Exercise # 6.1 Date

Q5. Find a matrix that generates the stated weighted inner product on R^2

$\langle u, v\rangle = 2u_1v_1 + 3u_2v_2$

$A = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{3} \end{bmatrix}$

$\langle u, v\rangle = \frac{1}{2}u_1v_1 + 5u_2v_2$

$A = \begin{bmatrix} \sqrt{1/2} & 0 \\ 0 & \sqrt{5} \end{bmatrix}$

Q7-8: use the inner product on R^2 generated by the A to find $\langle u, v\rangle$ for $u = (0, -3)$, $v = (6, 2)$

$$\begin{array}{r} 78 \\ -54 \\ \hline 24 \end{array} \qquad \begin{array}{r} 26 \\ \times 3 \\ \hline 78 \end{array}$$

$A = \begin{bmatrix} 4 & 1 \\ 2 & -3 \end{bmatrix}$

$\langle u, v\rangle = Au \cdot Av$

$= \begin{bmatrix} 4 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 6 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 0 - 3 \\ 0 + 9 \end{bmatrix} \cdot \begin{bmatrix} 24 + 2 \\ 12 - 6 \end{bmatrix} = \begin{bmatrix} -3 \\ 9 \end{bmatrix}\begin{bmatrix} 26 \\ 6 \end{bmatrix} = -78 + 54 = -24$

Q9-10: Compute the standard inner product on M_{22} of:

$u = \begin{bmatrix} 3 & -2 \\ 4 & 8 \end{bmatrix}$ $V = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$

$tr(u^T v) = (3)(-1) + (-2)(3) + (4)(1) + (8)(1)$

$= -3 - 6 + 4 + 8$

$= -9 + 12 = 3$

Q11-12: Find the standard inner product on P_2

$p = -2 + x + 3x^2$, $q = 4 - 7x^2$

$\langle p, q\rangle = (-2)(4) + (1)(0) + (3)(-7)$

$= -8 + 0 - 21 = -29$

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Q13-14: a weighted euclidean inner product on $\mathbb{R}^2$ is given for the vectors $u = (u_1, u_2)$ and $v = (v_1, v_2)$. Find a matrix that generates it.

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$\langle u,v \rangle = 3u_1v_1 + 5u_2v_2$ $\qquad$ $\langle u,v \rangle = 4u_1v_1 + 6u_2v_2$

$$= \begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{5} \end{bmatrix} \qquad = \begin{bmatrix} \sqrt{4} & 0 \\ 0 & \sqrt{6} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & \sqrt{6} \end{bmatrix}$$

Q15-16: sample points are given. use the evaluation inner product on P_3 to find $\langle p, q \rangle$

$$p = x + x^3 \quad \text{and} \quad q = 1 + x^2$$

$$x_0 = -2, \quad x_1 = -1, \quad x_2 = 0, \quad x_3 = 1$$

$$\langle p,q \rangle = p(x_0)q(x_0) + p(x_1)q(x_1) + p(x_2)q(x_2) + p(x_3)q(x_3)$$
$$= p(-2)q(-2) + p(-1)q(-1) + p(0)q(0) + p(1)q(1)$$
$$= (-10)(5) + (-2)(0) + (0)(1) + (2)(2)$$
$$= -50 + 0 + 0 + 4$$
$$= -46$$

Q17-18: Find $\|u\|$ and $d(u,v)$ when $\langle u,v \rangle = 2u_1v_1 + 3u_2v_2$

$$u = (-3, 2), \quad v = (1, 7)$$

$$\cancel{\|u\| = \sqrt{u \cdot u} = \sqrt{3^2 + 2^2} = \sqrt{9+4} = \sqrt{13}}$$ $\quad 18 + 12 = 30$

$$\cancel{d(u,v) = u - v = (-4, -5)}$$

$$\|u\| = \sqrt{2(-3)^2 + 3(2)^2} = \sqrt{2(9) + 3(4)} = \sqrt{18 + 12} = \sqrt{30}$$

$$\cancel{d(u,v) = 2(u_1 - v_1) + 3(u_2 - v_2)}$$
$$\cancel{= 2(-4) + 3(-5) = -8 - 18 = -23}$$

$$d(u,v) = u - v = (-4, -5)$$

$$\|u - v\| = \sqrt{2(-4)^2 + 3(-5)^2} = \sqrt{2(16) + 3(25)}$$
$$= \sqrt{107}$$

Q19-20 : Find $\|p\|$ and $d(p,q)$ Date

$p = -2 + x + 3x^2 \quad , \quad q = 4 - 7x^2$

$\|p\| = \sqrt{4 + 1 + 9} = \sqrt{14}$

$d(p,q) = p - q = (-2-4, 1-0, 3+7)$

$= (-6, 1, 10)$

$\|p-q\| = \sqrt{6^2 + 1^2 + 10^2} = \sqrt{36 + 1 + 100} = \sqrt{137}$

Q21-22 : Find $\|U\|$ and $d(u,v)$ relative to the standard inner product on M_{22}.

$U = \begin{bmatrix} 3 & -2 \\ 4 & 8 \end{bmatrix} \qquad V = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$

$\begin{array}{r} 64 \\ 16 \\ +13 \\ \hline 93 \end{array}$

$\|U\| = \sqrt{9 + 4 + 16 + 64} = \sqrt{93}$

$\begin{array}{r} 49 \\ 25 \\ 25 \end{array} \qquad \begin{array}{r} 49 \\ +50 \\ \hline 99 \end{array}$

$d(U,V) = U - V = \begin{bmatrix} 4 & -5 \\ 3 & 7 \end{bmatrix}$

$\|U-V\| = \sqrt{16 + 25 + 9 + 49} = \sqrt{99}$