National University of Computer and Emerging Sciences, Lahore Campus

Course:	**Discrete Structures**	**Course Code:**	**CS1005**
Program:	**BS- Computer Science**	**Semester:**	**Fall 21**
Duration:	**180 mins**	**Total Marks:**	**70**
Paper Date:	**10-01-2022**	**Weightage**	**50**
Section:	**All Sections**	**Page(s):**	
Exam:	**Final**	**Roll No:**	

Instruction/Notes:	**Attempt All the Questions**

Q.1. (6 marks)

- **(a)** What is the number of 5-element subsets of a 50-element set?
- **(b)** What is the number of functions from the set $\{1, 2, \ldots, 100\}$ to the set $\{a, b\}$?
- **(c)** What is the number of 5-letter upper-case words that contain the letter A?

Q.2. (4 marks) Find the coefficient of x^{10} in the expansion of $\left(2x + \frac{1}{x}\right)^{100}$?

Q.3. (5 marks) How many different ways are there to choose a dozen candies from the 21 varieties at a candy shop?

Q.4. (5 marks) Let m be an integer with m > 1. Define the congruence relation R = {(a, b) | a ≡ b (mod m)} on the set of integers. Is the relation *R* equivalence relation?

Q.5. (5 marks) show that (0, 1) and [0, 1] have the same cardinality.

Q.6. (4 marks) Find each of these values.

- (a) (177 **mod** 31 + 270 **mod** 31) **mod** 31
- (b) (177 **mod** 31 · 270 **mod** 31) **mod** 31

Q.7. (8 marks) Determine the smallest positive integer that gives a remainder of 2 upon division by 3, a remainder of 1 upon division by 5, and a remainder of 6 upon division by 7.

Q.8. (8 marks) Let *P(n)* be the statement that $n! < n^n$, where n is an integer greater than 1.

- **(a)** What is the statement *P(2)*?
- **(b)** Show that *P(2)* is true, completing the basis step of the proof.
- **(c)** What is the inductive hypothesis?
- **(d)** What do you need to prove in the inductive step?
- **(e)** Complete the inductive step.
- **(f)** Explain why these steps show that this inequality is true whenever n is an integer greater than 1.

Q.9. (10 marks) How many different messages can be transmitted in n microseconds using three different signals if one signal requires 1 microsecond for transmittal, the other two signals require 2 microseconds each for transmittal, and a signal in a message is followed immediately by the next signal?

Q.10. (5 marks) Are the simple graphs with the following adjacency matrices isomorphic? justify you answer.

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} . \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Q.11. (5 marks) Let *T (x, y)* mean that student x likes cuisine y, where the domain for x consists of all students at your school and the domain for y consists of all cuisines. Express each of these statements by a simple English sentence.

- **(a)** ¬T (Abdullah, Chinese)
- **(b)** ∃x T (x, Korean) ∧ ∀x T (x, Mexican)
- **(c)** ∃y(T (Ali, y) ∨ T (Usman, y))
- **(d)** ∀x∀z ∃y ((x ≠ z) → ¬(T (x, y) ∧ T (z, y)))
- **(e)** ∃x∃z∀y (T (x, y) ↔ T (z,y))

Q.12. (5 marks)

- **(a)** What rules of inference are used in this argument? "No man is an island. Manhattan is an island. Therefore, Manhattan is not a man."
- **(b)** What is the negation of the proposition? Sobia has an MP3 player.
- **(c)** What is the truth value of the proposition " There is no pollution in Lahore".
- **(d)** Find the domain of the function that assigns to each bit string the number of ones in the string minus the number of zeros in the string
- **(e)** Find the range of the function that assigns to each bit string the number of ones in the string minus the number of zeros in the string