National University of Computer and Emerging Sciences, Karachi

FAST School of Computing

Midterm 1 Examination Fall-2021

October 15, 2021, 09:00 am – 10:00 am

Course Code: CS1005	**Course Name: Discrete Structures**
Instructor Names: Mr. Shoaib Raza, Mr. Musawar Ali and Ms. Safia	
Student Roll No:	**Section No:**

Instructions:

- Return the question paper together with the answer script. Read each question completely before answering it. There are **3 questions** on **2 pages.**
- In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.
- Attempt all the questions in the given sequence of the question paper.

Total Time: 60 minutes **Maximum points:** 26

Question # 1 (Propositional Logic and Rules of Inference) **[5x2=10 points]**

(i) Let p and q be the propositions:

p: Swimming at the New Jersey shore is allowed. q: Sharks have been spotted near the shore.

Write these propositions using p and q and logical connectives (including negations):

a) Swimming at the New Jersey shore is not allowed and either Swimming at the New Jersey shore is allowed or sharks have not been spotted near the shore.

b) Swimming at the New Jersey shore is allowed iff sharks have not been spotted near the shore.

(ii) Prove the following logical equivalence using the laws of logic:

$$\neg[c \vee (b \wedge (\neg c \rightarrow \neg a))] \cong \neg c \wedge (a \vee \neg b)$$

(iii) Determine using truth table that whether the following statement is a tautology, contradiction or a contingency:

$$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$$

(iv) What relevant conclusion or conclusions can be drawn from the following premises? Also, explain the rules of inference used to obtain each conclusion from these premises.

"The file is either a binary file or a text file." "My program won't accept the file if it's a binary file."

"My program will accept the file."

Assume,

p = "The file is a binary file." q = "The file is a text file." r = "My program will accept the file."

(v) Write the negation of the following sentences in English.

a) If Jaffar lives in Pakistan, then he lives in Karachi.

b) If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

Question # 2 (Predicate and Quantifiers) **[3x2=6 points]**

(i) Let $F(a, b)$ means "$a + 3b = ab$", where a and b are Positive integers. Determine the truth value of the statement.

a) $\forall a \exists b\, F(a, b)$.

b) $\forall b \exists a \neg F(a, b)$.

(ii) Let $B(x)$ be the statement "**x has an Internet connection**" and $C(x, y)$ be the statement "**x and y have chatted over the Internet,**" where the domain for the variables x and y consists of all students in your class. Write the statement in good English without using variables in your answers.

a) $\neg \forall x\, B(x)$. Ex 7

b) $\exists x\, B(x) \wedge \forall y \neg C(x, y)$

(iii) Express the following sentences using logical expression with nested quantifiers:

a) The Sum of two negative integers numbers is negative.

b) The difference of two positive integers is not necessarily positive.

Question # 3 (Functions and Set theory) **[5x2=10 points]**

(i) Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be defined by the formula $f(x) = 4x-1\ \forall x \in R$. Is f a bijective function? If no, give reason why? If yes, find its inverse.

(ii) Let $f: Z \rightarrow Z$ and $g: Z \rightarrow Z$ be defined by $f(n) = n+1$ for $n \in Z$ and $g(n) = n^2$ for $n \in Z$.

a) Find the compositions $g \circ f$ and $f \circ g$.

b) Is $g \circ f = f \circ g$?

(iii) Draw Venn Diagrams of the following relationships between the sets:

a) $(A \cup B) - (A \cap B)$

b) $\bar{A} \cup \bar{B}$

(iv) Using Set Identities, prove or disprove the following set operations:

$A - (B \cap C) = (A - B) \cap (A - C)$.

(v) Among a group of 165 students, 8 are taking calculus, psychology, and computer science; 33 are taking calculus and computer science; 20 are taking calculus and psychology; 24 are taking psychology and computer science; 79 are taking calculus; 83 are taking psychology; and 63 are taking computer science. How many are taking none of the three subjects?

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