ROP

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Questions

$$1)\quad \forall s, \quad \sum_v x_{svt} = 1, \quad \forall t, \quad \sum_t x_{svt} = 1, \quad \forall v. \tag{1}$$

$$2)\quad y_{sv} \geq 0, \tag{2}$$

$$y_{sv} \geq \sum_t t \cdot x_{svt} - \Delta_s - \sum_t t \cdot x_{(s+1)vt}. \tag{3}$$

3) To ensure that $f_{s\ell t}$ captures whether a lot change occurs between positions t and $t+1$, we introduce the following constraints:

Variables

- $f_{s\ell t}$: Binary variable that equals 1 if $U_\ell(\sigma_s(t)) \neq U_\ell(\sigma_s(t+1))$, and 0 otherwise.
- $u_{s,t,\ell,k}$: Binary variable that equals 1 if the vehicle at position t in shop s belongs to lot k for constraint ℓ, and 0 otherwise.

1. Vehicle-to-Lot Mapping

To ensure $u_{s,t,\ell,k}$ represents whether the vehicle at position t belongs to lot k, we use:

$$u_{s,t,\ell,k} = \sum_{v \in V_k^\ell} x_{s,t,v}$$

where V_k^ℓ is the set of vehicles belonging to lot k under constraint ℓ.

2. Detect Lot Change

To determine if there is a lot change between positions t and $t+1$, we introduce the constraint:

$$f_{s\ell t} \geq u_{s,t,\ell,k} - u_{s,t+1,\ell,k} \quad \forall k.$$

This ensures $f_{s\ell t}$ equals 1 if the lot at position t differs from the lot at position $t+1$.

3. Bound on $f_{s\ell t}$

To ensure that $f_{s\ell t}$ does not exceed 1, we add:

$$f_{s\ell t} \leq \sum_{k} \big(u_{s,t,\ell,k} - u_{s,t+1,\ell,k}\big).$$

4. Lot Consistency

The following constraints ensure consistency of the lots over time:

$$z_{srt} \geq 0, \tag{4}$$

$$z_{srt} \geq \sum_{t'=t}^{t+w_r-1} q_{srt'} - M_r, \tag{5}$$

where:

$$q_{srt} \geq x_{svt} \quad \forall v \in V_r, \tag{6}$$

$$q_{srt} \leq \sum_{v \in V_r} x_{svt}. \tag{7}$$

5. New Variable p

Introduce p with the following constraints:

$$p_{stb} \geq x_{stb} \quad \forall v \in V_b, \tag{8}$$

$$p_{stb} \leq \sum_{v \in V_b} x_{svt}, \tag{9}$$

where p_{stb} indicates if $\sigma_s(t) \in V_b$.

Next, we impose the following constraints involving the variable p:

$$n_{sbtt'} \geq (1 - p_{stb}) \quad \text{if } t \geq 2, \tag{10}$$

$$m_{sbtt'} \geq p_{skb} \quad \text{if } k \geq t \text{ and } k \leq t', \tag{11}$$

$$l_{sbtt'} \geq (1 - p_{st'b}) \quad \text{if } t' \leq n-1, \tag{12}$$

$$g_{sbtt'} \geq n_{sbtt'} + m_{sbtt'} + l_{sbtt'} - 2 \tag{13}$$

6. Objective Function

The objective function is to minimize the sum of costs associated with different parameters. The optimization problem is:

$$\min_{x \in X} \; C_s(b) + C_l(f) + C_r(z) + C_b(g)$$

subject to the following:

$$x_{svt} \in \{0,1\}$$

$$\forall s,t, \quad \sum_v x_{svt} = 1$$

$$\forall s,v, \quad \sum_t x_{svt} = 1.$$

$$y_{sv} \geq 0$$

$$y_{sv} \geq \sum_t t \cdot x_{svt} - \Delta_s - \sum_t t \cdot x_{(s+1)vt}$$

$$b_{sv} \geq \frac{y_{sv}}{M}, \quad \forall s,v$$

$$b_{sv} \in \{0,1\}$$

where M is an upper bound on the values of y_{sv}, typically chosen as $M = \max(y_{sv})$.

The remaining constraints are:

$$u_{s,t,\ell,k} = \sum_{v \in V_k^\ell} x_{s,t,v}$$

$$f_{s\ell t} \geq u_{s,t,\ell,k} - u_{s,t+1,\ell,k} \quad \forall k.$$

$$f_{s\ell t} \leq \sum_k \big(u_{s,t,\ell,k} - u_{s,t+1,\ell,k}\big).$$

$$f_{slt} \in \{0,1\}$$

The constraints for lot consistency are:

$$z_{srt} \geq 0, \tag{14}$$

$$z_{srt} \geq \sum_{t'=t}^{t+w_r-1} q_{srt'} - M_r, \tag{15}$$

where:

$$q_{srt} \geq x_{svt} \quad \forall v \in V_r, \tag{16}$$

$$q_{srt} \leq \sum_{v \in V_r} x_{svt}. \tag{17}$$

For the new variable p, we have the following constraints:

$$p_{stb} \geq x_{svt} \quad \forall v \in V_b, \tag{18}$$

$$p_{stb} \leq \sum_{v \in V_b} x_{svt}, \tag{19}$$

Finally, the following constraints are introduced:

$$n_{sbtt'} \geq (1 - p_{stb}) \quad \text{if } t \geq 2, \tag{20}$$

$$m_{sbtt'} \geq p_{skb} \quad \text{if } k \geq t \text{ and } k \leq t', \tag{21}$$

$$l_{sbtt'} \geq (1 - p_{st'b}) \quad \text{if } t' \leq n - 1, \tag{22}$$

$$g_{sbtt'} \geq n_{sbtt'} + m_{sbtt'} + l_{sbtt'} - 2 \tag{23}$$

$$g_{sbtt'} \in \{0, 1\}$$

The cost functions are defined as:

$$C_s(b) = \sum_{s=1}^{S-1} \sum_{v \in [n]} c_s b_{sv}$$

$$C_l(f) = \sum_{s=1}^{S} \sum_{\ell \in L_s} \sum_{t=1}^{n-1} c_l f_{slt}$$

$$C_r(z) = X \sum_{s=1}^{S} \sum_{r \in R_s} \sum_{t=1}^{n-w_r+1} z_{srt}$$

$$C_b(g) = \sum_{s=1}^{S} \sum_{b \in B_s} \sum_{t=1}^{N-1} \sum_{t'=t}^{N} g_{sbtt'} \gamma_b(t' - t + 1)$$

$$\gamma_b^b(k) = c_b^b \max(0, m_b - k, k - M_b)$$