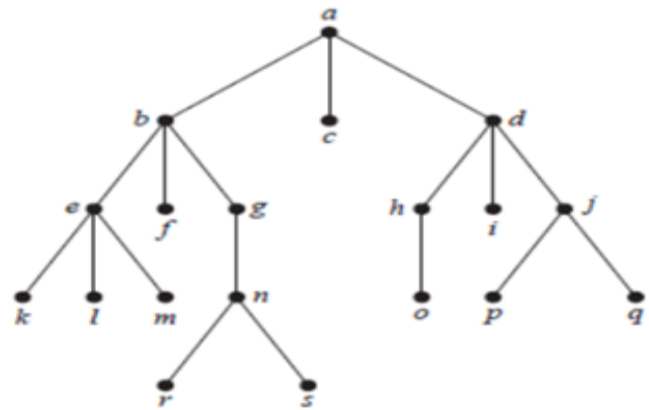


figure G6

- Form a binary search tree for the words mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry using alphabetical order. [2]
- Evaluate the postfix expression $521 - -314 + + *$ [2]
- Represent the compound proposition using ordered rooted tree and write the expression in prefix and infix notation $\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$. [2]
- Find the minimum spanning tree of figure G7. Indicate the order in which edges are added to form the tree using Prim's algorithm. [2]

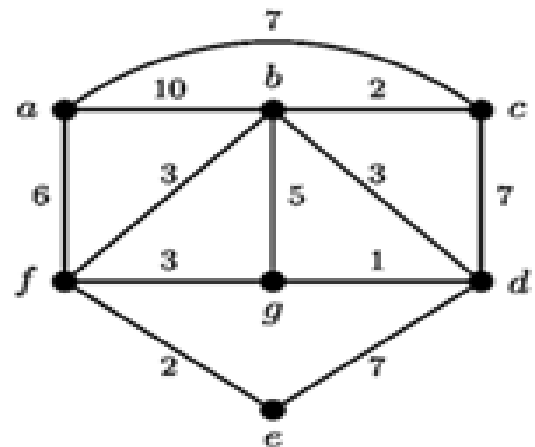


figure G7

WISH YOU ALL THE BEST