National University Of Computer & Emerging sciences (FAST)

(Karachi Campus)

Final Semester-Examination.2010

Course Code: MT104	**Linear Algebra**	Date: 28/12/2010
Time allowed: 2h 45min	Fall 2010	Max. Marks: 90

Instructions:

1) Attempt all questions. Show all necessary working and calculations.

2) Use blue or black marker to mention the question number on answer sheet.

Q1. Find the necessary and sufficient conditions for the existence of a solution to the following system. **[08]**

$$x + y + 2z = a_1$$
$$-2x - z = a_2$$
$$x + 3y + 5z = a_3$$

Q2. Solve the following system by Gauss Jordan elimination method. **[10]**

$$\begin{bmatrix} 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -10 & 6 & 12 & 28 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{bmatrix}$$

Q3. Find the Eigen values and Orthonormal basis for the Eigen space of A where $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ **[10]**

Q4. Find Rank and nullity of the matrix $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$ then verify that the values obtained satisfy the dimension theorem for matrices. **[10]**

Q5. Find the standard matrix for the composition of transformations when a vector in R^3 is rotated counter clock wise about x-axis through an angle $60°$, followed by a reflection in *yz-plane*, followed by a orthogonal projection on xz-*plane*. **[08]**

Q6. In each part use the given inner product on R^2 to find the $\|w\|$, where $w = (-1, 3)$. **[08]**

a) The weighted Euclidean product $\langle u, v \rangle = 3u_1v_1 + 2u_2v_2$ where $u = (u_1, u_2)$ and $v = (v_1, v_2)$.

b) The Inner product generated by the matrix $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

Q7. Given $A = \begin{bmatrix} 2 & -1 & 1 \\ 4 & 1 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, Evaluate determinant of A, and $\det(A^{-1})$. What is the relation between $\det(A)$ and $\det(A^{-1})$. **[10]**

Q8. a) Write the vector $V = (1, -2, 5)$ as a linear combination of the vectors $p = (1, -1, 1)$, $q = (1, 2, 3)$ and $r = (2, -1, 1)$ **[08]**

b) Determine whether the given vectors in R^3 are linearly independent or linearly dependent?
$(1, -2, 3)$, $(5, 6, -1)$, $(3, 2, 1)$

Q9. Given $u_1 = (1, 1, 1)$ $u_2 = (-1, 1, 0)$ and $u_3 = (1, 2, 1)$ on R^3 have the Euclidean inner product. Apply *Gram-Schmidt process* to transform the basis $\{u_1, u_2, u_3\}$ in to Orthonormal basis but at first Define Orthogonal and Orthonormal basis. **[10]**

Q10. V be the space spanned by $v_1 = Cos^2 x$ $v_2 = Sin^2 x$ and $v_3 = Cos2x$. Show that $S = \{v_1, v_2, v_3\}$ is not a basis for V and then find a basis for V. **[08]**

Best of luck