1. Show that the following vectors are linearly independent in $\mathbb{R}^4$
   $v_1 = (1, 2,2, -1), v_2 = (4,9,9, -4), v_3 = (5,8,9, -5)$

2. Determine whether the polynomials are linearly dependent or independent in P_2

   $P_1 = 1 - x\,, \quad P_2 = 5 + 3x - 2x^2\,, \quad P_3 = 1 + 3x - x^2$

3. Determine whether the set S spans M_{22}
   Where $S = \left\{ \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\}$

4. Consider the vectors $v = (2, -1, 3\,),\ v_1 = (1, 0, 0), v_2 = (2, 2, 0), v_3 = (3, 3, 3)$
   I. Check weather v is in span $\{v_1, v_2, v_3\}$
   II. Find the coordinate vector of v relative to the basis $S = \{v_1, v_2, v_3\}$ of R^3

5. Write the parametric solution if consistant and find a basis and dimension fir the system

$$\begin{aligned} 3x_1 + x_2 + x_3 + x_4 &= 0 \\ 5x_1 - x_2 + x_3 - x_4 &= 0 \end{aligned}$$

6. Consider the matrix $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$

   I. Write the general solution of $AX = 0$ and find the Rank and nullity of A
   II. Find a basis and dimension of the null space of A
   III. Find a basis for the column and row space of A

7.

   Consider the matrix $A = \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ -3 & 6 & -1 & 0 & -7 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$

   Find
   i. The basis for row & column space of A.
   ii. The basis for null space of A.
   iii. Verify dimension theorem of given matrix A.

8. Consider the matrices $A = \begin{bmatrix} -14 & 12 \\ -20 & 17 \end{bmatrix}$ and B= $\begin{bmatrix} 4 & 0 & -1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

   I. Find the eigen values and the bases for the eigen space of given matrices.
   II. If A is diagolizable then find the matrix P that diagonalize A
   III. If B is diagolizable then find the matrix P that diagonalize B