National University of Computer and Emerging Sciences

Karachi Campus

Linear Algebra (MT2004)

Final Exam

Total Time (Hrs.):	**3**
Total Marks:	**80**
Total Questions:	**5**

Date: December 26th ,2024

Course Instructor(s)

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__________ Roll No   __________ Section   __________ Student Signature

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Attempt all the questions.

CLO#1: Interpreting and finding the solutions of linear equations in detail. **[10]**

Q1(a) : For which values of c Does $A = \begin{bmatrix} 1 & 0 & -c \\ -1 & 3 & 1 \\ 0 & 2c & -4 \end{bmatrix}$ have an inverse? [3]

Q1(b): Find value of x_3 only use Cramer's rule. [3]

$$\begin{aligned} x_1 + \quad\quad + 2x_3 &= 6 \\ -3x_1 + 4x_2 + 6x_3 &= 30 \\ -x_1 - 2x_2 + 3x_3 &= 8 \end{aligned}$$

Q1(c): For the matrix $A = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}$, Prove that $(A^2)^{-1} = (A^{-1})^2$ [4]

CLO#1: Interpreting and finding the solutions of linear equations in detail. **[10]**

Q2(a): Find the basis for the solution space of the following given homogeneous linear system and find the dimension of that space [5]

$$\begin{aligned} x + y + z &= 0 \\ 3x + 2y - 2z &= 0 \\ 4x + 3y - z &= 0 \\ 6x + 5y + z &= 0 \end{aligned}$$

Q2(b): Let V be the set of 2×2 matrices with real entries, and take the vector space operations V to be the usual operations of matrix addition and scalar multiplication. [5]
Verify, Associative property for addition, Distributive properties & Associative property for scalar multiplication. Also find the additive identity of the given set.

CLO#2: Understanding the core concepts of Euclidean vector spaces and matrix transformations. **[20]**

Q3(a): The characteristic equation of a matrix having Eigen values : $\lambda_1 = 1$, $\lambda_2 = 2$ & $\lambda_3 = 3$. Determine: (i) the size of the matrix (ii) The geometric and algebraic multiplicity of for each of the Eigen values (iii) The possible dimensions of its Eigen spaces. [1+2+2]

Q3(b): Find the cosine of the angle between A and B with respect to the standard inner product on M_{22}.

$$A = \begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix}, \; B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$$ [5]

Q3(c): Let R^3 have the Euclidean inner product . (i) Use the Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis: $u_1 = (1,0,1), u_2 = (0,1,2), u_3 = (2,1,0)$.
(ii) Also, find the QR decomposition of the given matrix. [5+5]

CLO#1: Interpreting and finding the solutions of linear equations in detail. **[20]**

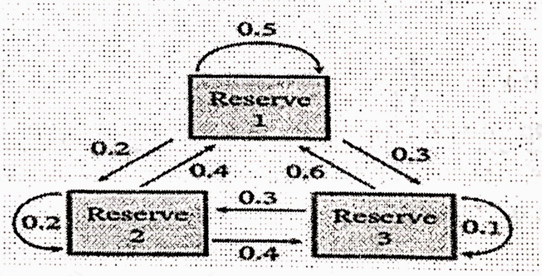
Q4: Let A be the matrix $A = \begin{bmatrix} -4 & -5 & 5 \\ -5 & -4 & -5 \\ 5 & -5 & -4 \end{bmatrix}$

a) Prove that $\det(\lambda I - A) = (9+\lambda)^2(6-\lambda)$. [3]
b) Orthogonally diagonalizes the matrix A. Compute an orthogonal matrix P and a diagonal matrix D such that $A = PDP^t$. [12]
c) Write down the quadratic form associated with matrix A using variables x_1, x_2 *and* x_3. [3]
d) Find the expression for A^k. [2]

CLO #3: Applying the basic linear algebra concepts in computer science. **[20]**

Q5(a): Find a singular value decomposition (SVD) of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$. [12]

Q5(b): See the following figure. Suppose that a tagged lion can migrate over adjacent game reserve in search of food. Reserve 1, Reserve 2, and Reserve 3. Based on data about the food resources, researchers conclude that the monthly migration pattern of the lion can be modeled by a Markov [8] chain with transition matrix.



$$P = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.5 & 0.4 & 0.6 \\ 0.2 & 0.2 & 0.3 \\ 0.3 & 0.4 & 0.1 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}$$

Assuming that t is in months and the lion is released in Reserve 2 at time $t = 0$, track its probable locations over a six- month period.
Find (i) the reserve in which it is most likely to be at the end of that period. (ii)the stochastic column.
