Discrete Mathematics

Set Theory

- Sets: Definition, Universal Set, Complement, Cardinality
- Subset and Power Set
- Sets Operations
- Set Equality
- Characteristic Vectors: Sets as Bit-Vectors
- Multisets

Set Theory 1/1

Sets as bit-strings (bit vectors)

- Sets stored in an unordered fashion in memory
- Union/Intersection etc. are computationally expensive
- lacktriangle When |U| is small compared to computer memory, then we can do set operations efficiently
- Impose any fixed ordering on elements of *U*
- $U = \{DM, Cal, Chem, Bio, Phy, Pro\}$ (in order)
- \blacksquare Sets (subsets of U) are represented by bit-string of length 6
- Each bit signifies whether the corresponding element is in the set
- Called bit-vector representation of sets or characteristic vector of a set

Set Theory 2/1

Sets as bit-strings (bit vectors)

DM	Calc	Chem	Bio	Phy	Prog

The set $\{Calc, Chem, Phy\}$ is

0	1 1	0	1	0
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The set $\{Prog, DM, Calc, Phy\}$ is

ICP 4-28 What is the characteristic vector of the set

$$\{Chem, DM\}$$
?

ICP 4-29 | What is the characteristic vector of the set

{ Calc, DM, Chem, Phy, Prog, Bio }?

Set Theory

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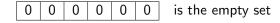
Sets as bit-strings (bit vectors)

DM	Calc	Chem	Bio	Phy	Prog

The set



The set



ICP 4-30 What is the set corresponding to the characteristic vector

Set Theory 4/1

Sets operations using bit-strings

$$A \cup B = \{x | x \in A \lor x \in B\}$$

$A = \{ Calc, Chem, Phy \}$	0	1	1	0	1	0	
$B = \{Prog, DM, Calc, Phy\}$	1	1	0	0	1	1	
$A \cup B$			<i>A</i> \	/ B			
$\{DM, Calc, Chem, Prog, Phy\}$	1	1	1	0	1	1	

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Sets operations using bit-strings

$$A \cap B = \{x | x \in A \land x \in B\}$$

$A = \{ Calc, Chem, Phy \}$	0	1	1	0	1	0	
$B = \{Prog, DM, Calc, Phy\}$	1	1	0	0	1	1	
$A\cap B$			<i>A</i> /	\ <i>B</i>			
$\{Calc, Phy\}$	0	1	0	0	1	0	

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Sets operations using bit-strings

$$A \oplus B = \{x | x \in A \oplus x \in B\}$$

$A = \{ Calc, Chem, Phy \}$	0	1	1	0	1	0	
$B = \{Prog, DM, Calc, Phy\}$	1	1	0	0	1	1	
$A\oplus B$			A ∈	∌ <i>B</i>			
$\{DM, Chem, Prog\}$		0	1	0	0	1	

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Sets as bit-vectors: Summary

- Sets can be represented as bit vectors, when universal set is 'small'
- Also called characteristic vectors of sets
- Order of U is critical
- Sets operations can be performed using bit-wise operators of programming language
- More suitable for computer implementations
- lacksquare Only feasible when U is small

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