1. Question 1

1. *Each position t in shop s is occupied by exactly one vehicle:*

$$\sum_{v=1}^{n} x_{s,v,t} \;=\; 1 \quad \forall\, s \in \{1,\dots,S\},\ \forall\, t \in \{1,\dots,n\}.$$

2. *Each vehicle v appears exactly once in each shop s:*

$$\sum_{t=1}^{n} x_{s,v,t} \;=\; 1 \quad \forall\, s \in \{1,\dots,S\},\ \forall\, v \in \{1,\dots,n\}.$$

3. *Binary domain:*

$$x_{s,v,t} \;\in\; \{0,1\}.$$

2. Question 2

We want

$$y_{s,v} \;\geq\; \max\Bigl(0,\ \sigma_s^{-1}(v) \;-\; \Delta_s \;-\; \sigma_{s+1}^{-1}(v)\Bigr),$$

where $\sigma_s^{-1}(v)$ is the *position* of vehicle v in shop s. So:

$$y_{s,v} \;\geq\; \sum_{t=1}^{n} t\, x_{s,v,t} \;-\; \sum_{t=1}^{n} t\, x_{s+1,v,t} \;-\; \Delta_s, \quad y_{s,v} \;\geq\; 0.$$

In other words, $\sum_{t=1}^{n} t\, x_{s,v,t}$ is the position of v in shop s, and similarly for shop $s+1$.

3. Question 3

For each shop s, lot-change constraint $\ell \in L_s$, and position $t \in \{1,\dots,n-1\}$, let

$$f_{s,\ell,t} \;\in\; \{0,1\}.$$

We want:

$$f_{s,\ell,t} \;\geq\; 1\bigl(U_\ell(\sigma_s(t)) \;\neq\; U_\ell(\sigma_s(t+1))\bigr).$$

The function $U_\ell(\cdot)$ gives the "lot" of each vehicle under constraint ℓ. If ℓ partitions vehicles into subsets $U_{\ell,1}, U_{\ell,2}, \ldots$, we can force

$$f_{s,\ell,t} \;\geq\; \sum_{v \in U_{\ell,i}} x_{s,v,t} \;+\; \sum_{v' \in U_{\ell,j}} x_{s,v',\,t+1} \;-\; 1 \quad \text{for every distinct } i \neq j.$$

This ensures $f_{s,\ell,t} = 1$ if the vehicle at position t is in a different lot-partition than the vehicle at position $t+1$.

4. Question 4

For each rolling-window constraint $r \in R_s$ (with window size w_r, threshold M_r, subset V_r), and each starting position $t \in \{1, \ldots, n - w_r + 1\}$, let

$$z_{s,r,t} \;\geq\; 0.$$

We want

$$z_{s,r,t} \;\geq\; \max\Bigl(0, \sum_{p=t}^{t+w_r-1} \sum_{v \in V_r} x_{s,v,p} \;-\; M_r\Bigr).$$

This is encoded via:

$$z_{s,r,t} \;\geq\; \sum_{p=t}^{t+w_r-1} \sum_{v \in V_r} x_{s,v,p} \;-\; M_r, \quad z_{s,r,t} \;\geq\; 0.$$

5. Question 5

For the batch-size constraint $b \in B_s$, we consider positions t, t' with $1 \leq t \leq t' \leq n$. Define $g_{s,b,t,t'}$ so that:

$$g_{s,b,t,t'} \;\geq\; 1 \quad \text{if} \begin{cases} \sigma_s(t-1) \notin V_b, & \text{if } t \geq 2, \\ \sigma_s(t), \ldots, \sigma_s(t') \in V_b, & \\ \sigma_s(t'+1) \notin V_b, & \text{if } t' \leq n-1. \end{cases}$$

That means the block of positions t through t' is exactly a contiguous run of vehicles from V_b.

