Exercise 1. Evaluate the following double integrals

1) $\iint_{[-1,1]\times[0,1]} (x+y)\,dy\,dx$, 2) $\iint_{[-1,1]\times[-x,1]} (x+y)\,dy\,dx$, 3) $\iint_{[-1,1]\times[-1,1]} |x+y|\,dy\,dx$,

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Exercise 2. Using Fubini's theorem, evaluate the integrals :

1. $\iint_D \frac{xy}{1+x^2+y^2}\,dx\,dy$, $D = \left\{(x,y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1, 0 \leq y \leq 1, 1 \leq x^2+y^2\right\}$.
2. $\iint_D x\cos(y)\,dx\,dy$, where D is delimited by $y=0$, $y=x^2$, and $x=2$.
3. $\iint_D xy\,dx\,dy$, $D = \left\{(x,y) \in \mathbb{R}^2 \mid 0 \leq x, 0 \leq y, x+y \leq 2\right\}$

Exercise 3. Evaluate the following double integrals by using change of variable.

1. $\iint_D (x+y)^2\,dx\,dy$, where $D = \left\{(x,y) \in \mathbb{R}^2 \mid x^2+y^2-x \leq 0, x^2+y^2-y \geq 0, 0 \leq y\right\}$.
2. $\iint_D \frac{y^2}{x^2+y^2}\,dx\,dy$, where $D = \left\{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 < 4, y > 1\right\}$.
3. $\iint_D x^2y^4\,dx\,dy$, where $D = \left\{(x,y) \in \mathbb{R}^2 \mid 9x^2+4y^2 < 36, x > 0, y > 0\right\}$.
4. $\iint_D \exp\left(\frac{x-y}{x+y}\right)\,dx\,dy$, where $D = \left\{(x,y) \in \mathbb{R}^2 \mid \frac{1}{2} < x+y < 1, x > 0, y > 0\right\}$.

Exercise 4. Calculate the triple integrals $\iiint_\Omega f(x,y,z)\,dx\,dy\,dz$, in each of the following cases

1. $\iiint_\Omega y\,dx\,dy\,dz$, where Ω is defined as $x \geq 0, y \geq 0, x^2+y^2 \leq z \leq 1$.
2. $\iiint_\Omega y\,dx\,dy\,dz$, where Ω is defined as $x \geq 0, y \geq 0, x^2+y^2 \leq z^2, 1 \leq z \leq 2$.
3. $\iiint_\Omega z\exp\left(x^2+y^2\right)\,dx\,dy\,dz$, where Ω is defined as $1 \leq x^2+y^2 \leq 4, 0 \leq z \leq 1$.
4. $\iiint_\Omega (x+y+z)^{3/2}\,dx\,dy\,dz$, where Ω is defined as $x^2+y^2+z^2 \leq 12, x^2+y^2 \leq 4z$.
5. $\iiint_\Omega \frac{1}{\sqrt{x^2+y^2+z^2}^3}\,dx\,dy\,dz$, where Ω is defined as $x^2+y^2+z^2 \leq 12, x^2+y^2 \leq z^2$.
6. $\iiint_\Omega \frac{1}{\sqrt{x^2+y^2+z^2}}\,dx\,dy\,dz$, where Ω is defined as $4 \leq x^2+y^2+z^2 \leq 16$