# Quiz Solution

**Q** Show that
(A-B) – (B-C) = A-B (Proof using Set Identities)

**Q** Use the inference rules and find the conclusion. Also, name the rule. (5 marks)

- "It is not sunny this afternoon and it is colder than yesterday."
- "We will go swimming only if it is sunny."
- "If we do not go swimming, then we will take a canoe trip."
- "If we take a canoe trip, then we will be home by sunset."

Using the inference rules, construct a valid argument for the conclusion:
"We will be home by sunset."

**<u>Solution</u>**
p: "It is sunny this afternoon."
r: "We will go swimming."
t: "We will be home by sunset."
q: "It is colder than yesterday."
s: "We will take a canoe trip

$\neg p \wedge q$ (Specialization)
$\neg p$
$R \rightarrow p$ (Modus tollens)
$\neg r$
$\neg r \rightarrow \neg s$ (Modus ponens)
S
$S \rightarrow t$ (Modus ponens)
t

Simplify the following expression

$$\neg p \; v \; \neg q \; v \; (p \;\wedge q \wedge \neg r)$$

p=a ; q= b and c=r

Ans:

$= [\neg a \vee \neg b] \vee (a \wedge b \wedge \neg c)$

$=([\neg a \vee \neg b] \vee a) \wedge ([\neg a \vee \neg b] \vee b) \wedge ([\neg a \vee \neg b] \vee \neg c)$

$=([\neg a \vee a] \vee \neg b) \wedge ([\neg b \vee b] \vee \neg a) \wedge (\neg a \vee \neg b \vee \neg c)$

$=(\boldsymbol{T} \vee \neg b) \wedge (\boldsymbol{T} \vee \neg a) \wedge (\neg a \vee \neg b \vee \neg c)$

$=(\boldsymbol{T}) \wedge (\boldsymbol{T}) \wedge (\neg a \vee \neg b \vee \neg c)$

$= \neg a \vee \neg b \vee \neg c$

Q Compute (f o g)(x) and (g o f)(x) for $f(x) = x^2 + 3, g(x) = \sqrt{5 + x^2}$

Solution: $(fog)(x) = f[g(x)] = f[\sqrt{5 + x^2}] = (\sqrt{5 + x^2})^2 + 3 = 8 + x^2$

$(gof)(x) = g[f(x)] = g[x^2 + 3] = \sqrt{5 + (x^2 + 3)^2} = \sqrt{x^4 + 6x^2 + 14}$

Find (f o g)(x) (1)

9

Find (g o f)(x)(3)

Sqrt(149)

Q If f : Q ⇢ Q is given by f(x) = x2 , then find f-1(16).

*Let $f^{-1}(16) = x$*

*$f(x) = 16$*

*$\Rightarrow x^2 = 16$*

*$\Rightarrow x = \pm 4$*

*Thus, $f^{-1}(16) = \{-4, 4\}$*

Q Prove that $(p \vee \neg q) \wedge (\neg p \vee \neg q)$ is logically equivalent to $\neg q$

**10: prove that** (p ∨ ~q) ∧ (~p ∨ ~q) is logically equivalent to **¬q**

(~q ∨ p) ∧ (~q ∨ ~p)

~q ∨ (p ∧ ~p)

~q ∨ c

~q

Q You are about to leave for school in the morning and discover that you don't have your glasses. You know the following statements are true:

- If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.
- If my glasses are on the kitchen table, then I saw them at breakfast.
- I did not see my glasses at breakfast.

- I was reading the newspaper in the living room or I was reading the newspaper in the kitchen.
- If I was reading the newspaper in the living room then my glasses are on the coffee table.

Where are the glasses?

**Solution:**

Let,

RK = I was reading the newspaper in the kitchen.

GK =My glasses are on the kitchen table.

SB =I saw my glasses at breakfast.

RL =I was reading the newspaper in the living room.

GC =My glasses are on the coffee table.

1. RK →GK by(a)

   GK →SB by(d)

   ∴ RK →SB by transitivity

2. RK →SB by the conclusion of (1)

~SB by (c)

∴ ~RK by modus tollens

3. RL V RK by(d)

~RK by the conclusion of (2)

∴ RL by elimination

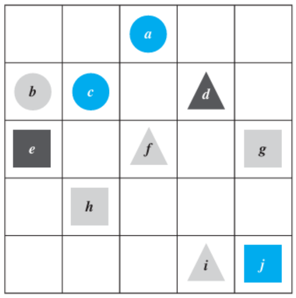
4. RL →GC by(e)

   RL by the conclusion of (3)

