

Priority Queue

- A kind of queue (dequeue, enqueue)
- dequeue gets element with the highest priority
- priority is based on a comparable value of each object (smaller value higher priority or higher value higher priority)

Applications in computer

operating system: run the shortest job first
printer: print the shortest document first

Minimum Priority Queue

We use a data structure named heap to implement a priority queue.

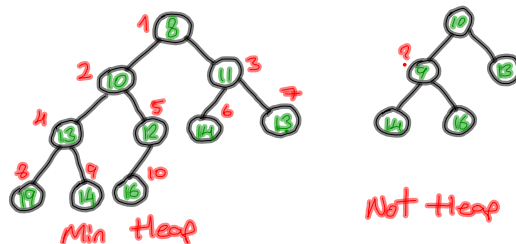
A Binary Heap

Structural property:

Binary tree & completely filled (bottom level is filled from left to right).

Heap order property:

Any node is smaller than (or equal to) all of its children.



Implementation of a binary heap

	0	1	2	3	4	5	6	7	8	9	10
min heap		8	10	11	13	12	14	13	19	14	16

Heap as a Tree

Root of tree: first element in the array after index

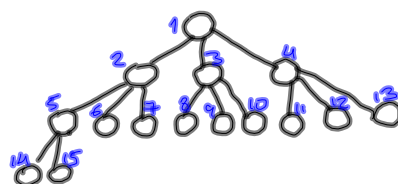
$$\text{parent}(i) = \left\lfloor \frac{i}{2} \right\rfloor \rightarrow \text{floor}$$

$$\text{left}(i) = 2i$$

$$\text{right}(i) = 2i+1$$

mapping

Question: What is the mapping if we use a 3-ary tree as a heap.



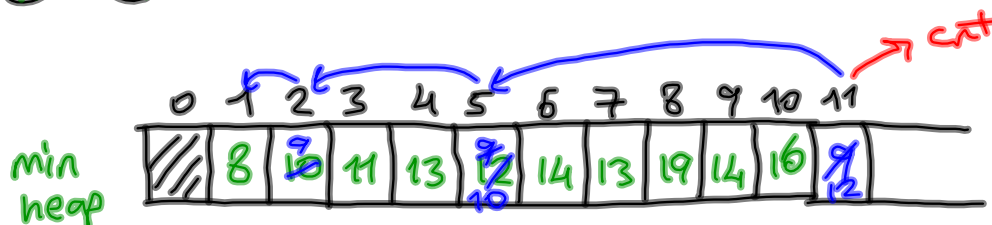
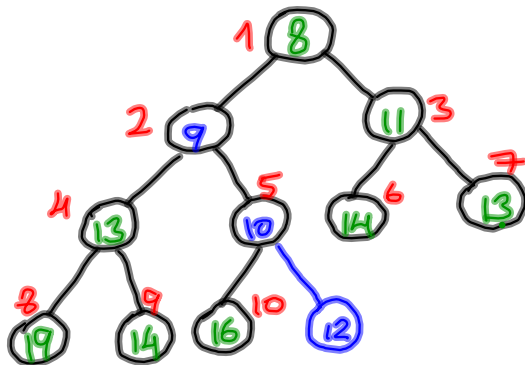
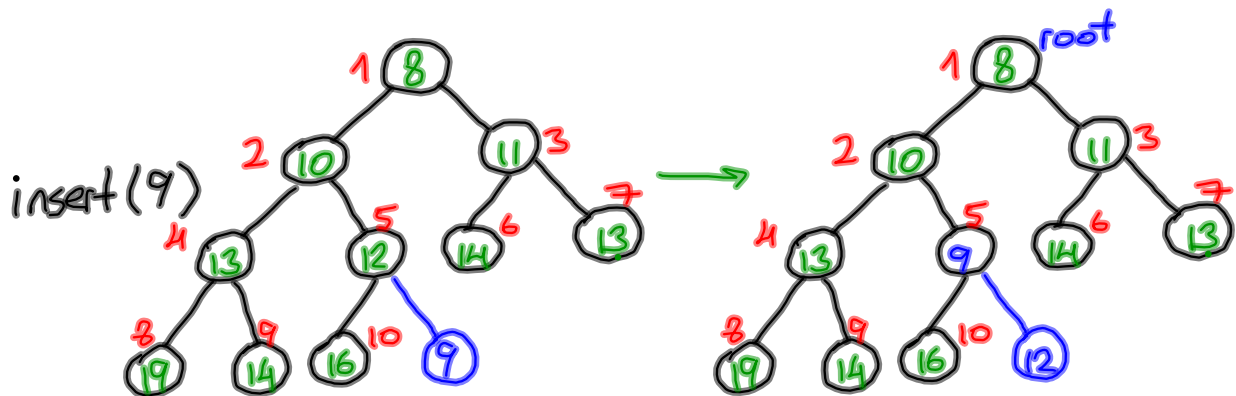
$$\begin{aligned} \text{parent}(i) &= \left\lfloor \frac{i+1}{3} \right\rfloor \\ \text{left}(i) &= 3i-1 \\ \text{middle}(i) &= 3i \\ \text{right}(i) &= 3i+1 \end{aligned}$$

$$\begin{array}{ll} 2 \rightarrow 1 & 11 \rightarrow 4 \\ 3 \rightarrow 1 & 12 \rightarrow 4 \\ 4 \rightarrow 1 & 13 \rightarrow 4 \end{array}$$

$$i=13 \\ \frac{13+1}{3} = \frac{14}{3} = 4.33$$

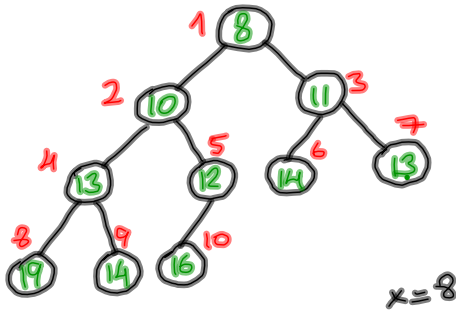
Insert

- Insert into the next available location
- Move it up (swap with its parent) until there is no violation for heap order property. This procedure is called percolate up.

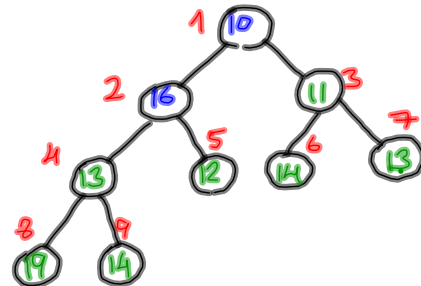
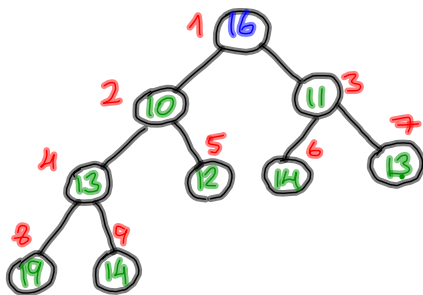


Delete (return the min and delete it)

- Keep the min. Move the last element in the heap to the first place.
- Move the first element down until there is no violation for heap order property. This process is called percolate down. Finally, we return the min which is stored.



delete() call returns 8 and it rearranges the heap



return x

