

Course Code: MT1004	Course Name: Linear Algebra
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Instructions:

- Read each question completely before answering it. There are 06 Questions and 02 pages.
- Graphical Calculator is not allowed.
- Use pen for solution.
- Return the question paper at the end of exam.

Time: 180 minutes**Max Marks: 100****Question 01****[CLO-1]****[5+10=15]**(a) Suppose that A is 3×3 matrix such that

$$\langle Ax, x \rangle = x_1^2 + 5x_2^2 - 3x_3^2 + 6x_1x_2 - 4x_1x_3 + 8x_2x_3$$

For all $x \in \mathbb{R}^3$. Then $A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$ where, $a = \underline{\hspace{1cm}}$, $b = \underline{\hspace{1cm}}$, $c = \underline{\hspace{1cm}}$, $d = \underline{\hspace{1cm}}$
 $e = \underline{\hspace{1cm}}$, $f = \underline{\hspace{1cm}}$.

(b) Consider

$$Q = 2x_1^2 + 2x_2^2 + 5x_3^2 - 2x_1x_2$$

Find an orthogonal change of variable that eliminates the cross product terms in the quadratic form and express Q in terms of the new variables.

Question 02**[CLO-2]****[10+5=15]**

(a) Let $Z = \{b_1, b_2, b_3\}$ represents the basis for \mathbb{R}^3 and $Z' = \{d_1, d_2, d_3\}$ also represents the basis for \mathbb{R}^3 , where

$$b_1 = (2, 1, 1), \quad b_2 = (2, -1, 1), \quad b_3 = (1, 2, 1) \text{ And}$$

$$d_1 = (3, 1, -5), \quad d_2 = (1, 1, -3), \quad d_3 = (-1, 0, 2)$$

- Find the transition matrix from Z to Z' .
- Compute the coordinate vector $[w]_Z$ where $w = (-5, 8, -5)$
- Find $[w]_{Z'} = P_{Z \rightarrow Z'}[w]_Z$

(b) A 3×3 Jordan block is a matrix of the form $J_c = \begin{bmatrix} c & 1 & 0 \\ 0 & c & 1 \\ 0 & 0 & c \end{bmatrix}$, where c is some constant.

- Find all eigenvalues of J_c , and calculate the corresponding eigenspace.
- Prove that J_c is not diagonalizable.

Question 03**[CLO-2]****[6+3+6=15]**

Let \mathbb{R}^3 have the Euclidean inner product and $\{u_1, u_2, u_3\}$ forms the basis for \mathbb{R}^3 .

$$u_1 = (1, 1, 0), u_2 = (2, 1, 3), u_3 = (1, 1, 1)$$

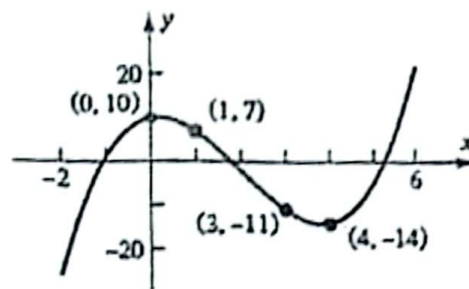
- Use the Gram-Schmidt process to transform the basis into an orthogonal basis.
- Find the coordinate vector for $b = (1, 2, 0)$ corresponding to orthogonal basis.
- If u_1, u_2, u_3 and u_4 are the column vectors of Matrix A , Find the QR decomposition of A .

Question 04

[CLO-3]

[12+3=15]

- (a) Find the coefficients a, b, c , and d so that the curve shown in the accompanying figure is the graph of the equation $y = ax^3 + bx^2 + cx + d$.
- Construct the linear System.
 - Find the solution of linear system.



(b) Let $A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$, and $P = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

Confirm that P diagonalizes A , and then compute A^8

Question 05

[CLO-2]

[9+3+8=20]

- (a) Use the standard inner product on $M_{2 \times 2}$ to calculate the following

If $m = \begin{bmatrix} -1 & 2 \\ 6 & 1 \end{bmatrix}$, $n = \begin{bmatrix} 1 & 0 \\ 3 & 3 \end{bmatrix}$ and $k = 3$.

- $\|m - kn\|$
 - $\langle 2m - 4n, m - 3n \rangle$
 - The Angle θ between m and n
- (b) Suppose $T: R^2 \rightarrow R^3$ is a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix} \text{ And } T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

- What is the standard matrix of T ?
- What is $T\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right)$?

- (c) Let $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$. Prove that the expression

$$\langle u, v \rangle = 2u_1v_1 + 3u_2v_2 + u_3v_3$$

Defines an inner product on R^3 by showing the inner product axioms hold.

Question 06

[CLO-3]

[3+3+4+4+6=20]

Mr. Asim, a regular consumer of coffee has three brands to choose from. Nescafe, Caffè Decaffeinato and Red Berry Roasters. His records show that if he chooses Nescafe a given month, then there is a 50% chance that he will choose Nescafe the next month, and 25% chance that he will choose Caffè Decaffeinato next month. If he chooses Caffè Decaffeinato a month, then there is a 30% chance that he will choose Nescafe next month, and 60% chance that he will choose Red Berry Roasters next month. And If he chooses Red Berry Roasters a month, then there is a 50% chance that he will choose Nescafe next month, and 50% chance that he will choose Caffè Decaffeinato next month.

- Find a transition matrix
- Draw the transition diagram.
- If Mr. Asim chooses Nescafe this month, what is the probability that he will choose the same next month.
- If Mr. Asim chooses Red Berry Roasters this month, what is the probability that he will choose the same five months from now.
- Discuss the long term priorities of Mr. Asim.