

Assignment Number 1 (Linear Algebra-Fall 2024)

Topic: *System of Linear Equations*

Question Number 1

Determine if the following system is consistent. Do not completely solve the system

$$\begin{array}{rcl} x_1 & + 3x_3 & = 2 \\ x_2 & - 3x_4 & = 3 \\ -2x_2 + 3x_3 + 2x_4 & = 1 \\ 3x_1 & + 7x_4 & = -5 \end{array}$$

Question Number 2

Determine the value(s) of h such that the matrix is the augmented matrix of a consistent non-homogeneous linear system.

a) $\begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix}$

b) $\begin{bmatrix} 1 & h & -3 \\ -2 & 4 & 6 \end{bmatrix}$

Question Number 3

In the following, find the elementary row operation that transforms the first matrix into the second, and then find the reverse row operation that transforms the second matrix into the first.

a) $\begin{bmatrix} 1 & 3 & -4 \\ 0 & -2 & 6 \\ 0 & -5 & 9 \end{bmatrix}, \begin{bmatrix} 1 & 3 & -4 \\ 0 & 1 & -3 \\ 0 & -5 & 9 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 4 & -1 & 3 & -6 \end{bmatrix}, \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 0 & 7 & -1 & -6 \end{bmatrix}$

Question Number 4

Find the interpolating polynomial $p(t) = a_0 + a_1t + a_2t^2$ for the data $(1,12), (2,15), (3,16)$. That is, find a_0, a_1 , and a_2 such that

$$a_0 + a_1(1) + a_2(1)^2 = 12$$

$$a_0 + a_1(2) + a_2(2)^2 = 15$$

$$a_0 + a_1(3) + a_2(3)^2 = 16$$

Question Number 5

Determines the conditions on the b_i 's, if any, in order to guarantee that the linear system is consistent.

a) $\begin{array}{l} 6x_1 - 4x_2 = b_1 \\ 3x_1 - 2x_2 = b_2 \end{array}$

b) $\begin{array}{l} x_1 - x_2 + 3x_3 + 2x_4 = b_1 \\ -2x_1 + x_2 + 5x_3 + x_4 = b_2 \\ -3x_1 + 2x_2 + 2x_3 - x_4 = b_3 \\ 4x_1 - 3x_2 + x_3 + 3x_4 = b_4 \end{array}$

Question Number 6

Solve the linear systems. Using the given values for the b's solve the system together by reducing an appropriate augmented matrix to reduced row echelon form.

$$-x_1 + 4x_2 + x_3 = b_1$$

$$x_1 + 9x_2 - 2x_3 = b_2$$

$$6x_1 + 4x_2 - 8x_3 = b_3$$

a) $b_1 = 0, b_2 = 1, b_3 = 0$

b) $b_1 = -3, b_2 = 4, b_3 = -5$

Question Number 7

Use the inversion algorithm to find the inverse of the matrix, if it exists

$$\begin{bmatrix} \frac{1}{5} & \frac{1}{5} & -\frac{2}{5} \\ \frac{2}{5} & \frac{3}{5} & -\frac{3}{10} \\ \frac{1}{5} & -\frac{4}{5} & \frac{1}{10} \end{bmatrix}$$

Question Number 8

Find a row operation and the corresponding elementary matrix that will restore the given elementary matrix to the identity matrix

a) $\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix}$

Question Number 9

Find the product by inspection.

a) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} -3 & 2 & 0 & 4 & -4 \\ 1 & -5 & 3 & 0 & 3 \\ -6 & 2 & 2 & 2 & 2 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 4 & -1 & 3 \\ 1 & 2 & 0 \\ -5 & 1 & -2 \end{bmatrix} \begin{bmatrix} -3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Question Number 10

Find the standard matrix for the transformation T defined by the formula:

$$T(x_1, x_2, x_3, x_4) = (7x_1 + 2x_2 - x_3 + x_4, x_2 + x_3, -x_1)$$