Set Theory

- Sets: Definition, Universal Set, Complement, Cardinality
- Subset and Power Set
- Sets Operations
- Set Equality
- Characteristic Vectors: Sets as Bit-Vectors
- Multisets

Sets as bit-strings (bit vectors)

- Sets stored in an unordered fashion in memory
- Union/Intersection etc. are computationally expensive
- When $|U|$ is small compared to computer memory, then we can do set operations efficiently
- Impose any fixed ordering on elements of U
- $U = \{DM, Cal, Chem, Bio, Phy, Pro\}$ (in order)
- Sets (subsets of U) are represented by bit-string of length 6
- Each bit signifies whether the corresponding element is in the set
- Called bit-vector representation of sets or characteristic vector of a set

Sets as bit-strings (bit vectors)

DM	Calc	Chem	Bio	Phy	Prog

The set $\{Calc, Chem, Phy\}$ is

0	1	1	0	1	0

The set $\{Prog, DM, Calc, Phy\}$ is

1	1	0	0	1	1

ICP 4-28 What is the characteristic vector of the set

$$\{Chem, DM\}?$$

ICP 4-29 What is the characteristic vector of the set

$$\{Calc, DM, Chem, Phy, Prog, Bio\}?$$

Sets as bit-strings (bit vectors)

DM	Calc	Chem	Bio	Phy	Prog

The set | 1 | 0 | 0 | 0 | 0 | 1 | is $\{DM, Prog\}$

The set | 0 | 0 | 0 | 0 | 0 | 0 | is the empty set

ICP 4-30 What is the set corresponding to the characteristic vector

1	1	1	1	1	1

Sets operations using bit-strings

$$A \cup B = \{x | x \in A \vee x \in B\}$$

$A = \{Calc, Chem, Phy\}$		0	1	1	0	1	0	
$B = \{Prog, DM, Calc, Phy\}$		1	1	0	0	1	1	
$A \cup B$		$A \vee B$						
$\{DM, Calc, Chem, Prog, Phy\}$		1	1	1	0	1	1	

Sets operations using bit-strings

$$A \cap B = \{x | x \in A \wedge x \in B\}$$

$A = \{Calc, Chem, Phy\}$		0	1	1	0	1	0	
$B = \{Prog, DM, Calc, Phy\}$		1	1	0	0	1	1	
$A \cap B$		$A \wedge B$						
$\{Calc, Phy\}$		0	1	0	0	1	0	

Sets operations using bit-strings

$$A \oplus B = \{x | x \in A \oplus x \in B\}$$

$A = \{Calc, Chem, Phy\}$		0	1	1	0	1	0	
$B = \{Prog, DM, Calc, Phy\}$		1	1	0	0	1	1	
$A \oplus B$		$A \oplus B$						
$\{DM, Chem, Prog\}$		1	0	1	0	0	1	

Sets as bit-vectors: Summary

- Sets can be represented as bit vectors, when universal set is *'small'*
- Also called characteristic vectors of sets
- Order of U is critical
- Sets operations can be performed using bit-wise operators of programming language
- More suitable for computer implementations
- Only feasible when U is small