Data Structures & Algorithms 2
Tutorial 6

Priority Queues

Exercise 1

1. Show the result of inserting 10, 12, 1, 14, 6, 5, 8, 15, 3, 9, 7, 4, 11, 13, and 2, one at a time, into an initially empty binary min heap.

2. Show the result of using the linear-time algorithm to build a binary min heap using the same input

Exercise 2

Show the result of performing three deleteMin operations in the heap of the previous exercise (Question 1).

Exercise 3

a) Prove that for the perfect binary tree of height h containing $\mathbf{2^{h+1}-1}$ nodes, the sum of the heights of the nodes is $\mathbf{2^{h+1} - 1 - (h + 1)}$.

b) using the theorem proved in (a) , determine the time complexity of the Build-Heap algorithm for a binary heap.

Exercise 4

a)- Suppose a priority queue implemented with an ordered array. Ignoring array resizing, what is the worst-case running-time cost for inserting items into the priority queue and for removing the maximum element? (*You may assume that larger values correspond to a larger priority*)

b)- A heap is represented as an array whose value at index 0 is null.

- Find the index of both the parent and right child of the element at index 21
- If the index of a node is n, what is the index of its grandparent?

Exercise 5

Suppose we want to add the decreaseAllKeys(delta) operation to the heap repertoire. The result of this operation is that all keys in the heap have their value decreased by

an amount delta. For the heap implementation of your choice, explain the necessary modifications so that all other operations retain their running times and decreaseAllKeys runs in O(1)

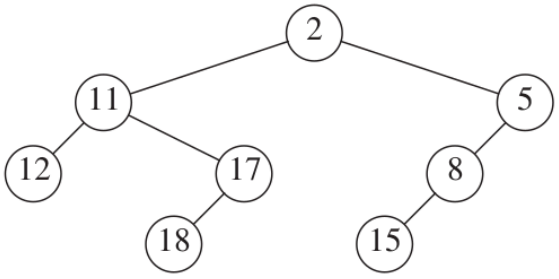
Exercise 6

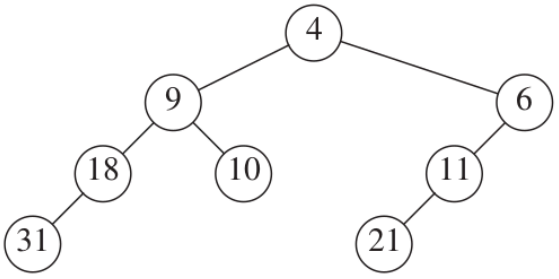
If a d-heap is stored as an array, for an entry located in position i, where are the parents and children?

Exercise 7

Merge the following :

1. two leftist heaps
2. two skew heaps





Exercise 8

Show the result of inserting keys 1 to 15 in order into :

1. an initially empty leftist heap.
2. a skew heap.