   ∴ GC by modus ponens

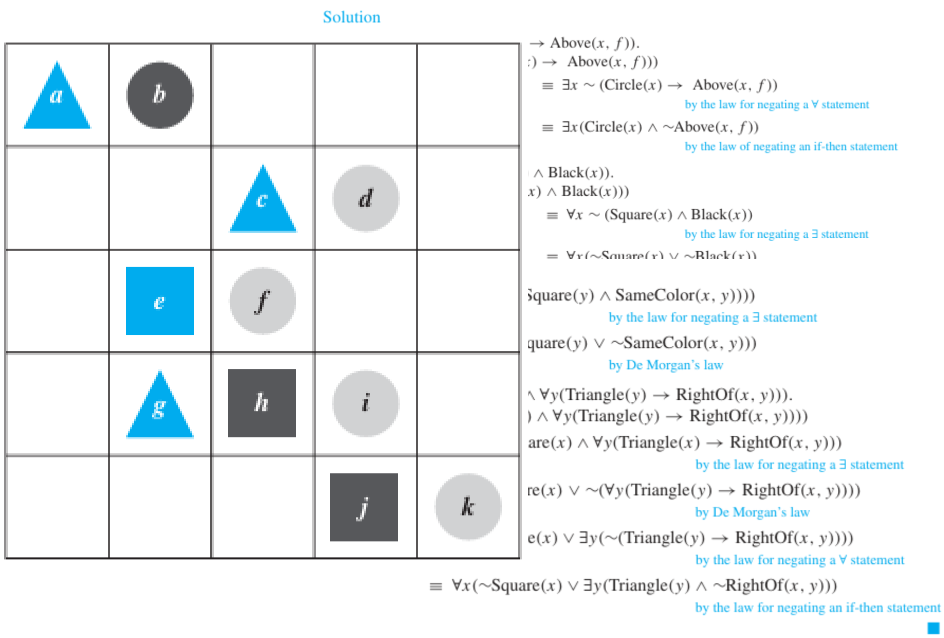
   Thus the glasses are on the coffee table.

Q Let Triangle(x), Circle(x), and Square(x) mean “x is a triangle,” “x is a circle,” and “x is a square”; let Blue(x), Gray(x), and Black(x) mean “x is blue,” “x is gray,” and “x is black”; let RightOf(x, y), Above(x, y), and SameColorAs(x, y) mean “x is to the right of y,” “x is above y,” and “x has the same color as y”; and use the notation x = y to denote the predicate “x is equal to y”. Let the common domain D of all variables be the set of all the objects in the Tarski world. Use formal, logical notation to write each of the following statements, and write a formal negation for each statement.




- For all circles x, x is above f .
- There is a square x such that x is black
- For all circles x, there is a square y such that x and y have the same color.
- There is a square x such that for all triangles y, x is to the right of y.

**Solution**




Q Write formal negations for the following statements:

- ∀ primes p, p is odd.
- ∃ a triangle T such that the sum of the angles of T equals 200◦.

**Solution**

∃ a prime p such that p is not odd
∀ triangles T, the sum of the angles of T does not equal 200◦.

Q The program for Tarski's World provides pictures of blocks of various sizes, shapes, and colors, which are located on a grid. Shown in Figure 3.1.1 is a picture of an arrangement of objects in a two-dimensional Tarski world. The configuration can be described using logical operators and—for the two-dimensional version—notation such as Triangle(x), meaning "x is a triangle," Blue(y), meaning "y is blue," and RightOf(x, y), meaning "x is to the right of y (but possibly in a different row)." Individual objects can be given names such as a,b, or c

Determine the truth or falsity of each of the following statements. The domain for all variables is the set of objects in the Tarski world shown above.

- ∀t, Triangle(t) → Blue(t).
- ∀x,Blue(x) → Triangle(x).
- ∃y such that Square(y)∧ RightOf(d, y).
- ∃z such that Square(z)∧ Gray(z)

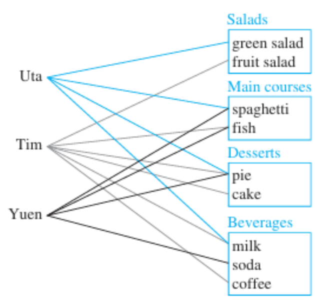
Solution

a. This statement is true: All the triangles are blue.

b. This statement is false. As a counterexample, note that *e* is blue and it is not a triangle.

c. This statement is true because *e* and *h* are both square and *d* is to their right.

d. This statement is false: All the squares are either blue or black. ■




**Q** A college cafeteria line has four stations: salads, main courses, desserts, and beverages. The salad station offers a choice of green salad or fruit salad; the main course station offers spaghetti or fish; the dessert station offers pie or cake; and the beverage station offers milk, soda, or coffee. Three students, Uta, Tim, and Yuen, go through the line and make the following choices: Uta: green salad, spaghetti, pie, milk Tim: fruit salad, fish, pie, cake, milk, coffee Yuen: spaghetti, fish, pie, soda.

