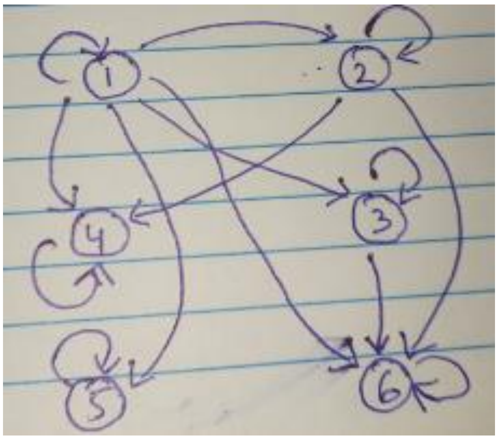
Sessional-II Solution

Q1: a.

(i) Solution:
R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)}.



(ii) Solution:
- It is not symmetric since (1, 2) is present but (2, 1) is not present. OR no bidirectional edges in digraph.
- It is transitive since (a, b) ∊ R and (b, c) ∊ R, then (a, c) ∊ R, for all a, b, c ∊ R.
- It is antisymmetric since (a, b) ∊ R and (b, a) ∊ R, then a = b, for all a, b ∊ R. (No bidirectional edges in digraph)
- It is not asymmetric since (a, a) ∊ R or it is not irreflexive. (Loops are present in digraph).

(iii) Solution:
For an equivalence relation, relation must be reflexive, symmetric, and transitive.
- It is reflexive since (a, a) ∊ R for all a ∊ S. (all nodes have a loop)
- It is not symmetric since (1, 2) is present but (2, 1) is not present. OR no bidirectional edges in digraph.
- It is transitive since (a, b) ∊ R and (b, c) ∊ R, then (a, c) ∊ R, for all a, b, c ∊ R.
  Hence, its not an equivalence relation.

(iv) Solution:
For a partial order relation, relation must be reflexive, antisymmetric, and transitive.
- It is reflexive since (a, a) ∊ R for all a ∊ S. (all nodes have a loop)
- It is antisymmetric since (a, b) ∊ R and (b, a) ∊ R, then a = b, for all a, b ∊ R. (No bidirectional edges in digraph)
- It is transitive since (a, b) ∊ R and (b, c) ∊ R, then (a, c) ∊ R, for all a, b, c ∊ R.
  Hence, it's a partial order relation.

b. (i) Solution:

(a)

1. Given the third term $a_3 = 324$ and the sixth term $a_6 = 96$, use the formula for the n-th term of a geometric sequence: $a_n = a_1 \cdot r^{(n-1)}$
2. Set up the equations: $a_3 = a_1 \cdot r^2$ and $a_6 = a_1 \cdot r^5$
3. Find the common ratio r by dividing the two equations: $\frac{a_6}{a_3} = r^3 = \frac{96}{324} = \frac{2}{3}$

(b)

1. Use the common ratio $r = \frac{2}{3}$ and the third term $a_3 = 324$ to find the first term a_1: $a_1 = \frac{a_3}{r^2} = \frac{324}{(\frac{2}{3})^2} = 729$

(ii) Solution:
$a_1 = 11$ $a_2 = 27$, $a_3 = 59$ and $a_4 = 123$.

(iii) Solution:

Proof. Suppose mn is odd. Since the product of an even number with any other integer is even, it must be the case that both m and n are odd. Thus $m = 2k + 1$ and $n = 2j + 1$ for some $k, j \in \mathbb{Z}$. It follows that $m + n = 2k + 1 + 2j + 1 = 2(k + j + 1)$ and since $k + j + 1 \in \mathbb{Z}$, we have that $m + n$ is even.

(iv) Solution:

If x is an even integer, then $x^2 - 6x + 5$ is odd. [Contrapositive of the given Proposition

Then $x = 2a$ for some $a \in \mathbb{Z}$, by definition of an even integer.

So $x^2 - 6x + 5 = (2a)^2 - 6(2a) + 5 = 4a^2 - 12a + 5 = 4a^2 - 12a + 4 + 1 = 2(2a^2 - 6a + 2) + 1$.

Therefore we have $x^2 - 6x + 5 = 2b + 1$, where $b = 2a^2 - 6a + 2 \in \mathbb{Z}$.

Consequently $x^2 - 6x + 5$ is odd, by definition of an odd number.

Q2: a. (i) Solution:

F: V(G1) →V(G2) defined by f(a) = d, f(b) = c, f(c) = e, f(d) = b, f(e) = a.

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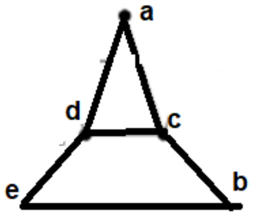
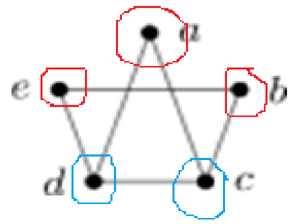


(ii) Solution:

Yes, It's a planar graph. By using Euler formula, r = e – v + 2 = 6 - 5 + 2 = 3.



(iii) Solution:

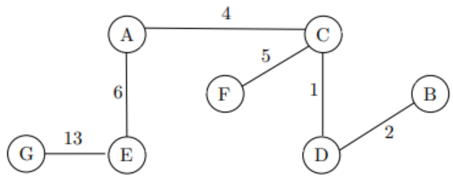
It's not a bipartite Graph.

As b and e are adjacent vertices with same color.

(iv) Solution:

(a) Since vertex a and b have an odd degree and all other vertices have an even degree so Euler path exists.
Euler path: a, b, c, d, a, e, c, f, b.

(b) Euler Circuit exists since all vertices have an even degree.
Euler Circuit: a, b, c, e, b, d, a.



b. (i) Solution

Minimum Cost = 31

Prim's sequence of Edges is: A-C, C-D, D-B, F-C, A-E, G-E

(ii) Solution:

N	D(B)	D(C)	D(D)	D(E)	D(F)	D(G)
A	∞	4,A	∞	6,A	8,A	∞
AC	7,C		5,C	6,A	8,A	∞
ACD	7,C			6,A	8,A	∞
ACDE	7,C				8,A	19,E
ACDEB					8,A	19,E
ACDEBF						19,E
ACDEBFG						

(iii) Solution:

prefix expression: + + a * b c * d + e f

= + + 1 * 2 3 * 4 + 5 6

= + + 1 * 2 3 * 4 11

= + + 1 * 2 3 44

= + + 1 6 44

= + 7 44

= 51