We can use that:

$$\text{“all in } V_b \text{ from } t \text{ to } t'\text{”} \quad \longleftrightarrow \quad \sum_{i=t}^{t'} \sum_{v \in V_b} x_{s,v,i} \;=\; (\,t' - t + 1\,).$$

And

$$\text{“neighbors not in } V_b\text{”} \quad \longleftrightarrow \quad \sum_{v \in V_b} x_{s,v,t-1} \;=\; 0 \text{ (if } t > 1\text{), and } \sum_{v \in V_b} x_{s,v,t'+1} \;=\; 0 \text{ (if } t' < n\text{)}.$$

$$g_{s,b,t,t'} \;\geq\; \frac{\displaystyle\sum_{i=t}^{t'} \sum_{v \in V_b} x_{s,v,i}}{(\,t' - t + 1)} \;+\; \sum_{v \notin V_b} \sum_{i=t}^{t-1} x_{s,v,i} \;+\; \sum_{i=t'+1}^{n} \sum_{v \in V_b} x_{s,v,i} \;-\; 2,$$

(assuming $t > 1$ and $t' < n$, with small modifications on boundaries). That yields $g_{s,b,t,t'} \geq 1$ if all conditions hold.

6. Full MILP (No Two-Tone, Linear Lot-Change)

- **Permutation constraints:**

$$\sum_{v=1}^{n} x_{s,v,t} = 1, \quad \sum_{t=1}^{n} x_{s,v,t} = 1, \quad x_{s,v,t} \in \{0,1\}, \quad \forall\, s,t,v.$$

- **Resequencing lags:**

$$y_{s,v} \;\geq\; \sum_{t=1}^{n} t\, x_{s,v,t} - \sum_{p=1}^{n} p\, x_{s+1,v,p} - \Delta_s, \quad y_{s,v} \;\geq\; 0, \quad \forall\, s \in \{1,\ldots,S{-}1\}, \forall\, v.$$

- **Lot-change constraints:**

$$f_{s,\ell,t} \;\geq\; \sum_{v \in U_{\ell,i}} x_{s,v,t} + \sum_{v' \in U_{\ell,j}} x_{s,v',t+1} - 1, \quad \forall\, i \neq j,\ \forall\, \ell,\ \forall\, t \in \{1,\ldots,n{-}1\}.$$

$$f_{s,\ell,t} \in \{0,1\}.$$

- **Rolling-window constraints:**

$$z_{s,r,t} \;\geq\; \sum_{p=t}^{t+w_r-1} \sum_{v \in V_r} x_{s,v,p} - M_r, \quad z_{s,r,t} \;\geq\; 0, \quad \forall\, t \in \{1,\ldots,n{-}w_r{+}1\}.$$

- **Batch-size constraints:** For $g_{s,b,t,t'} \geq 1$ to hold:

$$g_{s,b,t,t'} \geq \frac{\sum_{i=t}^{t'} \sum_{v \in V_b} x_{s,v,i}}{t' - t + 1} + \sum_{v \notin V_b} \sum_{i=t}^{t-1} x_{s,v,i} + \sum_{i=t'+1}^{n} \sum_{v \in V_b} x_{s,v,i} - 2, \quad \forall\, t, t' \in \{1, \ldots, n\},\, t \leq t'.$$

  Additionally:

$$g_{s,b,t,t'} \in \{0, 1\}.$$

Objective function: The goal is to minimize the sum of all penalties:

$$\min \quad \sum_{s=1}^{S-1} \sum_{v=1}^{n} c^s y_{s,v} + \sum_{s=1}^{S} \sum_{\ell=1}^{|L_s|} \sum_{t=1}^{n-1} c_\ell f_{s,\ell,t} + \sum_{s=1}^{S} \sum_{r=1}^{|R_s|} \sum_{t=1}^{n-w_r+1} c_r z_{s,r,t} + \sum_{s=1}^{S} \sum_{b=1}^{|B_s|} \sum_{t=1}^{n} \sum_{t'=t}^{n} c_b g_{s,b,t,t'}.$$