Write each of following statements informally and find its truth value.

- ∃ an item I such that ∀ students S, S chose I.
- ∃ a student S such that ∀ items I, S chose I.
- ∃ a student S such that ∀ stations Z,∃ an item I in Z such that S chose I.
- ∀ students S and ∀ stations Z,∃ an item I in Z such that S chose I

Solution

a. There is an item that was chosen by every student. This is true; every student chose pie.

b. There is a student who chose every available item. This is false; no student chose all nine items.

c. There is a student who chose at least one item from every station. This is true; both Uta and Tim chose at least one item from every station.

d. Every student chose at least one item from every station. This is false; Yuen did not choose a salad. ■

Q Show that (A - B) – C = A – (B ∪ C).

**Solution:**

$(A - B) - C = (A - B) \cap \bar{C}$ $\quad\quad$ $(P - Q = P \cap \bar{Q})$

$= (A \cap \bar{B}) \cap \bar{C}$

$= A \cap (\bar{B} \cap \bar{C})$ $\quad\quad$ (Associative)

$= A \cap (\overline{B \cup C})$ $\quad\quad$ (De Morgan's law)

**Q** Use inference rules to reach to a conclusion.

- Larry is a student at the university.
- Hubert is a student at the university.
- Larry and Hubert are taking Boolean Logic.
- Any student who takes Boolean Logic can take Algorithms.
- ∴ Larry and Hubert can take Algorithms

**Solution:**

B(x): x is taking Boolean Logic

A(x): x can take Algorithms

| | |
|---|---|
| • Larry is a student at the university. | Hypothesis |
| • Hubert is a student at the university. | Hypothesis |
| • B(Larry) ∧ B(Hubert) | Hypothesis |
| • B(Larry) | Simplification,3 |
| • ∀x (B(x) → A(x)) | Hypothesis |
| • B(Larry) → A(Larry) | Universal instantiation, 1, 5 |
| • A(Larry) | Modus ponens, 4, 6 |
| • B(Hubert) | Simplification, 3 |
| • B(Hubert) → A(Hubert) | Universal instantiation, 2, 5 |
| • A(Hubert) | Modus ponens, 8, 9 |
| • A(Larry) ∧ A(Hubert) | Addition, 7, 10 |

- Prove that A∩B=B-(B-A)

| | |
|---|---|
| $A \cap B = B - (B \cap \overline{A})$ | Definition of difference |
| $= B \cap \overline{(B \cap \overline{A})}$ | Definition of difference |
| $= B \cap (\overline{B} \cup \overline{\overline{A}})$ | DeMorgan's law |
| $= B \cap (\overline{B} \cup A)$ | Complementation law |
| $= (B \cap \overline{B}) \cup (B \cap A)$ | Distributive law |
| $= \varnothing \cup (B \cap A)$ | Complement law |
| $= (B \cap A)$ | Identity law |
| $= A \cap B$ | Commutative law |

**Claim:**

$(A-B)-(B-C)=A-B$

**Proof:**

| | |
|---|---|
| $(A-B)-(B-C) = (A \cap B') \cap (B \cap C')'$ | Definition of Set Difference |
| $= (A \cap B') \cap (B' \cup C)$ | De Morgan's Law |
| $= ((A \cap B') \cap B') \cup ((A \cap B') \cap C)$ | Distributive Law |
| $= (A \cap (B' \cap B')) \cup (A \cap (B' \cap C))$ | Associative Law |
| $= (A \cap B') \cup (A \cap (B' \cap C))$ | Idempotent Law |
| $= A \cap (B' \cup (B' \cap C))$ | Distributive Law |
| $= A \cap B'$ | Absorption Law |
| $= A - B'$ | Definition of Set Difference |

Prove : $\sim((A \cap B) \cup \sim B) = B \cap \sim A$

| | | | |
|---|---|---|---|
| [1] | $\sim((A \cap B) \cup \sim B)$ | | |
| [2] | $\sim(A \cap B) \cap \sim(\sim B)$ | Set De Morgan | [1] |
| [3] | $\sim(A \cap B) \cap B$ | Set Double Negation | [2] |
| [4] | $(\sim A \cup \sim B) \cap B$ | Set De Morgan | [3] |
| [5] | $B \cap (\sim A \cup \sim B)$ | Set Commutativity | [4] |
| [6] | $(B \cap \sim A) \cup (B \cap \sim B)$ | Set Distributivity | [5] |
| [7] | $(B \cap \sim A) \cup \varnothing$ | Set Computation | [6] |
| [8] | $B \cap \sim A$ | Set Computation | [7] |