National University of Computer & Emerging Sciences, Karachi
Department of Computer Science
Mid Term Exam ,Summer-2022
19th July 2022, 9:00 am – 11:00 am

Course Code: MT-1004	Subject: Linear Algebra
Instructor Name: Mr.Muhammad Jamil,	
Student Roll No:	Section:

Instructions:

- **Solve the entire questions on answer sheet and return the question paper.**
- **Read each question completely before answering it. There are 6 questions and 2 pages.**
- **All the answers must be solved according to the sequence given in the question paper.**
- **All questions having same marks and show all the necessary steps.**
- **Scientific calculator is allowed.**

Time: 120 minutes Max Marks: 60 points

Q-1(i) Consider the following reduce echelon form of the non homogenous system.Detemine the value of a for which the system has no solution , unique solution or infinite many solution.

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & a^2-16 & a-4 \end{bmatrix}$$

ii) Find the standard matrix for the transformation $T: R^4 \to R^2$ defined by

$$w_1 = 2x_1 + 3x_2 - 5x_3 - x_4$$
$$w_2 = x_1 - 5x_2 + 2x_3 - 3x_4$$

and then compute $T(1,-1,2,4)$

Q-2 Write the genral solution and find a basis and dimension for the solution space of homogeneous system

$$3x_1 + x_2 + x_3 + x_4 = 0$$
$$5x_1 - x_2 + x_3 - x_4 = 0$$

OR

Find cofactor of A , adjoint(A) and A^{-1} of matrix $A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & 4 & 5 \end{bmatrix}$

Q-3 Consider $v = (2, -1, 3), v_1 = (1,0,0), v_2 = (2,2,0),\ v_3 = (3,3,3)$

I. Find the Cordinate vector of v relative to the basis $S = \{v_1, v_2, v_3\}\ for\ R^3$
II. Check the set of vectors $\{v_1, v_2, v_3\}$ form a basis or not ?

Q-4 Consider the matrix $A = \begin{bmatrix} -14 & 12 \\ -20 & 17 \end{bmatrix}$

I. Find the eigen values and the bases for the eigen space of matrix A
II. If A is diagolizable then find the matrix P that diagonalize A
III. Verify *P* diagonalize A using $P^{-1}AP = D$, $where\ D\ is\ diagonal\ matrix$

Q-5 Consider the matrix $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$

I. Write the general solution and find the Rank and nullity of A
II. Find a basis and dimension of the null space of A
III. Find a basis for the column and row space of A

Q-6 i) Find the LU decomposition of coefficient matrix, use any method then solve the system

$$\begin{bmatrix} -3 & 12 & -6 \\ 1 & -2 & 2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -33 \\ 7 \\ -1 \end{bmatrix}$$

ii) Express matrix $A\ in\ the\ form\ of\ A = LDU$, Where L is lower triangular,U is upper triangular and D is diagonal matrix

WISH YOU ALL THE BEST